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Robust Economic-Statistical Design of Acceptance Control Chart

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Abstract– *Acceptance control charts (ACC), as an effective tool for monitoring highly capable processes, establish control limits based on specification limits when the fluctuation of the process mean is permitted or inevitable. For designing these charts by minimizing economic costs subject to statistical constraints, an economic-statistical model is developed in this paper. However, the parameters of some processes are in practice uncertain. Such uncertainty could be an obstacle to getting the best design. Therefore, the parameters are investigated by a robust optimization approach. For this reason, a solution procedure utilizing a genetic algorithm (GA) is presented. The algorithm procedure is illustrated based on numerical studies. Additionally, sensitivity analysis and some comparisons are carried out for more investigations. The results indicate better performance of the proposed approach in designing ACC and more reliable solutions for practitioners.*

Keywords– *Acceptance control chart, Economic-statistical design (ESD), Genetic algorithm, Robust optimization.*

I. INTRODUCTION

Statistical Process Control (SPC) is an industry-standard methodology for measuring, controlling, and improving the quality and productivity of manufacturing and service enterprises. Control charts, as the most popular tool in SPC, graphically show plotted quality data of a process in time order. These charts are widely used to stabilize and monitor the main characteristics of a process over time (Montgomery, 2009).

Some processes naturally experience unavoidable fluctuations in their mean value while being still capable of satisfying the voice of customers based on specification limits. To protect such processes against preventing the nonconforming fraction from exceeding the desired value, acceptance control charts (ACCs) were introduced to monitor the mean of high capable processes distributed according to a Normal distribution (Freund, 1957). Let USL and LSL be the upper and lower specification limits, respectively. The simplest capability index is defined as Cp=(USL-LSL)/6σ where σ is a standard deviation. Now, consider a process with $Cp \geq 2$. The difference between specification limits can be 6 standard deviations or more. Therefore, it may be acceptable to allow the process mean to vary over a specific range without producing undesirable amounts of nonconforming products.

The aim of ACC is different from that of classic control charts in which stability behavior of a process mean overtime is of importance. ACC has been largely neglected in the literature and there exist only a few studies about its developments

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or applications. In Wesolowsky's study (1990), simultaneous ACC was proposed to jointly control two positively or negatively correlated quality characteristics; sample sizes and control limits were optimally found by minimization of cost or maximum-sample-size criteria. Wesolowsky (1992) proposed another simultaneous ACC for monitoring two or more independent processes creating a product, or two or more characteristics of a product. By extending the simultaneous ACC introduced by Wesolowsky (1990), the general case of multiple correlated processes or characteristics was considered in the study by Steiner & Wesolowsky (1994). Moreover, a convex nonlinear optimization model was applied to find optimal solutions using a multi-dimensional search. Holmes and Mergen (2000) combined ACC concept with the exponentially weighted moving average (EWMA). For non-normal data or measurements, Chou et al. (2005) used Burr distribution to propose Burr acceptance control chart (BACC), and Tsai and Chiang (2008) developed a Skew Normal ACC (SNACC). Recently, Taherian and Balouchestani Asl (2016) used Shewhart and acceptance control charts in 6-Sigma applications as well as the capability analysis. They provided a case study of a pharmaceutical company to clarify the mentioned concepts. In this study, the design of ACC was considered through parameters related to them.

Generally, the design of a control chart requires specifying sample size (n), sampling interval (h), and control limits width (k). Duncan (1956) proposed the first economic design (ED) for X-bar chart by optimizing the expected cost per hour. Subsequently, many researchers (e.g., Lorenzen and Vance (1986) and Celano and Fichera (1999)) further addressed obtaining the optimal design parameters through an economic approach. For ACC, Mohammadian and Paynabar (2008) proposed an economic model as well.

However, EDs have been criticized for their poor statistical characteristics such as weakness in detecting shifts compared to statistical designs. Thus, Saniga (1989) introduced an economic-statistical model, which minimized Duncan's cost model (1956) subject to statistical constraints. Some recent studies on ESD of various control charts can be found in the studies of Amiri and Jafarian-Namin (2015), Naderi et al. (2018), Fallahnezhad et al. (2018), and Jafarian-Namin et al. (2019). Also, Mohammadian and Amiri (2012) implemented ESD approach to ACC.

All the researches performed to design ACC have considered certainty for model parameters until now. Nevertheless, the parameters of some processes are not exactly defined or estimated in practice. Such uncertainty could be an obstacle to getting the best design. Therefore, the parameters are investigated by a robust optimization approach, based on Bertsimas and Sim (2004), which is built on the notion of the budget of uncertainty. In this approach, range estimates for some imprecise parameters are proposed instead of point estimates. Accordingly, Safaei et al. (2015) recently proposed a robust economic-statistical design (RESD) of X-bar control chart considering the cost function introduced by Lorenzen and Vance (1986). The optimization procedure in their study was based on Genetic algorithm (for more information about meta-heuristic algorithms, refer to Tavakkoli-Moghaddam et al. (2013)).

Since there is no work in the literature performed on the robust design of ACC, a model is provided in which Duncan's cost function (1956) is minimized subject to statistical constraints. Uncertainty in model parameters is investigated by a robust optimization approach. For this reason, a solution procedure utilizing a GA is presented. In this regard, the rest of the paper is planned as follows. In Section II, ACC is briefly reviewed. After making some assumptions and introducing the economic model, the proposed robust model for ACC is presented in Section III. Then, employing a solution procedure based on a genetic algorithm (GA) for solving the proposed model is illustrated in the next section. In Section V, a numerical example is presented to investigate the best design of the proposed model. Subsequently, a sensitivity analysis is performed to study the effects of various values of parameters on optimal solutions. A comparison among statistical design (SD), ED, ESD, and RESD of ACC is carried out in Section VII. Finally, conclusions and final remarks are provided.

