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# Stock Evaluation under Mixed Uncertainties Using Robust DEA Model

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***Abstract–*** *Data Envelopment Analysis (DEA) is one of the popular and applicable techniques for assessing and ranking the stocks or other financial assets. It should be noted that in the financial markets, most of the times, the inputs and outputs of DEA models are accompanied by uncertainty. Accordingly, in this paper, a novel Robust Data Envelopment Analysis (RDEA) model, which is capable to be used in the presence of discrete and continuous uncertainties, is presented. The proposed novel RDEA model in the paper was implemented in a real case study of Tehran Stock Exchange (TSE). The results showed that the proposed new RDEA model was effective in the assessment and ranking of the stocks under different scenarios with interval values.*

***Keywords–*** *Robust data envelopment analysis, Stock performance measurement, Convex uncertainty set, Scenario based robust optimization.*

## I. INTRODUCTION

Evaluating stock to identify the desired portfolio is one of the most important problems in financial markets. Data Envelopment Analysis (DEA) is a non-parametric performance measurement technique used to estimate relative efficiency of Decision Making Units (DMUs) using inputs and outputs (Peykani et al., 2019).

With respect to the fact that one of the most important features of financial markets is their uncertainty, it is important to preserve the robustness of the solution obtained by the DEA model. Otherwise, the efficiency and ranking of the concerned stocks may be unreliable and, consequently, significant costs may be imposed on different stakeholders. To prevent such undesirable outcome, robust optimization methods can be employed, which do not need significant historical data and can be applied to almost all of the real-life DEA problems such as stock evaluation.

Sadjadi and Omrani (2008) were the pioneer researchers that worked on Robust Data Envelopment Analysis (RDEA) considering uncertainty of output parameters in performance evaluation of Iranian electricity distribution companies. Roghanian and Foroughi (2010) applied RDEA model to measuring the efficiency of Iranian regional airports. Sadjadi et al. (2011) presented a robust super-efficiency DEA model for ranking the Iranian provincial gas companies. Foroughi and Esfahani (2012) introduced robust AHP-DEA approach to efficiency measurement of Iranian airports.Hafezalkotob et al. (2015) proposed RDEA model under discrete uncertain data for performance measurement of Iranian electricity distribution companies by applying the scenario-based robust optimization. Lu (2015) presented the robust DEA approach to evaluating algorithmic performance.

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Peykani et al. (2016) introduced robust DEA models for measuring efficiency of stock under convex set uncertainty. Atıcı and Gülpınar (2016) employed RDEA models for performance assessment of olive oil production.Esfandiari et al. (2017) proposed robust two-stage DEA models in the presence of discrete uncertain data. Rabbani et al. (2017) presented a bootstrap interval RDEA for estimating efficiency and ranking the hospitals. Zahedi-Seresht et al. (2017) proposed a robust DEA model by applying scenario-based robust optimization approach. Peykani et al. (2018a) presented a robust DEA based on Variant of Radial Measure (VRM) model for stock efficiency measurement. Yousefi et al. (2018) used RDEA model and Failure Modes and Effect Analysis (FMEA) method for Health, Safety, and Environment (HSE) risk prioritization.

In this paper, novel RDEA models with constant return to scale and variable return to scale assumptions are presented, which can be used in the presence of discrete and continuous uncertainties for stock evaluation. The main characteristics of some papers on robust DEA and their differences from the RDEA model developed in this study are summarized in Table I.

**TABLE I. Review of some existing robust DEA models**

| ***Reference Article*** | ***Type of Uncertainty*** | | ***Robust Optimization Approach*** | |
|---|---|---|---|---|
| | ***Continuous*** | ***Discrete*** | ***Uncertainty Set*** | ***Scenario Based*** |
| Sadjadi and Omrani (2008) | ● | | ● | |
| Roghanian and Foroughi (2010) | ● | | ● | |
| Sadjadi et al. (2011) | ● | | ● | |
| Foroughi and Esfahani (2012) | ● | | ● | |
| Hafezalkotob et al. (2015) | | ● | | ● |
| Lu (2015) | ● | | ● | |
| Peykani et al. (2016) | ● | | ● | |
| Atıcı and Gülpınar (2016) | ● | | ● | |
| Esfandiari et al. (2017) | | ● | | ● |
| Rabbani et al. (2017) | ● | | ● | |
| Zahedi-Seresht et al. (2017) | | ● | | ● |
| Peykani et al. (2018a) | ● | | ● | |
| Yousefi et al. (2018) | ● | | ● | |
| Our Work | ● | ● | ● | ● |

