Journal of Quality Engineering and Production Optimization

Vol. 3, No. 1, PP. 43-66, 2018

A Multi-Criteria Analysis Model under an Interval Type-2 Fuzzy Environment with an Application to Production Project Decision Problems

Yahya Dorfeshan[1], Seyed Meysam Mousavi[1 *], Behnam Vahdani[2]

1. *Department of Industrial Engineering, Faculty of Engineering, Shahed University, Tehran, Iran*

2. *Department of Industrial Engineering, Faculty of Industrial and Mechanical Engineering, Qazvin Branch, Islamic Azad University, Qazvin, Iran*

***Corresponding Author:** Seyed Meysam Mousavi (E-mail: sm.mousavi@shahed.ac.ir)*

***Abstract-** Using Multi-Criteria Decision-Making (MCDM) to solve complicated decisions often includes uncertainty, which could be tackled by utilizing the fuzzy sets theory. Type-2 fuzzy sets consider more uncertainty than type-1 fuzzy sets. These fuzzy sets provide more degrees of freedom to illustrate the uncertainty and fuzziness in real-world production projects. In this paper, a new multi-criteria analysis model is introduced based on new compromise ratio and relative preference relation methods by vicinity to positive ideal and distance from negative ideal concepts under an interval type-2 fuzzy environment. Also, qualitative criteria are expressed as linguistic variables. Relative preference relation is more reasonable than defuzzification, because defuzzification cannot provide preference degree between two fuzzy numbers and cannot keep all the information. In this paper, an extended relative preference relation over the average is presented to deal with numeral values. Finally, a real application to designing and manufacturing of small electronic components, particularly for the aviation, defense, and space industries, is adopted from the literature and solved to determine the critical path by considering efficient criteria such as time, cost, risk, and quality.*

***Keywords:** Multi-criteria decision-making (MCDM), Interval type-2 fuzzy sets (IT2FSs), Relative preference relation, Compromise ratio method, Production projects, Critical path selection problem*

I. INTRODUCTION

The aim of multi-criteria decision-making is to choose the best candidate from a set of alternatives by means of evaluating multiple criteria of the alternatives. In recent years, many extensions of MCDM methods have been presented to project management problems (Chen & Chen, 2003; Yager & Xu, 2006; Fu, 2008). VIKOR method is categorized as one of the multi-criteria decision making approaches. Vahdani et al. (2010) proposed a compromise solution method based on traditional VIKOR method and the interval-valued fuzzy concept, aiming at solving MCDM problems in which the weights of criteria were unequal. Cristobal (2011) presented VIKOR method for the selection of a renewable energy project in Spain. Tavana et al. (2016) presented an extended VIKOR method using stochastic data and subjective judgments.

In the classical MCDM approaches, ratings and weights of criteria are known precisely. In other words, the problem is considered under certain environment. In real conditions of production projects, the decision environment is not certain and it has vagueness and ambiguity. To address uncertainty, evaluation ratings and criteria weights in fuzzy MCDM (FMCDM) problems are expressed by imprecision and vagueness. Furthermore, experts and decision makers (DMs) can utilize linguistic variables by their own knowledge and experience. With this approach, they can have more realistic and reasonable judgments and feelings. Sanayei et al. (2010) used fuzzy VIKOR for supplier selection by group decision-making process. Shemshadi et al. (2011) presented a fuzzy VIKOR method for supplier selection based on entropy measure for objective weights. Yücenur and Demirel (2012) expressed an extension of VIKOR method under a fuzzy environment for group decision process in insurance company selection problem. Vahdani et al. (2010) developed an interval-valued fuzzy VIKOR (IVF-VIKOR) to solve MCDM problems, in which the performance rating values as well as the weights of criteria were linguistic terms that could be taken in interval-valued fuzzy numbers (IVFNs).

Manuscript Received:07 -March-2017 and revised: 20-June-2017 *ISSN: 2423-3781*
Accepted: 23-Sep-2017

The membership degree of type-1 sets is a crisp number between [0, 1]. However, we are usually encountered with a situation where it is difficult to distinguish the precise membership function for a fuzzy set. Type-1 fuzzy sets are not appropriate to use in such cases. To confront this problem, Zadeh (1975) suggested type-2 fuzzy sets (T2FSs), which were the extension of type-1 fuzzy sets. T2FSs are qualified by means of both primary and secondary memberships to present more degrees of freedom and flexibility, and they are regarded to be three-dimensional. T2FSs have the merit of modeling uncertainty more accurately than type-1 fuzzy sets. IT2FSs can be viewed as a special case of general T2FSs in which all the values of secondary membership are equal to 1. Accordingly, it not only demonstrates uncertainty better than type-1 fuzzy sets do, but also simplifies the computation compared with T2FSs. Chen and Lee (2010) presented fuzzy multiple attributes group decision-making based on the ranking values and the arithmetic operations of IT2FSs; also, Chen and Wang (2013) devised a fuzzy decision making system based on IT2FSs. Furthermore, Han et al. (2016) introduced a group MCDM problem under triangular T2F numbers.

Defuzzification or fuzzy generalization has been utilized to generalize classical MCDM methods under fuzzy environments to solve FMCDM problems. In fact, defuzzification causes loss of fuzzy messages. Fuzzy preference relation is a new approach to resolve the above consideration, because it satisfies a total ordering relation for fuzzy numbers and is more reasonable than defuzzification in ranking fuzzy numbers. In other words, defuzzification cannot present preference degree between two fuzzy numbers and loses some messages of fuzziness (Wang, 2015a). Relative preference relation has been used in several applications. For instance, Wang (2014) expressed a criteria-weighting approach that combined the concepts of fuzzy quality function deployment and relative preference relation. With the relative preference relation for fuzzy numbers, it was not necessary to multiply two fuzzy numbers to derive criteria weights in fuzzy quality function deployment. Alternatively, adjusted criteria weights would substitute for original criteria weights through relative preference relation (Wang, 2014). Additionally, Wang (2015b) developed a fuzzy MCDM model based on simple additive weighting method and relative preference relation. FMCDM has widely been applied in the related studies to project management; however, given the advantages of the relative preference relation, this concept is still new in this field and has not been properly applied.

In regard to the new research on fuzzy production project management, Lin and Yao (2003) presented a fuzzy critical path method based on signed-distance ranking and statistical confidence-interval estimates. Lacouture (2009) presented construction project scheduling with time, cost, and material restrictions using fuzzy mathematical model and critical path method. Khalaf (2013) introduced fuzzy project scheduling based on a ranking function and applied the method to Al-SAMA project. Madhuri and Chandan (2016) applied a fuzzy critical path method to manufacturing tugboat, in which linear programing model was used for determining critical path. Mehlawat and Gupta (2016) presented a new MCDM method based on fuzzy preference relation to specify critical path for their case study, which was designing and manufacturing of small electronic components, particularly for the aviation, defense, and space industries. Mohagheghi et al. (2016) applied type-2 fuzzy sets to address R&D project portfolio selection. Mohagheghi et al. (2017) developed type-2 fuzzy sets to address high and new technology project evaluation and selection.

In this paper, in order to improve the existing literature on FMCDM methods, a method is developed under IT2F environment to address more uncertainty in real-world production projects. In other words, the rating of each activity and the weight of each criterion are described by IT2FSs. Furthermore, defuzzification has several disadvantages; in fact, not only it cannot present preference degree between two fuzzy numbers, but also it loses some messages of fuzziness. In this paper, relative preference relation is presented to deal with the above-mentioned disadvantage of defuzzification. In fact, in order to better address the uncertainty in ratings and weights of criteria in real-world projects, type-2 fuzzy sets are used. Memberships of type-1 fuzzy set are crisp numbers, whereas the memberships of type-2 fuzzy sets are type-1 fuzzy sets. Nevertheless, type-2 fuzzy sets provide additional degrees of freedom to tackle uncertainty. Moreover, MCDM approach is used that presents a new compromise solution method by a group of experts or DMs to effectively solve the evaluation and selection problems under a fuzzy environment. This method develops an appropriate solution that helps the DMs to reach an acceptable compromise between the maximum group utility of the majority and the minimum individual regret of the opponent. In other words, this MCDM method provides more degrees of freedom for the DM in project management decisions. Also, in this paper, relative preference relation is developed under type-2 fuzzy sets and applied to weights of criteria to avoid multiplying two fuzzy matrices. As a matter of fact, by means of relative preference relation, the computation is done easier and faster. Table I shows the priorities of the proposed method in comparison with the recent research.

The novelties of this paper are the following:

- To properly address the existing uncertainty in production project environments, IT2FSs are applied. This provides more flexibility and ability in expressing uncertainty and addressing vague decision making processes.
- A new extension of compromise ratio method under IT2FS uncertainty is expressed. This approach provides decision-making problems with better ability in expressing uncertainty.
- To avoid disadvantages of defuzzification, such as loss of information, a new version of relative preference relation is presented to determine weights of criteria.

The paper proceeds as follows. Section II discusses the preliminary information and definitions of the applied methods and tools, in addition to the developed IT2FSs based preference relation method. Section III introduces the proposed method. Section IV presents a case study from the literature to illustrate applicability of the proposed method. In Section V, sensitivity analysis is provided and, finally, Section VI concludes the study.

II. PRELIMINARIES

A. Type-2 fuzzy sets

Type-2 fuzzy sets have a measure of dispersion, which depicts inherent uncertainties. These sets are especially useful in uncertain situations where presenting the exact membership function of a fuzzy set is very difficult (Mendel, 2007). General T2FSs are computationally intensive and this feature has made the application of IT2FSs more common (Kilic & Kaya, 2015). In this section, the required basic knowledge of interval type-2 fuzzy sets is presented.

TABLE I. Literature review

Researcher	**Year**					**Uncertainty approach**		**Modeling approach**					**Weighting approach**	
		Time	**Cost**	**Risk**	**Quality**	**Classic fuzzy sets**	**Interval type-2 fuzzy sets**	**Group decision making**	**Compromise ratio**	**New MCDM method based on preference relation**	**Mathematical modeling**	**TOPSIS**	**Linguistic variable**	**Relative preference relation**
Lin and Yao	2003	✓				✓					✓			
Zammori et al.	2009	✓	✓	✓		✓					✓	✓	✓	
Lacouture et al.	2009	✓	✓			✓								
Amiri and Golozari	2011	✓	✓	✓	✓	✓						✓	✓	
Khalaf	2013	✓				✓					✓			
Madhuri and Chandan	2016	✓				✓					✓			
Mehlawat and Gupta	2016	✓	✓	✓	✓	✓		✓		✓			✓	
Proposed model		✓	✓	✓	✓		✓	✓	✓				✓	✓

$$\tilde{\tilde{A}} = \int_{x\in X} \int_{u\in j_X} \mu_{\tilde{\tilde{A}}}(x,u)/(x,u) \tag{1}$$

where $J_X \subseteq [0,1]$ and $\iint$ denote union over all admissible *x* and *u*. An interval type-2 fuzzy set $\tilde{\tilde{A}}$ is a special case of a type-2 fuzzy set as given in Eq. (2):

$$\tilde{\tilde{A}} = \int_{x \in X} \int_{u \in j_X} 1/(x,u) \tag{2}$$

where $J_X \subseteq [0,1]$.

The upper and lower membership functions of an interval type-2 fuzzy set are type-1 membership functions (Mendel et al., 2006).

