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Optimal Process Adjustment under Inspection Errors Considering the Cycle Time of Production

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Abstract– *Process adjustment, also known as process targeting, is one of the classical problems in the field of quality control and production economics. In the process adjustment problem, it is assumed that process parameters are variables and the aim is to determine these parameters such that certain economic criteria are optimally satisfied. The aim of this paper is to determine the optimal process adjustment in a two-stage production system with rework loops. An absorbing Markov chain model is developed in which all items are inspected for conformance with their specification limits. The cycle time of production process is included in the model for optimizing total profit of the system. Also, effects of inspection errors are investigated.*

Keywords– *Quality control, Rework loops, Markov chain, Optimal process mean, Inspection errors.*

I. INTRODUCTION

Quality control has become a major concern in competitive industrial environment, and industrial engineers are seeking to make process adjustments which will optimize production efficiency and improve product quality. In the process adjustment problem, it is assumed that the process or machine parameters are variable. The aim of this problem is to determine the process or machine parameters such that certain economic criteria are optimally satisfied. Each quality characteristic of the produced item should be adjusted at a special mean. During production of items in a production process, certain specifications limits are considered for inspection of the produced items. After production process, there is an inspection stage in which the items are examined. Inspection process is usually done 100% to reduce the amount of waste. By measuring and comparing the produced items with these specifications, the decision is made about accepting the item. If the value of quality characteristic is within the predetermined limits, then the item will be accepted and sold; otherwise, it is reworked or scrapped. If the item needs to be reworked, it is returned to production process and a reworking action is performed on it. For example, the item is reworked if the value of its quality characteristic falls above an upper specification limit and it is scrapped when the value of its quality characteristic falls below a lower specification limit.

Many models have recently been presented for optimal process adjustments. Lee and Elsayed (2002) considered the problem of optimum process mean and inspection limits by allocating alternative variables for inspecting quality characteristics in one of the two stages of the process. Optimum process mean in their research was obtained through profit maximization with the objective functions of sale, production costs, inspection cost, and scrapping costs. Al-sultan and Pulak (2000) presented a mathematical model for obtaining optimal adjustment point in a two-stage production system; they considered only lower inspection limits. Zinlong and Enriuedel (2006) obtained mean and

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variance of process through cost function. They minimized the sum of costs, including the costs of deviation from target and the costs of fixed adjustments. Jinshyang et al. (2000) considered lower control limit for product adaption evaluation and emphasized that optimal mean was affected by production line and raw materials. They assumed that production cost of an item was a linear function of the raw materials used in the production. Wang et al. (2004) presented a method of optimal adjustment and optimal control based on integrated control. Duffuaa and Gaally (2012) developed a multi-objective optimization model, which included profit function and income and used Taguchi loss function. Shokri and Walid (2011) presented a loss model to maximize profit function for obtaining process mean in continuous production systems. Park et al. (2011) obtained mean and inspection limits through maximization of profit function using the frequent method of Gauss-seidl. Chung and Hui (2009) and Lee et al. (2007) investigated different aspects of optimal process adjustment problem. Duffuaa and Gaaly (2017) presented a multi-objective model for the process targeting problem by incorporating the measurement errors in the inspection system. They considered the situation in which there were two markets with different cost/price structures and used the concept of cutoff points to counter and reduce the impact of inspection errors. Mohammadi et al. (2018) developed a robust bi-objective mixed-integer linear programming model for planning an inspection process used to detect nonconforming products and malfunctioning processors. Their objectives were the minimization of (1) internal and (2) external costs. Rezaei-Malek et al. (2018) presented a mixed-integer mathematical model for the integrated planning of the part quality inspection and preventive maintenance activities in deteriorating serial multi-stage manufacturing systems. Rasay et al. (2018) presented an integrated mathematical model for coordinating the decisions associated with maintenance management and statistical process control to optimize the profit of a series production system. Finally, Rasay et al. (2017) applied multivariate control charts in an integrated model of condition-based maintenance and process adjustment.

In the current research, similar to Bowling et al. (2004), the flow of material in a discrete production process is modeled using absorbing Markov chain with a transition probability matrix. At each stage of production, the item is inspected and if it does not conform to its specifications, it is either scrapped or reworked. The reworked item will be inspected again; thus, we use rework loops for reworking. Similar models have been presented by Fallahnezhad and Niaki (2010) and Fallahnezhad and Hosseininasab (2012).

