Performance of Target Detection in Phased-MIMO Radars

S. Jebali and H. Keshavarz
Department of Communication Engineering, University of Sistan and Baluchestan, Zahedan, Iran
safieh.jebali@pgs.usb.ac.ir , hkeshavarz@ieee.org
Corresponding author: H. Keshavarz

***Abstract*- In this paper, the problem of target detection in phased-MIMO radars is considered and target detection performance of phased-MIMO radars is compared with MIMO and phased-array radars. Phased-MIMO radars combine advantages of the MIMO and phased-array radars. In these radars, the transmit array will be partitioned into a number of subarrays that are allowed to overlap and each subarray transmits a waveform which is orthogonal to the waveform transmitted by other subarrays. In this paper, target detection performance of phased-MIMO radars is analyzed with two detectors theoretically in addition to the investigation of simulation results. First, it is assumed that the transmitted waveforms are ideally orthogonal and secondly the transmitted waveforms are considered to be correlated (not fully coherent or ideally orthogonal). The Generalized likelihood ratio test (GLRT) and the likelihood ratio test (LRT) are used for target detection. The closed-form expressions of the false alarm and detection probabilities in presence of Gaussian noise are obtained. Simulation results validate the theoretical analysis.**

***Index Terms*-** Multiple-input multiple-output (MIMO) radar, Phased-MIMO radar, Neyman-Pearson criterion, likelihood ratio test (LRT), Generalized likelihood ratio test (GLRT).

I. INTRODUCTION

In the last decade, multiple-input multiple-output radars have become the focus of attention of researchers [1]-[3]. In this radar systems, each transmit antenna is able to create an independent waveform. The MIMO radars employ multiple antennas for transmitting several orthogonal waveforms and multiple antennas for receiving signals reflected by the target [4], [5]. Based on antenna configurations, MIMO radars can be classified into two types: (i) MIMO radars with widely-separated antennas, and (ii) MIMO radars with co-located antennas [6].

The first type is known as the statistical MIMO radars or the distributed MIMO radars, where its antennas are separated far from each other such that a target can be viewed from different spatial directions to achieve spatial diversity gain [7], [8]. The second type is known as the co-located MIMO

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radars, where the transmitter and receiver antennas are closely spaced to transmit a beam towards a certain direction in the space [9], [10]. In the phased-array radars, if the signal coherency is preserved, the signal can be processed coherently at the transmit/receive antenna arrays [11].

Recently, adding the coherent processing gain to the MIMO radars with co-located antennas has been called phased-MIMO radars. The essence of this technique is on partitioning the transmitting array to a number of overlapped subarrays with smaller sizes such that each subarray operates in the phased-array mode. Hence, phased-MIMO radars exploit jointly the benefits of the phased-array and MIMO radars [11]-[14].

The phased-MIMO radar has all the advantages of the MIMO radar, in other words; it is capable of detecting a higher number of targets, improving parameter identifiability, improving angular resolution, and extending the array aperture. Furthermore, the phased-MIMO radar provides the means for designing the overall beam pattern of the virtual array and applies beamforming techniques at both the transmitting and the receiving sides. This radar provides a trade-off between angular resolution and robustness against beam-shape loss and offers improved robustness against strong interference [11].

In [15], [16], modern optimization algorithms are introduced with the orthogonal waveform design in MIMO radars. Hence, an algebraic method is presented to create poly-phase orthogonal sequences with small Doppler shift in [17]. In [18], it is assumed that transmitted waveforms are ideally orthogonal and fully coherent in target detection and localization for MIMO radars. It is shown that the optimal performance of target detection in MIMO radars is achieved when orthogonal signals are transmitted [19].

In this paper, at the first target detection is analysed with GLRT and LRT detectors in phased-MIMO radars, when all subarrays transmit orthogonal waveforms and secondly, target detection is considered with LRT detector when subarrays transmitted waveforms are correlated (not fully coherent or orthogonal). Furthermore, a comparison is made among target detection methods in phased-array, MIMO and phased-MIMO radars and then, the effect of the detector type is analysed on the target detection probability. The optimal detector based on the Neyman-Pearson criterion is extended and applied to phase-MIMO radars, when subarrays transmitted waveforms are not fully coherent or orthogonal. Numerical results are shown to validate the theoretical analysis and to compare performance of this detector in MIMO and phased-MIMO radars.

This paper is organised as follows: firstly, the signal models for MIMO and phased-MIMO radars are presented in Section II. Section III presents the problem of target detection in phased-MIMO radars with orthogonal and correlated (not fully coherent or ideally orthogonal) transmitted waveforms. In Section IV, simulation results are shown. Finally, Section V concludes the paper.

II. SIGNAL MODEL

In this section, co-located MIMO and phased-MIMO radars are briefly introduced.

