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Determining Economic Order Quantity (EOQ) with Increase in a Known Price under Uncertainty through Parametric Non-Linear Programming Approach

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Abstract– *Constant unit procurement cost is one of the main assumptions in the classic inventory control policies. In the realistic world and practice, suppliers sometimes face increase in the price of a known item. In this paper, an inventory model for items with a known one-time-only price increase under fuzzy environment is presented by employing trapezoidal fuzzy numbers to find the optimal solution. We developed three different policies on the basis of methods such as α-cuts, for defuzzification of internal parameters before solving the model, and Vujosevic, for difuzzification of the external parameters after solving it. In the first policy, we integrated α-cuts method and Parametric Non-Linear Programming (PNLP) problems to attain the Membership Functions (MFs) of external variables in the primary model for achieving the optimal solution. These variables were reached by internal parameters through two-phase maximum/minimum non-linear programming problems and the external variables were approximate fuzzy numbers. Under the other two policies, we used defuzzification techniques of Centroid of Gravity (COG), Signed Distance (SD), and the Maximum Degree of Membership (MDOM) to attain crisp numbers. The optimal order policies by the three methods were compared and numerical computations showed that the efficiency of the first method (i.e., the presented one) was considerably better than that of the other two methods. In fact, the first method selected the optimal and attractive strategies by allocating membership functions to different α-cuts and provided the Decision Maker (DM) with great information to decide and select the best strategies. The methods were validated by a numerical example. The main aim of this model was determining the special ordering range and net costs saving quantity (involving ordering, holding, and purchasing costs). The time of ordering for positive net costs saving was calculated.*

Keywords– *α-cuts, Fuzzy theory, Parametric non-Linear programming (PNLP), Special order, Zadeh's extension principle.*

I. INTRODUCTION AND LITERATURE REVIEW

Offering incentives by a supplier to consumers in order for encouraging them to procure a more substantial lot size than the usual one with one-time-offer support is a common practice and frequent suggestion (Cárdenas-Barrón et al., 2010a). There are several reasons and facts in a process of manufacturing why a producer might opt to offer consumers a brief discount, e.g., increasing cash flow, decreasing inventory levels of the products and goods, boosting and improving market share, and simply trying to retain and maintain customers. Experts have been developing lot-sizing inventory models (EOQ/EPQ) for inventory management since the "Economic Order Quantity (EOQ)" inventory model management was introduced in 1913. Sometimes, as a permanent increase in brief discount, "one-time-only (OTO)" discount in the unit purchase cost of an item is announced by a supplier. On the other hand, the consumer can lower the

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total purchase cost by assigning a major-specific order quantity. The presented study aims to extend the EOQ-type model with a special price discount under fuzzy environment constraint by considering parametric non-linear programming approach. Before explaining the inventory system in detail, a brief literature review will be presented in the following.

An inventory model different from that of Naddor (1966) was studied by Brown (1967). An EOQ system per increase in buying cost by considering a finite planning horizon was investigated by Lev and Soyster (1979). Another EOQ in which at least one of the demands or cost parameters would vary in the future was presented by Lev et al (1981) In this system, any variation in the prices had a direct influence on the consuming rate. An inventory lot size system with periods of increase in buying cost, which followed a probability function, was studied by Lev et al. (1981). Another paper was presented by Brown (1982) which illustrated no significant difference between the proposed models. A lot sizing model via an anticipated procurement increase was supposed by Yanasse (1990) for determining the optimal order quantity. An Economic Production Quantity (EPQ) system with price increase was developed by Tersine (1996) in which there were shortages and a constant fraction of the items produced by the manufacturer would be defective. More recently, a profit-maximizing inventory model was investigated by Arcelus et al. (2001). They took into account both delay in payments and discount price with price-dependent demand. This permitted passing a portion of the benefit from the buyer to the consumers, which induced a higher demand. Ghosh (2003) developed two different EOQ models with backlogging by considering an announced price increase. Afterwards, a study was carried out by Arcelus et al. (2003), which took into account perishable products in the inventory model management and made promotion in profit maximization for the buyer. The theories of Zadeh’s extension principle and α-cuts for the process of constructing the MF of a structure was applied by Huang (2006) by considering paired NLP models. Also, combining the theories of Zadeh’s extension principle and α-cuts in the queuing system to construct membership functions was previously proposed by Chen (2005). An economic order system with price discount to find the optimum strategies for different situations was developed by Sarker and Al Kindi (2006). The applicability of a developed integration of interval and fuzzy programming was investigated by Qin et al. (2007). They addressed uncertainties in the right/upper and left/lower-hand sides of the nonlinear models. A Mixed-Integer Nonlinear-Programing (MINLP) method, which constructed the MF of the objective value of a queueing problem with fuzzy trapezoidal arrival rates, was proposed by Chen (2007). They applied the α-cuts concept and Zadeh’s extension principle approach. This topic was first introduced by Naddor (1966). An EOQ-type model for a buyer who operated an inventory policy was improved by Cárdenas-Barrón et al. (2010a) with planned linear backorders. In this model, the supplier offered a temporary fixed-percentage discount for determining the optimal policies.

A discussion of the Max-Cut problem with fuzzy coefficients and a designed Genetic Algorithm (GA) were presented by Wang and Wang (2010) by combining fuzzy simulation techniques formulated for the general fuzzy Max-Cut problem. Cárdenas-Barrón et al. (2010b) investigated an inventory strategy operation for the buyer based on an EOQ system in which planned backorders as temporary fixed-percentage discounts were offered by the supplier. An inventory model with imperfect quality items was developed by Hsu and Yu (2011) under an announced price increase. Defective items were classified by a full screening procedure. Hu et al. (2011) considered two EPQs with fuzzy defective rate (LR-fuzzy number) based on different separating rates. Moreover, Model-I was developed with a classifying rate lower than the production rate and model-II with a classifying rate greater than the production rate. Besides, a signed distance and a sample algebraic method were utilized to find the optimal production quantity so that the total cost per unit time for the manufacturer in the fuzzy sense had a minimum value. Liu and Zheng (2012) expanded the classical EOQ management (or EOQ inventory system) with imperfect products, inspection errors, and shortages back-ordered. The classifying process of the items contained two types of errors in the inspection process. First, the defective item might be screened and defined as non-defective and second, the non-defective item might be screened and defined as defective. A triangular fuzzy number was considered for the fraction of defective items. Profit maximization function for the classical inventory system by considering backorder under promotional effort was studied by De and Sana (2013). They considered the output variables in the fuzzy mode. Mahata and Goswami (2013) proposed an inventory model for imperfect items and shortages (backorder) by employing two types of fuzzy numbers. The input parameters were triangular and trapezoidal in fuzzy environments. EOQ management (or EOQ inventory system) with price increase by considering two scenarios, namely the time of occurs of the increase and partial backordering, was presented by Taleizadeh and Pentico (2013). The managerial issues of a multi-period manufacturing system with capacity constraints for the plant with its own multiple delivery/shop systems and several producers were studied by De and Sana (2014). The system was run in several periods and it used triangular intuitionistic fuzzy numbers. Jana et al. (2014) proposed some multiple-product inventory management systems in a planning horizon in the stochastic fuzzy mode by considering time value of money, inflation, shortages, partial backlogging, fuzzy-deterioration items, and budget constraints. Samal and Pratihar (2014) studied an EOQ system for minimizing inventory cost, that is, holding cost of inventory system management under fuzzy environment. A fuzzy lot-sizing problem by incorporating human learning into a fuzzy EOQ-type system was proposed by Kazemi et al. (2015). An EPQ-type inventory system by considering deteriorating items under fuzzy environment and multiple markets for the supply chain system was investigated by Das et al. (2015). An aggregation linear assignment method and fuzzy arithmetic for multiple-attribute group decision making inventory classification were studied by Baykasoğlu et al. (2016). Investigation into optimization of a Vendor-Managed-Inventory (VMI) system with a discount using fuzzy demand and shortage by a hybrid imperialist competitive algorithm was carried out by Sadeghi et al. (2016).