II. ACCEPTANCE CONTROL CHART

The proposed ACC, introduced in Freund (1957), is applied to monitor the high capable processes when the natural spread of the process is too tighter than the spread of specification limits. To describe the principles of this chart, let *USL* and *LSL* be the upper and lower specification limits, respectively. The process mean is permitted to fluctuate over a range, known as acceptance process levels (*APL*s). Assuming normal distribution for the quality characteristic, *APL* limits are obtained using the following equations:

$$\begin{aligned} APL_U &= USL - Z_{AQL}\sigma \\ APL_L &= LSL + Z_{AQL}\sigma \end{aligned} \tag{1}$$

where σ is known the standard deviation and Z_{AQL} is a standard normal value associated with the probability *AQL*. On the other hand, rejectable process levels (*RPL*s) are proposed to preserve process against possible assignable causes. Given that Z_{RQL} is a standard normal value associated with the probability *RQL*, these limits are defined according to the following equations:

$$\begin{aligned} RPL_U &= USL - Z_{RQL}\sigma \\ RPL_L &= LSL + Z_{RQL}\sigma \end{aligned} \tag{2}$$

Fig. (1) shows different zones defined by *AQL*, *APL*, *RQL*, and *RPL* for distribution of *x* in ACC. Note that in the indifference zone, the process is neither accepted nor rejected.

Commonly, a control chart has an upper control limit (*UCL*) and a lower control limit (*LCL*) to display the range of expected variation. The control limits of ACC, derived from Montgomery (2009, pp. 454–458), can statistically be computed in three different ways:

1. Based on the desired Type I error (α-risk), *APL*, and specified sample size (*n*), the control limits of ACC are obtained as follows:

$$\begin{aligned} UCL_\alpha &= APL_U + Z_\alpha \frac{\sigma}{\sqrt{n}} \\ LCL_\alpha &= APL_L - Z_\alpha \frac{\sigma}{\sqrt{n}} \end{aligned} \tag{3}$$

   where Z_α is a standard normal value corresponding to the probability α.
2. Based on the desired Type II error (β-risk), *RPL*, and specified sample size, *UCL* and *LCL* can be calculated according to the following equations:

$$\begin{aligned} UCL_\beta &= RPL_U - Z_\beta \frac{\sigma}{\sqrt{n}} \\ LCL_\beta &= RPL_L + Z_\beta \frac{\sigma}{\sqrt{n}} \end{aligned} \tag{4}$$

   where Z_β is a standard normal value corresponding to the probability β.
3. In this method, a sample size, which satisfies the specified Type I and Type II errors, *APL*, and *RPL*, is determined. By equating the upper control limits from Eqs. (3-4), the sample size (rounded upward to an integer) is calculated as follows:

$$n = \left[\frac{Z_\alpha + Z_\beta}{Z_{AQL} - Z_{RQL}} \right]^2 \tag{5}$$

Substituting the determined sample size in Eq. (3) or (4), the corresponding control limits are set.

It must be noted that, despite three different ways of setting the limits, only one chart type is generated for monitoring the situation of a process. In this paper, the control limits obtained using the first method are considered. Moreover, *LSL* is assumed to be active and therefore, all computations are under this limit. Notice that similar studies could be easily extended to *USL* as well.

Fig. (2) shows α and β corresponding to the control limit. These errors are defined as follows:

- Type I error: the probability of indicating an unacceptable status when the process mean is at an *APL*,
- Type II error: the probability of indicating an acceptable status when the process mean is at an *RPL*.

Using the following equations respectively, the values of risks based on *LCL* can be obtained ($f_z(z)$ is the probability destiny function of the standard normal distribution) (Mohammadian & Amiri, 2012):

$$\alpha = P(\bar{X} \le LCL | \mu = APL_L) = \int_{-\infty}^{-Z_\alpha} f_Z(z)dz \tag{6}$$

$$\beta = 1 - P(\bar{X} \le LCL | \mu = RPL_L) = 1 - \int_{-\infty}^{Z_{AQL}\sqrt{n} - Z_\alpha - Z_{RQL}\sqrt{n}} f_Z(z)dz \tag{7}$$

Since the values of risks corresponding to *UCL* are almost zero, these parts of calculations are neglected. In practice, the production process is aimed to be permanently sampled with size *n* in specific time intervals of *h* hours. The sampled data, via control charts with the coefficient *k* considered for standard deviation, are inspected and the results are interpreted to monitor the state of the process.

In the next section, a model is proposed for optimally selecting the design parameters, i.e., *n*, *h*, and *k*. It is worth noting that with the attitude of quality improvement, one assignable cause or multiple assignable causes of variation in the process must be discovered and eliminated to reach a stable and predictable process. In this study, a single assignable cause is assumed to occur.