A. Signal model for co-located MIMO radar

Consider a co-located MIMO radar with a transmit array consisting of M_t co-located antennas and a receive array consisting of M_r co-located antennas. In co-located MIMO radars, both transmit and receive arrays are assumed to be spatially close to each other; therefore, the target direction is almost the same for all array antennas. In the transmit array, the *m*th antenna emits the *m*th waveform. Vector $\mathbf{\Phi}(t) = [\Phi_1(t), \dots, \Phi_{M_t}(t)]^T$ contains the waveforms transmitted from different elements of the array where t denotes the time index within the radar pulse, and $(.)^T$ stands for the matrix transpose operator. As it is known, the orthogonal condition is given by

$$\int_{T_0} \mathbf{\Phi}(t)\mathbf{\Phi}^H(t)\, dt = \mathbf{I}_{M_t \times M_t} \tag{1}$$

where T_0 is the radar pulse duration, $\mathbf{I}_{M_t \times M_t}$ is the $M_t \times M_t$ identity matrix and $(.)^H$ denotes Hermitian transpose operator. The $M_r \times 1$ received signal vector can be written as

$$\mathbf{x}(t) = \mathbf{x}_s(t) + \mathbf{x}_i(t) + \mathbf{n}(t) \tag{2}$$

where $\mathbf{x}_s(t)$, $\mathbf{x}_i(t)$, and $\mathbf{n}(t)$ refer to the source/target signal, interference/jamming, and sensor noise, respectively. The target signal is given by

$$\mathbf{x}_s(t) = \beta_s(\mathbf{a}^T(\theta_s)\mathbf{\Phi}(t))\mathbf{b}(\theta_s) \tag{3}$$

where β_s is the target reflection coefficient, θ_s is the target direction, $\mathbf{a}(\theta)$ is the actual transmit steering vector associated with the direction θ, and $\mathbf{b}(\theta)$ is the actual receive steering vector associated with the direction θ. The uniform linear arrays (ULA) are used for the transmission and reception then $\mathbf{a}(\theta)$ and $\mathbf{b}(\theta)$ can be expressed by

$$\mathbf{a}(\theta) = [1, \exp(-j2\pi d_t \sin\theta), \dots, \exp(-j2\pi(M_t - 1)d_t \sin\theta)]^T \tag{4}$$

$$\mathbf{b}(\theta) = [1, \exp(-j2\pi d_r \sin\theta), \dots, \exp(-j2\pi(M_r - 1)d_r \sin\theta)]^T \tag{5}$$

where d_t and d_r are the interelement spacing measured in wavelength for the transmit and receive arrays, respectively. After receiving the signals, they are passed through filters matched to $\{\Phi_m(t)\}_{m=1}^{M_t}$, the mth transmitted waveform is recovered as

$$\mathrm{X}_m(t) \triangleq \int_{T_0} \mathbf{x}(t)\Phi_m^*(t)\,, \quad m = 1, \dots, M_t \tag{6}$$

where $(.)^*$ denotes the conjugate operator. The virtual data vector is written as

$$\mathbf{y} = [\mathbf{x}_1^T, \dots, \mathbf{x}_{M_t}^T]^T = \beta_s \mathbf{V}(\theta_s) + \mathbf{y}_{i+n} \tag{7}$$

where $\mathbf{V}(\theta_s) \triangleq \mathbf{a}(\theta_s) \otimes \mathbf{b}(\theta_s)$ is the steering vector associated with the virtual array, $\otimes$ stands for the Kronker product and $\mathbf{y}_{i+n}$ represents the interference-plus-noise components [11]-[13].

B. Signal model for phased-MIMO radar

In the phased-MIMO radars, the transmit array is partitioned into K overlapping subarrays ($1 \leq K \leq M_t$) such that no subarray is exactly the same as another subarray. All antennas of the *k*th subarray emit the signal $\Phi_k(t)$ coherently; therefore, a beam is formed towards the target direction. Different waveforms are transmitted by different subarrays at the same time, Fig. 1.

The output signal of the *k*th subarray is modeled as

$$\mathrm{s}_k(t) = \sqrt{\frac{M_t}{K}}\Phi_k(t)\mathbf{w}_k^*\,, \quad k = 1, \dots, K \tag{8}$$

where $\mathbf{w}_k$ is the *k*th subarray beamforming weight vector. It is worth noting that energy of the *k*th subarray is $\frac{M_t}{K}$ within one radar pulse and it is equal to

$$E_k = \int_{T_0} \mathrm{s}_k^H(t)\mathrm{s}_k(t)dt = \frac{M_t}{K} \tag{9}$$

Hence, the total transmitted energy of the phased-MIMO radar equals M_t . The signal reflected by the target is given by

$$\mathbf{r}(t,\theta) = \sqrt{\frac{M_t}{K}}\beta(\theta)\big(\mathbf{c}(\theta) \odot \mathbf{d}(\theta)\big)^T \Phi_K(t) \tag{10}$$

where

$$\mathbf{c}(\theta) = [\mathbf{w}_1^H \mathbf{a}_1(\theta), \dots, \mathbf{w}_K^H \mathbf{a}_K(\theta)]^T \tag{11}$$

$$\mathbf{d}(\theta) = [e^{-j\tau_1(\theta)}, \dots, e^{-j\tau_K(\theta)}]^T \tag{12}$$

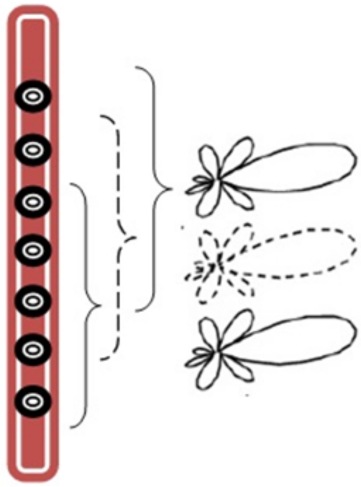


Fig. 1. A phased-MIMO radar with 3 subarrays.