A trapezoidal IT2FS is illustrated as $\tilde{\tilde{A}}_i = (\tilde{A}_i^U, \tilde{A}_i^L) = ((a_{i1}^u, a_{i2}^u, a_{i3}^u, a_{i4}^u; H_1(\tilde{A}_i^U), H_2(\tilde{A}_i^U)),$ $(a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L; H_1(\tilde{A}_i^L), H_2(\tilde{A}_i^L)))$ where $\tilde{A}_i^L$ and $\tilde{A}_i^U$ are in fact type-1 fuzzy sets. $a_{i1}^u, a_{i2}^u, a_{i3}^u, a_{i4}^u, a_{i1}^L, a_{i2}^L, a_{i3}^L$ and a_{i4}^L are the reference points of the interval type-2 fuzzy set $\tilde{\tilde{A}}_i$, $H_j(\tilde{A}_i)$ denotes the membership value of the element $a_{j(j+1)}^U$ in the upper trapezoidal membership function $\tilde{A}_i^U, 1 \le j \le 2, H_j(A_i^L)$ denotes the membership value of the element $a_{j(j+1)}^L$ in the lower trapezoidal membership function $\tilde{A}_i^L, 1 \le j \le 2, H_1(\tilde{A}_i^U) \in [0,1], H_2(\tilde{A}_i^U) \in [0,1], H_1(\tilde{A}_i^L) \in [0,1], H_2(\tilde{A}_i^L) \in [0,1]$ and $1 \le i \le n$ (Chen & Lee, 2010). The membership function of a trapezoidal IT2FS is depicted in Fig (1).

Let A_1 and A_2 be two trapezoidal interval type-2 fuzzy numbers:

$$A_1 = (A_1^U, A_1^L) = \left(\left(a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; H_1(A_1^U), H_2(A_1^U)\right), \left(a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L; H_1(A_1^L), H_2(A_1^L)\right)\right) \tag{3}$$

$$A_2 = (A_2^U, A_2^L) = \left(\left(a_{21}^U, a_{22}^U, a_{23}^U, a_{24}^U; H_1(A_2^U), H_2(A_2^U)\right), \left(a_{21}^L, a_{22}^L, a_{23}^L, a_{24}^L; H_1(A_2^L), H_2(A_2^L)\right)\right) \tag{4}$$

The addition operation between them is defined as follows (Hu et al., 2013).

$$\begin{aligned} A_1 \oplus A_2 = {} & (A_1^U, A_1^L) + (A_2^U, A_2^L) \\ = {} & \Big[\Big(a_{11}^U + a_{21}^U, a_{12}^U + a_{22}^U, a_{13}^U + a_{23}^U, a_{14}^U + a_{24}^U; H_1(\tilde{A}_1^U) + H_1(\tilde{A}_2^U) \\ & - H_1(\tilde{A}_1^U).H_1(\tilde{A}_2^U), H_1(\tilde{A}_1^U) + H_2(\tilde{A}_2^U) - H_2(\tilde{A}_1^U).H_2(\tilde{A}_2^U)\Big), \Big(a_{11}^L + a_{21}^L, a_{12}^L \\ & + a_{22}^L, a_{13}^L + a_{23}^L, a_{14}^L + a_{24}^L; H_1(\tilde{A}_1^L) + H_1(\tilde{A}_2^L) - H_1(\tilde{A}_1^L).H_1(\tilde{A}_2^L), H_1(\tilde{A}_1^L) \\ & + H_2(\tilde{A}_2^L) - H_2(\tilde{A}_1^L).H_2(\tilde{A}_2^L)\Big)\Big] \end{aligned} \tag{5}$$

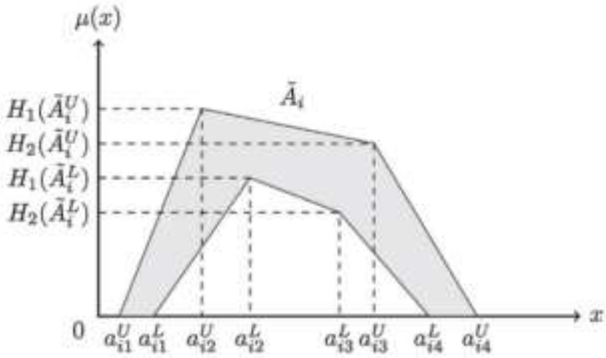

Fig 1. Membership function of a trapezoidal interval type-2 fuzzy set

The subtraction operation is defined as follows (Hu et al., 2013):

$$A_1 \ominus A_2 = (A_1^U, A_1^L) - (A_2^U, A_2^L) = \Big[\Big(a_{11}^U - a_{24}^U, a_{12}^U - a_{23}^U, a_{13}^U - a_{22}^U, a_{14}^U - a_{21}^U; H_1(\tilde{A}_1^U) + H_1(\tilde{A}_2^U) - H_1(\tilde{A}_1^U).H_1(\tilde{A}_2^U), H_2(\tilde{A}_1^U) + H_2(\tilde{A}_2^U) - H_2(\tilde{A}_1^U).H_2(\tilde{A}_2^U)\Big), \Big(a_{11}^L - a_{24}^L, a_{12}^L - a_{23}^L, a_{13}^L - a_{22}^L, a_{14}^L - a_{21}^L; H_1(\tilde{A}_1^L) + H_1(\tilde{A}_2^L) - H_1(\tilde{A}_1^L).H_1(\tilde{A}_2^L), H_2(\tilde{A}_1^L) + H_2(\tilde{A}_2^L) - H_2(\tilde{A}_1^L).H_2(\tilde{A}_2^L)\Big)\Big] \quad (6)$$

The multiplication operation is defined as follows (Hu et al., 2013):

$$A_1 \otimes A_2 = (A_1^U, A_1^L) \otimes (A_2^U, A_2^L) = \begin{pmatrix} \left(x_{11}^U, x_{12}^U, x_{13}^U, x_{14}^U; H_1(A_1^U).H_1(A_2^U), H_2(A_1^U).H_2(A_2^U)\right), \\ \left(x_{11}^L, x_{12}^L, x_{13}^L, x_{14}^L; H_1(A_1^L).H_1(A_2^L), H_2(A_1^L).H_2(A_2^L)\right) \end{pmatrix} \quad (7)$$

Where $x_{1i}^T = \min\left(a_{1i}^T a_{2i}^T, a_{1i}^T a_{2(5-i)}^T, a_{1(5-i)}^T a_{2i}^T, a_{1(5-i)}^T a_{2(5-i)}^T\right), T \in \{U, L\}, i \epsilon \{1,2\}$

$x_{1j}^T = \max\left(a_{1(5-j)}^T a_{2(5-j)}^T, a_{1(5-j)}^T a_{2j}^T, a_{1j}^T a_{2(5-j)}^T, a_{1j}^T a_{2j}^T\right), T \in \{U, L\}, j \epsilon \{3,4\}$

The multiplication operation between a crisp value (λ) and A_1 is defined as follows (Hu et al., 2013):

$$\lambda A_1 = \left[\left((\lambda a_{11}^U, \lambda a_{12}^U, \lambda a_{13}^U, \lambda a_{14}^U);\ 1 - \left(1 - H_1(\tilde{A}_1^U)\right)^{\lambda}, 1 - \left(1 - H_2(\tilde{A}_1^U)\right)^{\lambda}\right), \left((\lambda a_{11}^L, \lambda a_{12}^L, \lambda a_{13}^L, \lambda a_{14}^L);\ 1 - \left(1 - H_1(\tilde{A}_1^L)\right)^{\lambda}, 1 - \left(1 - H_2(\tilde{A}_1^L)\right)^{\lambda}\right)\right] \quad (8)$$

Division by an ordinary nonzero number (k) is defined as follows:

$if\ k > 0$

$$A_1/k = \left((a_{11}^U/k, a_{12}^U/k, a_{13}^U/k, a_{14}^U/k); H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U), (a_{11}^L/k, a_{12}^L/k, a_{13}^L/k, a_{14}^L/k); H_1(\tilde{A}_1^L), H_2(\tilde{A}_1^L)\right) \quad (9)$$

$if\ k < 0$

$$A_1/k = \left((a_{14}^U/k, a_{13}^U/k, a_{12}^U/k, a_{11}^U/k); H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U), (a_{14}^L/k, a_{13}^L/k, a_{12}^L/k, a_{11}^L/k); H_1(\tilde{A}_1^L), H_2(\tilde{A}_1^L)\right) \quad (10)$$

The division operation is depicted as follows (Chen & Lee, 2010):

$$A_1 \oslash A_2 = (A_1^U, A_1^L) \oslash (A_2^U, A_2^L) = \begin{pmatrix} \left(Y_{11}^U, Y_{12}^U, Y_{13}^U, Y_{14}^U; H_1(A_1^U).H_1(A_2^U), H_2(A_1^U).H_2(A_2^U)\right), \\ \left(Y_{11}^L, Y_{12}^L, Y_{13}^L, Y_{14}^L; H_1(A_1^L).H_1(A_2^L), H_2(A_1^L).H_2(A_2^L)\right) \end{pmatrix} \quad (11)$$

Where $Y_{1i}^T = \min\left(a_{1i}^T/a_{2i}^T, a_{1i}^T/a_{2(5-i)}^T, a_{1(5-i)}^T/a_{2i}^T, a_{1(5-i)}^T/a_{2(5-i)}^T\right), T \in \{U, L\}, i \epsilon \{1,2\}$

$Y_{1j}^T = \min\left(a_{1(5-j)}^T/a_{2(5-j)}^T, a_{1(5-j)}^T/a_{2j}^T, a_{1j}^T/a_{2(5-j)}^T, a_{1j}^T/a_{2j}^T\right), T \in \{U, L\}, j \epsilon \{3,4\}$

B. Relative preference relation for fuzzy numbers

In the previous studies, fuzzy ranking methods have commonly been classified into two categories. The first category is based on defuzzification, in which fuzzy numbers are defuzzified into crisp numbers. Various methods of defuzzification have been proposed. Then, the ranking is done based on the crisp numbers. Although it is easy to compute, the main drawback of this group is that defuzzification tends to lose some information and, thus, is unable to grasp the sense of uncertainty. The other category is based on fuzzy preference relation. The advantage of this group is that uncertainties of fuzzy numbers are kept during ranking process. Yuan (1991) proposed criteria for measuring ranking method. Lee (2001) proposed a new fuzzy ranking method based on fuzzy preference relation satisfying all the criteria proposed by Yuan (1991). Also, Lee (2005) introduced a fuzzy multi-criteria decision making model for the selection of distribution centers by means of fuzzy preference relation.

However, fuzzy pairwise comparison by preference relation is complex and difficult. To avoid the above shortcomings, the relative preference relation adopts the strengths of defuzzification and fuzzy preference relation. Notably, the relative preference relation expresses preference degrees of several fuzzy numbers over average same as the fuzzy preference relation does, and ranks fuzzy numbers by relative crisp values as defuzzification does. Thus, utilizing the relative preference relation ranks fuzzy numbers easily and quickly, and it is able to reserve fuzzy information (Wang, 2015a).

To present relative preference relation for fuzzy numbers, related definitions are provided as follows (Zadeh, 1965; Zimmermann, 1991).

Definition 1. A fuzzy preference relation *R* is a fuzzy subset of *R*R* with membership function $\mu_p(A,B)$ representing preference degree of fuzzy number *A* over *B*.

(1) *R* is reciprocal if $\mu_R(A,B)=1-\mu_R(B,A)$ for all fuzzy numbers *A* and *B*.

(2) *R* is transitive if $\mu_R(A,B)\geq(1/2)$ and $\mu_R(B,C)\geq(1/2)\Rightarrow\mu_R(A,C)\geq(1/2)$ for all fuzzy numbers *A*, *B*, and *C*.

(3) *R* is a fuzzy total ordering relation if *R* is both reciprocal and transitive.

Additionally, *A* is preferred to *B* if $\mu_R(A,B)\geq(1/2)$ and *A* is equal to *B* if $\mu_R(A,B)=(1/2)$.