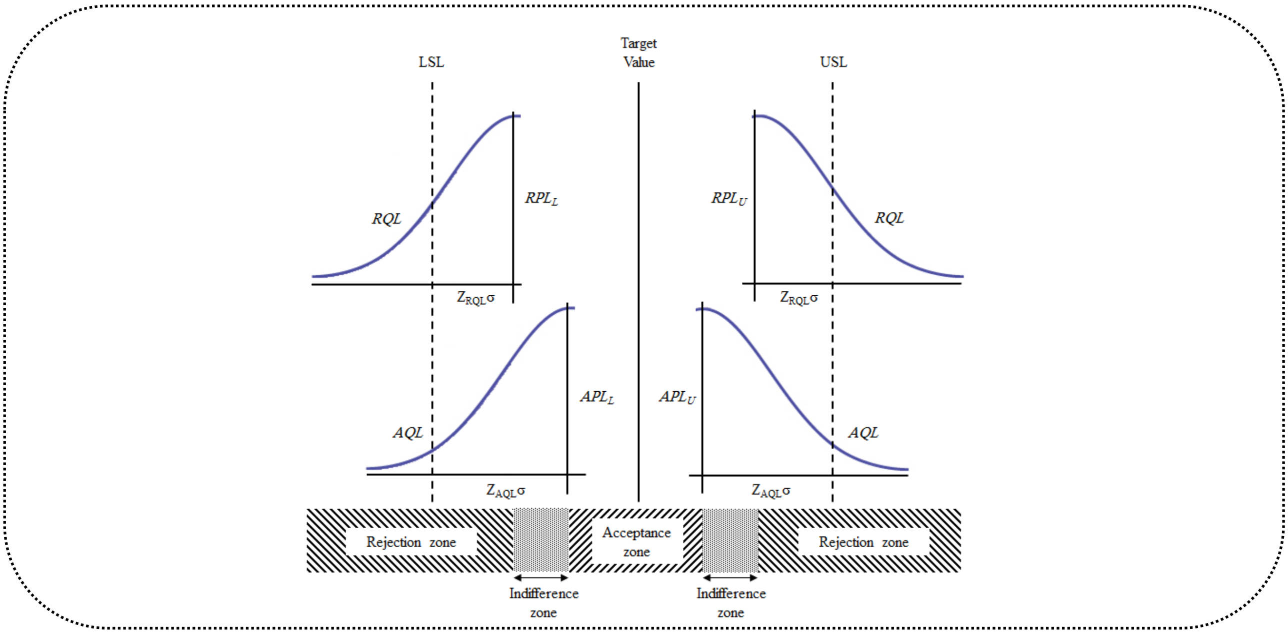

Fig. 1. Different zones defined for distribution of x in an ACC

III. ECONOMIC-STATISTICAL DESIGN OF ACC

In this section, we look at how optimal cost and design parameters are obtained using the models. Thus, we begin by making some assumptions about the model. Then, the economic cost function is introduced. The proposed model is presented in the last subsection.

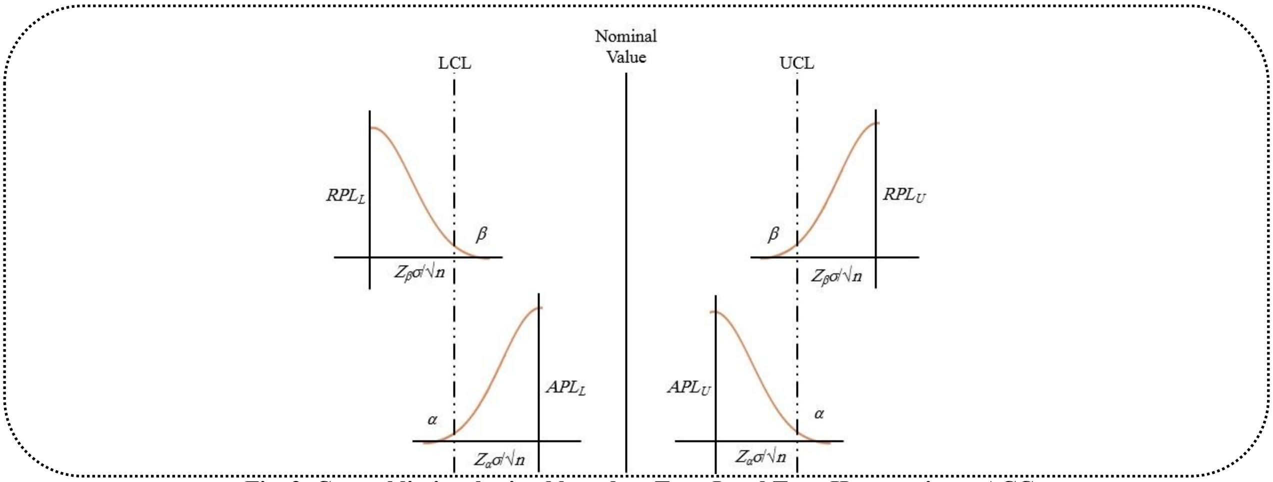

Fig. 2. Control limits obtained based on Type I and Type II errors in an ACC

A. Assumptions of the model

To simplify the mathematical analysis, some assumptions are stated to be held as follows:

1. The quality characteristic follows a normal distribution with known σ,
2. Only the lower specification limit is considered to be active,
3. The control limits based on the desired probability of Type I error, *APL*, and specified sample size are applied for monitoring the process,
4. Only the lower specification limit is considered for calculations,
5. The process is accepted when the mean alters inside the acceptable process level, i.e., $APL_L \leq \mu \leq APL_U$,
6. The occurrence of a random assignable cause leads to upward or downward shifts in the process mean. In these states, the process is rejected when $\mu \leq RPL_L$ or $\mu \geq RPL_U$,
7. The occurrence of an assignable cause has an exponential distribution with meantime $1/\lambda$,
8. The process is allowed to continue during the search and repair.

B. Economic cost function

To propose a constrained nonlinear model for finding the best design parameters of ACC with economic-statistical considerations, the traditional definition of the economic objective function is presented in this subsection. This function is based on a single quality control cycle of monitoring the production process. According to Duncan's definition (1956) of the expected hourly cost, an expected cycle length and an expected cost of the cycle can be formulated in an economic function. As a ratio of the expected cost during a cycle to the expected cycle time length, the expected hourly cost, adapted for ACC by applying the probability of Type I and Type II errors, is as follows:

$$EHC = \frac{f + vn}{h} + \frac{\lambda(W + TA + MB)}{1 + \lambda B} \tag{8}$$

where

$A = \alpha / (e^{\lambda h} - 1)$,

$$B = h/P - \tau + gn + D,$$

and

$$\tau = 1 - (1+\lambda h)e^{-\lambda h} / (\lambda - \lambda e^{-\lambda h}).$$

The parameters of the model, presented in *Eq.* (8), are defined in Table I. For different sets of design parameters *n*, *h*, and *k*, the optimal setting in the ED of ACC is selected with a minimum cost function. However, this optimal set is provided based on a single scenario, which is a single-point estimate of parameters. In practice, these estimates are often not accurately known. In the proposed model, we aim to discover a robust setting given the set of scenarios that could potentially be realized within the bounds of the uncertain parameters. Suppose that for each scenario $s \in S$, vectors of cost parameters $C^s = \{f^s, v^s, W^s, T^s, M^s\}$ and process parameters $P^s = \{\lambda^s, g^s, D^s\}$ are defined. Therefore, different values of the expected hourly cost EHC^s will be obtained for different scenarios of cost and process parameters.