where $\mathbf{c}(\theta)$, $\mathbf{d}(\theta)$, and $\mathbf{a}(\theta)$ are the transmit coherent processing vector, waveform diversity vector, and actual transmit steering vector, respectively; $\beta(\theta)$ is the target reflection coefficient, $\tau_K(\theta)$ is the time delay for the signal to travel between the first antenna of the *k*th subarray and the first antenna of the transmit array, and $\odot$ denotes the Hadamard product operator. Hence, the received vector of array observations is written as

$$\mathbf{x}(t) = \mathbf{r}(t, \theta_s)\mathbf{b}(\theta_s) + \mathbf{n}(t) \tag{13}$$

where $\mathbf{n}(t)$ and $\mathbf{b}(\theta_s)$are the noise term and the actual receive steering vector associated with direction θ_s, respectively. Assuming that the target is observed in the presence of D interferers with reflection coefficients $\{\beta_i\}_{i=1}^{D}$ and directions$\{\theta_i\}_{i=1}^{D}$, the array received signal is written as

$$\mathbf{x}(t) = \mathbf{r}(t, \theta_s)\mathbf{b}(\theta_s) + \sum_{i=1}^{D} \mathbf{r}(t, \theta_i)\mathbf{b}(\theta_i) + \mathbf{n}(t) \tag{14}$$

Then, the virtual data vector after matched-filtering is converted into

$$\mathbf{y} = [\mathbf{x}_1^T, \dots, \mathbf{x}_K^T]^T = \sqrt{\frac{M_t}{K}}\beta_s\mathbf{u}(\theta_s) + \sum_{i=1}^{D}\sqrt{\frac{M_t}{K}}\beta_i\mathbf{u}(\theta_i) + \mathbf{n} \tag{15}$$

where $\mathbf{n}$ is the noise term with covariance matrix $\mathrm{R}_n = \sigma_n^2\mathbf{I}_{KM_r}$ and σ_n^2 is noise power, and $\mathbf{u}(\theta) \triangleq (\mathbf{c}(\theta)\odot\mathbf{d}(\theta))\otimes\mathbf{b}(\theta)$ is the virtual steering vector associated with direction θ [11]-[13]. Using signals modeled at (7) and (15) for MIMO and phased-MIMO radars, respectively, $\mathbf{V}(\theta)$ and $\mathbf{U}(\theta)$ are defined as

$$\mathbf{V}(\theta) \triangleq \sqrt{\frac{M_t}{K}}\beta_s \mathbf{v}(\theta_s) \tag{16}$$

$$\mathbf{U}(\theta) \triangleq \sqrt{\frac{M_t}{K}}\beta_s \mathbf{u}(\theta_s) \tag{17}$$

If the number of subarrays is equal to the number of transmitter antennas, the virtual data vector without interference is written as

$$\mathbf{y} = \begin{cases} \mathbf{V}(\theta) + \mathbf{n} & \text{for MIMO Radar} \\ \mathbf{U}(\theta) + \mathbf{n} & \text{for Phasd} - \text{MIMO Radar} \end{cases} \tag{18}$$

III. TARGET DETECTION

A. Orthogonal waveforms

In this section, two detectors, namely the LRT and GLRT detectors, are presented. The optimal detector in the Neyman-Pearson criterion is the likelihood ratio test [22].

1) The GLRT detector

In [20], the radar detection problem is formulated as follows:

$$\begin{cases} H_0 : \mathbf{y} = \mathbf{n} \\ H_1 : \mathbf{y} = \sqrt{\frac{M_t}{K}}\boldsymbol{\alpha} + \mathbf{n} \end{cases} \tag{19}$$

where $\mathbf{y}$ is a $M_t M_r \times 1$ complex vector whose entries correspond to the output of the each matched filter at every receiver, $\mathbf{n}$ is a $M_t M_r \times 1$ white Gaussian noise vector and $\boldsymbol{\alpha}$ is a $M_t M_r \times 1$ complex vector defined as

$$\boldsymbol{\alpha} = \begin{cases} \beta_s \mathbf{a}(\theta_s) \otimes \mathbf{b}(\theta_s) & \text{for MIMO Radar} \\ \beta_s \big(\mathbf{c}(\theta) \odot \mathbf{d}(\theta)\big) \otimes \mathbf{b}(\theta) & \text{for Phased} - \text{MIMO Radar} \end{cases} \tag{20}$$

Assume that the distribution of β_s is known and equal to $\beta_s \sim CN(0,1)$. Although the distribution of β_s is known, the angles of direction, θ_s and θ, are unknown. As a result, the distribution of $\boldsymbol{\alpha}$ cannot be known exactly, so in this detection problem, generalized likelihood ratio test (GLRT) can be employed by replacing the unknown coefficient vector $\boldsymbol{\alpha}$ by its ML estimate then the likelihood ratio test can be written as

$$\frac{\max_{\boldsymbol{\alpha}} P(\mathbf{y}|H_1, \sigma_n^2, \boldsymbol{\alpha})}{P(\mathbf{y}|H_0, \sigma_n^2)} \lessgtr_{H_0}^{H_1} \tag{21}$$