The novelties of the presented model can be expressed as follows: most of the models in the literature optimize profit per item, but it is known that the adjustment of process affects the number of reworking actions performed on the item; therefore, it affects the time spent on production of an item. This fact is usually ignored in these types of problems (Bowling et al., 2004). Therefore, we try to consider the cycle time of production process as well as the cost of the model, and try to develop a general model for the two-stage production process. Similar models can be developed for multi-stage production processes. The cycle time of production is considered in profit objective function. It is the time between the production of two successive items, which is computed based on the time of the bottle-neck stage.

The rest of the paper is organized as follows: we first present the notation in Section II. The model development comes next in Section III. Numerical demonstration of the application of the proposed methodology is given in Section IV. Sensitivity analysis of parameters comes in Section V. The inspection errors are investigated in Section VI. Finally, we conclude the paper in Section VII.

II. NOTATION

The notation is:

U_i : The upper specification limit in the i^{th} stage of production, $i=1,2$

L_i : The lower specification limit in the i^{th} stage of production, $i=1,2$

P_{ij} : The probability of going from state *i* to state *j* in a single step

f_{ij} : The long run probability of going from a non-absorbing state (*i*) to an absorbing state (*j*)

$E(PR)$: The expected profit per item
$E(RV)$: The expected revenue per item
$E(PC)$: The expected processing cost per item
$E(SC)$: The expected scrapping cost per item
$E(RC)$: The expected reworking cost per item
$E(QC)$: The expected value of Taguchi loss function
$E(PCO)$: The expected penalty cost
SP : The selling price of an item
TP : Total profit
A_1: Coefficient of Taguchi loss function for the first quality characteristics
A_2: Coefficient of Taguchi loss function for second quality characteristics
τ_1: Target value for the first quality characteristics
τ_2 : Target value for the second quality characteristics
PC_i : The processing cost of the i^{th} stage, $i = 1, 2$
SC_i : The scrapping cost of the i^{th} stage, $i = 1, 2$
RC_i : The reworking cost of the i^{th} stage, $i = 1, 2$
PCO : The penalty cost of selling a non-conforming item
$\mathbf{P}$: The transition probability matrix
$\mathbf{Q}$: The transition probability matrix of going from a non-absorbing state to another non-absorbing state
$\mathbf{R}$: A matrix containing all probabilities of going from a non-absorbing state to another absorbing state (i.e., accepted or rejected item)
$\mathbf{I}$: The identity matrix
$\mathbf{O}$: A matrix with zero elements
$\mathbf{M}$: The fundamental matrix
$\mathbf{F}$: The absorption probability matrix
T_1: Operation time in the first stage per item
T_2: Operation time in the second stage per item
C : Cycle time of production
H : Total production time in the period of decision making

III. MODEL DEVELOPMENT

Consider a two-stage serial production system in which in each stage the items are 100% inspected. We assume that there is an inspection stage after each production stage in production line and rework loops are applied to the inspection system where the items are reworked in production line and then, inspected again. The item is then reworked, accepted, or scrapped. Raw materials come into the production system and finally, the finished items are produced. A Markov chain can represent different states of the raw materials, i.e., reworking, scrapping, and accepting. Among the states, some are transient and the others absorbing. A Markov chain with one or more absorbing states is known as absorbing Markov chain (Pillai & Chandrasekharan, 2008).

The expected profit per item in the two-stage production system under consideration can be expressed as follows:

$$E(PR) = [E(RV) - E(PC) - E(SC) - E(RC) - E(QC)] \tag{1}$$

Thus, total profit can be obtained as follows:

$$TP = \frac{H}{C} E(PR), \tag{2}$$

where $\frac{H}{C}$ is the total number of items produced in each production period. Consider a two-stage production system with the following states:

State 1: An item is being processed in the first stage of the production process;
State 2: An item is being processed in the second stage of the production process;
State 3: An item is accepted as the finished item;
State 4: An item is scrapped.

The quality characteristic of an item in the first stage follows a normal distribution with unknown mean μ_1 and standard deviation σ_1 and the quality characteristic of an item in the second stage follows a normal distribution with unknown mean μ_2 and standard deviation σ_2. Transition probability matrix can be expressed as follows:

$$\mathbf{P} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} P_{11} & P_{12} & 0 & P_{14} \\ 0 & P_{22} & P_{23} & P_{24} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}, \tag{3}$$

where P_{11} and P_{22} are the probabilities of reworking an item in the first and second stages, respectively, P_{23} is the probability of accepting an item as finished product, and P_{14} and P_{24} are the probabilities of scrapping an item in the first and the second stages, respectively.