Definition 2. Let *A* and *B* be two fuzzy numbers, where *A* is in an interval [a_l, a_r] and *B* is in an interval [b_l, b_r]. According to Wang & Lee (2010) and Lee (2005), a fuzzy preference relation is a fuzzy subset of *R*R* with membership function $\mu_p(A,B)$ representing preference degree of *A* over *B*. Let μ_p be defined as follows:

$$\mu_P(A,B)=\frac{1}{2}\left(\frac{\int_0^1((A-B)_\alpha^L+(A-B)_\alpha^U)d\alpha}{\|T\|}+1\right) \tag{12}$$

where

$$\|T\|=\begin{cases}\int_0^1((T^+-T^-)_\alpha^L+(T^+-T^-)_\alpha^U)d\alpha & if\ \ t_l^+\geq t_r^- \\ \int_0^1((T^+-T^-)_\alpha^L+(T^+-T^-)_\alpha^U)+2(t_r^--t_l^+) & if\ \ t_l^+\prec t_r^-\end{cases} \tag{13}$$

T^+ is in an interval $[t_l^+,t_r^+]$, T^- is in an interval $[t_l^-,t_r^-]$, and

$$t_l^+=\max\{a_l,b_l\},t_r^+=\max\{a_r,b_r\},t_l^-=\min\{a_l,b_l\},t_r^-=\min\{a_r,b_r\} \tag{14}$$

Definition 3. Let $S=\{X_1,X_2,...,X_n\}$ indicate a set consisting of *n* trapezoidal fuzzy numbers in case $X_j=(x_{jl},x_{jh},x_{jg},x_{ju})\in S$, where *j*=1, 2, ...,*n*. By extension principle, $\bar{X}=(1/n)\otimes(X_1,X_2,...,X_n)$

$=(\bar{x}_l,\bar{x}_h,\bar{x}_g,\bar{x}_u)$ is assumed to be the average of $X_1,X_2,...,X_n$. Then, the relative preference relation p^* is defined with membership function $\mu_{p^*}(X_j,\bar{X})$ representing relative preference degree of X_j over $\bar{X}$ in *S*, where $j=1,2,...,n$ (Wang, 2014):

$$\mu_{p^*}(X_j,\bar{X})=\frac{1}{2}\left(\frac{(x_{jl}-\bar{x}_u)+(x_{jh}-\bar{x}_g)+(x_{jg}-\bar{x}_h)+(x_{ju}-\bar{x}_l)}{2\|T_S\|}+1\right) \tag{15}$$

where

$$\|T_S\|=\begin{cases}\dfrac{(t_{sl}^+-t_{su}^-)+(t_{sh}^+-t_{sg}^-)+(t_{sg}^+-t_{sh}^-)+(t_{su}^+-t_{sl}^-)}{2} & if\ \ t_{sl}^+\geq t_{su}^- \\ \dfrac{(t_{sl}^+-t_{su}^-)+(t_{sh}^+-t_{sg}^-)+(t_{sg}^+-t_{sh}^-)+(t_{su}^+-t_{sl}^-)}{2}+2(t_{su}^- - t_{sl}^+) & if\ \ t_{sl}^+\prec t_{su}^-\end{cases} \tag{16}$$

$$\begin{aligned}&t_{sl}^+=\max_j\{x_{jl}\},t_{sh}^+=\max_j\{x_{jh}\},t_{sg}^+=\max_j\{x_{jg}\},t_{su}^+=\max_j\{x_{ju}\},t_{sl}^-=\min_j\{x_{jl}\},\\&t_{sh}^-=\min_j\{x_{jh}\},t_{sg}^-=\min_j\{x_{jg}\},t_{su}^-=\min_j\{x_{ju}\},\quad j=1,2,...,n.\end{aligned} \tag{17}$$

III. PROPOSED NEW MULTI-CRITERIA ANALYSIS MODEL

In this paper, IT2FSs is used to address uncertainty to solve complicated decision making. Also, relative preference relation has several advantages over defuzzification, which is why relative preference relation is developed under IT2FSs to deal with the uncertainty in real-world production projects. Furthermore, the compromise ratio method for fuzzy group decision-making problems is developed under IT2FSs to address uncertainty. This approach provides better ability in expressing uncertainty. Furthermore, this model is applied to determine critical path for production projects by considering different and conflicting criteria such as time, cost, risk, and quality. This paper focuses on a case study from literature (Mehlawat & Gupta, 2016), which is related to designing and manufacturing of small electronic components, particularly for the aviation, defense, and space industries. Fig (2) illustrates the proposed framework.

Step 1: Form a team of production experts who are responsible to determine the best alternative considering the evaluating criteria. Also, experts' judgments in terms of each criterion are gathered as linguistic variables and, finally, experts' judgments on qualitative criteria and weights are converted to their equivalent IT2FNs presented in Table II.

TABLE II. Linguistic terms and their corresponding interval type-2 fussy sets (Chen & Lee, 2010)

Linguistic terms	Interval type-2 fuzzy sets
Very Low (VL)	((0,0,0,0.1;1,1),(0,0,0,0.05;0.9,0.9))
Low (L)	((0,0.1,0.1,0.3;1,1),(0.05,0.1,0.1,0.2;0.9,0.9))
Medium Low (ML)	((0.1,0.3,0.3,0.5;1,1),(0.2,0.3,0.3,0.4;0.9,0.9))
Medium (M)	((0.3,0.5,0.5,0.7;1,1),(0.4,0.5,0.5,0.6;0.9,0.9))
Medium High (MH)	((0.5,0.7,0.7,0.9;1,1),(0.6,0.7,0.7,0.8;0.9,0.9))
High (H)	((0.7,0.9,0.9,1;1,1),(0.8,0.9,0.9,0.95;0.9,0.9))
Very High (VH)	((0.9,1,1,1;1,1),(0.95,1,1,1;0.9,0.9))

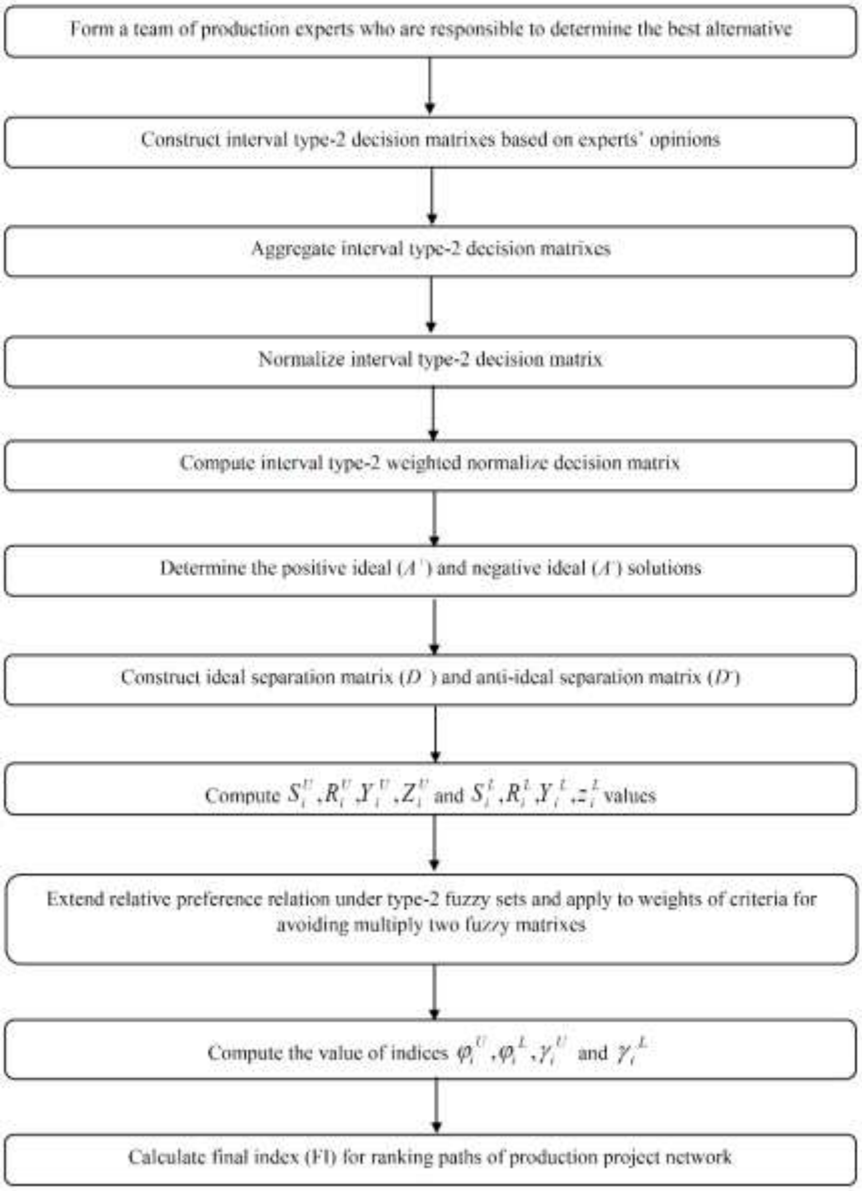

Fig. 2. Proposed framework

Step 2: Construct the decision matrix Y_p of the pth decision maker and the average decision matrix $\overline{Y}$, respectively. The aggregated fuzzy information on the alternative for each criterion is obtained via Eq. (18). Note that each alternative is a possible path for the production project network.

Integration method is as follows:

$$Y_P = (\tilde{\tilde{f}}_{ij}^{\,p})_{m\times n} = \begin{array}{c} \\ Path\,1 \\ \vdots \\ Path\;m \end{array} \begin{array}{c} \begin{array}{cccc} time & \cos t & risk & quality \end{array} \\ \begin{pmatrix} \tilde{\tilde{f}}_{1,time}^{\,p} & \tilde{\tilde{f}}_{1,\cos t}^{\,p} & \tilde{\tilde{f}}_{1,risk}^{\,p} & \tilde{\tilde{f}}_{1,quality}^{\,p} \\ \vdots & & \ddots & \vdots \\ \tilde{\tilde{f}}_{m,time}^{\,p} & \tilde{\tilde{f}}_{m,\cos t}^{\,p} & \tilde{\tilde{f}}_{m,risk}^{\,p} & \tilde{\tilde{f}}_{m,quality}^{\,p} \end{pmatrix} \end{array} \quad (18)$$

$$\bar{Y} = (\tilde{\tilde{f}}_{ij})_{m\times n}$$

and

$$w_p = (\tilde{\tilde{w}}_j^{\,p})_{1\times n} = \left(\tilde{\tilde{w}}_1^{\,p}, \tilde{\tilde{w}}_2^{\,p}, \dots, \tilde{\tilde{w}}_n^{\,p}\right)$$

$$\bar{w} = (\tilde{\tilde{w}}_j)_{1\times n} \tag{19}$$

where $\tilde{\tilde{w}}_j = (\frac{\tilde{\tilde{w}}_j^{\,1} \oplus \tilde{\tilde{w}}_j^{\,2} \oplus \dots \oplus \tilde{\tilde{w}}_j^{\,k}}{k})$, $\tilde{\tilde{w}}_j$ is an IT2FS, and $1 \le j \le n, 1 \le p \le k$.

$\tilde{\tilde{f}}_{ij} = (\frac{\tilde{\tilde{f}}_{ij}^{\,1} \oplus \tilde{\tilde{f}}_{ij}^{\,2} \oplus \dots \oplus \tilde{\tilde{f}}_{ij}^{\,k}}{k})$; $\tilde{\tilde{f}}_{ij}$ is an IT2FS and $1 \le i \le m, 1 \le j \le n, 1 \le p \le k$. Also, k denotes the number of decision makers; $A_1, A_2, \dots, A_m$ are possible paths; and $C_1, C_2, \dots, C_n$ are evaluation criteria.