The probability distribution of $\mathbf{y}$ under H_1 can be written as

$$P(\mathbf{y}|H_1,\sigma_n^2,\boldsymbol{\alpha}) = \frac{1}{\pi^{M_tM_r}\sigma_n^{2M_tM_r}} \exp\left[-\frac{\left[\mathbf{y}-\sqrt{\frac{M_t}{K}}\boldsymbol{\alpha}\right]^H\left[\mathbf{y}-\sqrt{\frac{M_t}{K}}\boldsymbol{\alpha}\right]}{\sigma_n^2}\right] \tag{22}$$

After differentiating natural logarithm of (22) with respect to $\boldsymbol{\alpha}$ and equating the result to 0, the ML estimate of $\boldsymbol{\alpha}$ can be found as

$$\hat{\boldsymbol{\alpha}}_{ML} = \sqrt{\frac{K}{M_t}}\mathbf{y} \tag{23}$$

If $\hat{\boldsymbol{\alpha}}_{ML}$ is replaced with $\boldsymbol{\alpha}$ in (22), the conditional probability becomes

$$P(\mathbf{y}\big|H_1,\sigma_n^2,\hat{\boldsymbol{\alpha}}_{ML}) = 1/(\pi^{M_tM_r}\sigma_n^{2M_tM_r}) \tag{24}$$

The probability distribution of $\mathbf{y}$ conditioned on H_0 is given by

$$P(\mathbf{y}|H_0,\sigma_n^2) = \frac{1}{\pi^{M_tM_r}\sigma_n^{2M_tM_r}} \exp\left[-\frac{\mathbf{y}^H\mathbf{y}}{\sigma_n^2}\right]. \tag{25}$$

Hence, the log likelihood ratio can be written as

$$\ln\left[\frac{P(\mathbf{y}|H_1,\sigma_n^2,\hat{\boldsymbol{\alpha}}_{ML})}{P(\mathbf{y}|H_0,\sigma_n^2)}\right] = -\frac{\mathbf{y}^H\mathbf{y}}{\sigma_n^2} \tag{26}$$

and the likelihood ratio test becomes

$$\|\ \mathbf{y}\ \|^2 \lessgtr_{H_0}^{H_1} T' \tag{27}$$

where T' is the accordingly modified version of T and $\|.\|$ represents the Fobenious norm.

2) The LRT detector

Based on [19], the radar detection problem is formulated as follows

$$\begin{cases} \mathrm{H}_0\text{: Target dose not exists} \\ \mathrm{H}_1\text{: Target exists} \end{cases} \tag{28}$$

Using two hypotheses represented in (28) and (18) is replaced by

$$\begin{cases} \mathbf{y}_0 = \begin{cases} \mathbf{n} & \text{for MIMO Radar} \\ \mathbf{n} & \text{for Phased} - \text{MIMO Radar} \end{cases} \\ \mathbf{y}_1 = \begin{cases} \mathbf{V}(\theta) + \mathbf{n} & \text{for MIMO Radar} \\ \mathbf{U}(\theta) + \mathbf{n} & \text{for Phased} - \text{MIMO Radar} \end{cases} \end{cases} \tag{29}$$

Now, assume R denotes the data field satisfying the target existing assumption, the detection probability and the false alarm probability are respectively given by

$$\begin{cases} P_D = \int_R \ P_y(\mathbf{y}|H_1)dy \\ P_{FA} = \int_R \ P_y(\mathbf{y}|H_0)dy \end{cases} \tag{30}$$

where $P_y(\mathbf{y}|H_1)$ and $P_y(\mathbf{y}|H_0)$ denote the probability density function of measured data with assumption 1 and the probability density function of measured data with assumption 0, respectively. Consider additive white Gaussian noise $\mathbf{n} \sim \mathrm{N}(0, \sigma_n^2)$, then the virtual data vector is Gaussian and can be rewritten as

$$\mathbf{y} = \begin{cases} \mathrm{N}(\mathbf{V}(\theta), \sigma_\mathrm{n}^2) & \text{for MIMO Radar} \\ \mathrm{N}(\mathbf{U}(\theta), \sigma_\mathrm{n}^2) & \text{for Phased} - \text{MIMO Radar} \end{cases} \tag{31}$$

The likelihood function is represented by

$$\mathrm{p}(\mathbf{y};\theta) = \frac{1}{(2\pi)^{\frac{\mathrm{M_r K}}{2}} \det^{\frac{1}{2}}[\mathrm{C}]}$$

$$\times \exp[-\tfrac{1}{2}(\mathbf{y} - \mathbf{U}(\theta))^H \mathrm{C}^{-1}(\mathbf{y} - \mathbf{U}(\theta))] \tag{32}$$

where C is covariance matrix $\mathbf{C} = \sigma_n^2 \mathbf{I}_{M_r N}$, $\mathbf{I}_{M_r N}$ denotes an $M_r N \times M_r N$ identity matrix, H and N stands for the Hermitian operation and the number of samples in a radar pulse, respectively.