Since the quality characteristic of an item in each stage follows a normal distribution with means μ_1 and μ_2 and standard deviations σ_1 and σ_2, respectively, transition probabilities can be obtained as follows:

$$P_{11} = \int_{U_1}^{+\infty} \frac{1}{\sigma_1\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x_1-\mu_1}{\sigma_1}\right)^2} dx_1 = 1 - \Phi_1(U_1), \tag{4}$$

$$P_{12} = \int_{L_1}^{U_1} \frac{1}{\sigma_1\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x_1-\mu_1}{\sigma_1}\right)^2} dx_1 = \Phi_1(U_1) - \Phi_1(L_1), \tag{5}$$

$$P_{22} = \int_{U_2}^{+\infty} \frac{1}{\sigma_2\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x_2-\mu_2}{\sigma_2}\right)^2} dx_2 = 1 - \Phi_2(U_2), \tag{6}$$

$$P_{14}=\int_{-\infty}^{L_1}\frac{1}{\sigma_1\sqrt{2\pi}}e^{-\frac{1}{2}(\frac{x_1-\mu_1}{\sigma_1})^2}dx_1=\Phi_1(L_1), \tag{7}$$

$$P_{24}=\int_{-\infty}^{L_2}\frac{1}{\sigma_2\sqrt{2\pi}}e^{-\frac{1}{2}(\frac{x_2-\mu_2}{\sigma_2})^2}dx_2=\Phi_2(L_2), \tag{8}$$

$$P_{23}=\int_{L_2}^{U_2}\frac{1}{\sigma_2\sqrt{2\pi}}e^{-\frac{1}{2}(\frac{x_2-\mu_2}{\sigma_2})^2}dx_2=\Phi_2(U_2)-\Phi_2(L_2). \tag{9}$$

The term $\Phi_1(.)$ is the cumulative function of the normal distribution with mean μ_1 and standard deviation σ_1, and $\Phi_2(.)$ is the cumulative function of the normal distribution with mean μ_2 and standard deviation σ_2. Transition probability matrix should be rearranged in the form of $\mathbf{P}=\begin{pmatrix}\mathbf{Q} & \mathbf{R}\\ \mathbf{O} & \mathbf{I}\end{pmatrix}$, in which,

$$\mathbf{Q}=\begin{pmatrix}P_{11} & P_{12}\\ 0 & P_{22}\end{pmatrix},\ \mathbf{R}=\begin{pmatrix}0 & P_{14}\\ P_{23} & P_{24}\end{pmatrix},\ \mathbf{O}=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix},\ \mathbf{I}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}. \tag{10}$$

Using mathematical formulations of absorbing Markov chains, the following is obtained:

$$\mathbf{M}=(\mathbf{I}-\mathbf{Q})^{-1}, \tag{11}$$

where the elements of fundamental matrix are the expected number of transitions from any non-absorbing state to any other non-absorbing state before absorption occurs (Bowling et al., 2004),

$$\mathbf{M}=\begin{pmatrix}\frac{1}{1-P_{11}} & \frac{P_{12}}{(1-P_{11})(1-P_{22})}\\ 0 & \frac{1}{(1-P_{22})}\end{pmatrix}=\begin{pmatrix}m_{11} & m_{12}\\ 0 & m_{22}\end{pmatrix}. \tag{12}$$

Also, the absorption probabilities are obtained as follows:

$$\mathbf{F}=\mathbf{M}\times\mathbf{R}=\begin{pmatrix}\frac{P_{12}P_{23}}{(1-P_{11})(1-P_{22})} & \frac{P_{14}(1-P_{22})+P_{12}P_{24}}{(1-P_{11})(1-P_{22})}\\ \frac{P_{23}}{(1-P_{22})} & \frac{P_{24}}{(1-P_{22})}\end{pmatrix}=\begin{pmatrix}f_{13} & f_{14}\\ f_{23} & f_{24}\end{pmatrix}. \tag{13}$$

Denoting the cycle time with parameter C, the following is obtained:

$$C = Max\left\{T_1 m_{11}, T_2 m_{22}\frac{P_{12}}{1-P_{11}}\right\}. \tag{14}$$

The parameter m_{ii} represents the expected number of times that the transient state i is occupied before absorption occurs and T_i is the production time in transient state i . Also, the probability of going from state 1 to state 2 for each item (the probability of processing each item in stage 2) is $\frac{P_{12}}{1-P_{11}}$, the expected value of operation time for each item in stage 1 of production is $T_1 m_{11}$, and the expected value of operation time for each item in stage 2 of production is $T_2 m_{22}\frac{P_{12}}{1-P_{11}}$. Considering the fact that the cycle time of a production line is equal to the maximum operation time of all stages, Eq. (14) is obtained.