Table I. Notations of Duncan's model

f	fixed cost of sampling	*A*	average number of false alarms per cycle
v	variable cost of sampling	*M*	hourly loss due to poor quality
n	sample size	*B*	average time of the process being in out-of-control state
h	sampling interval	*α*	probability of false alarm or Type I error
λ	Poisson distribution parameter for process failure rate	*p*	detection power, which is equal to (1-*β*)
W	average cost to detect an assignable cause	*g*	time required to sample, inspect, and interpret the results
T	cost of verifying a false alarm	*D*	time to discover and repair the assignable cause

C. The proposed model for ACC

In ESD, some statistical constraints are added to the cost function to obtain proper statistical properties for ACC. According to Saniga (1989), these statistical constraints under scenario *s* include: 1) the probability of false alarm (α^s), which is not allowed to be more than a maximum value, 2) the detection power (p^s), which is confined by a minimum value, and 3) a maximum value to limit average time to signal (ATS_1^s) when an assignable cause occurs. These extreme values, i.e., α_{max}, p_{min}, and ATS_1^{max}, may be determined as desired bounds by the decision-maker (DM) or quality engineers. Thus, taking into account such properties, the proposed model is formulated as follows:

$$\begin{aligned}
&\min_{x\in X}\max_{s\in S} EHC^s \\
&s.t. \\
&\alpha^s \le \alpha_{\max} \quad \forall s \in S \\
&p^s \ge p_{\min} \quad \forall s \in S \\
&ATS_1^s \le ATS_1^{\max} \quad \forall s \in S \\
&n_{\min} \le n \le n_{\max} \\
&h_{\min} \le h \le h_{\max} \\
&k_{\min} \le k \le k_{\max}
\end{aligned} \tag{9}$$

where the design parameters *n*, *h*, and *k* are set between lower and upper bounds as well. Briefly speaking, we altered the model presented by Safaei et al. (2015) as follows: 1) optimizing the model for ACC, 2) changing the cost function according to Duncan's definition (1956), and 3) substituting some constraints from the study by Saniga (1989). In the proposed model, the parameters are divided into two classes: 1) with uncertainty, and 2) with nominal value or point estimation. Therefore, different scenarios can be generated by putting lower and upper bounds on uncertain parameters. The scenario generation procedure is described in the next section. Despite largely disturbed uncertainties for parameters, the feasibility of the solution is ensured through this RESD.

The design of a control chart requires the specification of three decision variables (i.e., *n*, *h*, and *k*). These variables are determined by optimization of the robust counterpart of the proposed model. As noted, uncertainty in the model means that some of the parameters in the planning phase are not exactly defined. In the next section, a methodology is introduced to tackle the uncertainty in the model. Then, a procedure is provided for optimization.

IV. SOLUTION ALGORITHM

In the study by Ben-Tal et al. (2006), a robust optimization methodology was proposed to tackle the uncertainty in mathematical models. In such cases, the uncertainty set is defined in which possible values of the uncertain parameters are determined. Indeed, uncertain parameters are unknown but bounded. Furthermore, to avoid having an infinite number of constraints, the set of uncertain values is separated into a limited set of scenarios.

The concept of "budget of uncertainty" is developed by Bertsimas et al. (2011) to avoid excessive conservatism and it enables the designer to trade risk-off (robustness) for performance. Parameter Γ, as budget of uncertainty, is an integer number ($\Gamma \in [0, m]$), which is equal to a maximum number of uncertain parameters in each scenario, and m is the number of uncertain parameters in the proposed model. If Γ=0, all parameters take nominal values and no uncertainty is considered in the model. In contrast, if Γ=*m*, all uncertain parameters take values between their lower and upper bounds and the results are quite conservative. If $\Gamma \in [0, m]$, a trade-off is made between performance and the degree of conservatism of the solution.

Due to the non-linear nature of our model, a metaheuristic approach can be used. For ESD of control charts, Niaki et al. (2011) investigated the use of four well-known algorithms, namely genetic algorithm (GA), simulated annealing (SA), differential evolution (DE), and particle swarm optimization (PSO). Their results approved the best performance of GA among the others. Moreover, Safaei et al. (2015) used GA in the optimization of a model for determining design parameters of *X*-bar control chart. Thus, a GA procedure, shown in *Fig*. (3), is developed in this section for getting the best solutions.

A. Scenario generation

The first step to solve the problem is scenario generation based on the value of Γ. As mentioned earlier, the value of Γ (budget of uncertainty) is equal to the maximum number of uncertain parameters in each scenario. The scenarios are randomly generated and in each scenario, the number of Γ uncertain parameters is selected and the values are taken within their given bounds. The scenario generation procedure is described as follows:

Procedure 1: Scenario generation

Step 1: generate $Sc\binom{m}{\Gamma}$ "random scenarios" in each of them; Γ uncertain parameters take values within their bounds randomly and (*m*-Γ) parameters take their nominal values. *Sc* is the number of times that $\binom{m}{\Gamma}$ scenarios are selected.

Step 2: generate $2^{\Gamma}\binom{m}{\Gamma}$ "extreme scenarios" in each of them; Γ uncertain parameters take extreme values of their bounds and (*m*-Γ) parameters take their nominal values.

Step 3: total number of scenarios (*S*) is equal to $S = Sc\binom{m}{\Gamma} + 2^{\Gamma}\binom{m}{\Gamma}$.

B. Proposed genetic algorithm

Steps of the proposed GA are presented in *Fig*. (3). In each iteration, offspring are generated using crossover and mutation operators as follows:

Procedure 2: crossover operator

```
Begin
 Choose 2 parents from the population (P1, P2)
   If rand<0.5 (arithmetical crossover)
     r=rand ();
     offspring1=r*P1+ (1-r)*P2
     offspring2= (1-r)*P1+r*P2
   Else (uniform crossover)
     For i=1:3
       If rand<0.5
         offspring1 (i)=P1(i)
         offspring2 (i)=P2(i)
       Else
         offspring1 (i)=P2(i)
         offspring2 (i)=P1(i)
       End
     End
   End
 End
```

Procedure 3: mutation operator

```
Begin
 Choose 1 parent from the population (P)
   If rand<0.5 (uniform mutation)
     Select one gen in P and substitute its value with a random number within the lower and upper bounds of the
   variable
    Else (boundary mutation)
     For i=1:3
       If rand<0.5
         Offspring (i)= upper or lower bound of the decision variable (with equal chance)
       Else
         Offspring (i)=P(i)
       End
     End
   End
 End
```

In each iteration, evaluation of all chromosomes (parents and offspring) is done to select the next generation based on the following procedure.

Procedure 4: evaluation

Step 1: calculate the objective function of each chromosome in all scenarios ($EHC_{X,S}$). The fitness of each chromosome is equal to *Maximum* ($EHC_{X,S}$).

Step 2: based on the constraints, if a chromosome is infeasible (in one or more scenarios), penalize its fitness by 10 times.