As can be seen in Table I, the presented model in this research is the only RDEA model that is capable to be applied under mixed continuous and discrete uncertainties. In other words, the novel robust DEA model could be employed under different scenarios with interval values.

The rest of the paper is organized as follows. Modeling and formulations of the DEA models and robust optimization approach used in this study are explained in Section II and the novel RDEA models are proposed in Section III. Then, the proposed models in this study are implemented for a case study of Tehran Stock Exchange (TSE) and the results are evaluated in Section IV. Finally, the conclusions of the study and some directions for future research are given in Section V.

## II. BACKGROUND

### *A. DEA*

DEA is one of the prominent non-parametric performance measurement techniques that uses multiple inputs to produce multiple outputs for measuring efficiency and ranking homogeneous Decision Making Units (DMUs). This methodology was proposed by Charnes et al. (1978) for the first time based on Farrell's (1957) idea. Charnes et al. (1978) proposed the first DEA model based on the Constant Returns to Scale (CRS) assumption and called it CCR model.

There are $n$ homogenous decision making units, $DMU_j \ (j=1,...,n)$, that convert $m$ inputs, $x_{ij} \ (i=1,...,m)$, into $s$ outputs, $y_{rj} \ (r=1,...,s)$, and $DMU_0$ is an under-evaluation $DMU$. The non-negative weights $v_i \ (i=1,...,m)$ and $u_r \ (r=1,...,s)$ are assigned as inputs and outputs, respectively. The input-oriented multiplier CCR (CCR-IO) model is as follows:

*The CCR Model*

$$\begin{aligned} \text{Max} \quad & \sum_{r=1}^{s} y_{r0} u_r && (1) \\ \text{S.t.} \quad & \sum_{i=1}^{m} x_{i0} v_i = 1 \\ & \sum_{r=1}^{s} y_{rj} u_r - \sum_{i=1}^{m} x_{ij} v_i \le 0, \quad \forall j \\ & v_i, u_r \ge 0, \quad \forall i, r \end{aligned}$$

Banker et al. (1984) developed a CCR model based on the VRS assumption and called it BCC model. The Input-Oriented multiplier BCC (BCC-IO) model is as follows:

*The BCC Model*

$$\begin{aligned} \text{Max} \quad & \sum_{r=1}^{s} y_{r0} u_r + w_0 && (2) \\ \text{S.t.} \quad & \sum_{i=1}^{m} x_{i0} v_i = 1 \\ & \sum_{r=1}^{s} y_{rj} u_r - \sum_{i=1}^{m} x_{ij} v_i + w_0 \le 0, \quad \forall j \\ & v_i, u_r \ge 0, \quad \forall i, r \end{aligned}$$

CCR and BCC models are radial projection constructs. With respect to the fact that CCR and BCC are basic and popular DEA models, CCR-IO and BCC-IO are used in this research.

### *B. Robust Optimization*

Robust Optimization (RO) is one of the applicable and popular methods that can be used to deal with uncertainty in optimization problems. In general, uncertainties in the real-world optimization problems can be classified into discrete and continuous uncertainties. It should be noted that scenario based and convex uncertainty set based approaches are used for dealing with discrete and continuous uncertainties, respectively.

Mulvey et al. (1995) presented the scenario based robust optimization model by considering a benefit-cost analysis between feasibility robustness and optimality robustness. Also, Soyster (1973), Ben-Tal and Nemirovski (2000), and Bertsimas and Sim (2004) presented a popular robust optimization approach to convex uncertainty set. The RO approach of Soyster (1973) was too conservative. Ben-Tal and Nemirovski (2000) presented an RO approach with a Non-Linear Programming (NLP) robust counterpart, which could be problematic in the real-world problems, but it was capable to adjust the conservatism by setting the parameters. Bertsimas and Sim (2004) presented a robust approach that could flexibly adjust the level of conservatism of the robust solutions by setting the parameters. The robust counterpart in the approach was Linear Programming (LP).