Also, according to Eq. (1), the following is obtained:

$$TP = \frac{H}{C}\begin{pmatrix} f_{13}SP - (m_{11}-1)RC_1 - (m_{22}-1)\dfrac{P_{12}}{(1-P_{11})}RC_2 \\ -(\dfrac{P_{14}}{1-P_{11}})SC_1 - f_{24}SC_2 - PC_1 - (\dfrac{P_{12}}{1-P_{11}})PC_2 - E(QC) \end{pmatrix}, \tag{15}$$

where $\frac{H}{C}$ is the expected number of produced items and $E(RV)$ is a selling price per item (SP) multiplied by the probability of accepting an item (f_{13}),

$$E(RV) = SP(f_{13}). \tag{16}$$

$E(PC)$ is the expected processing cost per item at stage 1 (PC_1) plus the expected processing cost at stage 2 (PC_2) multiplied by the probability of processing the item in stage 2,

$$E(PC) = PC_1 + (\frac{P_{12}}{1-P_{11}})PC_2. \tag{17}$$

$E(SC)$ is the scrapping cost (SC_1) per item multiplied by the probability of scrapping the item in stage 1 plus (SC_2) multiplied by probability of scrapping the item in stage 2 (f_{24}),

$$E(SC) = \frac{P_{14}}{1-P_{11}}(SC_1) + f_{24}(SC_2). \tag{18}$$

$E(RC)$ is the reworking cost (RC_1) per item multiplied by expected number of reworking actions in stage 1 $(m_{11}-1)$ plus RC_2 multiplied by expected number of reworking actions in stage 2 $(m_{22}-1)$ multiplied by the probability of processing the item in stage 2 $(\frac{P_{12}}{1-P_{11}})$,

$$E(RC)=(m_{11}-1)RC_1+(m_{22}-1)(\frac{P_{12}}{1-P_{11}})RC_2. \tag{19}$$

The above objective function is derived from Bowling et al. (2004) with some revision. This revision has completely been elaborated on by Fallahnezhad and Niaki (2010), Fallahnezhad et al. (2013), and Fallahnezhad and Ahmadi (2014). The cost of quality QC is considered in the model using quadratic Taguchi loss function for each accepted item as follows:

$$QC=A_1\frac{\int_{L_1}^{U_1}(x_1-\tau_1)^2 f(x_1)dx_1}{\int_{L_1}^{U_1} f(x_1)dx_1}+A_2\frac{\int_{L_2}^{U_2}(x_2-\tau_2)^2 f(x_2)dx_2}{\int_{L_2}^{U_2} f(x_2)dx_2}, \tag{20}$$

where the values of Taguchi loss function in the first and second stages are obtained as follows:

$$E\left(loss\,|L_1<X_1<U_1\right)=\frac{E(loss,L_1<X_1<U_1)}{P(L_1<X_1<U_1)}=A_1\frac{\int_{L_1}^{U_1}(x_1-\tau_1)^2 f(x_1)dx_1}{\int_{L_1}^{U_1} f(x_1)dx_1}, \tag{21}$$

$$E(loss\,|L_2<X_2<U_2)=\frac{E(loss,L_2<X_2<U_2)}{P(L_2<X_2<U_2)}=A_2\frac{\int_{L_2}^{U_2}(x_2-\tau_2)^2 f(x_2)dx_2}{\int_{L_2}^{U_2} f(x_2)dx_2}. \tag{22}$$

Since the Taguchi loss function should be considered for all sold items, $E(QC)$ is the value of Taguchi loss function multiplied by the probability of accepting an item (f_{13}),

$$E(QC)=QC(f_{13}). \tag{23}$$

In the next section, a numerical application of the proposed methodology is presented.

IV. NUMERICAL EXAMPLE

In this section, a numerical example is solved in order to illustrate the applicability of the proposed model. It is assumed that the production system has two operation stages and the parameters are as follows:

$$SP = \$120, PC_1 = \$25, PC_2 = \$20, SC_1 = \$15, SC_2 = \$12, RC_1 = \$10,$$
$$RC_2 = \$17, T_1 = 80s, T_2 = 50s, L_1 = 8$$
$$L_2 = 13, U_1 = 12, U_2 = 17, H = 1000, A_1 = A_2 = 1, \tau_1 = 10, \tau_2 = 15, \sigma_1 = \sigma_2 = 1.0.$$

The optimal solution to the proposed models is determined by solving the related nonlinear optimization model using MATLAB R2015a software and applying a grid search procedure. The expected profit is maximized at $\mu_1^* = 10.45, \mu_2^* = 15.075$. The expected profit of production is $TP^* = 836.7864$. *Fig.* (1) denotes the plotted values of the function TP versus decision variables μ_1, μ_2. As can be seen, the expected profit is a concave function of the process means.