The EOQ inventory system with backorder under fuzzy mode was developed by De and Mahata (2017). Mojaveri and Moghimi (2017) considered one supplier and one retailer to perform a multi-product business under VMI approach with uncertain demand. Dey (2017) suggested single-buyer single-vendor inventory system with defects under a mixed environment with randomness and fuzziness. Taleizadeh et al. (2017) provided an inventory model for a selling system with probabilistic replenishment intervals and partial backordering. An inventory system with defects was studied wherein shortages were permitted under fuzzy environment by Kumar (2018). An inventory system with deteriorating products and permissible delay in payments under fuzzy mode was studied by Shaikh et al. (2018). An EPQ-type system with defects, inspection errors (Types 1 and 2), partial backlogging, and preventive maintenance under uncertainty was developed by Taheri-Tolgari et al. (2018). A fuzzy EOQ system for a high-technology product under trial-repeat procurement demand criterion was surveyed by Chanda et al. (2018).

An uncertain inventory system with shortage via graded mean/GM integration value was proposed by Saranya an Varadarajan (2018). A fuzzy geometric programming problem by considering decision variables, the right-hand sides, and the constraint coefficients as fuzzy numbers was presented by Bharani (2018). A Lagrangian relaxation for a fuzzy random EPQ system with redundancy allocation and shortages was studied by Sadeghi et al. (2018). Both fuzzy and crisp EOQ systems with proportionate discount and defects were studied by Patro et al. (2019). Rani et al. (2019) investigated a fuzzy Green Supply Chain (GSC) including collection of reverse logistics, waste products, remanufacturing, and deterioration products. Garai et al. (2019) proposed a multi-objective inventory system with both holding cost and stock-dependent demand rate under fuzzy random mode.

This main aim of this study is to achieve optimal special order inventory considering known price increase by parametric non-linear programming. An inventory model for items with a known one-time-only price increase under fuzzy environments is considered by employing trapezoidal fuzzy numbers to find out the optimal solution procedure. Shortages are not considered. The paper is organized as follows: the next section presents definition of the problem and assumptions in a crisp state. In section III, the proposed solution procedure is given. Section IV presents the fuzzy mathematical inventory model. In Section V, the proposed parametric nonlinear programs approach is developed. Section VI gives the solution procedure for the fuzzy model. A numerical example is presented in Section VII. Finally, conclusions are made in Section VIII. In order to highlight the contributions of this research, Table I provides a complete literature review in the field of research.

Table I. Characteristics of the most recent and related published models

Reference(s)	*System type*	*Defuzzification method*			*Model input*	*Model output*	*Solution procedure*
		COG	*MDOM*	*SD*			
Chen (2005) & Chen (2007)	Queuing system	✓			Fuzzy	Crisp	Parametric nonlinear programming
Wang & Wang (2010)	Max-Cut problem		✓		Fuzzy	Crisp	Genetic algorithm
Cárdenas-Barrón et al. (2010a)	EOQ-type				Crisp	Crisp	Derivative
Hu et al. (2011)	EPQ-type	✓			Fuzzy	Crisp	Algebraic method
Taleizadeh & Pentico (2013)	EOQ-type				Fuzzy	Crisp	Derivative
Dey (2017)	EPQ-type	✓			Fuzzy	Crisp	Derivative
Mojaveri & Moghimi (2017)	EPQ-type	✓			Fuzzy	Crisp	Particle swarm optimization
De & Mahata (2017)	EOQ-type	✓			Fuzzy	Crisp	Derivative
Sadeghi et al. (2018)	EPQ-type	✓			Fuzzy	Crisp	Meta-heuristics
Bharani (2018)	EPQ-type				Fuzzy	Crisp	Geometric programming
Saranya & Varadarajan (2018)	EOQ-type	✓			Fuzzy	Crisp	Derivative
Chanda et al. (2018)	EOQ-type	✓			Fuzzy	Crisp	Derivative
Taheri-Tolgari et al. (2018)	EPQ-type	✓		✓	Fuzzy	Fuzzy-crisp	Karush-Kuhn-Tucker
Shaikh et al. (2018)	EOQ-type		✓		Fuzzy	Crisp	Particle swarm optimization
Kumar (2018)	EOQ-type	✓		✓	Fuzzy	Crisp	Derivative
Rani et al. (2019)	EPQ-type			✓	Fuzzy	Crisp	Derivative
Patro et al. (2019)	EOQ-type			✓	Fuzzy	Crisp	Derivative
Proposed model	EOQ-type	✓	✓	✓	Fuzzy	Fuzzy-crisp	Parametric nonlinear programming

* COG: Centroid of gravity

* SD: Signed distance

* MDOM: Maximum degree of membership

II. PROBLEM DEFINITION AND ASSUMPTIONS IN THE CRISP STATE

In a production and inventory control system with one-time special order option, it should be decided whether or not a special order will be made. Consider a condition in which a distributer, supplier, manufacture, or buyer states that a price or procurement increase for an item will take place at or before the next scheduled ordering time of the buyer. A logical response is to order additional special items to take advantage of the lower price or procurement prior to or at the regular replenishment time. Hence, the Decision Maker (DM) should decide whether to place a special order quantity or not; and if yes, they should determine the quantity of the special order. If the unit price or procurement of an item is $u_1 u_1$ at a specific time, then the unit cost before that special time will still be $u_0 u_0$ ($u_1 > u_0$). Therefore, the buyer or purchaser either orders a special quantity Q to take advantage of the lower price or ignores this prospect and uses EOQ for all the future orders. If a special order quantity is to be placed, then the buyer or purchaser manager should determine the optimal value of Q and the lot size of the next order, after which all future orders will be of the of size EOQ. Our assumptions are basically similar to those of Tersine (1994), but improved to reflect that the buyer or purchaser is interested in taking advantage of the current lower price before it increases rather than buying at a sale price. The assumptions of the proposed model are as follows:

Assumptions:

- Holding costs are applied to average inventory;
- The model is for only one product and the planning horizon is one year;
- Shortages are not allowed;
- All orders placed after time t^* will be at the new higher cost per unit.

The parameters used in this model are as follows:

Parameters:

u_0	Current unit purchase cost of an item before t^*;
u_1	Unit purchase cost after the increase in price after t^*;
t_f	Latest time of order before increase in item price;
t_p	Moment of increase in item price;
C	Fixed order cost;
D	Demand quantity for product per period;
t_b	Moment of a one-time special inventory cycle at the current price;
t_a	Moment of finishing the latest order;
h_c	Inventory fixed carrying cost per unit per time unit;
i	Inventory carrying cost rate (percent per period).

The decision variables in this model are as follows:

Decision variables:

Q	Quantity of a one-time special order at the current price.

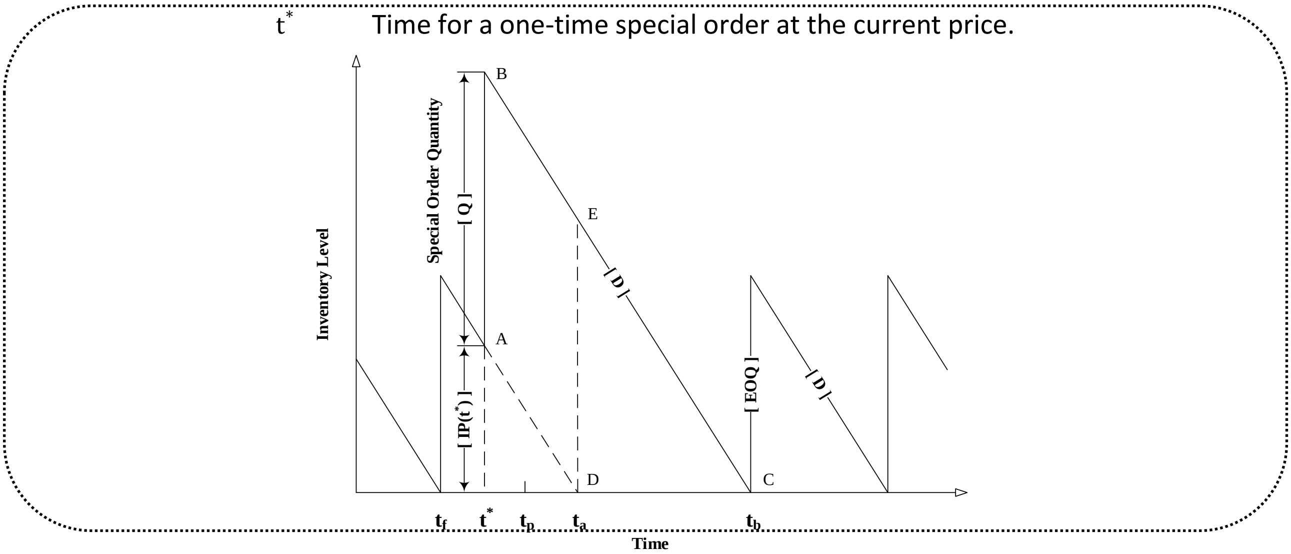


Figure 1. EOQ model with increased price and no shortage.