Applying the logarithm operation on both sides,

$$\log p(\mathbf{y};\theta) = -\frac{\mathrm{M_r K}}{2}\log(2\pi) - \frac{1}{2}\log(\det[\mathrm{C}])$$

$$-\tfrac{1}{2\sigma_n^2}(\mathbf{y}^H\mathbf{y} - 2\mathbf{U}^H(\theta)\mathbf{y} + \mathbf{U}^H(\theta)\mathbf{U}(\theta)) \tag{33}$$

To estimate θ using the virtual data vector, according to the Neyman-Fisher factorization [21] for the phased-MIMO radars, the log-likelihood function is calculated by

$$\log p(\mathbf{y};\theta) = h(\mathbf{y}) + \mathrm{g}_1(\mathbf{y},\theta) + \mathrm{g}_2(\theta) \tag{34}$$

The optimal detector in the Neyman-Pearson criterion is the likelihood ratio test [22] and its log-form is defined as

$$T = \log(p_y(\mathbf{y}|H_1)/p_y(\mathbf{y}|H_0)) \gtrless_{H_0}^{H_1} \tag{35}$$

Where ξ refers to the threshold and is equivalent to the desired probability of the false alarm. According to (34), T is calculated by

$$T = \log p_y(\mathbf{y}|H_1) - \log p_y(\mathbf{y}|H_0)$$

$$= h(\mathbf{y}|H_1) + \mathrm{g}_1(\mathbf{y},\theta|H_1) - h(\mathbf{y}|H_0) - \mathrm{g}_1(\mathbf{y},\theta|H_0) \tag{36}$$

According to the signal model, we have $\mathbf{y} \triangleq \begin{cases}\mathbf{y}_1 = \mathbf{U}(\theta) + \mathbf{n} & ; H_1 \\ \mathbf{y}_0 = \mathbf{n} & ; H_0\end{cases}$. From (29) and (35), the log-likelihood ratio is given by

$$T = h(\mathbf{y}|H_1) - h(\mathbf{y}|H_0) + \mathrm{g}_1(\mathbf{y},\theta|H_1) - \mathrm{g}_1(\mathbf{y},\theta|H_0)$$

$$= -\frac{1}{2\sigma_n^2}\mathbf{y}_1^H\mathbf{y}_1 + \frac{1}{2\sigma_n^2}\mathbf{y}_0^H\mathbf{y}_0$$

$$+(\mathbf{U}^H(\theta)\mathbf{y}_1)/\sigma_n^2 - (\mathbf{U}^H(\theta)\mathbf{y}_0)/\sigma_n^2 \tag{37}$$

Note that $-\frac{1}{2\sigma_n^2}\mathbf{y}_1^H\mathbf{y}_1$ and $\frac{1}{2\sigma_n^2}\mathbf{y}_0^H\mathbf{y}_0$ are constant. Hence, the new detector is defined as

$$\eta = \mathbf{U}^H(\theta)\mathbf{y} = \begin{cases}\mathbf{U}^H(\theta)\mathbf{U}(\theta) + \mathbf{U}^H(\theta)\mathbf{n} & ;\ H_1 \\ \mathbf{U}^H(\theta)\mathbf{n} & ;\ H_0\end{cases} \tag{38}$$

and the optimal detector is given by

$$\eta \lessgtr \xi' \ \ ; \ \left\{\xi' = \left(\xi - \frac{1}{2\sigma_n^2}\mathbf{y}_0^H\mathbf{y}_0 + \frac{1}{2\sigma_n^2}\mathbf{y}_1^H\mathbf{y}_1\right).\sigma_n^2\right\} \tag{39}$$

where ξ' is the new threshold. Since, the equation of radar system Gaussian noise was modeled by $\mathbf{n} \sim N(0,\sigma_n^2)$, so, the following equation is obtained as

$$\begin{cases}\mathbf{U}^H(\theta)\mathbf{U}(\theta) + \mathbf{U}^H(\theta)\mathbf{n} \sim \mathrm{N}\left(\mathbf{U}^H(\theta)\mathbf{U}(\theta), \sigma_n^2\mathbf{U}^H(\theta)\mathbf{U}(\theta)\right) \\ \mathbf{U}^H(\theta)\mathbf{n} \sim \mathrm{N}\left(0, \sigma_n^2\mathbf{U}^H(\theta)\mathbf{U}(\theta)\right)\end{cases} \tag{40}$$

According to the distribution of the test statistic,

$$\eta/\sqrt{\sigma_n^2\mathbf{U}^H(\theta)\mathbf{U}(\theta)} \triangleq \eta' \sim \begin{cases} N(\sqrt{\mathbf{U}^H(\theta)\mathbf{U}(\theta)/\sigma_n^2}\,,1) \\ N(0,1) \end{cases} \tag{41}$$

and the threshold is replaced by

$$\xi'' = \xi'/\sqrt{\sigma_n^2\mathbf{U}^H(\theta)\mathbf{U}(\theta)} \tag{42}$$