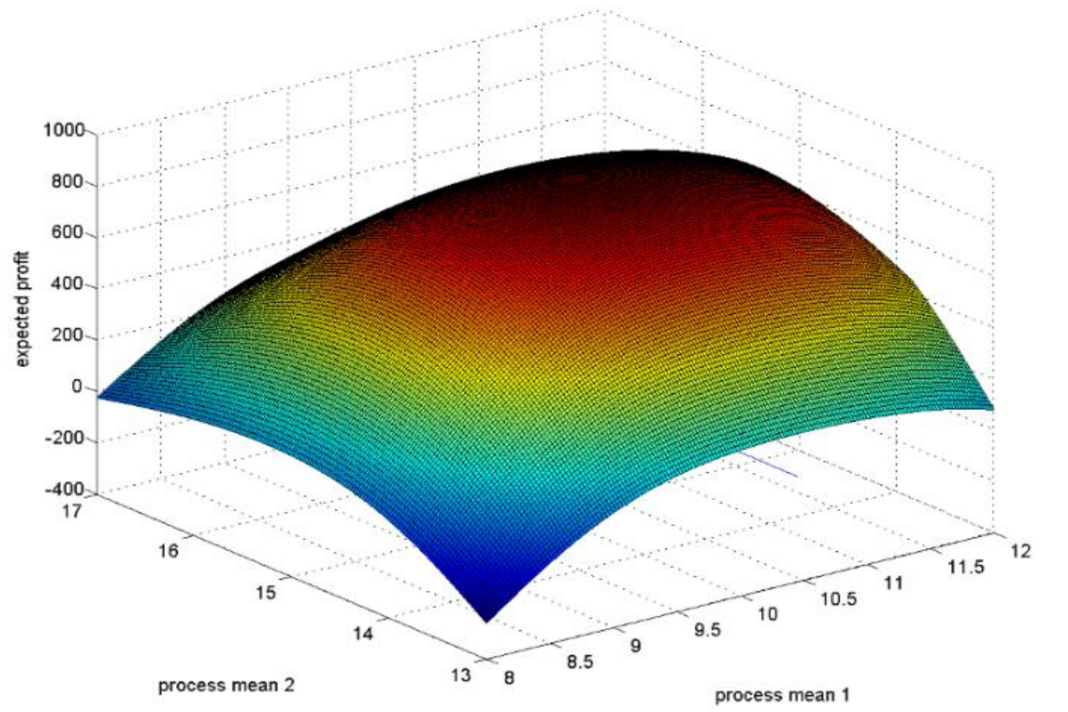

Fig.1. The expected profit of the production system versus process means

V. SENSITIVITY ANALYSIS

A sensitivity analysis is performed to analyze the effects of changing the parameters on the optimal process mean and the optimal expected profit. All parameters vary in this production system and the results for the analysis of their effects are provided in this section. Table I shows the values of the optimal process mean and the optimal expected profit with the variation of parameters.

Table I. Behaviors of optimal mean and expected profit with the variation of parameters

Parameter	Case #	Value	μ_1^*	μ_2^*	TP^*
SC_1	1	15	10.45	15.075	836.7864
	2	20	10.45	15.10	835.6031
	3	30	10.45	15.10	833.3699
	4	100	10.45	15.225	820.0480
	5	200	10.45	15.325	805.4858
SC_2	6	12	10.45	15.075	836.7864
	7	25	10.475	15.075	835.6174
	8	50	10.50	15.075	833.5409
	9	100	10.575	15.075	829.9441
	10	200	10.675	15.075	824.1023
RC_1	11	10	10.45	15.075	836.7864
	12	20	10.45	15.05	833.4879
	13	40	10.45	15.00	827.4512
	14	80	10.45	14.925	817.0339
	15	100	10.45	14.90	812.4350
RC_2	16	17	10.45	15.075	836.7864
	17	25	10.375	15.075	831.1719
	18	40	10.275	15.075	822.5729
	19	100	10.05	15.075	798.9337
	20	120	10	15.075	792.9229
PC_1	21	15	10.45	15.05	958.4879
	22	25	10.45	15.075	836.7864
	23	45	10.45	15.15	594.1276
	24	70	10.45	15.275	292.9010
	25	100	10.45	15.625	-60.9506
PC_2	26	5	10. 45	15.075	1.0156e+03
	27	20	10.45	15.075	836.7864
	28	50	10.45	15.10	479.1875
	29	90	10.45	15.275	3.6293
	30	110	10.45	17.00	-185.9810
A_1	1	1	10.45	15.075	836.7864
	2	5	10.45	15.075	799.0365
	3	10	10.45	15.075	751.8492
A_2	1	1	10.45	15.075	836.7864
	2	5	10.35	15.075	797.2527
	3	10	10.275	15.075	749.3937