When the one-time-only discount occurs at t^*, the manufacturer or buyer system has two options: placing a special order quantity of Q units at the original equipment manufacturer or taking no action, i.e., adopting the original normal manufacturing policy. If the manufacturer manager chooses to place a special order quantity, they will resume the original manufacturing policy after the ordered products are depleted. The optimum decision depends on whether they can achieve profits from the special order policy. The optimal ordering quantity Q should be determined by maximizing the cost saving incurred by the policy, which can be expressed as follows. The behavior of the inventory level over time is illustrated in *Fig.* (1). In the figure, we have:

$$t_a = t^* + \frac{IP(t^*)}{D} \tag{1}$$

$$t_b = t_a + \frac{Q}{D} \tag{2}$$

According to the above description of the model and *Fig.* (1), the Net Saving (NS) by the special order policy is given by:

$$NS = Q.(u_1\text{-}u_0) + \sqrt{2.C.D.(h_c + i.u_1)}.(t_b\text{-}t_a)\text{-}(h_c + i.u_0).\left(\frac{Q.IP(t^*)}{D} + \frac{Q^2}{2.D}\right)\text{-}C \tag{3}$$

$$S_{ABCD} = S_{ABED} + S_{DEC} = Q.\left(t_a\text{-}t^*\right) + \frac{Q}{2}.(t_b\text{-}t_a) = Q.\frac{IP(t^*)}{D} + \frac{Q^2}{2.D} \tag{4}$$

$$Q^* = \frac{(u_1\text{-}u_0).D}{u_c + i.u_0} + \frac{u_c + i.u_1}{u_c + i.u_0}.\sqrt{\frac{2.C.D}{u_c + i.u_1}}\text{-}IP(t^*) \tag{5}$$

III. SOLUTION PROCEDURE FOR THE CRISP MODEL

Step1. Determine Q^* considering $t^* = t_p$, and replace it with Q_p by *Eq.* (5);

Step 2. Determine Q^* considering $t^* = t_f$, and replace it with Q_f by *Eq.* (5);

Step 3. Calculate the net saving for $t^* = t_p$, and replace it with NS_p by *Eq.* (3);

Step 4. Calculate the net saving for $t^* = t_p$, and replace it with NS_p by *Eq.* (3);

Step 5. If $\max\{NS_f + C, NS_p\} > 0$, go to Step 4. Otherwise, do not order the special quantity when an increase in the purchasing price occurs;

Step 6. If $NS_f + C \le NS_p$, special quantity is ordered in t_p; otherwise, it is in t_f.

IV. FUZZY MATHEMATICAL INVENTORY MODEL

The basic parameters, variables, and conditions involved in reality or practice are cost vectors and demand vectors. These vectors are flexible in nature. Consequently, to improve the practice model, we will fuzzify the parameters as follows.

A. Basic fuzzy calculus (cf. Zimmermann (2011))

We consider the so called α-cuts for the definition of fuzzy numbers.

Definition 1. If X is a collection of objects denoted universally by x, then a fuzzy set F in X is a set of ordered pairs $\tilde{F} = \{(x, \mu_{\tilde{F}}(x)) | x \in X\}$ and $\mu_{\tilde{F}}(x)$ is called the membership function or grade of membership of x in $\tilde{F}$, which maps X onto the membership space.

Definition 2. A continuous fuzzy number or interval μμ is any pair (μ^-, μ^+) of functions $\mu^{\mp}: [0,1] \to R^+$ satisfying the following conditions:

- $\mu^-: \alpha \to \mu_\alpha^- \in R^+$ is a bounded monotonic increasing (non-decreasing) continuous function, $\forall \alpha \in [0,1]$.
- $\mu^+: \alpha \to \mu_\alpha^+ \in R^+$ is a bounded monotonic decreasing (non-increasing) continuous function, $\forall \alpha \in [0,1]$.
- $\mu_\alpha^- \le \mu_\alpha^+, \alpha \in [0,1]$; if $\mu_\alpha^- \le \mu_\alpha^+$, we have a fuzzy interval and if $\mu_1^- \le \mu_1^+$, we have a fuzzy number. $\mu_\alpha = [\mu_\alpha^-, \mu_\alpha^+]$ explicitly denotes the α-cuts of μ.

Definition 3. Addition: $(\mu + \rho)_\alpha = (\mu_\alpha^- + \rho_\alpha^-, \mu_\alpha^+ + \rho_\alpha^+), \alpha \in [0,1]$.

Definition 4. Scalar multiplication: for any real m, $(m\mu)_\alpha = [\min(m\mu_\alpha^-, m\mu_\alpha^+), \max(m\mu_\alpha^-, m\mu_\alpha^+)], \alpha \in [0,1]$.

Definition 5. Subtraction: $(\mu\text{-}\rho)_\alpha = (\mu_\alpha^- \text{-} \rho_\alpha^-, \mu_\alpha^+ \text{-} \rho_\alpha^+), \alpha \in [0,1]$.

Definition 6. Multiplication: $(\mu\rho)_\alpha = [(\mu\rho)_\alpha^-, (\mu\rho)_\alpha^+] = \begin{bmatrix} \min(\mu_\alpha^-\rho_\alpha^-, \mu_\alpha^-\rho_\alpha^+, \mu_\alpha^+\rho_\alpha^-, \mu_\alpha^+\rho_\alpha^+), \\ \max(\mu_\alpha^-\rho_\alpha^-, \mu_\alpha^-\rho_\alpha^+, \mu_\alpha^+\rho_\alpha^-, \mu_\alpha^+\rho_\alpha^+) \end{bmatrix}, \alpha \in [0,1]$.

Definition 7. Division: If 0 does not belong to $(\rho_\alpha^-, \rho_\alpha^+)$, then $\forall \alpha \in [0,1]$ and we have: $(\mu/\rho)_\alpha = ((\mu/\rho)_\alpha^-, (\mu/\rho)_\alpha^+) = \left[\min\left(\frac{\mu_\alpha^-}{\rho_\alpha^+}, \frac{\mu_\alpha^-}{\rho_\alpha^-}, \frac{\mu_\alpha^+}{\rho_\alpha^-}, \frac{\mu_\alpha^+}{\rho_\alpha^+}\right), \max\left(\frac{\mu_\alpha^-}{\rho_\alpha^+}, \frac{\mu_\alpha^-}{\rho_\alpha^-}, \frac{\mu_\alpha^+}{\rho_\alpha^-}, \frac{\mu_\alpha^+}{\rho_\alpha^+}\right)\right]$.

Definition 8. Zadeh's extension principle: $\mu_{p(\hat{\lambda},\hat{\mu})}(z) = \sup_{x \in X, y \in Y} \min\{\mu_{\hat{\lambda}}(x), \mu_{\hat{\mu}}(y) | z = p(x,y)\}$.

Throughout, we use the following variables and parameters in order to simplify the treatment of the fuzzy special order inventory model.

B. Fuzzification by trapezoidal fuzzy numbers

In this sub-section, we use the concepts of fuzzy set and Zadeh's extension principle to extend the crisp model in presented in Section II by fuzzifying the following parameters:

$\widetilde{u_0} = \{(u_0, \mu_{\widetilde{u_0}}(x)) | x \in X\}$: Fuzzy unit purchase cost of an item before t^*;

$\widetilde{u_1} = \{(u_1, \mu_{\widetilde{u_1}}(y)) | y \in Y\}$: Fuzzy unit purchase cost after the increase in price after t^*;

$\tilde{D} = \{(D, \mu_{\tilde{D}}(u)) | u \in U\}\}$: Fuzzy demand quantity of product per period;

$\widetilde{h_c} = \{(h_c, \mu_{\widetilde{h_c}}(v)) | v \in V\}\}$: Fuzzy inventory fixed carrying cost per unit per time unit;

$\tilde{i} = \{(i, \mu_{\tilde{i}}(z)) | z \in Z\}$: Fuzzy inventory carrying cost rate (percent per period);

$\tilde{C} = \{(C, \mu_{\tilde{C}}(w)) | w \in W\}$: Fuzzy fixed order cost;

$\widetilde{t_p} = \{(t_p, \mu_{\widetilde{t_p}}(j)) | j \in J\}$: Fuzzy moment time for increase in price item.