After evaluation of all chromosomes, the best chromosome with minimum fitness is selected for the next generation (elitism) and the rest of the chromosomes of the next generation are selected using the roulette wheel selection operator.

It is necessary to determine the appropriate values for GA operators, i.e., crossover and mutation. Based on the results of Safaei et al. (2015), who used the Taguchi method to set the parameters of GA, we set the value of crossover to 0.6, mutation to 0.3, and pop-size to 150 in this study. Also, the probability of changing any gene in the mutation operator is assumed to be 0.05. Moreover, since 100 replicates are sufficient for the convergence of the algorithm, it is considered as the stop condition.

V. AN ILLUSTRATIVE EXAMPLE

In this section, an illustrative example is presented to compare the economic-statistical design versus the economic design of ACC by optimization of the robust counterpart. The parameter values shown in Table II are point estimates or nominal values. Also, a range estimate of parameters is needed to describe the uncertainty corresponding to various quality and cost effects that can lead to a loss for manufacturer and customer. Cost parameters $C^s = \{f^s, v^s, W^s, T^s, M^s\}$ and process parameters $P^s = \{\lambda^s, g^s, D^s\}$ are considered as uncertain parameters for each scenario $s \in S$. Deviation of these parameters from their nominal values includes two scenarios: 1) 10% shift scenarios from nominal estimates, and 2) 20% shift scenarios from the nominal estimates. The maximum number of uncertain parameters (m) is set to 8 considering the 8 uncertain parameters. Hence, we can consider all combinations of expected uncertain parameters, i.e., $\Gamma = 0, \ldots, 8$.

Moreover, feasible space is determined by considering [l, 20] items for the sample size, [0.1, 8] hours for the sampling interval, and [1, 4] standard deviation coefficients. The right-hand-side values of constraints in *Eq.* (9) are respectively set to 0.005, 0.98, and 4. All calculations have been facilitated under a program coded in MATLAB (version R2016b) environment in this study.

Table III shows the results for different scenarios based on budgets $\Gamma = 0, \ldots, 8$. Additionally, the trends for the model cost under different shift scenarios and budgets of uncertainty are presented in *Fig*. (4). For the nominal scenario, when $\Gamma=0$, the best plan for ACC is obtained as $n=17$, $h=2.25$, and $k=2.61$. The expected hourly cost is 5.18. For different scenarios; it is obvious that design parameters are almost unchanged. The only sampling time interval is slightly variable, about 24 minutes when designs for $\Gamma=0$ and $\Gamma=3$ are compared for the 20% shift scenario. This value for h decreases to 15 minutes under 10% shift scenario.

Interesting results can be obtained from *Fig*. (4) when comparing various RESD with the nominal design. The least expected hourly cost is calculated for the nominal design. It leads to 10% lower cost than the least expensive RESD in 10% shift scenario and 13% lower cost than that in 20% shift scenario.

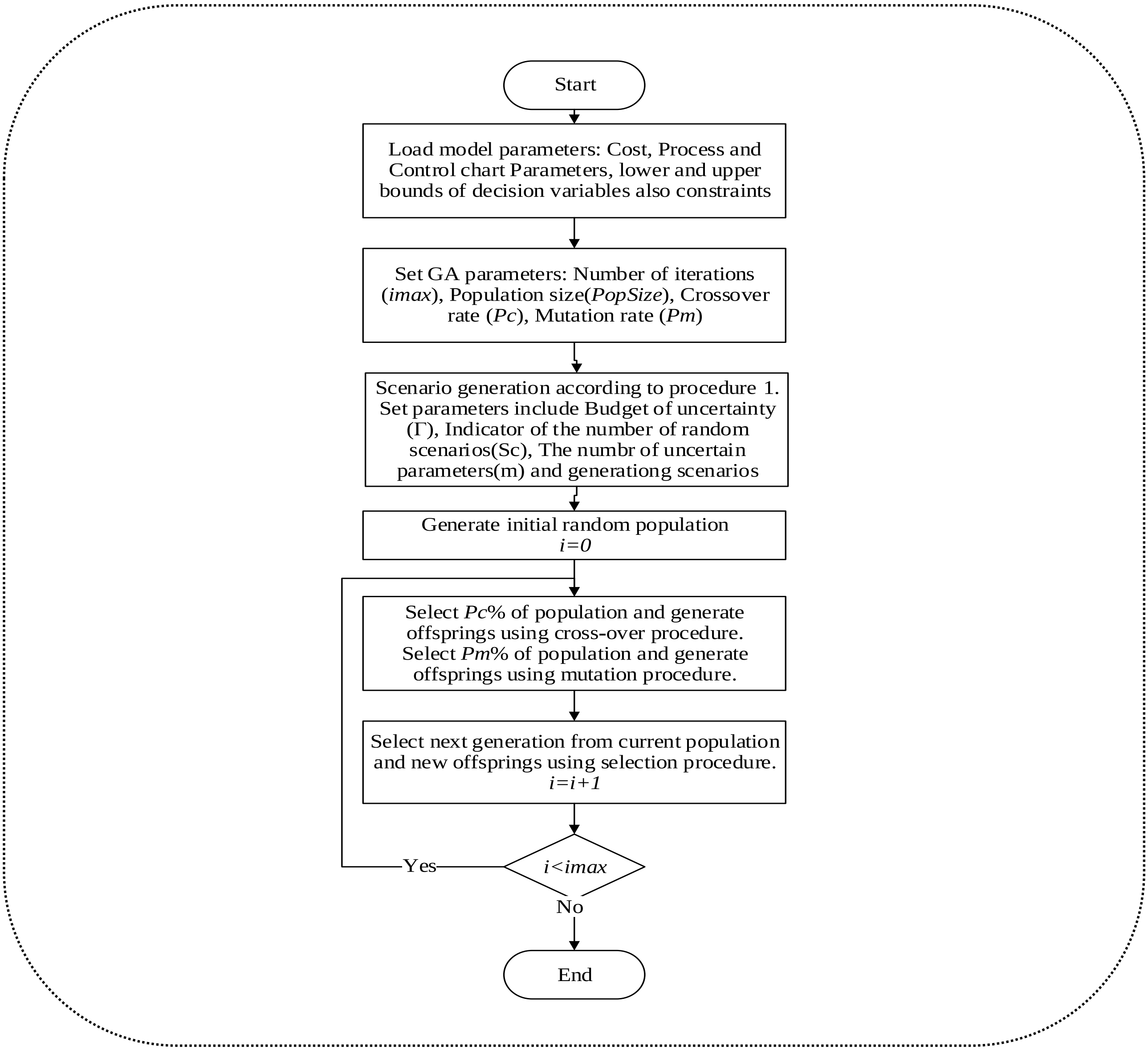

Fig. 3. Framework of the proposed solution algorithm

Table II. Nominal values of parameters derived from the study by Steiner & Wesolowsky (1994)

f=0.5	v=0.1	M=100	T=50	W=25
λ=0.01	g=0.05	D=2	AQL=0.001	RQL=0.025
μ=10	σ=0.01	USL=10.5	LSL=9.5	