To implement the robust approach of Bertsimas and Sim (2004) to dealing with uncertainty in data, consider a particular constraint $i$ of a nominal model and let $J_i$ represent the set of coefficients in constraint $i$ that are subject to uncertainty. It should be noted that each entry $a_{ij}, j \in J_i$ is modeled as a symmetric and bounded random variable, which takes values in $\left[a_{ij} - \hat{a}_{ij}, a_{ij} + \hat{a}_{ij}\right]$. The center of this interval at the point $a_{ij}$ is a nominal value and $\hat{a}_{ij}$ is perturbation of uncertain parameters $a_{ij}, j \in J_i$. Finally, the robust counterpart of constraint $i$ is given in *Eq.* (3):

$$Robust\ Counterpart\ of\ \left\{\sum_j \tilde{a}_{ij}\,\varphi_j \le b_i\ ,\ \ \forall i\right\} \Rightarrow \begin{cases} \sum_j a_{ij}\,\varphi_j + Z_i\,\Gamma_i + \sum_{j \in J_i} P_{ij} \le b_i\ , & \forall i \\ Z_i + P_{ij} \ge \hat{a}_{ij}\,\phi_j\ , & \forall i, j \in J_i \\ -\phi_{ij} \le \varphi_j \le \phi_{ij}\ , & \forall i, j \in J_i \\ Z, P, \phi \ge 0 \end{cases} \tag{3}$$

It should be noted that the parameter $\Gamma$ adjusts robustness of the model to solve the conservative level. With respect to this feature and linearity of the robust counterpart of the model, the robust approach of Bertsimas and Sim (2004) is one of the popular RO ones.

## III. NOVEL RDEA MODEL

In the real world, we are faced with uncertain data and one of the most important features of financial markets is their uncertainty (Peykani and Mohammadi, 2018). Also, one of the most important assumptions in DEA is that the measured data are certain and conclusive (Peykani et al., 2018b). However, a small bias or deviation in the values of the data can lead to significant differences in the final results. In the worst case, we are faced with infeasible solutions. Therefore, ranking the results can be invalid, especially when efficiency of a unit is close to another one (Sadjadi & Omrani, 2008).

In this section, the novel RDEA models with CRS and VRS assumptions, which are capable to be used in the presence of discrete and continuous uncertainties for Stock Performance Measurement (SPM), are presented. In order to be familiar with the hybrid scenario based and interval based uncertainty, see Fig. (1).

As illustrated in Fig. (1), the uncertain parameter $\tilde{\Lambda}$ takes different values for different scenarios and for each scenario, an interval is considered instead of a nominal value. Thus, for dealing with this hybrid uncertainty, RO scenario based and RO convex uncertainty set based approaches will be applied. By considering the uncertainty in outputs, Models (1) and (2) will be converted to Models (4) and (5):