By the extension principle, the membership of the fuzzy net saving function is given by

$\mu_{\tilde{E}}(NS) = \underbrace{SUP}_{\sigma} \ \min\{\mu_{\widetilde{u_0}}(x), \mu_{\widetilde{u_1}}(y), \mu_{\tilde{D}}(u), \mu_{\widetilde{h_c}}(v), \mu_{\tilde{i}}(z), \mu_{\tilde{C}}(w)\} | NS = Q.(u_1\text{-}u_0) +$

$\sqrt{2.C.D.(h_c + i.u_1)}.(t_b\text{-}t_a)\text{-}(h_c + i.u_0).\left(\frac{Q.IP(t^*)}{D} + \frac{Q^2}{2.D}\right)\text{-}C\}.$

(6)

with $\sigma = \{x \in X, y \in Y, u \in U, v \in V, z \in Z, w \in W | x > 0, y > 0, u > 0, v > 0, z > 0, w > 0\}$.

MF in *Eq.* (6) is not in the usual form for concrete use. Therefore, it very hard to imagine or determine its shape. Therefore, in this paper, we approach the problem using a mathematical programming technique. Parametric Nonlinear Programs (NLPs) are established to find the α-cuts of $NS(\widetilde{u_0}, \widetilde{u_1}, \widetilde{h_c}, \tilde{D}, \tilde{i}, \tilde{C})$ based on the extension principle.

V. PARAMETRIC NONLINEAR PROGRAMS APPROACH

One approach to creating the MF $\mu_{NS(\widetilde{u_0},\widetilde{u_1},\widetilde{h_c},\tilde{D},\tilde{i},\tilde{C})}$ is to derive the α-cuts of $\mu_{NS(\widetilde{u_0},\widetilde{u_1},\widetilde{h_c},\tilde{D},\tilde{i},\tilde{C})}$. Therefore, the α-cuts or α-level sets of $\widetilde{u_0}$, $\widetilde{u_1}$, $\widetilde{h_c}$, $\tilde{D}$, $\tilde{i}$, and $\tilde{C}$ are defined as follows:

$$u_0(\alpha) = \{x \in X \,|\mu_{\widetilde{u_0}}(x) \geq \alpha\} \tag{7}$$

$$u_1(\alpha) = \{y \in Y \,|\mu_{\widetilde{u_1}}(y) \geq \alpha\} \tag{8}$$

$$D(\alpha) = \{u \in U \,|\mu_{\tilde{D}}(u) \geq \alpha\} \tag{9}$$

$$h_c(\alpha) = \{v \in V \,|\mu_{\widetilde{h_c}}(v) \geq \alpha\} \tag{10}$$

$$i(\alpha) = \{z \in Z|\mu_{\tilde{i}}(z) \geq \alpha\} \tag{11}$$

$$C(\alpha) = \{w \in W \,|\mu_{\tilde{C}}(w) \geq \alpha\} \tag{12}$$

The α-level sets of $\widetilde{u_0}$, $\widetilde{u_1}$, $\widetilde{h_c}$, $\tilde{D}$, $\tilde{i}$, and $\tilde{C}$ defined in (7)-(12) are crisp intervals which can be expressed in another forms:

$$u_0(\alpha) = [\min_{x\in X}\{x|\mu_{\widetilde{u_0}}(x) \geq \alpha\}, \max_{x\in X}\{x|\mu_{\widetilde{u_0}}(x) \geq \alpha\}] \tag{13}$$

$$u_1(\alpha) = [\min_{y\in Y}\{y|\mu_{\widetilde{u_1}}(y) \geq \alpha\}, \max_{y\in Y}\{y|\mu_{\widetilde{u_1}}(y) \geq \alpha\}] \tag{14}$$

$$D(\alpha) = [\min_{u\in U}\{u|\mu_{\tilde{D}}(u) \geq \alpha\}, \max_{u\in U}\{u|\mu_{\tilde{D}}(u) \geq \alpha\}] \tag{15}$$

$$h_c(\alpha) = [\min_{v\in V}\{v|\mu_{\widetilde{h_c}}(v) \geq \alpha\}, \max_{v\in V}\{v|\mu_{\widetilde{h_c}}(v) \geq \alpha\}] \tag{16}$$

$$C(\alpha) = [\min_{w\in W}\{w|\mu_{\tilde{C}}(w) \geq \alpha\}, \max_{w\in W}\{w|\mu_{\tilde{C}}(w) \geq \alpha\}] \tag{17}$$

$$i(\alpha) = [\min_{z\in Z}\{z|\mu_{\tilde{i}}(z) \geq \alpha\}, \max_{z\in Z}\{z|\mu_{\tilde{i}}(z) \geq \alpha\}] \tag{18}$$

To derive the MF $\mu_{NS(\widetilde{u_0},\widetilde{u_1},\tilde{i},\widetilde{h_c},\tilde{D},\tilde{C},Q)}(\vartheta)$, we need at least one of the following to satisfy $\mu_{NS(\widetilde{u_0},\widetilde{u_1},\tilde{i},\widetilde{h_c},\tilde{D},\tilde{C},Q)}(\vartheta)=\alpha$.

$$\{\mu_{\widetilde{u_0}}(x) = \alpha, \mu_{\widetilde{u_1}}(y) \geq \alpha, \mu_{\tilde{D}}(u) \geq \alpha, \mu_{\widetilde{h_c}}(v) \geq \alpha, \mu_{\tilde{i}}(z) \geq \alpha, \mu_{\tilde{C}}(z) \geq \alpha\} \tag{19}$$

$$\{\mu_{\widetilde{u_0}}(x) \geq \alpha, \mu_{\widetilde{u_1}}(y) = \alpha, \mu_{\tilde{D}}(u) \geq \alpha, \mu_{\widetilde{h_c}}(v) \geq \alpha, \mu_{\tilde{i}}(z) \geq \alpha, \mu_{\tilde{C}}(z) \geq \alpha\} \tag{20}$$

$$\{\mu_{\widetilde{u_0}}(x) \geq \alpha, \mu_{\widetilde{u_1}}(y) \geq \alpha, \mu_{\widetilde{D}}(u) = \alpha, \mu_{\widetilde{h_c}}(v) \geq \alpha, \mu_{\tilde{\imath}}(z) \geq \alpha, \mu_{\widetilde{C}}(z) \geq \alpha\} \tag{21}$$

$$\{\mu_{\widetilde{u_0}}(x) \geq \alpha, \mu_{\widetilde{u_1}}(y) \geq \alpha, \mu_{\widetilde{D}}(u) \geq \alpha, \mu_{\widetilde{h_c}}(v) = \alpha, \mu_{\tilde{\imath}}(z) \geq \alpha, \mu_{\widetilde{C}}(z) \geq \alpha\} \tag{22}$$

$$\{\mu_{\widetilde{u_0}}(x) \geq \alpha, \mu_{\widetilde{u_1}}(y) \geq \alpha, \mu_{\widetilde{D}}(u) \geq \alpha, \mu_{\widetilde{h_c}}(v) \geq \alpha, \mu_{\tilde{\imath}}(z) = \alpha, \mu_{\widetilde{C}}(z) \geq \alpha\} \tag{23}$$

$$\{\mu_{\widetilde{u_0}}(x) \geq \alpha, \mu_{\widetilde{u_1}}(y) \geq \alpha, \mu_{\widetilde{D}}(u) \geq \alpha, \mu_{\widetilde{h_c}}(v) \geq \alpha, \mu_{\tilde{\imath}}(z) \geq \alpha, \mu_{\widetilde{C}}(z) = \alpha\} \tag{24}$$

Clearly, the membership function of $NS(\widetilde{u_0}, \widetilde{u_1}, \widetilde{h_c}, \widetilde{D}, \tilde{\imath}, \widetilde{C})$ defined in the extension principle is also parameterized by α. Accordingly, we can use its α-cut to construct its membership function. This can be accomplished via parametric NLP techniques. The NLP to determine the lower and upper bounds of the α-cut of variables are:

$$E_\alpha^l = \min_{\sigma\epsilon R} \sqrt{\frac{2.w.u}{v+z.x}}, E_\alpha^u = \max_{\sigma\epsilon R} \sqrt{\frac{2.w.u}{v+z.x}} \tag{25}$$

where $\sigma = w \in [w_\alpha^l, w_\alpha^u], u \in [u_\alpha^l, u_\alpha^u], v \in [v_\alpha^l, v_\alpha^u], z \in [z_\alpha^l, z_\alpha^u], x \in [x_\alpha^l, x_\alpha^u]$.