Table I. Behaviors of optimal mean and expected profit with the variation of parameters

Parameter	Case #	Value	μ_1^*	μ_2^*	TP^*
T_1	1	50	10.325	15.125	1.3367e+03
	2	80	10.45	15.075	836.7864
	3	110	10.45	15.075	608.5719
T_2	1	40	10.45	15.075	836.7864
	2	50	10.45	15.075	836.7864
	3	80	10.325	15.125	835.4216
	4	100	10.10	15.275	675.6110
τ_1	1	9	10.45	15.00	823.7668
	2	10	10.45	15.075	836.7864
	3	11	10.45	15.15	826.7935
τ_2	1	14	10.325	15.075	818.0380
	2	15	10.45	15.075	836.7864
	3	16	10.575	15.075	834.1747
σ_1	1	1/2	10.45	15.00	897.4576
	2	1	10.45	15.075	836.7864
	3	2	10.45	15.325	510.9285
σ_2	1	1/2	10.025	15.075	868.9137
	2	1	10.45	15.075	836.7864
	3	2	11.25	15.10	620.4906

In Table I, it is observed that the optimal expected profit decreases as scrapping and reworking costs increase. Also, with increase in the value of SC_1, the optimum value of μ_1^* remains constant, but the optimum value of μ_2^* increases and with increase in the value of SC_2, the optimum value of μ_2^* remains constant but the optimum value of μ_1^* increases, which means that probability of scrapping in the first stage decreases. When parameters of one production stage change, the optimal adjustment of that stage does not substantially differ, but the optimal adjustment of the other production stage changes. By increasing the value of RC_1, the optimum value of μ_1^* remains constant, but the optimum value of μ_2^* decreases and with increase in the value of RC_2, the optimum value of μ_2^* remains constant, but the optimum value of μ_1^* decreases; thus, the same conclusion is drawn. Furthermore, by increasing the value of PC_1, the optimum value of μ_1^* does not change, but the optimum value of μ_2^* increases; as a result, the expected number of reworking actions in the second stage increases. Also, by increasing the value of PC_2, the optimum value of μ_1^* does not change, but the optimum value of μ_2^* increases; as a result, the expected number of reworking actions tends to increase in the second stage.

It is observed that with increase in the value of A_1, the optimum values of μ_1^* and μ_2^* remain constant, but the optimum value of TP^* decreases. When A_2 increases, μ_2^* does not change, but μ_1^* and TP^* decrease. Also, by increasing the value of T_1, the optimum value of μ_1^* increases, but the values of μ_2^* and TP^* decrease. This result implies that the expected number of reworking actions in the first stage increases; therefore, the cycle time of production may increase. Moreover, when T_2 increases, μ_1^* and TP^* decrease, but μ_2^* increases. This result implies that the expected number of reworking actions decreases and the cycle time of production may decrease due to higher processing time in the first stage. Furthermore, the optimum value of μ_2^* increases as τ_1 increases, but μ_1^* remains constant and the optimum value of μ_1^* increases as τ_2 increases, but μ_2^* remains constant. Also, as σ_1 increases, the optimum value of μ_1^* remains constant, but μ_2^* increases and TP^* decreases. By increasing σ_2, the optimum values of μ_2^* and μ_1^* increase and the optimum value of TP^* decreases. Since increasing the standard deviation results in increase in the scrapping cost, the optimal process mean increases in order to decrease the effect of large scrapping costs (considering price of one sold item). In general, it is seen that when parameters of one production stage change, the optimal adjustment of that stage does not substantially differ, but the optimal adjustment of the other production stage completely changes in most of the cases, denoting that the optimal adjustments of two production stages are completely dependent on each other.