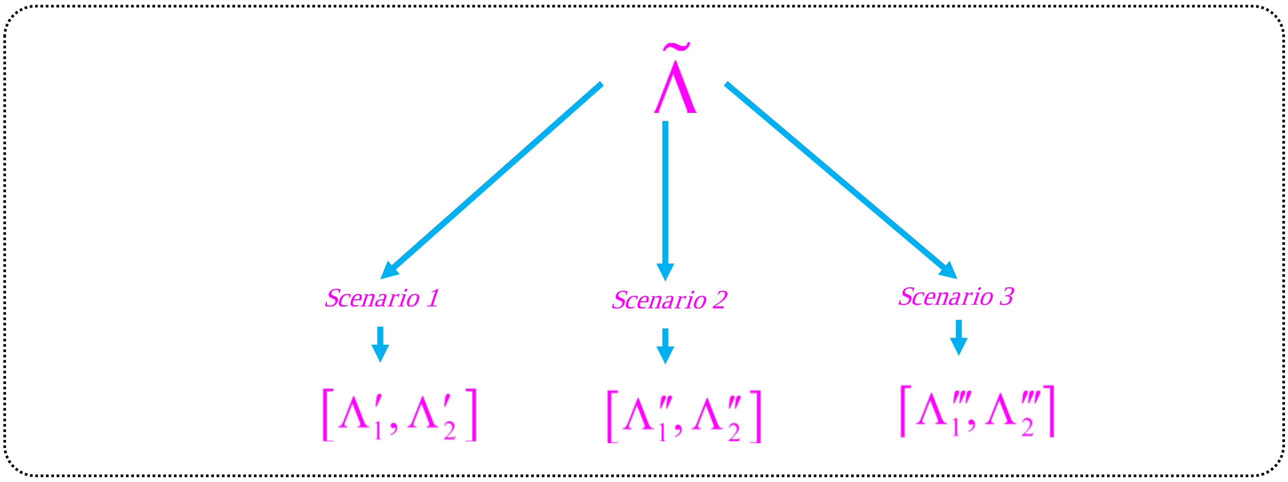


**Figure 1. Hybrid uncertainty (discrete and continuous uncertainties)**

*The Uncertain CCR Model*

$$\text{Max} \quad \Psi \tag{4}$$

$$\text{S.t.} \quad \sum_{r=1}^{s} \tilde{y}_{r0\xi} u_r \geq \Psi, \qquad \forall \xi$$

$$\sum_{i=1}^{m} x_{i0} v_i = 1$$

$$\sum_{r=1}^{s} \tilde{y}_{rj\xi} u_r - \sum_{i=1}^{m} x_{ij} v_i \leq 0, \qquad \forall j, \xi$$

$$v_i, u_r \geq 0, \qquad \forall i, r$$

*The Uncertain BCC Model*

$$\text{Max} \quad \Phi \tag{5}$$

$$\text{S.t.} \quad \sum_{r=1}^{s} \tilde{y}_{r0\xi} u_r + w_0 \geq \Phi, \qquad \forall \xi$$

$$\sum_{i=1}^{m} x_{i0} v_i = 1$$

$$\sum_{r=1}^{s} \tilde{y}_{rj\xi} u_r - \sum_{i=1}^{m} x_{ij} v_i + w_0 \leq 0, \qquad \forall j, \xi$$

$$v_i, u_r \geq 0, \qquad \forall i, r$$

Therefore, in order to deal with the hybrid uncertainty in uncertain CCR and BCC models, for each scenario, by applying Bertsimas and Sim's (2004) robust approach, the robust counterpart of the uncertain constraint will be given as Models (6) and (7):

*The Robust CCR Model*

$$\begin{aligned}
\text{Max} \quad & \Psi && (6)\\
\text{S.t.} \quad & -\sum_{r=1}^{s} y_{r0\xi} u_r + Z_{0\xi}\,\Gamma_{0\xi} + \sum_{r=1}^{s} P_{r0\xi} \le -\Psi, \quad \forall \xi\\
& Z_{0\xi} + P_{r0\xi} \ge \hat{y}_{r0\xi}\, u_r, \quad \forall\, r,\xi\\
& \sum_{i=1}^{m} x_{i0} v_i = 1\\
& \sum_{r=1}^{s} \tilde{y}_{rj\xi} u_r - \sum_{i=1}^{m} x_{ij} v_i + Z_{j\xi}\,\Gamma_{j\xi} + \sum_{r=1}^{s} P_{rj\xi} \le 0, \quad \forall j,\xi\\
& Z_{j\xi} + P_{rj\xi} \ge \hat{y}_{rj\xi}\, u_r, \quad \forall r, j,\xi\\
& P_{r0\xi}, P_{rj\xi} \ge 0, \quad \forall r, j,\xi\\
& Z_{0\xi}, Z_{j\xi} \ge 0, \quad \forall j,\xi\\
& v_i, u_r \ge 0, \quad \forall i, r
\end{aligned}$$