Besides, a consistent increase in cost is detected as the budget of uncertainty becomes larger. It is expected that incorporating more uncertainty in the model will lead to an increase in the optimal cost. On the other hand, the most conservative robust design, when $\Gamma = 8$, has the most expected hourly cost.

Table III. S of the proposed model under different shift scenarios and different budgets

Γ	10% shift scenario				20% shift scenario			
	n	h	k	EHC	n	h	k	EHC
0	17	2.25	2.61	5.18	17	2.25	2.60	5.18
1	17	2.17	2.60	5.46	17	2.08	2.61	5.96
2	17	2.08	2.61	5.99	17	1.85	2.61	6.85
3	17	2.02	2.61	6.21	17	1.85	2.61	7.37
4	17	2.08	2.60	6.31	17	1.92	2.61	7.59
5	17	2.08	2.61	6.39	17	2.00	2.61	7.76
6	17	2.08	2.61	6.41	17	2.02	2.61	7.82
7	17	2.17	2.60	6.44	17	2.08	2.61	7.87
8	17	2.17	2.60	6.45	17	2.08	2.60	7.89

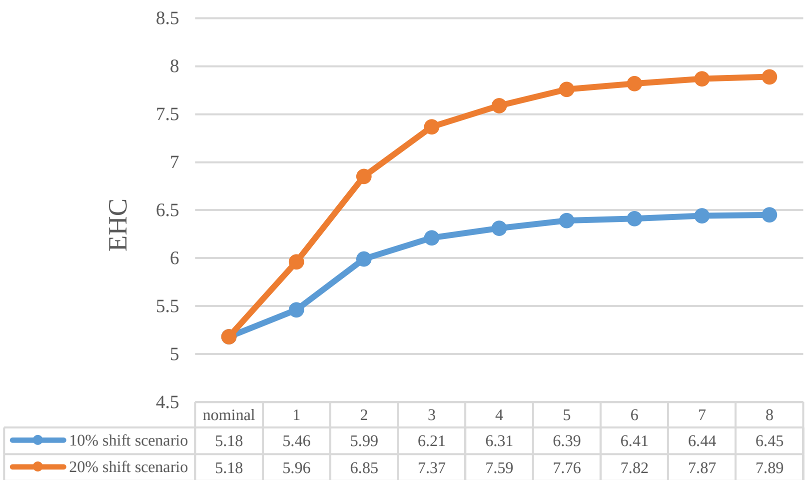

Fig. 4. Changes in cost under different scenarios (10% and 20%) and budgets of uncertainty

VI. SENSITIVITY ANALYSIS

In this section, the parameters of the model are changed to investigate their effects on the optimal solution. The results of the sensitivity analysis for ESD and RESD are respectively presented in

Table IV and V (note that valueless cells in both tables are duplicated in the first row). Accordingly, the following points are deduced:

- By increasing *AQL* and *RQL*, the sample size and the sampling interval are reduced. As a result, the expected cost declines. However, the trend of changes for *k* is not realizable.
- Increasing λ, despite making no change in sample size and control limit width, decreases *h* and thus, increases the expected hourly cost.
- When *m* decreases, only *h* increases and *EHC* decreases.
- The larger values of parameter *g* cause increase in sampling interval. However, it does not have any influence on *n* and *k*. This change, in turn, results in the growth of *EHC*. Similar results are observed for the increase in *D*.
- The simultaneous growth of *T* and *W* increases all design parameters to some extent and causes an increase in *EHC*.

- Increasing *f* and *v* separately leads to increase in *h* and decrease in *k*. Therefore, the expected hourly cost is raised.

Table IV. Results of the sensitivity analysis for ESD under 10% scenarios

Experiment no.	*RQL*	*AQL*	λ	*M*	*g*	*D*	*T*	*W*	*f*	*v*	*n*	*h*	*k*	*EHC*
1	0.025	0.001	0.01	100	0.05	2	50	25	0.5	0.1	17	2.25	2.61	5.18
2	0.050	0.002									15	2.10	2.72	4.97
3	0.075	0.003									13	2.02	2.66	4.80
4	0.100	0.004									12	1.94	2.69	4.69
5			0.02								17	1.60	2.61	8.84
6			0.03								17	1.35	2.61	12.10
7				1							17	3.92	2.61	0.90
8				10							17	3.92	2.61	1.32
9					0.5						17	2.42	2.61	11.74
10						20					17	2.60	2.61	19.29
11							5	2.5			17	2.17	2.58	4.88
12							500	250			20	2.58	3.00	7.78
13							5000	2500			20	3.92	3.00	30.85
14									5		17	3.83	2.59	6.67
15										1	17	3.92	2.58	9.43
16										10	17	3.92	2.58	48.50
17			0.02	50							17	2.33	2.61	5.26
18				10						1	13	3.92	2.66	4.20
19	0.050	0.002		12.87							15	3.92	2.72	1.38
20	0.050	0.002		128.7							15	1.85	2.72	6.02
21	0.050	0.002		12.87			500	250			19	3.92	3.32	3.66
22	0.050	0.002		12.87					5		15	3.92	2.72	2.53
23	0.050	0.002		12.87						1	15	3.92	2.72	4.82
24	0.075	0.003		2.25							13	3.92	2.66	0.85
25	0.075	0.003		225							13	1.35	2.66	9.01
26	0.075	0.003		2.25					5		13	3.92	2.66	1.99
27	0.075	0.003		2.25						1	13	3.92	2.66	3.83

Here, another investigation is carried out by depicting the effects of the right-hand-side (RHS) values of constraints on the expected hourly cost. Note that when a specific RHS value is altered, the others are assumed to remain unchanged. As shown in Figs. (5) – (7), the expected hourly cost increases when feasible space is limited by increasing α, decreasing *p*, and increasing *ATS*. On the other hand, when a constraint is relaxed, it is logical to expect lower EHC, which is confirmed here.