From (41), the probability of the false alarm is given by

$$P_{FA} = P(H_1;H_0) = \Pr\{\eta' > \xi'';H_0\} = \int_{\xi''}^{\infty} \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}t^2\right)dt$$

$$= Q(\xi'') \tag{43}$$

and

$$\xi'' = Q^{-1}(P_{FA}) \tag{44}$$

where $Q(.)$ and $Q^{-1}(.)$, denote the cumulative distribution function of the normal distribution and the inverse cumulative distribution function of the standard normal distribution, respectively. The detection probability is defined as

$$P_D = P(H_1;H_1) = \Pr\{\eta' > \xi'';H_1\}$$

$$= \int_{\xi''}^{\infty} \frac{1}{\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(t-\sqrt{\mathbf{U}^H(\theta)\mathbf{U}(\theta)/\sigma_n^2}\right)^2\right]dt$$

$$= Q\left[\xi''/\sqrt{\mathbf{U}^H(\theta)\mathbf{U}(\theta)/\sigma_n^2}\right] \tag{45}$$

B. Correlated waveforms

The coherence correlation matrix is defined by

$$\mathbf{R}_s = \frac{1}{N}\sum_{l=1}^{N}\mathbf{\Phi}(l)\mathbf{\Phi}^H(l)$$

$$= \begin{bmatrix} 1 & \beta_{12} & \cdots & \beta_{1M_t} \\ \beta_{21} & \beta_{22} & \cdots & \beta_{2M_t} \\ \cdots & \cdots & \cdots & \cdots \\ \beta_{M_{t1}} & \beta_{M_{t2}} & \cdots & 1 \end{bmatrix}$$

$$= \boldsymbol{I}_{M_t} + \boldsymbol{\beta}_{M_t} \tag{46}$$

where $\beta_{ij} = \frac{1}{N}\sum_{l=1}^{N} \Phi_i(l)\Phi_j^*(l)$ represents the complex correlation coefficient between the *i*th and *j*th waveform, $\boldsymbol{I}_{M_t}$ and, $\boldsymbol{\beta}_{M_t}$ denote the identity matrix and cross-correlation matrix, respectively. So, if the transmitted signals are ideally orthogonal, then $\beta_{ij} = 0$ and $\mathbf{R}_s = \boldsymbol{I}_{M_t}$. If the transmitted waveforms are fully coherent, then $\beta_{ij} = 1$ and $\mathbf{R}_s = \boldsymbol{I}_{M_t} + \boldsymbol{\beta}_{M_t}$. Because, there is a correlation between transmitter waveforms, the term $\mathbf{U}^H(\theta)\mathbf{U}(\theta)$ is written as

$$\mathbf{U}^H(\theta)\mathbf{U}(\theta) =$$

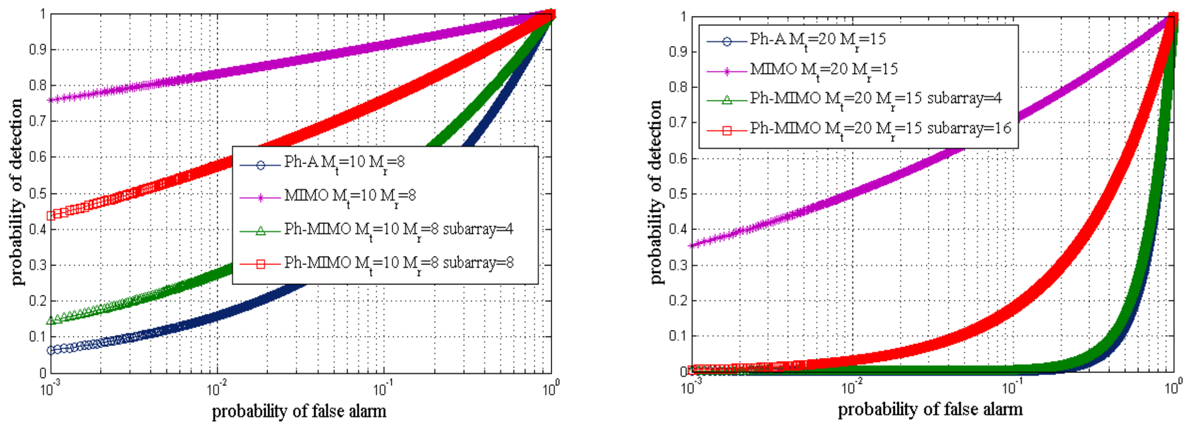
$$= \left[\sqrt{\frac{M_t}{K}}\beta_s \mathbf{u}^*(\theta_s)\right]\left[\sqrt{\frac{M_t}{K}}\beta_s \mathbf{u}(\theta_s)\right]$$

$$= \frac{M_t^2}{K} M_r N \beta_s^2 + \frac{M_t}{K} M_r N \beta_s^2 \sum_{i=1}^{M_t}\sum_{j=1,i\neq j}^{M_t} \mathbf{a}_t^*(i)\mathbf{a}_t(j)\beta_{ij} \tag{47}$$

Substituting (47) in (45), probability of detection can be obtained.