If both E_α^l and E_α^u are invertible with respect to α, a left shape function $L(EOQ_{u_0}) = (E_\alpha^l)^{-1}$ and a right shape function $U(EOQ_{u_0}) = (E_\alpha^u)^{-1}$ can be extracted, from which the membership function $\mu_{\widetilde{E}}\left(EOQ_{u_0}\right)$ is constructed:

$$\mu_{\widetilde{E}}(EOQ_{u_0}) = \begin{cases} L(EOQ_{u_0}), & EOQ_{u_{0_1}} \leq EOQ_{u_0} \leq EOQ_{u_{0_2}} \\ 1, & EOQ_{u_{0_2}} \\ U(EOQ_{u_0}), & EOQ_{u_{0_2}} \leq EOQ_{u_0} \leq EOQ_{u_{0_3}} \end{cases} \tag{26}$$

In most cases, the values of E_α^l and E_α^u cannot be determined analytically. Therefore, a closed-form membership function for EOQ_{u_0} cannot be obtained. However, the numerical solutions for E_α^l and E_α^u at different possibility levels can be collected to approximate the shapes of $L(EOQ_{u_0}) = (E_\alpha^l)^{-1}$ and $U(EOQ_{u_0}) = (E_\alpha^u)^{-1}$. That is, the set of intervals $\{[E_\alpha^l, E_\alpha^u] \in | \alpha \in [0,1]\}$ shows the shape of $\mu_{\widetilde{E}}$, although the exact function is not known explicitly.

Similarly, for t_a, we have:

$$F_\alpha^l = \min_{\sigma\epsilon R} \frac{q}{u}, F_\alpha^u = \max_{\sigma\epsilon R} \frac{q}{u} \tag{27}$$

where $q \in [q_\alpha^l, q_\alpha^u], u \in [u_\alpha^l, u_\alpha^u]$.

$$\mu_{\widetilde{F}}(t_a) = \begin{cases} L(t_a), & t_{a_1} \leq t_a \leq t_{a_2} \\ 1 & t_{a_2} \\ U(t_a), & t_{a_2} \leq t_a \leq t_{a_3} \end{cases} \tag{28}$$

Likewise, for t, we have:

$$G_\alpha^l = \min_{\sigma\epsilon R} \sqrt{\frac{2.w.u^{-\frac{1}{2}}}{v+z.x}}, G_\alpha^u = \max_{\sigma\epsilon R} \sqrt{\frac{2.w.u^{-\frac{1}{2}}}{v+z.x}} \tag{29}$$

where $\sigma = w \in [w_\alpha^l, w_\alpha^u], u \in [u_\alpha^l, u_\alpha^u], v \in [v_\alpha^l, v_\alpha^u], z \in [z_\alpha^l, z_\alpha^u], x \in [x_\alpha^l, x_\alpha^u]$.

Therefore, the time interval between orders is the following:

$$t_a, t+t_a, t+2.t_a, \ldots \tag{30}$$

Thus, we have:

$$t_f = \max_m (t_a + m.t) \le t_p \;, \quad \forall\, m = 0,1,2, \ldots \tag{31}$$

Using the above calculation values for $\mathrm{IP}(t_p)$ and $\mathrm{IP}(t_f)$, we have:

$$I_\alpha^l = \min_{\sigma\epsilon R}\left(\sqrt{\frac{2.w.u}{v+z.x}}\text{-}(j\text{-}k).u\right), I_\alpha^u = \max_{\sigma\epsilon R}\left(\sqrt{\frac{2.w.u}{v+z.x}}\text{-}(j\text{-}k).u\right) \tag{32}$$

Therefore, we have $\sigma = w \in [w_\alpha^l, w_\alpha^u], u \in [u_\alpha^l, u_\alpha^u], v \in [v_\alpha^l, v_\alpha^u], z \in [z_\alpha^l, z_\alpha^u], x \in [x_\alpha^l, x_\alpha^u], j \in [j_\alpha^l, j_\alpha^u], k \in [k_\alpha^l, k_\alpha^u]$. *Fig.* (1) clearly shows that $\mathrm{IP}(t_f)$ is equal to EOQ_{u_0}.

Thus, the values of the membership functions Q_p and Q_f are obtained below. First, we determine the membership function Q_p as follows:

$$H_\alpha^l = \min_{\sigma\epsilon R}\left(\frac{(y\text{-}x).u}{v+z.x} + \frac{v+z.y}{v+z.x}.\sqrt{\frac{2.w.u}{v+z.y}}\text{-}\sqrt{\frac{2.w.u}{v+z.x}} + (j\text{-}k).u\right),$$

$$H_\alpha^u = \max_{\sigma\epsilon R}\left(\frac{(y\text{-}x).u}{v+z.x} + \frac{v+z.y}{v+z.x}.\sqrt{\frac{2.w.u}{v+z.y}}\text{-}\sqrt{\frac{2.w.u}{v+z.x}} + (j\text{-}k).u\right) \tag{33}$$

Therefore, $\sigma = w \in [w_\alpha^l, w_\alpha^u], u \in [u_\alpha^l, u_\alpha^u], v \in [v_\alpha^l, v_\alpha^u], z \in [z_\alpha^l, z_\alpha^u], x \in [x_\alpha^l, x_\alpha^u], j \in [j_\alpha^l, j_\alpha^u], k \in [k_\alpha^l, k_\alpha^u]$. Then, we calculate Q_f as follows:

$$L_\alpha^l = \min_{\sigma\epsilon R}\left(\frac{(y\text{-}x).u}{v+z.x} + \frac{v+z.y}{v+z.x}.\sqrt{\frac{2.w.u}{v+z.y}}\text{-}\sqrt{\frac{2.w.u}{v+z.x}}\right),$$

$$L_\alpha^u = \max_{\sigma\epsilon R}\left(\frac{(y\text{-}x).u}{v+z.x} + \frac{v+z.y}{v+z.x}.\sqrt{\frac{2.w.u}{v+z.y}}\text{-}\sqrt{\frac{2.w.u}{v+z.x}}\right) \tag{34}$$

Hence, $\sigma = w \in [w_\alpha^l, w_\alpha^u], u \in [u_\alpha^l, u_\alpha^u], v \in [v_\alpha^l, v_\alpha^u], z \in [z_\alpha^l, z_\alpha^u], x \in [x_\alpha^l, x_\alpha^u]$.

Faintly, the membership functions of net saving in two modes of t_p and t_f are obtained, First, we determine the membership function NS_p as follows:

$$P_\alpha^l = \min_{\sigma\epsilon R}\Bigg(Q_p.(y\text{-}x) + \sqrt{2.w.u.(v+z.y)}.\left(\frac{y\text{-}x}{v+z.x} + \frac{v+z.y}{v+z.x}.\sqrt{\frac{2.w.u^{\text{-}1/2}}{v+z.y}}\text{-}t + j\text{-}k\right)\text{-}(v\;+z.x).\Bigg(\bigg(\frac{y\text{-}x}{v+z.x} + \frac{v+z.y}{v+z.x}.\sqrt{\frac{2.w.u^{\text{-}\frac{1}{2}}}{v+z.y}}\text{-}t +$$

$$j\text{-}k\bigg).\left(\mathrm{EOQ}_{u_0}\text{-}(j\text{-}k).u\right) + \frac{1}{2}.\left(\frac{(y\text{-}x).u^{1/2}}{v+z.x} + \frac{v+z.y}{v+z.x}.\sqrt{\frac{2.w}{v+z.y}}\text{-}\sqrt{\frac{2.w}{v+z.x}} + (j\text{-}k).u^{1/2}\right)^2\Bigg)\text{-}w\Bigg),$$

$$P_\alpha^u=\max_{\sigma\epsilon R}\left(Q_p.(y\text{-}x)+\sqrt{2.w.u.(v+z.y)}.\left(\frac{y\text{-}x}{v+z.x}+\frac{v+z.y}{v+z.x}.\sqrt{\frac{2.w.u^{-1/2}}{v+z.y}}\text{-}t+j\text{-}k\right)\text{-}(v+z.x).\left(\left(\left(\frac{y\text{-}x}{v+z.x}+\frac{v+z.y}{v+z.x}.\sqrt{\frac{2.w.u^{-\frac{1}{2}}}{v+z.y}}\text{-}t+j\text{-}k\right).\left(EOQ_{u_0}\text{-}(j\text{-}k).u\right)+\frac{1}{2}.\left(\frac{(y\text{-}x).u^{1/2}}{v+z.x}+\frac{v+z.y}{v+z.x}.\sqrt{\frac{2.w}{v+z.y}}\text{-}\sqrt{\frac{2.w}{v+z.x}}+(j\text{-}k).u^{1/2}\right)^2\right)\text{-}w\right)\right) \quad (35)$$