Moreover, it is observed that the results of RESD are costlier than those of ESD. As indicated before, they are consequences of applying uncertainty to the parameters.

Table V. Results of the sensitivity analysis for RESD under 10% scenarios

Experiment no.	*RQL*	*AQL*	*λ*	*M*	*g*	*D*	*T*	*W*	*f*	*v*	*n*	*h*	*k*	*EHC*
1	0.025	0.001	0.01	100	0.05	2	50	25	0.5	0.1	17	2.17	2.60	5.46
2	0.050	0.002									15	2.02	2.72	4.97
3	0.075	0.003									13	1.94	2.66	4.98
4	0.100	0.004									12	1.85	2.69	5.05
5			0.02								17	1.58	2.61	8.89
6			0.03								17	1.33	2.61	13.06
7				1							17	3.92	2.61	0.90
8				10							17	3.92	2.61	1.35
9					0.5						17	2.29	2.61	12.42
10						20					17	2.50	2.61	19.30
11							5	2.5			17	2.02	2.58	4.88
12							500	250			20	2.58	3.00	7.80
13							5000	2500			20	3.92	3.00	30.85
14									5		17	3.60	2.60	6.68
15										1	17	3.92	2.59	9.90
16										10	17	3.92	2.58	48.52
17			0.02	50							17	2.25	2.61	5.26
18				10						1	13	3.92	2.66	4.20
19	0.050	0.002		12.87							14	3.92	2.58	1.36
20	0.050	0.002		128.7							15	1.77	2.72	6.03
21	0.050	0.002		12.87			500	250			19	3.92	3.32	3.66
22	0.050	0.002		12.87					5		15	3.92	2.72	2.55
23	0.050	0.002		12.87						1	15	3.92	2.72	4.90
24	0.075	0.003		2.25							13	3.92	2.66	0.85
25	0.075	0.003		225							13	3.92	2.72	8.30
26	0.075	0.003		2.25					5		13	3.92	2.66	1.99
27	0.075	0.003		2.25						1	13	3.92	2.66	3.50

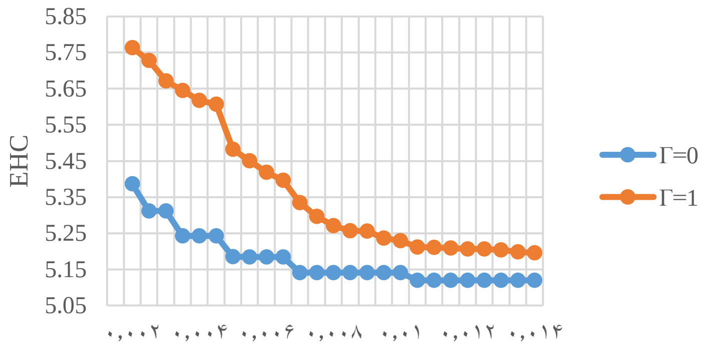

Fig. 5. Effect of changes in the RHS value of α constraint on EHC in ESD and RESD (under 10% scenarios)

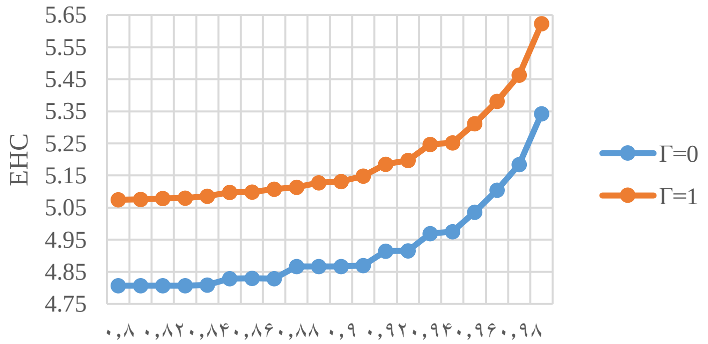

Fig. 6. Effect of changes in the RHS value of power constraint on EHC in ESD and RESD (under 10% scenarios)

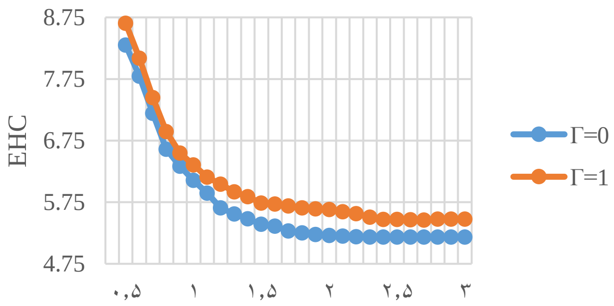

Fig. 7. Effect of changes in the RHS value of ATS constraint on EHC in ESD and RESD (under 10% scenarios)

VII. COMPARISON

In this section, the performances of SD, ED, ESD, and RESD of ACC are compared. The parameters for this study are assumed to be the same as shown in Section VI. For each experiment number, 3 values including the expected hourly cost, the probability of Type I error, and the probability of Type II error are calculated. Moreover, the bounds of constraints are α=0.005, P=0.98, and ATS=4. The results are summarized in Table VI.