IV. SIMULATION RESULT

In this section, simulation results are represented to compare the performances of target detection in phased-MIMO radars, phased-array radars and co-located MIMO radars. First, it is assumed that transmitted waveforms are ideally orthogonal and secondly, transmitted waveforms are correlated (not fully coherent or ideally orthogonal). Numerical results shown in this section are obtained by 10,000 Monte Carlo simulation runs. It is assumed both transmit and receive arrays are uniform linear. Both arrays are half-wavelength inter-element spaced, the transmit and receive beams are both directed to 60º angle. Moreover, the variance of white Gaussian noise used in these simulations is equal to $\delta_n^2 = 1$, and all received amplitudes are real and $\xi = 0.1$ (ξ refers to the threshold and is equivalent to the desired probability of the false alarm). The number of radar pulses is 20 and the number of samples within one radar pulse is equal to 40.



(a) (b)

Fig. 2. Comparison of receiver-operating-characteristic (ROC) curves in GLRT detectors with $(a) M_t$=10, M_r=8 , and (b) M_t=20, M_r=15

A. *Target detection with ideally orthogonal waveforms*

1) *GLRT Detector*

In this case, the probability of the false alarm is on the interval 10^{-4} to 1. Fig. 2 shows that probability of detection in phased-MIMO radars is less than co-located MIMO radars and higher than phased-array radars with different number of subarrays. Increasing the number of subarrays results in improving target detection performance in the phased-MIMO radars.

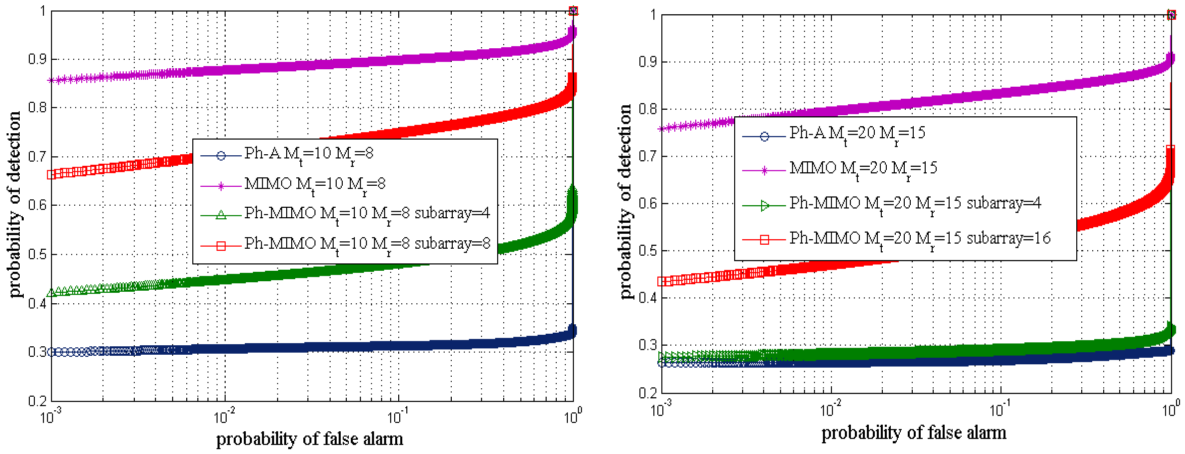
2) *LRT Detector in Neyman- Pearson Criterion*

In this case, the probability of the false alarm is on the interval 10^{-4} to 1. Fig. 3 shows that performance of target detection in phased-MIMO radars is between co-located MIMO radars and phased-array radars with different number of subarrays. Increasing the number of subarrays results in improving target detection performance in the phased-MIMO radars.

Comparing two detectors mentioned above, it can be seen that with the same probability of the false alarm, the probability of detection in LRT detectors is better than GLRT detectors in all three radars considered in this paper.

B. *Target detection with correlated waveforms*

In this case, the probability of the false alarm is assumed to be 10^{-4}, 10^{-5}, 10^{-6}, 10^{-7} and 10^{-8}. Figs. 4 present target detection performance for phased-MIMO radars transmitting correlated waveforms with cross-correlation coefficients being uniformly, given the fixed false alarm



(a) *(b)*

Fig. 3. Comparison of receiver-operating-characteristic (ROC) curves in LRT detector with $(a) M_t$=10, M_r=8, and (b) M_t=20, M_r=15

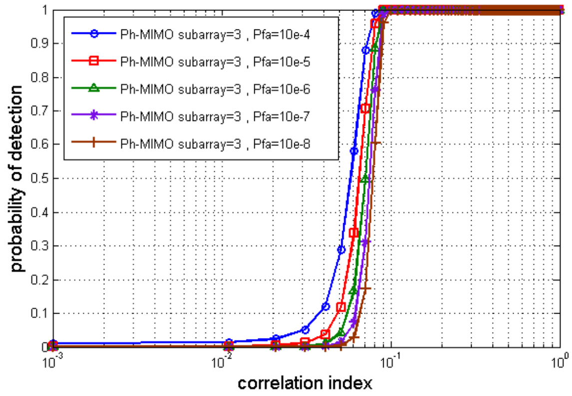
probabilities. Detection performance in the phased-MIMO radars grows with increasing the false alarm probability. It is assumed that the number of transmit and receive antennas equal 16 and 10 respectively.

Detection performance in the phased-MIMO radars grows with increasing the false alarm probability, increasing correlation coefficients leads to increasing target detection performance in the phased-MIMO radars, and also increasing number of subarrays leads to increasing target detection performance in the phased-MIMO radars, Fig. 4.