Accordingly, $w\in[w_\alpha^l,w_\alpha^u]$, $u\in[u_\alpha^l,u_\alpha^u]$, $v\in[v_\alpha^l,v_\alpha^u]$, $z\in[z_\alpha^l,z_\alpha^u]$, $x\in[x_\alpha^l,x_\alpha^u]$, $j\in[j_\alpha^l,j_\alpha^u]$, $k\in[k_\alpha^l,k_\alpha^u]$ and $y\in[y_\alpha^l,y_\alpha^u]$. Then, similarly, we determine the membership function NS_f as follows:

$$M_\alpha^l=\min_{\sigma\epsilon R}\left(\frac{(y\text{-}x).u}{v+z.x}+\frac{v+z.y}{v+z.x}.\sqrt{\frac{2.w.u}{v+z.y}}\text{-}\sqrt{\frac{2.w.u}{v+z.x}}\right).(y\text{-}x)+\sqrt{2.w.u.(v+z.y)}.\left(\frac{y\text{-}x}{v+z.x}+\frac{v+z.y}{v+z.x}.\sqrt{\frac{2.w.u^{-1/2}}{v+z.y}}\text{-}\sqrt{\frac{2.w.u^{-1/2}}{v+z.x}}\right)\text{-}(v+z.x).\left(\left(\left(\frac{y\text{-}x}{v+z.x}+\frac{v+z.y}{v+z.x}.\sqrt{\frac{2.w.u^{-1/2}}{v+z.y}}\text{-}\sqrt{\frac{2.w.u^{-1/2}}{v+z.x}}\right).\left(\sqrt{\frac{2.w.u}{v+z.x}}\right)+\frac{1}{2}.\left(\frac{(y\text{-}x).u^{1/2}}{v+z.x}+\frac{v+z.y}{v+z.x}.\sqrt{\frac{2.w}{v+z.y}}\text{-}\sqrt{\frac{2.w}{v+z.x}}\right)^2\right)\text{-}w,\right.$$

$$M_\alpha^u=\max_{\sigma\epsilon R}\left(\frac{(y\text{-}x).u}{v+z.x}+\frac{v+z.y}{v+z.x}.\sqrt{\frac{2.w.u}{v+z.y}}\text{-}\sqrt{\frac{2.w.u}{v+z.x}}\right).(y\text{-}x)+\sqrt{2.w.u.(v+z.y)}.\left(\frac{y\text{-}x}{v+z.x}+\frac{v+z.y}{v+z.x}.\sqrt{\frac{2.w.u^{-1/2}}{v+z.y}}\text{-}\sqrt{\frac{2.w.u^{-1/2}}{v+z.x}}\right)\text{-}(v+z.x).\left(\left(\left(\frac{y\text{-}x}{v+z.x}+\frac{v+z.y}{v+z.x}.\sqrt{\frac{2.w.u^{-1/2}}{v+z.y}}\text{-}\sqrt{\frac{2.w.u^{-1/2}}{v+z.x}}\right).\left(\sqrt{\frac{2.w.u}{v+z.x}}\right)+\frac{1}{2}.\left(\frac{(y\text{-}x).u^{1/2}}{v+z.x}+\frac{v+z.y}{v+z.x}.\sqrt{\frac{2.w}{v+z.y}}\text{-}\sqrt{\frac{2.w}{v+z.x}}\right)^2\right)\text{-}w\right. \quad (38)$$

Accordingly, $w\in[w_\alpha^l,w_\alpha^u]$, $u\in[u_\alpha^l,u_\alpha^u]$, $v\in[v_\alpha^l,v_\alpha^u]$, $z\in[z_\alpha^l,z_\alpha^u]$, $x\in[x_\alpha^l,x_\alpha^u]$, $j\in[j_\alpha^l,j_\alpha^u]$, $k\in[k_\alpha^l,k_\alpha^u]$ and $y\in[y_\alpha^l,y_\alpha^u]$.

VI. SOLUTION PROCEDURE

A. The first approach to solving the model

Step1. For parameter α, from zero to one with steps of 0.1, repeat Steps 2 through 14.

Step2. Calculate the following parameters.

$$\left\{\begin{array}{c} x_\alpha^l=(x_2\text{-}x_1).\alpha+x_1 \;\; ; \;\; x_\alpha^u=x_4\text{-}(x_4\text{-}x_3).\alpha \\ y_\alpha^l=(y_2\text{-}y_1).\alpha+y_1 \;\; ; \;\; y_\alpha^u=y_4\text{-}(y_4\text{-}y_3).\alpha \\ u_\alpha^l=(u_2\text{-}u_1).\alpha+u_1 \;\; ; \;\; u_\alpha^u=x_4\text{-}(u_4\text{-}u_3).\alpha \\ v_\alpha^l=(v_2\text{-}v_1).\alpha+v_1 \;\; ; \;\; v_\alpha^u=v_4\text{-}(v_4\text{-}v_3).\alpha \\ z_\alpha^l=(z_2\text{-}z_1).\alpha+z_1 \;\; ; \;\; z_\alpha^u=z_4\text{-}(z_4\text{-}z_3).\alpha \\ w_\alpha^l=(w_2\text{-}w_1).\alpha+w_1 \;\; ; \;\; w_\alpha^u=w_4\text{-}(w_4\text{-}w_3).\alpha \\ q_\alpha^l=(q_2\text{-}q_1).\alpha+q_1 \;\; ; \;\; q_\alpha^u=q_4\text{-}(q_4\text{-}q_3).\alpha \\ j_\alpha^l=(j_2\text{-}j_1).\alpha+j_1 \;\; ; \;\; j_\alpha^u=j_4\text{-}(j_4\text{-}j_3).\alpha \end{array}\right\}$$

Step3. Repeat Steps 3 to 14 for w_α^l to w_α^u;

Step4. Repeat Steps 3 to 14 for u_α^l to u_α^u;

Step5. Repeat Steps 3 to 14 for v_α^l to v_α^u;

Step6. Repeat Steps 3 to 14 for z_α^l to z_α^u;

Step7. Repeat Steps 3 to 14 for y_α^l to y_α^u;

Step8. Repeat Steps 3 to 14 for q_α^l to q_α^u;

Step9. Repeat Steps 3 to 14 for j_α^l to j_α^u;

Step10. Repeat Steps 3 to 14 for x_α^l to x_α^u.

Step11. Calculate the following values.

$$E_\alpha^l = \arg\{\min E(w,u,v,z,x)\}$$
$$E_\alpha^u = \arg\{\max E(w,u,v,z,x)\}$$
$$G_\alpha^l = \arg\{\min G(w,u,v,z,x)\}$$
$$G_\alpha^u = \arg\{\max G(w,u,v,z,x)\}$$
$$F_\alpha^l = \arg\{\min F(q,u)\}$$
$$F_\alpha^u = \arg\{\max F(q,u)\}$$
$$[k_\alpha^l, k_\alpha^u] = \max_{\forall:m}([F_\alpha^l, F_\alpha^u] + m.[G_\alpha^l, G_\alpha^u]) \leq [j_\alpha^l, j_\alpha^u]$$
$$I_\alpha^l = \arg\{\min I(w,u,v,z,x,j,k)\}$$
$$I_\alpha^u = \arg\{\max I(w,u,v,z,x,j,k)\}$$
$$H_\alpha^l = \arg\{\min H(w,u,v,z,x,y)\}$$
$$H_\alpha^u = \arg\{\max H(w,u,v,z,x,y)\}$$
$$L_\alpha^l = \arg\{\min L(w,u,v,z,x,y,j,k)\}$$
$$L_\alpha^u = \arg\{\max L(w,u,v,z,x,y,j,k)\}$$
$$P_\alpha^l = \arg\{\min P(w,u,v,z,x,y,j,k)\}$$
$$P_\alpha^u = \arg\{\max P(w,u,v,z,x,y,j,k)\}$$
$$M_\alpha^l = \arg\{\min M(w,u,v,z,x,y)\}$$
$$M_\alpha^u = \arg\{\max M(w,u,v,z,x,y)\}$$

Step12. Calculate upper and lower bound variable sets by repeating Step 11.