*The Robust BCC Model*

$$\begin{aligned}
\text{Max} \quad & \Phi && (7)\\
\text{S.t.} \quad & -\sum_{r=1}^{s} y_{r0\xi} u_r + w_0 + Z_{0\xi}\,\Gamma_{0\xi} + \sum_{r=1}^{s} P_{r0\xi} \le -\Phi, \quad \forall \xi\\
& Z_{0\xi} + P_{r0\xi} \ge \hat{y}_{r0\xi}\, u_r, \quad \forall\, r,\xi\\
& \sum_{i=1}^{m} x_{i0} v_i = 1\\
& \sum_{r=1}^{s} \tilde{y}_{rj\xi} u_r - \sum_{i=1}^{m} x_{ij} v_i + w_0 + Z_{j\xi}\,\Gamma_{j\xi} + \sum_{r=1}^{s} P_{rj\xi} \le 0, \quad \forall j,\xi\\
& Z_{j\xi} + P_{rj\xi} \ge \hat{y}_{rj\xi}\, u_r, \quad \forall r, j,\xi\\
& P_{r0\xi}, P_{rj\xi} \ge 0, \quad \forall r, j,\xi\\
& Z_{0\xi}, Z_{j\xi} \ge 0, \quad \forall j,\xi\\
& v_i, u_r \ge 0, \quad \forall i, r
\end{aligned}$$

It should be noted that $\tilde{y}$ takes values in $[y-\hat{y}, y+\hat{y}]$; the center of this interval at the point $y$ is a nominal value and $\hat{y}$ is perturbation of the uncertain parameter $\tilde{y}$ .

## IV. CASE STUDY AND NUMERICAL RE

In this section, the implementation of the novel robust DEA models based on the CRS and VRS assumptions by using real-world data from banks and credit institutions industry of TSE will be presented. Inputs and outputs of the DEA models are shown in Table II.

**Table II. Inputs and outputs of the DEA models**

| | *Financial Criterion* | *Symbol* | *Description* |
|---|---|---|---|
| Inputs | Current ratio | I (1) | Total current asset divided by total current liability |
| | Solvency ratio-II | I (2) | Total liability divided by shareholders equity |
| | Price to Earnings ratio (P/E) | I (3) | Stock price divided by net income per share |
| | Receivable turnover ratio | I (4) | Net receivable sales divided by average net receivable |
| Outputs | Return on Asset (ROA) | O (1) | Net income divided by the total assets |
| | Earnings per Share (EPS) | O (2) | Current quarters EPS divided by the previous quarters EPS minus one |

After choosing the inputs and outputs, financial data for 10 stocks of the banks and credit institutions industry are extracted. A summary of the real-world data for inputs and outputs that will be used in this research is presented in Tables III and IV, respectively.

**Table III. Summary of the real-world data for inputs**

| *Stock* | *I (1)* | *I (2)* | *I (3)* | *I (4)* |
|---|---|---|---|---|
| Stock 01 | 1.859 | 16.976 | 15.532 | 113.507 |
| Stock 02 | 0.919 | 13.446 | 6.709 | 236.925 |
| Stock 03 | 0.939 | 16.177 | 16.746 | 781.461 |
| Stock 04 | 1.071 | 5.855 | 4.486 | 183.996 |
| Stock 05 | 0.895 | 14.382 | 5.464 | 196.428 |
| Stock 06 | 2.527 | 4.859 | 8.519 | 197.886 |
| Stock 07 | 0.949 | 10.001 | 7.170 | 182.290 |
| Stock 08 | 7.742 | 46.409 | 6.312 | 105.361 |
| Stock 09 | 0.947 | 21.064 | 5.993 | 108.080 |
| Stock 10 | 1.723 | 6.608 | 9.025 | 66.271 |