Step 3: Compute the normalized IT2F decision matrix. The data are normalized using the following:

$$\tilde{\tilde{N}}_{ij} = \begin{bmatrix} (\frac{a_{ij1}^u}{d_j}, \frac{a_{ij2}^u}{d_j}, \frac{a_{ij3}^u}{d_j}, \frac{a_{ij4}^u}{d_j}; H_1(\tilde{A}_i^u), H_2(\tilde{A}_i^u)), \\ (\frac{a_{ij1}^L}{d_j}, \frac{a_{ij2}^L}{d_j}, \frac{a_{ij3}^L}{d_j}, \frac{a_{ij4}^L}{d_j}; H_1(\tilde{A}_i^L), H_2(\tilde{A}_i^L)) \end{bmatrix} \tag{20}$$

$$j = 1, 2, \dots, n, i = 1, 2, \dots, m$$

Where

$$d_j = \sqrt{\sum_{i=1}^{m}\sum_{p=1}^{4}(a_{ijp}^L)^2 + \sum_{i=1}^{m}\sum_{p=1}^{4}(a_{ijp}^u)^2},$$

$$p = \{1, 2, 3, 4\} \;\; for \;\; \forall j = 1, 2, \dots, n \tag{21}$$

Step 4: Construct the IT2F weighted normalized decision matrix. The fuzzy IT2F weighted normalized decision matrix is computed by multiplying each column of the matrix by the fuzzy weights of time, cost, risk, and quality criteria as follows:

$$\tilde{\tilde{V}}_{ij} = \tilde{\tilde{N}}_{ij} \otimes \tilde{\tilde{W}}_j = \begin{bmatrix} (v_{11}^U, v_{12}^U, v_{13}^U, v_{14}^U; H_1(\tilde{N}_{ij}^U).H_1(\tilde{W}_{ij}^U), H_2(\tilde{N}_j^U).H_2(\tilde{W}_j^U), \\ (v_{11}^L, v_{12}^L, v_{13}^L, v_{14}^L; H_1(\tilde{N}_{ij}^L).H_1(\tilde{W}_{ij}^L), H_2(\tilde{N}_j^L).H_2(\tilde{W}_j^L)) \end{bmatrix}$$

$$v_{1i}^T = \min(a_{1i}^T a_{2i}^T, a_{1i}^T a_{2(5-i)}^T, a_{1(5-i)}^T a_{2i}^T, a_{1(5-i)}^T a_{2(5-i)}^T), T \in \{U, L\}, i \in \{1, 2\} \tag{22}$$

$$v_{1j}^T = \max(a_{1(5-j)}^T a_{2(5-j)}^T, a_{1(5-j)}^T a_{2j}^T, a_{1j}^T a_{2(5-j)}^T, a_{1j}^T a_{2j}^T), T \in \{U, L\}, j \in \{3, 4\}$$

Step 5: Determine the positive ideal (A^+) and negative ideal (A^-) solutions. In this case, time, cost, and risk criteria are considered as benefit criteria and quality as cost criterion based on the basic definition of critical path, i.e., critical path is the path with the maximum time, in which one unit of delay in each activity causes one unit of delay in the completion time of the project.

$$\begin{aligned}
A^{+U} &= (\tilde{\tilde{V}}_1^{+U}, \tilde{\tilde{V}}_2^{+U}, \tilde{\tilde{V}}_3^{+U}, ..., \tilde{\tilde{V}}_n^{+U}) = \left\{ (\max_i \tilde{\tilde{V}}_{ij}^{U} \left| j \in J \right.), (\min_i \tilde{\tilde{V}}_{ij}^{U} \left| j \in J' \right.) \right\} \\
A^{+L} &= (\tilde{\tilde{V}}_1^{+L}, \tilde{\tilde{V}}_2^{+L}, \tilde{\tilde{V}}_3^{+L}, ..., \tilde{\tilde{V}}_n^{+L}) = \left\{ (\max_i \tilde{\tilde{V}}_{ij}^{L} \left| j \in J \right.), (\min_i \tilde{\tilde{V}}_{ij}^{L} \left| j \in J' \right.) \right\} \\
A^{-U} &= (\tilde{\tilde{V}}_1^{-U}, \tilde{\tilde{V}}_2^{-U}, \tilde{\tilde{V}}_3^{-U}, ..., \tilde{\tilde{V}}_n^{-U}) = \left\{ (\min \tilde{\tilde{V}}_{ij}^{U} \left| j \in J \right.), (\max_i \tilde{\tilde{V}}_{ij}^{U} \left| j \in J' \right.) \right\} \\
A^{-L} &= (\tilde{\tilde{V}}_1^{-L}, \tilde{\tilde{V}}_2^{-L}, \tilde{\tilde{V}}_3^{-L}, ..., \tilde{\tilde{V}}_n^{-L}) = \left\{ (\min \tilde{\tilde{V}}_{ij}^{L} \left| j \in J \right.), (\max_i \tilde{\tilde{V}}_{ij}^{L} \left| j \in J' \right.) \right\}
\end{aligned} \tag{23}$$

where

$$\begin{aligned}
\max_i \tilde{\tilde{V}}_{ij}^{U} &= (\max_i v_{ij1}^{U}, \max_i v_{ij2}^{U}, \max_i v_{ij3}^{U}, \max_i v_{ij3}^{U}) \\
\max_i \tilde{\tilde{V}}_{ij}^{L} &= (\max_i v_{ij1}^{L}, \max_i v_{ij2}^{L}, \max_i v_{ij3}^{L}, \max_i v_{ij3}^{L}) \\
\min_i \tilde{\tilde{V}}_{ij}^{U} &= (\min_i v_{ij1}^{U}, \min_i v_{ij2}^{U}, \min_i v_{ij3}^{U}, \min_i v_{ij3}^{U}) \\
\min_i \tilde{\tilde{V}}_{ij}^{L} &= (\min_i v_{ij1}^{L}, \min_i v_{ij2}^{L}, \min_i v_{ij3}^{L}, \min_i v_{ij3}^{L})
\end{aligned} \tag{24}$$

$J = 1, 2, ..., n \in$ *benefit criteria*

$J' = 1, 2, ..., n \in$ cos*t criteria*

Obviously, A^{+} indicates the most preferable alternative or the ideal solution and A^{-} illustrates the least preferable alternative or the anti-ideal solution.

Step 6: Construct ideal separation matrix (D^{+}) and anti-ideal separation matrix (D^{-}), which are introduced as follows:

$$D^{+U} = \left[d_{ij}^{+U} \right] = \begin{bmatrix} \left| \tilde{\tilde{v}}_{11}^{U} - \tilde{\tilde{v}}_{1}^{+U} \right| & \left| \tilde{\tilde{v}}_{12}^{U} - \tilde{\tilde{v}}_{2}^{+U} \right| & \cdots & \left| \tilde{\tilde{v}}_{1n}^{U} - \tilde{\tilde{v}}_{n}^{+U} \right| \\ \left| \tilde{\tilde{v}}_{21}^{U} - \tilde{\tilde{v}}_{1}^{+U} \right| & \left| \tilde{\tilde{v}}_{22}^{U} - \tilde{\tilde{v}}_{2}^{+U} \right| & \cdots & \left| \tilde{\tilde{v}}_{2n}^{U} - \tilde{\tilde{v}}_{n}^{+U} \right| \\ \vdots & \vdots & \ddots & \vdots \\ \left| \tilde{\tilde{v}}_{m1}^{U} - \tilde{\tilde{v}}_{1}^{+U} \right| & \left| \tilde{\tilde{v}}_{m2}^{U} - \tilde{\tilde{v}}_{2}^{+U} \right| & \cdots & \left| \tilde{\tilde{v}}_{mn}^{U} - \tilde{\tilde{v}}_{n}^{+U} \right| \end{bmatrix} \tag{25}$$

$$D^{+L} = \left[d_{ij}^{+L} \right] = \begin{bmatrix} \left| \tilde{\tilde{v}}_{11}^{L} - \tilde{\tilde{v}}_{1}^{+L} \right| & \left| \tilde{\tilde{v}}_{12}^{L} - \tilde{\tilde{v}}_{2}^{+L} \right| & \cdots & \left| \tilde{\tilde{v}}_{1n}^{L} - \tilde{\tilde{v}}_{n}^{+L} \right| \\ \left| \tilde{\tilde{v}}_{21}^{L} - \tilde{\tilde{v}}_{1}^{+L} \right| & \left| \tilde{\tilde{v}}_{22}^{L} - \tilde{\tilde{v}}_{2}^{+L} \right| & \cdots & \left| \tilde{\tilde{v}}_{2n}^{L} - \tilde{\tilde{v}}_{n}^{+L} \right| \\ \vdots & \vdots & \ddots & \vdots \\ \left| \tilde{\tilde{v}}_{m1}^{L} - \tilde{\tilde{v}}_{1}^{+L} \right| & \left| \tilde{\tilde{v}}_{m2}^{L} - \tilde{\tilde{v}}_{2}^{+L} \right| & \cdots & \left| \tilde{\tilde{v}}_{mn}^{L} - \tilde{\tilde{v}}_{n}^{+L} \right| \end{bmatrix} \tag{26}$$

$$D^{-U}=\left[d_{ij}^{-U}\right]=\begin{bmatrix} \left|\tilde{\tilde{v}}_{11}^{U}-\tilde{\tilde{v}}_{1}^{-U}\right| & \left|\tilde{\tilde{v}}_{12}^{U}-\tilde{\tilde{v}}_{2}^{-U}\right| & \cdots & \left|\tilde{\tilde{v}}_{1n}^{U}-\tilde{\tilde{v}}_{n}^{-U}\right| \\ \left|\tilde{\tilde{v}}_{21}^{U}-\tilde{\tilde{v}}_{1}^{-U}\right| & \left|\tilde{\tilde{v}}_{22}^{U}-\tilde{\tilde{v}}_{2}^{-U}\right| & \cdots & \left|\tilde{\tilde{v}}_{2n}^{U}-\tilde{\tilde{v}}_{n}^{-U}\right| \\ \vdots & \vdots & \ddots & \vdots \\ \left|\tilde{\tilde{v}}_{m1}^{U}-\tilde{\tilde{v}}_{1}^{-U}\right| & \left|\tilde{\tilde{v}}_{m2}^{U}-\tilde{\tilde{v}}_{2}^{-U}\right| & \cdots & \left|\tilde{\tilde{v}}_{mn}^{U}-\tilde{\tilde{v}}_{n}^{-U}\right| \end{bmatrix} \quad (27)$$

$$D^{-L}=\left[d_{ij}^{-L}\right]=\begin{bmatrix} \left|\tilde{\tilde{v}}_{11}^{L}-\tilde{\tilde{v}}_{1}^{-L}\right| & \left|\tilde{\tilde{v}}_{12}^{L}-\tilde{\tilde{v}}_{2}^{-L}\right| & \cdots & \left|\tilde{\tilde{v}}_{1n}^{L}-\tilde{\tilde{v}}_{n}^{-L}\right| \\ \left|\tilde{\tilde{v}}_{21}^{L}-\tilde{\tilde{v}}_{1}^{-L}\right| & \left|\tilde{\tilde{v}}_{22}^{L}-\tilde{\tilde{v}}_{2}^{-L}\right| & \cdots & \left|\tilde{\tilde{v}}_{2n}^{L}-\tilde{\tilde{v}}_{n}^{-L}\right| \\ \vdots & \vdots & \ddots & \vdots \\ \left|\tilde{\tilde{v}}_{m1}^{L}-\tilde{\tilde{v}}_{1}^{-L}\right| & \left|\tilde{\tilde{v}}_{m2}^{L}-\tilde{\tilde{v}}_{2}^{-L}\right| & \cdots & \left|\tilde{\tilde{v}}_{mn}^{L}-\tilde{\tilde{v}}_{n}^{-L}\right| \end{bmatrix} \quad (28)$$

Step 7: Compute $S_i^U, R_i^U, Y_i^U, Z_i^U$ and $S_i^L, R_i^L, Y_i^L, z_i^L$ values for *j=1,2,...,n* as follows:

$$\begin{aligned} S_i^U &= \sum_{j=1}^{n} w_j^U d_{ij}^{+U} = \sum_{j=1}^{n} \mu\,(w_j^U, \bar{w}^U) d_{ij}^{+U} \\ R_i^U &= \max_j w_j^U d_{ij}^{+U} = \max_j \mu\,(w_j^U, \bar{w}^U) d_{ij}^{+U} \end{aligned} \quad (29)$$

$$\begin{aligned} Y_i^U &= \sum_{j=1}^{n} w_j^U d_{ij}^{-U} = \sum_{j=1}^{n} \mu\,(w_j^U, \bar{w}^U) d_{ij}^{-U} \\ Z_i^U &= \max_j w_j^U d_{ij}^{-U} = \max_j \mu\,(w_j^U, \bar{w}^U) d_{ij}^{-U} \end{aligned} \quad (30)$$

$$\begin{aligned} S_i^L &= \sum_{j=1}^{n} w_j^L d_{ij}^{+L} = \sum_{j=1}^{n} \mu\,(w_j^L, \bar{w}^L) d_{ij}^{+L} \\ R_i^L &= \max_j w_j^L d_{ij}^{+L} = \max_j \mu\,(w_j^L, \bar{w}^L) d_{ij}^{+L} \end{aligned} \quad (31)$$

$$\begin{aligned} Y_i^L &= \sum_{j=1}^{n} w_j^L d_{ij}^{-L} = \sum_{j=1}^{n} \mu\,(w_j^L, \bar{w}^L) d_{ij}^{-L} \\ Z_i^L &= \max_j w_j^L d_{ij}^{-L} = \max_j \mu\,(w_j^L, \bar{w}^L) d_{ij}^{-L} \end{aligned} \quad (32)$$

where w_j^U, w_j^L are obtained by using relative preference relation as follows:

If $S=\left\{\tilde{\tilde{w}}_1, \tilde{\tilde{w}}_2, ..., \tilde{\tilde{w}}_n\right\}$ indicates a set consisting of *n* trapezoidal fuzzy numbers in case $\tilde{\tilde{w}}_j = ((w_{j1}^U, w_{j2}^U, w_{j3}^U, w_{j4}^U; H_1(\tilde{w}_j^U), H_2(\tilde{w}_j^U)), (w_{j1}^L, w_{j2}^L, w_{j3}^L, w_{j4}^L; H_1(\tilde{w}_j^L), H_2(\tilde{w}_j^L))) \in S$ where *j* =1, 2, ..., *n*. By extension principle, $\bar{\tilde{\tilde{w}}} = (1/n) \otimes (\tilde{\tilde{w}}_1, \tilde{\tilde{w}}_2, ..., \tilde{\tilde{w}}_n) = ((\bar{w}_{j1}^U, \bar{w}_{j2}^U, \bar{w}_{j3}^U, \bar{w}_{j4}^U; H_1(\tilde{w}_j^U),$ $H_2(\tilde{w}_j^U)), (\bar{w}_{j1}^L, \bar{w}_{j2}^L, \bar{w}_{j3}^L, \bar{w}_{j4}^L; H_1(\tilde{w}_j^L), H_2(\tilde{w}_j^L)))$ is assumed to be the average of $\tilde{\tilde{X}}_1, \tilde{\tilde{X}}_2, ..., \tilde{\tilde{X}}_n$. Now, the relative preference relation p^* with membership function $\mu_{p^*}(\tilde{\tilde{w}}_j, \bar{\tilde{\tilde{w}}})$, representing relative preference degree of $\tilde{\tilde{w}}_j$ over $\bar{\tilde{\tilde{w}}}$ in *S*, is defined where , $j=1,2,...,n,$:

$$\mu_{p^*}(\tilde{\tilde{w}}_j,\bar{\tilde{\tilde{w}}}) = \frac{1}{2}\left[\mu_{p^*}(\tilde{w}_j^U,\bar{\tilde{w}}^U) + \mu_{p^*}(\tilde{w}_j^L,\bar{\tilde{w}}^L)\right] =$$
$$\frac{1}{2}\left[\begin{array}{l}\frac{1}{2}\left(\frac{(w_{j1}^U-\bar{w}_4^U)+(w_{j2}^U-\bar{w}_3^U)+(w_{j3}^U-\bar{w}_2^U)+(w_{j4}^U-\bar{w}_1^U)}{2\left\|T_S^U\right\|}+1\right)+\\ \frac{1}{2}\left(\frac{(w_{j1}^L-\bar{w}_4^L)+(w_{j2}^L-\bar{w}_3^L)+(w_{j3}^L-\bar{w}_2^L)+(w_{j4}^L-\bar{w}_1^L)}{2\left\|T_S^L\right\|}+1\right)\end{array}\right] \tag{33}$$

where

$$\left\|T_S^U\right\| = \begin{cases}\frac{(t_{s1}^{+U}-t_{s4}^{-U})+(t_{s2}^{+U}-t_{s3}^{-U})+(t_{s3}^{+U}-t_{s2}^{-U})+(t_{s4}^{+U}-t_{s1}^{-U})}{2} & if\ \ t_{s1}^{+U}\ge t_{s4}^{-U}\\ \frac{(t_{s1}^{+U}-t_{s4}^{-U})+(t_{s2}^{+U}-t_{s3}^{-U})+(t_{s3}^{+U}-t_{s2}^{-U})+(t_{s4}^{+U}-t_{s1}^{-U})}{2}+2(t_{s4}^{-U}-t_{s1}^{+U}) & if\ \ t_{s1}^{+U}\prec t_{s4}^{-U}\end{cases} \tag{34}$$

$$\left\|T_S^L\right\| = \begin{cases}\frac{(t_{s1}^{+L}-t_{s4}^{-L})+(t_{s2}^{+L}-t_{s3}^{-L})+(t_{s3}^{+L}-t_{s2}^{-L})+(t_{s4}^{+L}-t_{s1}^{-L})}{2} & if\ \ t_{s1}^{+L}\ge t_{s4}^{-L}\\ \frac{(t_{s1}^{+L}-t_{s4}^{-L})+(t_{s2}^{+L}-t_{s3}^{-L})+(t_{s3}^{+L}-t_{s2}^{-L})+(t_{s4}^{+L}-t_{s1}^{-L})}{2}+2(t_{s4}^{-L}-t_{s1}^{+L}) & if\ \ t_{s1}^{+L}\prec t_{s4}^{-L}\end{cases} \tag{35}$$

$$\begin{array}{l} t_{S1}^{+L}=\max\limits_j\left\{w_{j1}^L\right\}, t_{s2}^{+L}=\max\limits_j\left\{w_{j2}^L\right\}, t_{s3}^{+L}=\max\limits_j\left\{w_{j3}^L\right\}, t_{s4}^{+L}=\max\limits_j\left\{w_{j4}^L\right\}\\ t_{s1}^{-L}=\min\limits_j\left\{w_{j1}^L\right\}, t_{sh2}^{-L}=\min\limits_j\left\{w_{j2}^L\right\}, t_{s3}^{-L}=\min\limits_j\left\{w_{j3}^L\right\}, t_{s4}^{-L}=\min\limits_j\left\{w_{j4}^L\right\}\\ t_{s1}^{+U}=\max\limits_j\left\{w_{j1}^U\right\}, t_{s2}^{+U}=\max\limits_j\left\{w_{j2}^U\right\}, t_{s3}^{+U}=\max\limits_j\left\{w_{j3}^U\right\}, t_{s4}^{+U}=\max\limits_j\left\{w_{j4}^U\right\}\\ t_{s1}^{-U}=\min\limits_j\left\{w_{j1}^U\right\}, t_{s2}^{-U}=\min\limits_j\left\{w_{j2}^U\right\}, t_{s3}^{-U}=\min\limits_j\left\{w_{j3}^U\right\}, t_{s4}^{-U}=\min\limits_j\left\{w_{j4}^U\right\}\end{array} \tag{36}$$

Step 8: Compute the values of indices $\varphi_i^U, \varphi_i^L, \gamma_i^U$, and γ_i^L as follows:

$$\varphi_i^U = \begin{cases}\frac{S_i^U - S^{+U}}{S^{-U}-S^{+U}} & if\ \ S^{-U}=S^{+U}\\ \frac{R_i^U - R^{+U}}{R^{-U}-R^{+U}} & if\ \ R^{-U}=R^{+U}\\ \left(\frac{R_i^U-R^{+U}}{R^{-U}-R^{+U}}\right)\lambda+\left(\frac{S_i^U-S^{+U}}{S^{-U}-S^{+U}}\right)(1-\lambda) & otherwise\end{cases} \tag{37}$$

$$\gamma_i^U = \begin{cases}\frac{Y_i^U - Y^{-U}}{Y^{+U}-Y^{-U}} & if\ \ Y^{+U}=Y^{-U}\\ \frac{Z_i^U - Z^{-U}}{Z^{+U}-Z^{-U}} & if\ \ Z^{+U}=Z^{-U}\\ \left(\frac{Z_i^U-Z^{-U}}{Z^{+U}-Z^{-U}}\right)\beta+\left(\frac{Y_i^U-Y^{-U}}{Y^{+U}-Y^{-U}}\right)(1-\beta) & otherwise\end{cases} \tag{38}$$

$$\varphi_i^L = \begin{cases} \dfrac{S_i^L - S^{+L}}{S^{-L} - S^{+L}} & if \;\; S^{-L} = S^{+L} \\ \dfrac{R_i^L - R^{+L}}{R^{-L} - R^{+L}} & if \;\; R^{-L} = R^{+L} \\ \left(\dfrac{S_i^L - S^{+L}}{S^{-L} - S^{+L}}\right)\lambda + \left(\dfrac{R_i^L - R^{+L}}{R^{-L} - R^{+L}}\right)(1-\lambda) & otherwise \end{cases} \tag{39}$$

$$\gamma_i^L = \begin{cases} \dfrac{Y_i^L - Y^{-L}}{Y^{+L} - Y^{-L}} & if \;\; Y^{+L} = Y^{-L} \\ \dfrac{Z_i^L - Z^{-L}}{Z^{+L} - Z^{-L}} & if \;\; Z^{+L} = Z^{-L} \\ \left(\dfrac{Z_i^L - Z^{-L}}{Z^{+L} - Z^{-L}}\right)\beta + \left(\dfrac{Y_i^L - Y^{-L}}{Y^{+L} - Y^{-L}}\right)(1-\beta) & \text{otherwise} \end{cases} \tag{40}$$

where

$$\begin{aligned} S^{+U} &= \min_i S_i^U, & R^{+U} &= \min_i R_i^U, & Y^{+U} &= \min_i Y_i^U, & Z^{+U} &= \min_i Z_i^U \\ S^{-U} &= \max_i S_i^U, & R^{-U} &= \max_i R_i^U, & Y^{-U} &= \max_i Y_i^U, & Z^{-U} &= \max_i Z_i^U \end{aligned} \tag{41}$$

And

$$\begin{aligned} S^{+L} &= \min_i S_i^L, & R^{+L} &= \min_i R_i^L, & Y^{+L} &= \min_i Y_i^L, & Z^{+L} &= \min_i Z_i^L \\ S^{-L} &= \max_i S_i^L, & R^{-L} &= \max_i R_i^L, & Y^{-L} &= \max_i Y_i^L, & Z^{-L} &= \max_i Z_i^L \end{aligned} \tag{42}$$

Also, λ and β are regarded as weights for the strategy of the majority criteria, while $(1-\lambda)$ and $(1-\beta)$ are the weights of individual regret. The values of λ and β are within the range of 0 to 1, and these strategies can be compromised by $\lambda = 0.5$ and $\beta = 0.5$.

Step 9: Calculate final index (FI) as follows:

$$FI_i^U = \varphi_i^U + \frac{1}{\gamma_i^U} + \phi_{i'}^U \tag{43}$$

$$FI_i^L = \varphi_i^L + \frac{1}{\gamma_i^L} + \phi_{i'}^L \tag{44}$$

where $\frac{1}{\gamma_i^U}$ and $\frac{1}{\gamma_i^L}$ refer to all i for which $\gamma_i^U \geq 0, \gamma_i^L \geq 0$, while $\phi_{i'}^U, \phi_{i'}^L$ refer to all i' for which $\gamma_i^U = 0, \gamma_i^L = 0$ and $\phi_{i'}^U = (\min_i \gamma_i^U)^{\min w_j^U}$, $\phi_{i'}^L = (\min_i \gamma_i^L)^{\min w_j^L}$.

Step 10: Aggregate FI_i^U and FI_i^L by using Eq. (45). The minimum value of *FI* illustrates the best performance for the alternative *i*. In other words, the minimum value indicates the highest rank in each critical path of the production project network.

$$FI_i = (FI_i^U + FI_i^L)/2 \tag{45}$$

IV. APPLICATION TO PRODUCTION PROJECT MANAGEMENT PROBLEMS

In this section, in order to determine critical path of production projects by considering time, cost, quality, and risk criteria, an existing case study from the literature (Flouris, 2008) is adopted and solved. To better illustrate the applicability and ability of the proposed approach, a real-world case of an aircraft components development company is expressed that designs and builds small electronic components, especially for the aviation, defense, and space industries (Flouris, 2008). It has an order to manufacture electronic control units that will be mounted in or near the engine bays of a new range of aircraft to be manufactured in both civil and military versions. These small units will include a number of electronic parts assembled on a printed circuit board, which in turn will be installed on an aluminum chassis. This assembly is to be encapsulated in epoxy resin to protect the components from harsh environmental conditions of the engine bay. A cable connector and a pressure switch will also be installed on the chassis to protrude outside the encapsulated block. The project presented here is for the design and environmental testing of a small prototype batch. Details of the activities along with dependencies are provided in Table III. The production project network is depicted in Fig (3). Note that in the network, the dotted lines indicate dummy activities (6−8, 14−15, 16−17) needed to define additional precedence relationships among activities.