Step13. If the following equation is satisfied, go to Step 14. Otherwise, a special order is not issued.

$$\max_\alpha\{[M_\alpha^l, M_\alpha^u] + [w_\alpha^l, w_\alpha^u], [P_\alpha^l, P_\alpha^u]\} > 0 \text{ or } [M_\alpha^u, M_\alpha^u] + [w_\alpha^l, w_\alpha^u] \vee [P_\alpha^l, P_\alpha^u] > 0.$$

Step14. If the following equation is satisfied, then the specific order in time t_p will be issued. Otherwise, it is issued in time t_f. $[M_\alpha^l, M_\alpha^u] + [w_\alpha^l, w_\alpha^u] \leq [P_\alpha^l, P_\alpha^u]$.

Step15. Stop.

B. The second approach to solving the model

This approach was developed by Vujošević et al. (1996). First, defuzzification is done by one of the methods of fuzzy input parameters to get into the crisp state. Finally, the problem is solved with crisp values. In fact, the crisp model is analyzed.

C. The third approach to solving the model

In this approach, the output parameters converted by one of the methods are difuzzified to crisp numbers. In this method, the final answer is a crisp number and the discussion on the different α-levels of the membership function parameters is not needed. The output of the model is converted to the crisp mode using defuzzification methods.

VII. NUMERICAL EXAMPLE

In this section, to illustrate the suggested fuzzy models numerically, the following example is considered (planning horizon is one year and ss denotes system inventory level at the beginning of the period). According to the definition of the input parameters of the model in Sections II and IV, the input parameters of the model presented in Table II are shown as trapezoidal fuzzy numbers.

Table II. Values of parameters

Parameter	***Fuzzy trapezoidal value***	***Parameter***	***Fuzzy trapezoidal value***
D	[200,235,260,275]	u_1	[240,248.263.270]
h_c	[1.1,1.7,2.2,2.5]	s	[115,121,128,130]
i	[0.003,0.008,0.012,0.015]	t_p	[2.5,2.8,3,3.2]
C	[1700,1850,2050,2100]	u_0	[180,188,205,210]

The results for the α-cut input parameters are summarized in the following tables. Now, using the procedure in sub-section A of the previous section, we calculate output parameters of the model. The results for these parameters are shown in Tables V to VIII. According to the data in Tables V to VIII, the membership function for each parameter is presented in *Fig.* (2). It is observed that the membership function parameters are triangular fuzzy numbers. The parameters in the figure are in the highest range for $\alpha = 0$. Therefore, the minimum and maximum values of the parameters are not lower or higher with other values of α, respectively. Therefore, by using this method, the decision maker at each level of alpha cut can determine the optimal policy.

Table III. The α-cuts of the input parameters for 11 α values

$\boldsymbol{\alpha}$	$\boldsymbol{j^l_\alpha}$	$\boldsymbol{j^u_\alpha}$	$\boldsymbol{z^l_\alpha}$	$\boldsymbol{z^u_\alpha}$	$\boldsymbol{w^l_\alpha}$	$\boldsymbol{w^u_\alpha}$	$\boldsymbol{y^l_\alpha}$	$\boldsymbol{y^u_\alpha}$
0	2.5	3.2	0.003	0.015	1700	2100	240	270
0.1	2.53	3.18	0.0035	0.0147	1715	2095	240.8	269.3
0.2	2.56	3.16	0.004	0.0144	1730	2090	241.6	268.6
0.3	2.59	3.14	0.0045	0.0141	1745	2085	242.4	267.9
0.4	2.62	3.12	0.005	0.0138	1760	2080	243.2	267.2
0.5	2.65	3.1	0.0055	0.0135	1775	2075	244	266.5
0.6	2.68	3.08	0.006	0.0132	1790	2070	244.8	265.8
0.7	2.71	3.06	0.0065	0.0129	1805	2065	245.6	265.1
0.8	2.74	3.04	0.007	0.0126	1820	2060	246.4	264.4
0.9	2.77	3.02	0.0075	0.0123	1835	2055	247.2	263.7
1	2.8	3	0.008	0.012	1850	2050	248	263

Table IV. The α-cuts of the input parameters for 11 α values

α	x_α^l	x_α^u	v_α^l	v_α^u	u_α^l	u_α^u	q_α^l	q_α^u
0	180	210	1.1	2.5	200	275	115	130
0.1	180.8	209.5	1.16	2.47	203.5	273.5	115.6	129.8
0.2	181.6	209	1.22	2.44	207	272	116.2	129.6
0.3	182.4	208.5	1.28	2.41	210.5	270.5	116.8	129.4
0.4	183.2	208	1.34	2.38	214	269	117.4	129.2
0.5	184	207.5	1.4	2.35	217.5	267.5	118	129
0.6	184.8	207	1.46	2.32	221	266	118.6	128.8
0.7	185.6	206.5	1.52	2.29	224.5	264.5	119.2	128.6
0.8	186.4	206	1.58	2.26	228	263	119.8	128.4
0.9	187.2	205.5	1.64	2.23	231.5	261.5	120.4	128.2
1	188	205	1.7	2.2	235	260	121	128

Table V. Optimized values obtained by the procedure in Section VI.A for trapezoidal membership function distributions

α	$E_\alpha^l(EOQ)$	$E_\alpha^u(EOQ)$	$F_\alpha^l(t_a)$	$F_\alpha^u(t_a)$
0	374.662	627.922	0.522	0.591
0.1	380.137	612.051	0.534	0.569
0.2	390.325	594.489	0.535	0.57
0.3	396.64	578.197	0.536	0.571
0.4	415.486	563.022	0.537	0.572
0.5	424.631	548.842	0.538	0.573
0.6	436.461	529.752	0.552	0.552
0.7	446.861	517.403	0.553	0.553
0.8	450.828	505.762	0.554	0.554
0.9	459.343	494.761	0.555	0.555
1	464.342	484.342	0.556	0.556

Table VI. Optimized values obtained by the procedure in Section VI.A for trapezoidal membership function distributions

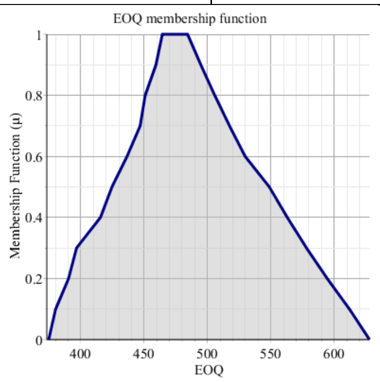
α	$G_\alpha^l(t)$	$G_\alpha^u(t)$	$I_\alpha^l(IP(t_p))$	$I_\alpha^u(IP(t_p))$
0	0.533	0.591	2.232	3.382
0.1	0.534	0.585	2.26	3.269
0.2	0.535	0.58	2.286	3.185
0.3	0.536	0.576	2.326	3.108
0.4	0.537	0.573	2.365	3.036
0.5	0.539	0.57	2.406	2.968
0.6	0.542	0.566	2.423	2.882
0.7	0.544	0.562	2.466	2.823
0.8	0.546	0.559	2.505	2.767
0.9	0.55	0.557	2.573	2.714
1	0.556	0.556	2.664	2.664

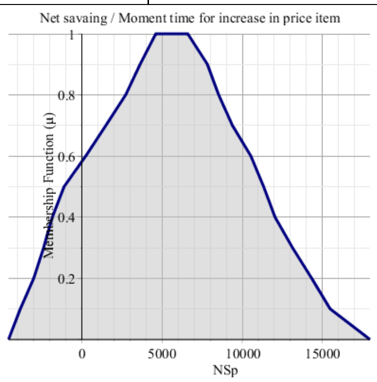
Table VII. Optimized values obtained by the procedure in Section VI.A for trapezoidal membership function distributions

α	$H_\alpha^l(Q_p)$	$H_\alpha^u(Q_p)$	$L_\alpha^l(Q_f)$	$L_\alpha^u(Q_f)$
0	0.254	305.622	24.75	80.929
0.1	3.381	270.926	30.888	78.043
0.2	7.339	251.839	33.525	69.478
0.3	18.896	226.62	37.141	65.571
0.4	29.954	205.198	38.744	63.345
0.5	43.591	193.528	39.338	60.154
0.6	65.609	185.354	40.909	55.354
0.7	85.695	171.486	41.967	52.407
0.8	99.268	149.268	44.057	51.057
0.9	112.347	132.347	47.179	50.179
1	124.991	124.991	49.333	49.333