**Table IV. Summary of the real-world data for outputs**

| *Stock* | | *O (1)* | | | *O (2)* | |
|---|---|---|---|---|---|---|
| | *Scenario (1)* | *Scenario (2)* | *Scenario (3)* | *Scenario (1)* | *Scenario (2)* | *Scenario (3)* |
| Stock 01 | [1.658, 2.026] | [0.951, 1.162] | [0.512, 0.625] | [429.300, 524.700] | [292.500, 357.500] | [163.800, 200.200] |
| Stock 02 | [1.297, 1.586] | [1.405, 1.718] | [1.160, 1.418] | [297.900, 364.100] | [366.300, 447.700] | [279.900, 342.100] |
| Stock 03 | [1.578, 1.929] | [0.159, 0.194] | [0.366, 0.448] | [466.200, 569.800] | [36.900, 45.100] | [100.800, 123.200] |
| Stock 04 | [3.738, 4.569] | [2.561, 3.130] | [1.947, 2.380] | [334.800, 409.200] | [225.000, 275.000] | [207.900, 254.100] |
| Stock 05 | [0.960, 1.173] | [0.468, 0.572] | [0.526, 0.643] | [180.900, 221.100] | [96.300, 117.700] | [118.800, 145.200] |
| Stock 06 | [3.472, 4.243] | [2.488, 3.041] | [2.960, 3.618] | [184.500, 225.500] | [190.800, 233.200] | [244.800, 299.200] |
| Stock 07 | [3.542, 4.330] | [1.420, 1.736] | [1.207, 1.476] | [435.600, 532.400] | [12.600, 15.400] | [204.300, 249.700] |
| Stock 08 | [0.417, 0.510] | [0.425, 0.520] | [0.069, 0.084] | [179.100, 218.900] | [114.300, 139.700] | [99.000, 121.000] |
| Stock 09 | [1.103, 1.348] | [1.140, 1.394] | [0.632, 0.772] | [378.000, 462.000] | [412.200, 503.800] | [223.200, 272.800] |
| Stock 10 | [3.007, 3.675] | [2.944, 3.598] | [1.951, 2.385] | [358.200, 437.800] | [392.400, 479.600] | [282.600, 345.400] |

Now, after collecting data, the novel robust CCR and BCC models for different values of Gamma $\Gamma$ will be run. The results of NRCCR and NRBCC models, which are presented in Models (6) and (7), are introduced in Tables V and VI, respectively.

**TABLE V. Results of the novel robust CCR model**

| *Stock* | *Classic CCR* | *NRCCR* | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | *Gamma* | | | | | | | | | | |
| | | *0%* | *10%* | *20%* | *30%* | *40%* | *50%* | *60%* | *70%* | *80%* | *90%* | *100%* |
| Stock 01 | 0.650 | 0.305 | 0.296 | 0.286 | 0.277 | 0.268 | 0.259 | 0.259 | 0.259 | 0.259 | 0.259 | 0.259 |
| Stock 02 | 1.000 | 0.671 | 0.647 | 0.624 | 0.602 | 0.579 | 0.556 | 0.556 | 0.556 | 0.556 | 0.556 | 0.556 |
| Stock 03 | 0.620 | 0.080 | 0.077 | 0.075 | 0.072 | 0.069 | 0.067 | 0.067 | 0.066 | 0.066 | 0.066 | 0.066 |
| Stock 04 | 1.000 | 0.621 | 0.597 | 0.573 | 0.551 | 0.529 | 0.514 | 0.508 | 0.508 | 0.508 | 0.508 | 0.508 |
| Stock 05 | 0.460 | 0.261 | 0.253 | 0.246 | 0.239 | 0.231 | 0.222 | 0.222 | 0.222 | 0.222 | 0.222 | 0.222 |
| Stock 06 | 1.000 | 0.772 | 0.749 | 0.726 | 0.705 | 0.685 | 0.671 | 0.658 | 0.650 | 0.644 | 0.638 | 0.632 |
| Stock 07 | 0.900 | 0.356 | 0.344 | 0.332 | 0.320 | 0.309 | 0.298 | 0.297 | 0.296 | 0.294 | 0.293 | 0.292 |
| Stock 08 | 0.380 | 0.239 | 0.234 | 0.229 | 0.224 | 0.218 | 0.213 | 0.213 | 0.213 | 0.213 | 0.213 | 0.213 |
| Stock 09 | 1.000 | 0.543 | 0.532 | 0.522 | 0.511 | 0.500 | 0.489 | 0.488 | 0.487 | 0.487 | 0.487 | 0.487 |
| Stock 10 | 1.000 | 0.720 | 0.694 | 0.669 | 0.658 | 0.651 | 0.644 | 0.638 | 0.631 | 0.625 | 0.619 | 0.614 |