VI. INSPECTION ERROR

There are two types of inspection error in the sampling plan; the first type is classifying a conforming item as non-conforming and the second type is classifying a non-conforming item as conforming. Therefore, the inspector rejects some conforming items and accepts some non-conforming. Assume that α is the probability of the first type of error and β is the probability of the second type of error. If P'_{ij} denotes the probability of going from state *i* to state *j* in this case, we have

$$P'_{12} = (1-\alpha)P(L_1 \le X_1 \le U_1) + \beta P(X_1 \le L_1) = (1-\alpha)P_{12} + \beta P_{14}, \tag{24}$$

$$P'_{14} = (1-\beta)P(X_1 \le L_1) + \alpha P(L_1 \le X_1 \le U_1) = (1-\beta)P_{14} + \alpha P_{12}. \tag{25}$$

Also, the following is obtained:

$$P'_{11} = 1 - P'_{12} - P'_{14} = P_{11}. \tag{26}$$

On the same basis, the following is obtained:

$$P'_{23} = (1-\alpha)P(L_2 \le X_2 \le U_2) + \beta P(X_2 \le L_2) = (1-\alpha)P_{23} + \beta P_{24}, \tag{27}$$

$$P'_{24} = (1-\beta)P(X_2 \le L_2) + \alpha P(L_2 \le X_2 \le U_2) = (1-\beta)P_{24} + \alpha P_{23}. \tag{28}$$

Therefore,

$$P'_{22} = P_{22}. \tag{29}$$

Now, we can derive the Markov chain of production process in this case as follows:

$$\mathbf{Q}=\begin{pmatrix} P'_{11} & P'_{12} \\ 0 & P'_{22} \end{pmatrix},\ \mathbf{R}=\begin{pmatrix} 0 & P'_{24} \\ P'_{23} & P'_{24} \end{pmatrix},\ \mathbf{O}=\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix},\ \mathbf{I}=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (30)$$

Now, we can evaluate the objective function in Eq. (13) and determine the optimal process adjustment under the presence of inspection errors. Table II presents a sensitivity analysis of the values of inspection errors.

Table II. Behaviors of optimal mean and expected profit with the variation of inspection errors

Inspection errors	Case #	Inspection errors (α,β)	μ_1^*	μ_2^*	TP^*
(α,β)	1	(0,0)	10.45	15.075	836.7864
	2	(0.05,0.1)	10.40	15.05	698.5106
	3	(0.05,0.2)	10.375	15.025	702.5705
	4	(0.1,0.1)	10.40	15.075	563.3825
	5	(0.1,0.2)	10.375	15.05	567.1774
	6	(0.1,0.3)	10.325	15.00	571.3197
	7	(0.3,0.1)	10.35	15.20	93.7542

It is observed in Table II that by increasing the value of β, the optimum values of μ_1^* and μ_2^* decrease, but the optimum value of TP^* increases. The total profit increases by ignoring the cost of non-conforming items sold in the market. Also, with increase in α, the optimum values of μ_1^* and TP^* decrease, but μ_2^* increases. Considering the first type of error in the model results in decrease in the total profit of the system as it is expected. We have modified the state of Markov chain in order to consider penalty cost of selling non-conforming items in the market. The new states are as follows:

State 1: An item is being processed in the first stage of the production process
State 2: An item is being processed in the second stage of the production process
State 3: An item is accepted as finished item
State 4: An item is scrapped
State 5: A conforming item is sold in the market

Transition probability matrix can be expressed as follows:

$$\mathbf{P}=\begin{pmatrix} P_{11} & P_{12} & 0 & P_{14} & 0 \\ 0 & P_{22} & P_{23} & P_{24} & P_{25} \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad (31)$$

where,

$$\begin{aligned}
&P_{12} = (1-\alpha)P(L_1 \le X_1 \le U_1) + \beta P(X_1 \le L_1),\\
&P_{15} = 0,\\
&P_{14} = (1-\beta)P(X_1 \le L_1) + (\alpha)P(L_1 \le X_1 \le U_1),\\
&P_{11} = P(X_1 > U_1),\\
&P_{23} = (1-\alpha)P(L_2 \le X_2 \le U_2),\\
&P_{25} = \beta P(X_2 \le L_2),\\
&P_{24} = (1-\beta)P(X_2 \le L_2) + (\alpha)P(L_2 \le X_2 \le U_2),\\
&P_{22} = P(X_2 > U_2).
\end{aligned} \tag{32}$$

Transition probability matrix in the form of $\mathbf{P} = \begin{pmatrix} \mathbf{Q} & \mathbf{R} \\ \mathbf{O} & \mathbf{I} \end{pmatrix}$ can be expressed as follows:

$$\mathbf{Q} = \begin{pmatrix} P_{11} & P_{12} \\ 0 & P_{22} \end{pmatrix}, \ \mathbf{R} = \begin{pmatrix} 0 & P_{14} & 0 \\ P_{23} & P_{24} & P_{25} \end{pmatrix}, \ \mathbf{O} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \ \mathbf{I} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{33}$$