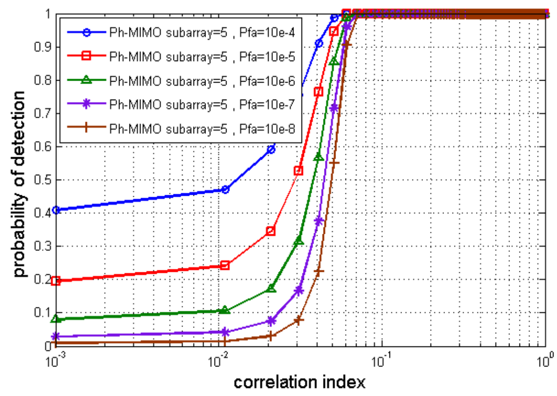
For example, if the number of subarrays equals the number of transmitter antennas, target detection performance of the phased-MIMO radars is similar to MIMO radars.

V. CONCLUSION

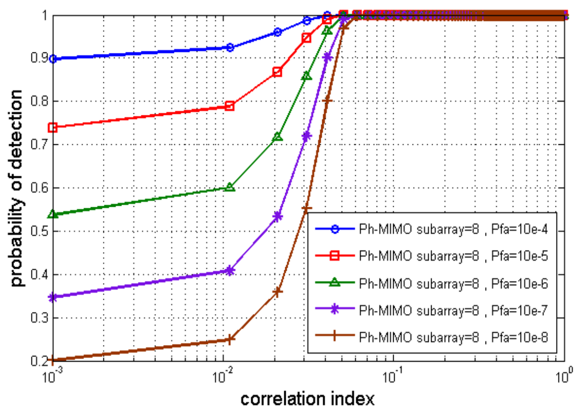
Phased-MIMO radars are recently introduced in the literature in order to improve parameter estimation capability of co-located MIMO radars. However, target detection performance of these radars has not been investigated yet. In this paper, the GLRT and LRT target detector are considered and target detection performance is analyzed in phased-MIMO radars transmitting fully orthogonal or correlated waveforms. Simulation results show that detection performance in the phased-MIMO radars is higher than the phased-array radars and lower than the MIMO radars with co-located antennas. Moreover, increasing the false alarm probability, correlation coefficients, and number of subarrays leads to growing target detection performance in the phased-MIMO radars. It can be seen that with the same probability of the false alarm, the probability of detection in LRT detectors is higher than GLRT detectors.



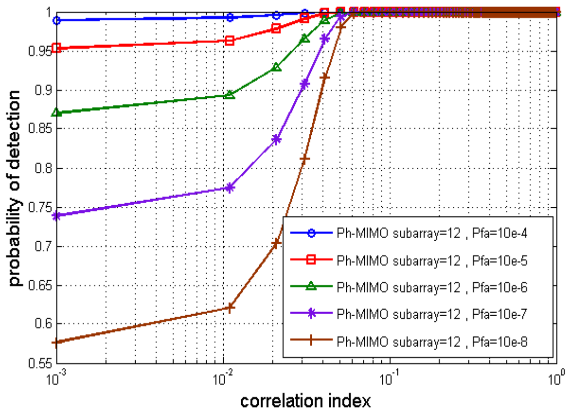
(a)



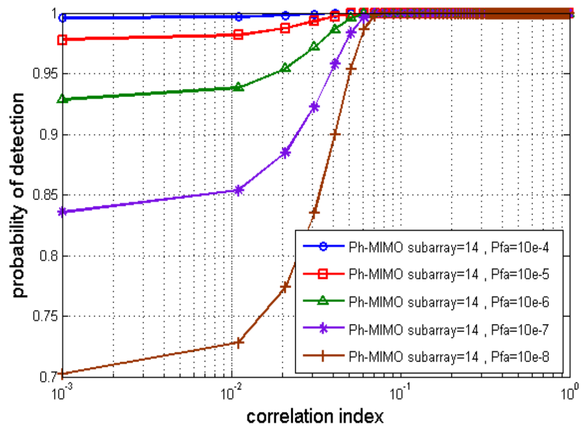
(b)



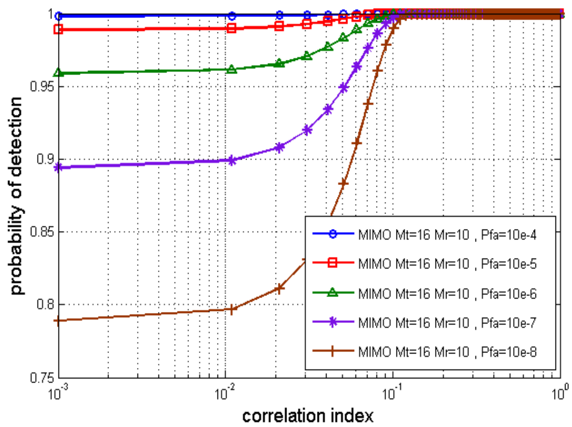
(c)



(d)



(e)



f)

Fig. 4. Probability of detection versus the correlation index in LRT detector for different number of subarrays in phased-MIMO radars: (a) with 3 subarrays (b) with 5 subarrays (c) with 8 subarrays (d) with 12 subarrays (e) with 14 subarrays f) with 16 subarrays.

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