**TABLE VI. Results of the novel robust BCC model**

| *Stock* | *Classic BCC* | *NRBCC* | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | *Gamma* | | | | | | | | | | |
| | | *0%* | *10%* | *20%* | *30%* | *40%* | *50%* | *60%* | *70%* | *80%* | *90%* | *100%* |
| Stock 01 | 0.750 | 0.748 | 0.748 | 0.748 | 0.748 | 0.748 | 0.748 | 0.748 | 0.748 | 0.748 | 0.748 | 0.748 |
| Stock 02 | 1.000 | 0.992 | 0.991 | 0.990 | 0.989 | 0.989 | 0.989 | 0.989 | 0.989 | 0.989 | 0.989 | 0.989 |
| Stock 03 | 0.960 | 0.953 | 0.953 | 0.953 | 0.953 | 0.953 | 0.953 | 0.953 | 0.953 | 0.953 | 0.953 | 0.953 |
| Stock 04 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| Stock 05 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| Stock 06 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| Stock 07 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| Stock 08 | 0.990 | 0.991 | 0.991 | 0.991 | 0.991 | 0.991 | 0.991 | 0.991 | 0.991 | 0.991 | 0.991 | 0.991 |
| Stock 09 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| Stock 10 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

After running NRDEA models, the rankings of all stocks in NRCCR and NRBCC models are presented in Tables VII and VIII, respectively.

**TABLE VII. Ranking of stocks in the novel robust CCR model**

| *Stock* | *Classic CCR* | *NRCCR* | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | *Gamma* | | | | | | | | | | |
| | | *0%* | *10%* | *20%* | *30%* | *40%* | *50%* | *60%* | *70%* | *80%* | *90%* | *100%* |
| Stock 01 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 |
| Stock 02 | 1 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| Stock 03 | 8 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 |
| Stock 04 | 1 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| Stock 05 | 9 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 |
| Stock 06 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Stock 07 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| Stock 08 | 10 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |
| Stock 09 | 1 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| Stock 10 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |

**TABLE VIII. Ranking of stocks in the novel robust BCC model**

| *Stock* | *Classic BCC* | *NRBCC* | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | *Gamma* | | | | | | | | | | |
| | | *0%* | *10%* | *20%* | *30%* | *40%* | *50%* | *60%* | *70%* | *80%* | *90%* | *100%* |
| Stock 01 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 |
| Stock 02 | 1 | 7 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 |
| Stock 03 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |
| Stock 04 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Stock 05 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Stock 06 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Stock 07 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Stock 08 | 8 | 8 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 |
| Stock 09 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Stock 10 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

As it is shown in Tables V and VI, as the budget of robustness $\Gamma$ increases from 0% to 100% for uncertain parameters, efficiency results get worse. Also, as expected, the results of Model (7) are greater than or equal to the results of Model (6). According to Tables VII and VIII, Stock 06, Stock 10, and Stock 02 are the best stocks, respectively. It should be noted that, according to Tables V to VIII, if classical DEA models are used and uncertainty is not taken into account, the results and rankings may be invalid. Also, the advantages of NRCCR and NRBCC models are the capability to be implemented in the presence of uncertain data, acceptable conservatism against uncertainty, and increase in the discrimination of the results.

## V. CONCLUSION AND DIRECTIONS FOR FUTURE RESEARCH

In this study, novel robust DEA models with CRS and VRS assumptions were presented, which had the capability to be used in the presence of discrete and continuous uncertainties for stock efficiency measurement. All uncertain parameters took different values for different scenarios and for each scenario, an interval was considered instead a nominal value. In other words, data were tainted by mixed uncertainties. Thus, for dealing with this hybrid uncertainty, RO scenario based and RO convex uncertainty set based approaches were used. Finally, for solving and validating the novel robust DEA models, a real case study of banks and credit institutions industry of Tehran Stock Exchange (TSE) was used. For the future studies, the novel robust DEA models could be applied to other real-word problems such as supply chain management and energy, transportation, power, communication, and health care. Also, the robust DEA model could be formulated based on network DEA models for performance measurement of DMUs with network structure.

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