Thus, the following is obtained:

$$\mathbf{M} = \begin{pmatrix} \frac{1}{1-P_{11}} & \frac{P_{12}}{(1-P_{11})(1-P_{22})} \\ 0 & \frac{1}{(1-P_{22})} \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ 0 & m_{22} \end{pmatrix}, \tag{34}$$

$$\mathbf{F} = \mathbf{M} \times \mathbf{R} = \begin{pmatrix} \frac{P_{12}P_{23}}{(1-P_{11})(1-P_{22})} & \frac{P_{14}(1-P_{22}) + P_{12}P_{24}}{(1-P_{11})(1-P_{22})} & \frac{P_{12}P_{25}}{(1-P_{11})(1-P_{22})} \\ \frac{P_{23}}{(1-P_{22})} & \frac{P_{24}}{(1-P_{22})} & \frac{P_{25}}{(1-P_{22})} \end{pmatrix} = \begin{pmatrix} f_{13} & f_{14} & f_{15} \\ f_{23} & f_{24} & f_{25} \end{pmatrix}.$$

Now, Eq. (1) can be modified as follows:

$$E(PR) = [E(RV) - E(PC) - E(SC) - E(RC) - E(QC) - E(PCO)]. \tag{35}$$

If PCO denotes penalty cost of selling a non-conforming item, according to Eq. (35), the following is obtained:

$$TP = \frac{H}{C}\begin{pmatrix} f_{13}SP - (m_{11}-1)RC_1 - (m_{22}-1)\frac{P_{12}}{(1-P_{11})}RC_2 \\ -(\frac{P_{14}}{1-P_{11}})SC_1 - f_{24}SC_2 - PC_1 - (\frac{P_{12}}{1-P_{11}})PC_2 - E(QC) - f_{15}PCO \end{pmatrix}. \tag{36}$$

Now, we can evaluate the objective function in Eq. (36) and determine the optimal process adjustment under the presence of inspection errors. Table III denotes a sensitivity analysis of the values of inspection errors

Table III. Effect of penalty cost on optimal mean and expected profit with the variation of inspection errors

PCO	Case #	Inspection errors (α, β)	μ_1^*	μ_2^*	TP^*
50	1	(0,0)	10.45	15.075	836.7864
	2	(0.05,0.1)	10.45	15.10	694.8661
	3	(0.05,0.2)	10.45	15.10	694.1502
	4	(0.1,0.1)	10.425	15.125	561.3574
100	1	(0,0)	10.45	15.075	836.7864
	2	(0.05,0.1)	10.45	15.10	693.2311
	3	(0.05,0.2)	10.47	15.05	692.1504
	4	(0.1,0.1)	10.45	15.10	559.8103

It is observed in Table III that by increasing the value of β, the optimal adjustment does not change in the case of $PCO = 50$, but in the case of $PCO = 100$, the value of μ_1^* increases and the value of μ_2^* decreases. Also, the value of TP^* decreases as it is expected. Moreover, by increasing the value of α, the optimal adjustment does not change in the case of $PCO = 100$, but in the case of $PCO = 50$, the value of μ_1^* decreases and the value of μ_2^* increases. Also, the value of TP^* decreases as it is expected.

It is obvious that considering the penalty cost of selling non-conforming items in the market leads to decrease in the total profit; thus, the total profit reported in Table III will be less than that in Table II. If the producer is responsible for non-conforming items sold in the market and exposed to the penalty cost of selling non-conforming items, the model with penalty cost is recommended. Otherwise, this model is not recommended.

VII. CONCLUSION AND FUTURE RESEARCH DIRECTIONS

In this paper, an absorbing Markov chain model with reworking loops was developed to determine the optimal process means for maximizing the expected profit of two-stage production systems in which all items were inspected to be classified as accepted, scrapped, or reworked. Also, performance of the proposed methodology under inspection errors was investigated. Numerical examples were provided to illustrate the applicability of the proposed model. In general, it was seen that when parameters of one production stage changed, the optimal adjustment of that stage did not substantially differ, but the optimal adjustment of the other production stage completely changed in most of the cases, denoting that the optimal adjustments of two production stages were completely dependent on each other. Moreover, the effect of inspection errors was analyzed and it was concluded that presence of inspection errors would affect the optimal adjustment.

As a future research direction, in cases with high inspection costs, the proposed model can be extended by considering the sampling plans instead of 100 % inspection policy and assuming that the standard deviation of the quality characteristic is unknown. In this condition, t distribution should be employed for evaluating the required probabilities. Another extension of this research is considering multi-stage production process rather than the two-stage problem. Another aspect that deserves further attention is considering the inherent uncertainty of the concerned parameters.

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