Table VIII. Optimized values obtained by the procedure in Section VI.A for trapezoidal membership function distributions

α	$P_\alpha^l(NS_p)$	$P_\alpha^u(NS_p)$	$M_\alpha^l(NS_f)$	$M_\alpha^u(NS_f)$
0	-4571.88	17962.36	-513.28	3411.47
0.1	-3838.53	15479.24	-443.60	2987.02
0.2	-3003.12	14329.04	-331.36	2724.74
0.3	-2386.48	13134.44	-290.17	2488.00
0.4	-1843.32	12046.41	-180.69	2250.28
0.5	-1110.05	11332.96	-105.67	2111.95
0.6	238.49	10538.49	100.47	1921.47
0.7	1489.93	9389.93	265.34	1755.34
0.8	2730.51	8530.51	512.16	1501.16
0.9	3628.72	7828.72	678.89	1328.89
1	4605.03	6605.03	768.46	118.46





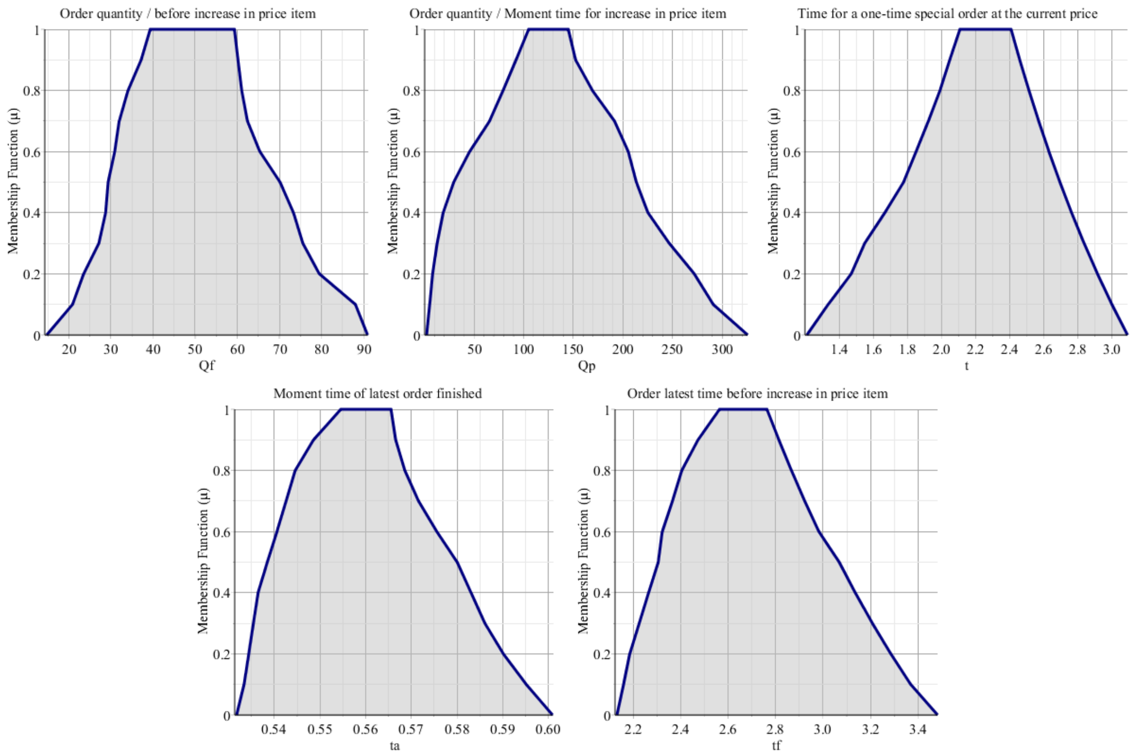


Figure 2. Membership functions for the example.

Table IX. The α-cuts of the special order for 11 values of α

α	*Order / No order*	*Special ordering time*
0	No order	-
0.1	No order	-
0.2	No order	-
0.3	No order	-
0.4	No order	-
0.5	No order	-
0.6	Special order	t_f
0.7	Special order	t_f
0.8	Special order	t_p
0.9	Special order	t_p
1	Special order	t_p

Table X. The α-cuts of the output parameters for 11 values of α

α	*Special order quantity*	*Net saving of special order*
0	-	-
0.1	-	-
0.2	-	-
0.3	-	-
0.4	-	-
0.5	-	-
0.6	[238.49,10538.49]	[46,206]
0.7	[1489.93, 9389.94]	[66,192]
0.8	[2730.51,8530.50]	[80,170]
0.9	[3628.72,7828.72]	[93,153]
1	[4605.03,6605.03]	[105,145]

According to Table VI and *Fig.* (2), for cuts 0 to 0.6, part membership function graph on the left is negative, which means extra cost to the organization in some orders.

The second approach to solving the model:

As described in the review of literature, the each parameter should be determined closest to its membership function (triangular, trapezoidal, Gaussian, etc.). Then, approximate the difuzzified membership function close to the actual crisp state.

The first method:

Difuzzification is done by using the Signed Distance (SD). If $\tilde{\mu} \in$ Fuzzy number , the signed distance from $\tilde{\mu}$ to $\tilde{0}$ is defined as:

$$d(\tilde{\mu}, \tilde{0}) = \int_0^1 d([\mu_\alpha^-, \mu_\alpha^+], \tilde{0}) d\alpha \qquad (39)$$

The second method:

Difuzzification is done by using the Maximum Degree of Membership (MDOM) (largest value of maximum). It is supposed that there is a plateau at the maximum value of the final function, which is the largest within the range.

The third method:

Centroid of Gravity (COG) method calculates the center of gravity for the area under the curve as follows

$$X^* = \frac{\int \mu_{\tilde{A}}(x).x \, dx}{\int \mu_{\tilde{A}}(x) \, dx} \qquad (40)$$

The results of these three methods in calculating the model parameters are given in Table XI.

Table XI. The α-cuts of the output parameters for 11 values of α by the first, second, and third methods

	Defuzzification method		
Variable	***SD***	***MDOM***	***COG***
EOQ	485	479	499
t_a	0.554	0.555	0.56
t_f	2.665	2.664	2.759
Q_f	49	50	52
NS_f	1023.71	968.464	1288.884
Q_p	125	125	144
NS_p	5485.397	5605.026	6331.835
Order time	t_p	t_p	t_p
Order quantity	125	125	144
Net saving	5485.397	5605.026	6331.835

As observed, these methods, unlike the first approach, cannot determine the optimal policy at different levels of alpha-cutting.

The third approach to solving the model:

This approach is based on the research performed by Chen (2005). In this approach, input parameters are difuzzified and then, placed in the objective functions to calculate variables of the model. In other word, we using the center of gravity.

Table XII. The α-cuts of the output parameters for 11 values of α by the third approach

Variable	***COG method***
EOQ	476
t_a	0.56
t_f	2.67
Q_f	50
NS_f	968.3
Q_p	124
NS_p	5517.6
Order time	t_p
Order quantity	124
Net saving	5517.6

This approach also determines the result for the total alpha-cuts. The model presented in this study provides the decision maker with a more realistic assessment than the models presented by Cárdenas-Barrón et al. (2010b) and Taheri-Tolgari et al. (2018) do. In other words, the decision maker faces a wider range of choices. Also, considering the input parameters of the model, which are trapezoidal fuzzy numbers, the proposed model is closer to reality. Finally, the proposed model at the cut level equal to 1 tends toward a crisp state.

VIII. CONCLUSION

In this paper, an economic order quantity model was presented with a known price increase under fuzzy environment constraint by parametric non-linear programming. We used three different policies based on methods such as α-cuts (defuzzification of internal parameters before solving the model) and Vujosevic (difuzzification of the external parameters after solving the model). Under the first policy, we integrated α-cuts method and parametric non-linear programming problems to attain the membership functions of external parameters for reaching the optimal solution. The optimal order policies by the three methods were compared and numerical computations showed that the efficiency of the first method (i.e., the presented method) was considerably higher than that of the other two methods. The first method provided a more realistic evaluation than the others for decision making in the management of the organization. Also, the organization had a wide range of alternatives to select for making the optimal decisions by the first method.

The present concept can be extended to multi-product manufacturing systems via shortage (backorder and backlog), quantity discount, imperfect items, etc. for future research. The fields of plication of the proposed model may include the ordering of components in the oil and gas industry, import ordering in the conditions of the sanctioning, and ordering in the pharmaceutical industry.

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