Table VI. Performance comparison among statistical, economic, economic-statistical, and robust economic-statistical design of ACC

Exp. no.	SD			ED			ESD			RESD		
	EHC	α	P	EHC	α	P	EHC	α	P	EHC	α	P
1	5.58	0.004	0.984	4.79	0.0069	0.823	5.18	0.0046	0.980	5.46	0.0046	0.980
2	5.37	0.004	0.983	4.63	0.0059	0.839	4.97	0.0032	0.980	4.97	0.0032	0.980
3	5.24	0.004	0.981	4.53	0.0048	0.867	4.80	0.0039	0.980	4.98	0.0039	0.980
4	5.16	0.004	0.982	4.45	0.0048	0.852	4.69	0.0035	0.980	5.05	0.0035	0.980
5	10.10	0.004	0.984	8.10	0.0078	0.788	8.84	0.0046	0.980	8.89	0.0046	0.980
6	14.26	0.004	0.984	11.04	0.0083	0.788	12.10	0.0046	0.980	13.06	0.0046	0.980
7	0.92	0.004	0.984	0.49	0.0038	0.549	0.90	0.0046	0.980	0.90	0.0046	0.980
8	1.34	0.004	0.984	1.14	0.0055	0.886	1.32	0.0046	0.980	1.35	0.0046	0.980
9	12.39	0.004	0.984	6.86	0.0146	0.418	11.74	0.0046	0.980	12.42	0.0046	0.980
10	19.49	0.004	0.984	18.99	0.0079	0.835	19.29	0.0046	0.980	19.30	0.0046	0.980
11	5.32	0.004	0.984	4.07	0.0663	0.847	4.88	0.0049	0.981	4.88	0.0050	0.981
12	8.15	0.004	0.984	7.41	0.0007	0.818	7.78	0.0013	0.980	7.80	0.0013	0.980
13	33.87	0.004	0.984	29.37	0.0001	0.837	30.85	0.0014	0.980	30.85	0.0013	0.980
14	6.73	0.004	0.984	6.51	0.0132	0.939	6.67	0.0047	0.981	6.68	0.0047	0.980
15	9.70	0.004	0.984	7.08	0.0450	0.797	9.43	0.0049	0.981	9.90	0.0049	0.981
16	50.86	0.004	0.984	11.48	0.1587	0.552	48.50	0.0049	0.981	48.52	0.0049	0.981
17	5.59	0.004	0.984	4.89	0.0072	0.827	5.26	0.0046	0.980	5.26	0.0046	0.980
18	46.19	0.004	0.984	1.87	0.0200	0.178	4.20	0.0200	0.178	4.20	0.0218	0.187
19	1.38	0.004	0.983	1.26	0.0044	0.900	1.38	0.0032	0.980	1.36	0.0050	0.980
20	6.69	0.004	0.983	5.58	0.0058	0.833	6.02	0.0032	0.980	6.03	0.0032	0.980
21	3.96	0.004	0.983	3.47	0.0004	0.902	3.66	0.0004	0.980	3.66	0.0004	0.980
22	2.53	0.004	0.983	1.88	0.0066	0.948	2.53	0.0032	0.980	2.55	0.0032	0.980
23	4.82	0.004	0.983	2.05	0.0372	0.762	4.82	0.0032	0.980	4.90	0.0402	0.768
24	0.85	0.004	0.981	0.58	0.0037	0.791	0.85	0.0039	0.980	0.85	0.0039	0.980
25	10.85	0.004	0.981	8.41	0.0058	0.826	9.01	0.0039	0.980	8.30	0.0039	0.980
26	1.99	0.004	0.981	1.14	0.0037	0.790	1.99	0.0039	0.980	1.99	0.0039	0.980
27	3.83	0.004	0.981	0.96	0.0197	0.417	3.83	0.0039	0.980	3.50	0.0039	0.980

For SD, the sample size was determined by *Eq.* (5), k was obtained from Z_α, and h was calculated by $ATS \times (1-\beta)$. As indicated by Mohammadian and Amiri (2012), the decimal values of n were rounded up to the next integer value for easier application in reality. Thus, the values of β and h should be computed again because of an increase in power values effected by larger sample sizes. For example, in the second experiment, the design parameters were obtained as n=14.09, h=3.92, and k=2.58 with the cost of 5.33 and statistical properties of α=0.0049, P=0.979, and ATS=4. When sample size was rounded up to 15, k remained the same as before. However, h changed to 3.94, the cost was raised to 5.37, and the power increased to 0.983.

In the following, some points inferred from Table VI are mentioned:

- For all experiments, the expected hourly costs for SD are the greatest among all three designs.
- In ED, the lowest expected hourly costs are obtained because only the economical features are of importance.
- Statistical features of ED get worse than those of SD. For example, in the second experiment, α increases to 0.0059 and power decreases to 0.839.
- To improve the statistical features of ED, the statistical constraints are introduced into the model to have ESD (or RESD with Γ=0 based on *Eq.* (9)). In the second experiment, for instance, α and p are improved respectively to 0.0032 and 0.98 for ESD in comparison with those for ED. However, the *EHC* of ESD gets higher than that of ED. In fact, despite marginally larger values of *EHC* in ESD, it is preferred to ED since the statistical features are satisfactory by applying the constraints.
- It is confirmed that $EHC_{\text{ED}} < EHC_{\text{ESD}} \leq EHC_{\text{SD}}$ for the whole experiments in the table.
- It is observed that the results of RESD are costlier than those of ESD. Indeed, this extra cost is the amount paid for applying uncertainty to parameters.

VII. CONCLUSION AND FUTURE RESEARCH

In designing a control chart, some parameters of some processes are uncertain due to measurement errors or other technical obstacles. To get the best design in such circumstances, the concept of budgets of uncertainty was considered in the robust framework to take into account the uncertainty of parameters in a systematic routine. Thus, the robust design of the acceptance control chart was presented in this paper. The provided model included an economic cost function subject to some statistical constraints. For this reason, a solution procedure utilizing a genetic algorithm (GA) was proposed.

A numerical example was presented to explain the proposed model under different shift scenarios and different budgets of uncertainty. Such results could help the practitioners to select the best design based on excellence in cost regarding statistical performance while providing the highest protection against uncertainties in parameters. Moreover, the features of statistical, economic, economic-statistical, and robust economic-statistical designs were compared for more investigations.

As indicated in Section II, the control limits of ACC can statistically be computed in three different ways. As in the study by Mohammadian and Amiri (2012), the control limits were calculated using the first method. Therefore, it is recommended as a future research direction to use the other methods for obtaining UCL and LCL and compare the results with deep discussion. Another potential subject is related to ACC for attributes. For example, by referring to Chou et al. (2005) and Tsai and Chiang (2008), the designs based on any proposed mathematical modeling can be investigated. Finally, other types of ACC can be developed similar to the study of Holmes and Mergen (2000), in which EWMA ACC was presented.

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