Step 1: First, a team of 3 experts is formed; then, experts' judgments about importance of criteria and rate of each activity under efficient criteria such as time, cost, risk, and quality are gathered. These judgments are depicted in Tables IV to VI. Note that time and cost criteria are expressed as numerical data and risk and quality are expressed by linguistic variables, which were introduced in Table IV.

TABLE III. Activities Along with dependencies

Activities	ID	Predecessors	Corresponding arrow
Get the customer's specification	GCS	-	0-1
Determine environmental parameters	DEP	GCS	2-8
Design and breadboard test circuitry	DBTC	GCS	2-3
Design printed circuit board layout	DPBL	DBTC	3-5
Specify and list electronic components	SLEC	DBTC	3-4
Design the chassis	DC	DPBL	5-6
Buy prototype circuit boards	BCB	DPBL	5-7
Buy components for prototypes	BCP	SLEC	4-7
Design encapsulation mold	DEM	DC	6-9
Make prototype chassis	MPC	DC	6-10
Assemble printed circuit boards	APCB	BCB, BCP	7-10
Write production test procedure	WPTP	DBTC	3-18
Design vibration testing clamp	DVTC	DEP, DC	8-12
Make molds	MM	DEM	9-11
Assemble and test the prototype units	ATPU	MPC, APCB	10-11
Cast (encapsulate) units	CU	MM, ATPU	11-13
Make vibration test clamp	MVTC	DVTC	12-15
Retest all units after casting	RUAC	CU	13-14
Vibration and shock tests	VSTU	MVTC, RUAC	15-17
Climatic tests	CTU	RUAC	14-17
Accelerated life tests	ALTU	RUAC	14-16
Assess test results	ATR	VSTU, CTU, ALTU	17-18
Finalize and issue production documents	FPD	WPTP, ATR	18-19

TABLE IV. IT2F information of activities by the time criterion (days)

ACT.	Experts		
	DM1	DM2	DM3
0-1	((2,4,7,9;1,1),(3,5,6,8;00.9,00.9))	((2,5,8,11;1,1),(3,6,7,10;00.9,00.9))	((2,4,7,9;1,1),(3,5,6,8;0.9,0.9))
2-8	((1,3,6,8;1,1),(2,4,5,7;0.9,0.9))	((1,3,6,8;1,1),(2,4,5,7;0.9,0.9))	((1,3,6,8;1,1),(2,4,5,7;0.9,0.9))
2-3	((3,6,9,12;1,1),(4,7,8,11;0.9,0.9))	((4,6,9,11;1,1),(5,7,8,10;0.9,0.9))	((2,5,8,11;1,1),(3,6,7,10;0.9,0.9))
3-5	((1,3,6,8;1,1),(2,4,5,7;0.9,0.9))	((1,3,6,8;1,1),(2,4,5,7;0.9,0.9))	((1,3,6,8;1,1),(2,4,5,7;0.9,0.9))
3-4	((1,3,6,8;1,1),(2,4,5,7;0.9,0.9))	((1,3,6,8;1,1),(2,4,5,7;0.9,0.9))	((1,3,6,9;1,1),(2,4,5,7;0.9,0.9))
5-6	((2,4,7,10;1,1),(3,5,6,9;0.9,0.9))	((2,4,7,9;1,1),(3,5,6,8;0.9,0.9))	((2,5,8,10;1,1),(3,6,7,9;0.9,0.9))
5-7	((8,11,14,17;1,1), (9,12,13,16;0.9,0.9))	((9,12,15,18;1,1), (10,13,14,17;0.9,0.9))	((9,13,16,19;1,1), (10,14,15,18;0.9,0.9))
4-7	((4,7,10,13;1,1), (5,8,9,12;0.9,0.9))	((5,8,11,14;1,1), (6,9,10,13;0.9,0.9))	((3,6,9,12;1,1), (4,7,8,11;0.9,0.9))
6-9	((1,3,6,8;1,1),(2,4,5,7;0.9,0.9))	((2,4,7,9;1,1),(3,5,6,8;0.9,0.9))	((1,3,6,8;1,1),(2,4,5,7;0.9,0.9))
6-10	((3,6,9,12;1,1),(4,7,8,11;0.9,0.9))	((4,6,9,11;1,1),(5,7,8,10;0.9,0.9))	((3,6,9,12;1,1),(4,7,8,10;0.9,0.9))
7-10	((1,3,6,8;1,1),(2,4,5,7;0.9,0.9))	((2,4,7,9;1,1),(3,5,6,8;0.9,0.9))	((1,3,6,8;1,1),(2,4,5,7;0.9,0.9))
3-18	((2,4,7,9;1,1),(3,5,6,8;0.9,0.9))	((2,4,7,9;1,1),(3,5,6,8;0.9,0.9))	((2,4,7,9;1,1),(3,5,6,8;0.9,0.9))
8-12	((1,3,6,8;1,1),(2,4,5,7;0.9,0.9))	((1,3,6,8;1,1),(2,4,5,7;0.9,0.9))	((1,3,6,9;1,1),(2,4,5,7;0.9,0.9))
9-11	((4,6,9,11;1,1),(5,7,8,10;0.9,0.9))	((5,7,10,13;1,1),(6,8,9,12;0.9,0.9))	((4,7,10,13;1,1),(5,8,9,12;0.9,0.9))
10-11	((3,6,9,11;1,1),(4,7,8,10;0.9,0.9))	((4,6,9,11;1,1),(5,7,8,10;0.9,0.9))	((3,6,9,11;1,1),(4,7,8,10;0.9,0.9))
11-13	((4,6,9,12;1,1),(5,7,8,11;0.9,0.9))	((5,7,10,12;1,1),(6,8,9,11;0.9,0.9))	((4,7,10,12;1,1),(5,8,9,11;0.9,0.9))
12-15	((2,4,7,10;1,1),(3,5,6,9;0.9,0.9))	((2,4,7,9;1,1),(3,5,6,8;0.9,0.9))	((3,5,8,10;1,1),(4,6,7,9;0.9,0.9))
13-14	((1,3,6,8;1,1),(2,4,5,7;0.9,0.9))	((1,3,6,8;1,1),(2,4,5,7;0.9,0.9))	((1,3,6,8;1,1),(2,4,5,7;0.9,0.9))
15-17	((1,3,6,8;1,1),(2,4,5,7;0.9,0.9))	((1,3,6,8;1,1),(2,4,5,7;0.9,0.9))	((1,3,6,8;1,1),(2,4,5,7;0.9,0.9))
14-17	((1,3,6,8;1,1),(2,4,5,7;0.9,0.9))	((2,4,7,9;1,1),(3,5,6,8;0.9,0.9))	((1,3,6,8;1,1),(2,4,5,7;0.9,0.9))
14-16	((2,4,7,10;1,1), (3,5,6,9;0.9,0.9))	((2,4,7,10;1,1), (3,5,6,9;0.9,0.9))	((2,4,7,10;1,1), (3,5,6,9;0.9,0.9))
17-18	((1,3,6,8;1,1),(2,4,5,7;0.9,0.9))	(1,3,6,8;1,1),(2,4,5,7;0.9,0.9))	((1,3,6,8;1,1),(2,4,5,7;0.9,0.9))
18-19	((1,3,6,8;1,1),(2,4,5,7;0.9,0.9)	(2,4,7,9;1,1),(3,5,6,8;0.9,0.9))	((1,3,6,8;1,1),(2,4,5,7;0.9,0.9))

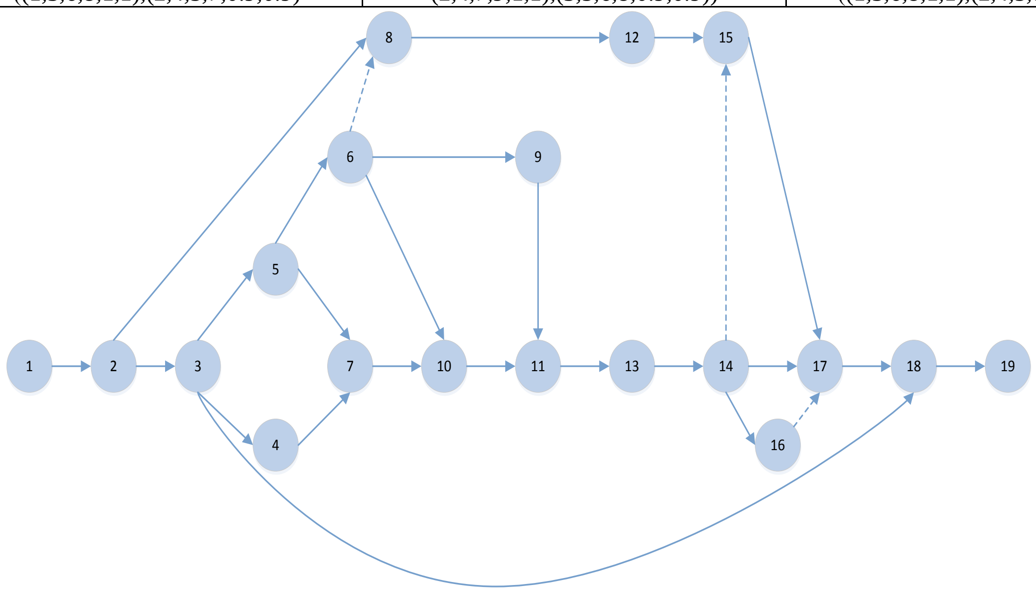

Fig 3. Production project network

TABLE V. IT2F information of acitivites by the cost criterion (US$)

ACT	Experts		
	DM1	**DM2**	**DM3**
0-1	((6,10,13,18;1,1), (7,11,12,17;0.9,0.9))	((6,9,12,15;1,1), (7,10,11,14;0.9,0.9))	((7,10,13,16;1,1), (8,11,12,15;0.9,0.9))
2-8	((2000,2500,3000,3500;1,1), (2100,2600,2900,3300;0.9,0.9))	((2000,2500,3000,3500;1,1), (2100,2600,2900,3300;0.9,0.9))	((2000,2500,3000,3500;1,1), (2100,2600,2900,3300;0.9,0.9))
2-3	((95,115,145,165;1,1), (105,125,135,155;0.9,0.9))	((85,105,135,155;1,1), (95,115,125,145;0.9,0.9))	((90,120,150,180;1,1), (100,130,140,170;0.9,0.9))
3-5	((70,95,125,150;1,1), (80,105,115,140;0.9,0.9))	((70,100,130,160;1,1), (80,110,120,150;0.9,0.9))	((75,90,120,135;1,1), (85,100,110,125;0.9,0.9))
3-4	((30,45,75,90;1,1), (40,55,65,80;0.9,0.9))	((25,45,75,105;1,1), (35,55,65,95;0.9,0.9))	((30,50,80,100;1,1), (40,60,70,90;0.9,0.9))
5-6	((140,170,200,230;1,1), (150,180,190,220;0.9,0.9))	((130,155,185,210;1,1), (140,165,175,200;0.9,0.9))	((140,160,190,210;1,1), (150,170,180,200;0.9,0.9))
5-7	((350,380,410,440;1,1), (360,390,400,430;0.9,0.9))	((360,400,430,470;1,1), (370,410,420,460;0.9,0.9))	((360,390,420,450;1,1), (370,400,410,440;0.9,0.9))
4-7	((35,55,85,105;1,1), (45,65,75,95;0.9,0.9))	((30,50,80,100;1,1), (40,60,70,90;0.9,0.9))	((35,60,90,115;1,1), (45,70,80,105;0.9,0.9))
6-9	((75,95,125,145;1,1), (85,105,115,135;0.9,0.9))	((75,95,125,145;1,1), (85,105,115,135;0.9,0.9))	((80,100,130,150;1,1), (90,110,120,140;0.9,0.9))
6-10	((270,300,330,360;1,1), (280,310,320,350;0.9,0.9))	((240,290,320,370;1,1), (250,300,310,360;0.9,0.9))	((250,280,310,340;1,1), (260,290,300,330;0.9,0.9))
7-10	((65,95,125,155;1,1), (75,105,115,145;0.9,0.9))	((60,90,120,150;1,1), (70,100,110,140;0.9,0.9))	((65,85,115,135;1,1), (75,95,105,125;0.9,0.9))
3-18	((1000,1200,1500,1800;1,1), (1100,1300,1400,1700;0.9,0.9))	((1000,1200,1500,1800;1,1), (1100,1300,1400,1700;0.9,0.9))	((1000,1200,1500,1800;1,1), (1100,1300,1400,1700;0.9,0.9))
8-12	((145,165,195,215;1,1), (155,175,185,205;0.9,0.9))	((140,170,200,230;1,1), (150,180,190,220;0.9,0.9))	((140,160,190,210;1,1), (150,170,180,200;0.9,0.9))
9-11	((290,330,360,400;1,1), (300,340,350,390;0.9,0.9))	((290,310,340,360;1,1), (300,320,330,350;0.9,0.9))	((280,310,340,370;1,1), (290,320,330,360;0.9,0.9))
10-11	((200,230,260,290;1,1), (210,240,250,280;0.9,0.9))	((190,220,250,280;1,1), (200,230,240,270;0.9,0.9))	((200,240,270,310;1,1), (210,250,260,300;0.9,0.9))
11-13	((280,300,330,350;1,1), (290,310,320,340;0.9,0.9))	((270,290,320,340;1,1), (280,300,310,330;0.9,0.9))	((285,305,335,355;1,1), (295,315,325,345;0.9,0.9))
12-15	((150,170,200,220;1,1), (160,180,190,210;0.9,0.9))	((140,160,190,210;1,1), (150,170,180,200;0.9,0.9))	((145,165,195,215;1,1), (155,175,185,205;0.9,0.9))
13-14	((35,55,85,105;1,1), (45,65,75,95;0.9,0.9))	((30,50,80,100;1,1), (40,60,70,90;0.9,0.9))	((35,60,90,115;1,1), (45,70,80,105;0.9,0.9))
15-17	((35,55,85,105;1,1), (45,65,75,95;0.9,0.9))	((35,50,80,95;1,1), (45,60,70,85;0.9,0.9))	((30,60,90,120;1,1), (40,70,80,110;0.9,0.9))
14-17	((65,95125,155;1,1), (75,105,115,145;0.9,0.9))	((60,90,120,150;1,1), (70,100,110,140;0.9,0.9))	((65,85,115,135;1,1), (75,95,105,125;0.9,0.9))
14-16	((10,25,55,70;1,1), (20,35,45,60;0.9,0.9))	((10,25,55,70;1,1), (20,35,45,60;0.9,0.9))	((50,70,100,120;1,1), (60,80,90,110;0.9,0.9))
17-18	((7,10,13,16;1,1), (8,11,12,15;0.9,0.9))	((8,11,14,17;1,1), (9,12,13,16;0.9,0.9))	((8,11,14,17;1,1), (9,12,13,16;0.9,0.9))
18-19	((320,350,380,410;1,1), (330,360,370,400;0.9,0.9))	((310,340,370,400;1,1), (320,350,360,390;0.9,0.9))	((320,345,375,400;1,1), (330,355,365,390;0.9,0.9))

TABLE VI. IT2F rating activities by the risk and quality criteria

ACT	Risk			Quality		
	D1	**D2**	**D3**	**D1**	**D2**	**D3**
0-1	ML	L	L	L	ML	ML
2-8	ML	ML	M	M	M	ML
2-3	ML	ML	M	MH	MH	M
3-5	M	H	ML	ML	ML	M
3-4	M	MH	MH	H	MH	H
5-6	ML	M	M	M	MH	M
5-7	ML	ML	ML	MH	MH	ML
4-7	MH	M	MH	M	ML	M
6-9	M	M	MH	MH	MH	M
6-10	MH	MH	H	M	MH	MH
7-10	MH	H	H	H	MH	H
3-18	MH	MH	MH	MH	MH	H
8-12	ML	ML	L	H	MH	H
9-11	M	MH	H	L	ML	ML
10-11	M	M	ML	M	M	ML
11-13	ML	M	ML	M	M	ML
12-15	ML	M	M	ML	M	ML
13-14	ML	M	ML	M	MH	M
15-17	MH	H	H	MH	MH	ML
14-17	ML	M	L	M	ML	M
14-16	MH	H	MH	H	MH	M
17-18	ML	L	ML	M	ML	M
18-19	ML	L	ML	MH	MH	ML

Step 2: Construct decision matrix by considering every existing path as an alternative, and consider time, cost, risk, and quality as efficient criteria for critical path selection of production project. In addition, experts' judgments are aggregated by using Eqs. (18) and (19).

Step 3: Normalize the decision matrix via Eqs. (20) and (21).

Step 4: Construct the IT2F weighted normalized decision matrix by using Eq. (22) shown in Tables VII-A and VIII-B.

Step 5: Specify positive ideal and negative ideal solutions via Eqs. (23) and (24) for benefit and cost criteria illustrated in Tables VII-A and VIII-B .

Step 6: Construct ideal separation matrix (D^+) and anti-ideal separation matrix (D^-) by using Eqs. (25-28) for upper and lower limits of IT2FSs.

Step 7: Compute $S_i^U, R_i^U, Y_i^U, Z_i^U$ and $S_i^L, R_i^L, Y_i^L, z_i^L$ values for $j=1,2,\ldots,n$ via Eqs. (29-36) shown in Table VIII.

Step 8: Compute the values of indices $\varphi_i^U, \varphi_i^L, \gamma_i^U$, and γ_i^L by using Eqs. (37-42) shown in Table VIII.

TABLE VII-A. Weighted normalized fuzzy decision matrix

Num.	Alternatives	Time	Cost
1	1-2-8-12-15-17-18-19	((0.006,0.021,0.04,0.06;1,1), (0.01,0.03,0.034,0.05;0.81,0.81))	((0.07,0.11,0.14,0.18;1,1), (0.09,0.12,0.13,0.16;0.81,0.81))
2	1-2-3-5-6-8-12-15-17-18-19	((0.01,0.03,0.05,0.09;1,1), (0.02,0.04,0.047,0.07;0.81,0.81))	((0.03,0.04,0.05,0.06;1,1), (0.03,0.042,0.045,0.054;0.81,0.81))
3	1-2-3-5-6-9-11-13-14-15-17-18-19	((0.015,0.04,0.07,0.11;1,1), (0.03,0.05,0.06,0.09;0.81,0.81))	((0.04,0.06,0.07,0.08;1,1), (0.04,0.058,0.062,0.07;0.81,0.81))
4	1-2-3-5-6-9-11-13-14-17-18-19	((0.015,0.04,0.07,0.11;1,1), (0.03,0.05,0.06,0.09;0.81,0.81))	((0.04,0.056,0.065,0.08;1,1), (0.045,0.059,0.062,0.075;0.81,0.81))
5	1-2-3-5-6-9-11-13-14-16-17-18-19	((0.016,0.04,0.07,0.12;1,1), (0.03,0.05,0.06,0.1;0.81,0.81))	((0.04,0.055,0.064,0.082;1,1), (0.045,0.058,0.06,0.073;0.81,0.81))
6	1-2-3-5-6-10-11-13-14-15-17-18-19	((0.016,0.04,0.07,0.11;1,1), (0.03,0.05,0.06,0.1;0.81,0.81))	((0.04,0.06,0.07,0.09;1,1), (0.05,0.06,0.065,0.08;0.81,0.81))
7	1-2-3-5-6-10-11-13-14-17-18-19	((0.016,0.04,0.07,0.12;1,1), (0.03,0.05,0.06,0.1;0.81,0.81))	((0.04,0.059,0.069,0.09;1,1), (0.05,0.06,0.066,0.08;0.81,0.81))
8	1-2-3-5-6-10-11-13-14-16-17-18-19	((0.017,0.05,0.08,0.12;1,1), (0.03,0.06,0.07,0.1;0.81,0.81))	((0.04,0.06,0.07,0.087;1,1), (0.05,0.062,0.065,0.078;0.81,0.81))
9	1-2-3-4-7-10-11-13-14-15-17-18-19	((0.016,0.04,0.07,0.12;1,1), (0.03,0.05,0.06,0.1;0.81,0.81))	((0.03,0.046,0.056,0.073;1,1), (0.04,0.05,0.053,0.065;0.81,0.81))
10	1-2-3-4-7-10-11-13-14-17-18-19	((0.016,0.04,0.07,0.12;1,1), (0.03,0.06,0.065,0.096;0.81,0.81))	((0.03,0.047,0.057,0.074;1,1), (0.04,0.05,0.053,0.065;0.81,0.81))
11	1-2-3-4-7-10-11-13-14-16-17-18-19	((0.017,0.05,0.08,0.12;1,1), (0.03,0.06,0.07,0.1;0.81,0.81))	((0.03,0.046,0.056,0.072;1,1), (0.04,0.049,0.52,0.064;0.81,0.81))
12	1-2-3-18-19	((0.006,0.016,0.027,0.04;1,1), (0.009,0.02,0.023,0.035;0.81,0.81))	((0.04,0.06,0.072,0.096;1,1), (0.048,0.063,0.068,0.086;0.81,0.81))
13	1-2-3-5-7-10-11-13-14-17-18-19	((0.02,0.05,0.08,0.12;1,1), (0.03,0.06,0.067,0.1;0.81,0.81))	((0.04,0.06,0.07,0.09;1,1), (0.05,0.064,0.067,0.08;0.81,0.81))
14	1-2-3-5-7-10-11-13-14-15-17-18-19	((0.019,0.05,0.08,0.12;1,1), (0.032,0.06,0.07,0.1;0.81,0.81))	((0.04,0.06,0.07,0.09;1,1), (0.05,0.06,0.066,0.08;0.81,0.81))
15	1-2-3-5-7-10-11-13-14-16-17-18-19	((0.02,0.05,0.08,0.124;1,1), (0.04,0.06,0.07,0.1;0.81,0.81))	((0.04,0.059,0.069,0.09;1,1), (0.05,0.06,0.066,0.08;0.81,0.81))
A^+		((0.006,0.016,0.027,0.04;1,1), (0.009,0.02,0.023,0.035;0.81,0.81))	((0.03,0.04,0.05,0.06;1,1), (0.03,0.042,0.045,0.054;0.81,0.81))
A^-		((0.02,0.05,0.08,0.124;1,1), (0.04,0.06,0.07,0.1;0.81,0.81))	((0.07,0.11,0.14,0.18;1,1), (0.09,0.12,0.13,0.16;0.81,0.81))

TABLE VIII-B. Weighted normalized fuzzy decision matrix (continued)

Alt.	Risk	Quality
1	((0.02,0.04,0.04,0.07;1,1), (0.03,0.04,0.04,0.06;0.81,0.81))	((0.01,0.04,0.04,0.07;1,1), (0.02,0.04,0.04,0.05;0.81,0.81))
2	((0.02,0.06,0.06,0.1;1,1), (0.04,0.06,0.06,0.08;0.81,0.81))	((0.02,0.05,0.05,0.09;1,1), (0.03,0.05,0.05,0.07;0.81,0.81))
3	((0.04,0.08,0.08,0.13;1,1), (0.06,0.08,0.08,0.11;0.81,0.81))	((0.02,0.06,0.06,0.11;1,1), (0.04,0.06,0.06,0.08;0.81,0.81))
4	((0.03,0.07,0.07,0.12;1,1), (0.05,0.07,0.07,0.1;0.81,0.81))	((0.022,0.05,0.05,0.1;1,1), (0.04,0.05,0.05,0.08;0.81,0.81))
5	((0.03,0.08,0.08,0.13;1,1), (0.05,0.08,0.08,0.11;0.81,0.81))	((0.02,0.06,0.06,0.11;1,1), (0.04,0.06,0.06,0.08;0.81,0.81))
6	((0.03,0.08,0.08,0.13;1,1), (0.06,0.08,0.08,0.11;0.81,0.81))	((0.02,0.06,0.06,0.11;1,1), (0.039,0.06,0.06,0.081;0.81,0.81))
7	((0.03,0.07,0.07,0.12;1,1), (0.05,0.07,0.07,0.1;0.81,0.81))	((0.02,0.06,0.06,0.11;1,1), (0.038,0.06,0.06,0.08;0.81,0.81))
8	((0.03,0.08,0.08,0.13;1,1), (0.05,0.08,0.08,0.1;0.81,0.81))	((0.3,0.06,0.06,0.11;1,1), (0.04,0.06,0.06,0.08;0.81,0.81))
9	((0.04,0.09,0.09,0.14;1,1), (0.06,0.09,0.09,0.11;0.81,0.81))	((0.03,0.06,0.06,0.11;1,1), (0.04,0.06,0.06,0.09;0.81,0.81))
10	((0.033,0.08,0.08,0.13;1,1), (0.05,0.08,0.08,0.1;0.81,0.81))	((0.03,0.06,0.06,0.11;1,1), (0.04,0.06,0.06,0.09;0.81,0.81))
11	((0.04,0.08,0.08,0.14;1,1), (0.05,0.08,0.08,0.11;0.81,0.81))	((0.03,0.07,0.07,0.12;1,1), (0.05,0.07,0.07,0.09;0.81,0.81))
12	((0.01,0.025,0.025,0.04;1,1), (0.02,0.025,0.025,0.03;0.81,0.81))	((0.01,0.02,0.02,0.04;1,1), (0.016,0.02,0.02,0.03;0.81,0.81))
13	((0.03,0.07,0.07,0.12;1,1), (0.05,0.07,0.07,0.1;0.81,0.81))	((0.02,0.06,0.06,0.11;1,1), (0.04,0.06,0.06,0.08;0.81,0.81))
14	((0.03,0.08,0.08,0.13;1,1), (0.05,0.08,0.08,0.1;0.81,0.81))	((0.02,0.06,0.06,0.11;1,1), (0.04,0.06,0.06,0.083;0.81,0.81))
15	((0.03,0.08,0.08,0.13;1,1), (0.05,0.08,0.08,0.1;0.81,0.81))	((0.03,0.06,0.06,0.12;1,1), (0.04,0.06,0.06,0.085;0.81,0.81))
A^+	((0.01,0.025,0.025,0.04;1,1), (0.02,0.025,0.025,0.03;0.81,0.81))	((0.03,0.07,0.07,0.12;1,1), (0.05,0.07,0.07,0.09;0.81,0.81))
A^-	((0.04,0.09,0.09,0.14;1,1), 0.06,0.09,0.09,0.11;0.81,0.81))	((0.01,0.02,0.02,0.04;1,1), (0.016,0.02,0.02,0.03;0.81,0.81))

TABLE VIX. Indices values

Alt.	S_1^U	R_1^U	Y_1^U	Z_1^U	S_1^L	R_1^L	Y_1^L	Z_1^L	φ_i^U	φ_i^L	γ_i^U	γ_i^L
1	0.188	0.079	0.211	0.082	0.138	0.067	0.146	0.056	0.86	0.347	0.974	0
2	0.152	0.061	0.237	0.079	0.102	0.042	0.171	0.067	0.43	0.758	0.361	1
3	0.195	0.083	0.228	0.074	0.138	0.06	0.154	0.06	0.941	0.388	0.891	0.361
4	0.19	0.077	0.231	0.074	0.133	0.054	0.158	0.06	0.842	0.444	0.771	0.435
5	0.195	0.082	0.229	0.074	0.138	0.059	0.155	0.06	0.935	0.412	0.878	0.389
6	0.198	0.082	0.228	0.073	0.14	0.059	0.153	0.058	0.945	0.347	0.895	0.296
7	0.193	0.076	0.231	0.074	0.135	0.053	0.158	0.058	0.846	0.454	0.775	0.37
8	0.197	0.081	0.229	0.073	0.139	0.059	0.155	0.059	0.938	0.371	0.882	0.324
9	0.193	0.086	0.232	0.077	0.135	0.063	0.158	0.064	0.965	0.6	0.908	0.635
10	0.189	0.08	0.235	0.077	0.13	0.057	0.162	0.063	0.866	0.655	0.787	0.71
11	0.193	0.086	0.233	0.078	0.134	0.062	0.159	0.064	0.958	0.623	0.895	0.66
12	0.116	0.042	0.223	0.086	0.079	0.028	0.168	0.063	0	0.727	0	0.789
13	0.194	0.074	0.231	0.074	0.136	0.052	0.157	0.058	0.838	0.465	0.77	0.332
14	0.199	0.08	0.228	0.072	0.141	0.058	0.153	0.058	0.937	0.322	0.89	0.259
15	0.198	0.08	0.229	0.072	0.14	0.057	0.154	0.058	0.93	0.347	0.878	0.287

Step 9: Obtain FI^L and FI^U for each path by Eqs. (43) and (44) illustrated in Table IX.

Step 10: Aggregate FI_i^U and FI_i^L by using Eq. (45). Note that the minimum value indicates the highest rank of each alternative, which is given in Table IX.

V. SENSITIVITY ANALYSIS

In this section, sensitivity analysis is performed by changing the amounts of weights of majority criteria, i.e., λ and β . The maximal group utility is 1, maximal regret is 0, and the combination of both is 0.5. By changing the amounts of λ and β between [0,1], new results are obtained, which are presented in Fig (4). It is obvious that path 12 is a critical path in most cases and this in turn illustrates that the critical path is insensitive to values of λ and β .

TABLE X. Final ranking

Alternatives	FI^U	FI^L	FI	Ranking
1	3.75	1.56	2.65	6
2	1.75	1.36	1.55	2
3	3.52	3.66	3.59	11
4	3.09	3.07	3.08	7
5	3.36	3.44	3.4	10
6	3.82	4.27	4.05	13
7	3.05	3.48	3.26	8
8	2.63	3.96	3.8	12
9	2.63	2.48	2.56	5
10	2.39	2.2	2.29	3
11	2.56	2.4	2.48	4
12	1.37	1.27	1.32	1
13	2.99	3.78	3.38	9
14	4.03	4.76	4.39	15
15	3.81	4.36	4.09	14

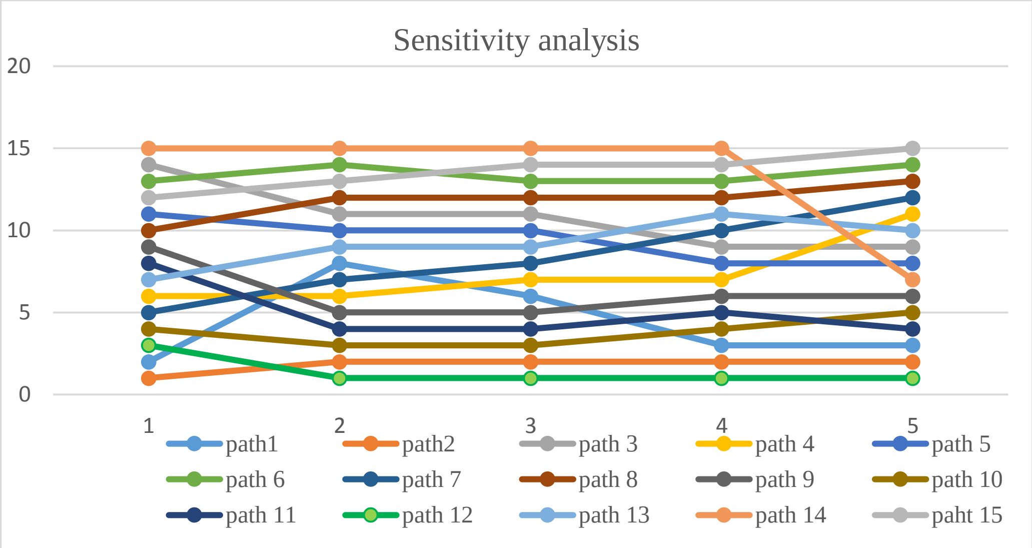

Fig 4. Sensitivity analysis

Also, to illustrate the importance of evaluation criteria and their effects on the critical path selection, first, a pair of criteria is considered for selecting critical path, in which time criterion, because of its importance, is considered in all experiments. Afterwards, triple combination of efficient criteria is considered. The results of all experiments are shown in Table X. The results demonstrate that by changing or ignoring some criteria, the critical path will be altered. In other words, critical path depends on the efficient criteria and all criteria should be considered in the critical path problem. For example, if time and cost criteria are considered, then critical path will change from path 12 to path 2, because other conflicting criteria have effects on determining critical path. This trend has been illustrated in Table X.

VI. CONCLUSION

In this paper, a new multi-criteria analysis model based on compromise ratio method for fuzzy group decision making problems was introduced under an IT2F-environment. Uncertainty is an inherent part of decision making in production projects. Under this condition, experts or DMs may not be able to assign a certain number between [0,1] to explain their assessment; in this paper, the importance of criteria (i.e., weights) and the rating of alternatives were described by linguistic terms and, then, converted to IT2F numbers. IT2FSs provide more degrees of freedom than type-1 fuzzy sets do. Furthermore, in order to deal with shortcomings of defuzzification, a new relative preference relation over the average was used. A real case study of critical path selection for production project with considering conflict criteria was adopted from the literature and solved. This analysis model was a beneficial procedure to solve multi-criteria decision-making problems in a more flexible and more intelligent manner owing to the fact that it used IT2FSs instead of type-1 fuzzy sets to demonstrate evaluating performance values and the weights of criteria. Moreover, the results of sensitivity analysis showed that critical path selection problem depended on efficient criteria and by ignoring some of them, the criticality of all paths would change. The critical path was insensitive to values of λ and β, but it was very sensitive to the efficient criteria. In fact, these criteria played an important role in the critical path selection problem of the production project. From the viewpoint of project management, by specifying the critical path based on conflicting criteria, the project manager could obtain useful information about manufacturing systems and concentrate the resources on critical activities to reduce the time of production. In this paper, a new multi-criteria compromise ratio model was developed under IT2FSs to better address the uncertainty of production projects in real-world conditions. Also, a new version of relative preference relation was presented and applied to compute the weights of criteria for avoiding multiplying of two fuzzy decision matrices; relative preference relation facilitated the calculations.

TABLE XI. Sensitivity analysis of efficient criteria

Path	Time & cost	Time & risk	Time & quality	Time, cost & risk	Time, cost & quality	Time, risk & quality
1	1.789929	1.613087	1.715149	8.125782	1.743917	1.604067
2	1.006394	2.616421	1.780799	1.872019	1.054628	1.976987
3	1.743174	11.18011	2.623988	10.22701	1.579028	4.315368
4	1.768587	4.517561	2.748314	5.21138	1.619031	2.535797
5	1.759152	9.572695	2.713733	9.032271	1.57718	3.740511
6	1.911133	9.498647	2.642481	2.59848	1.672641	3.50765
7	1.939972	4.270585	2.763915	7.086733	1.715268	2.297104
8	1.927251	8.343514	2.731843	18.32383	1.669561	3.156639
9	1.475951	1.220196	2.436713	4.623771	1.303837	4.504629
10	1.495623	6.612692	2.537893	3.646416	1.334414	2.622474
11	1.491692	5.06138	2.517974	4.415837	1.306442	3.886928
12	1.328045	1	2.136273	1.023019	2.041381	2
13	2.123984	4.280029	3.475988	10.0395	1.816589	2.262216
14	2.091411	9.579538	3.283374	9.5973	1.771609	3.425285
15	2.107683	8.403044	3.403268	3.38962	1.767425	3.091805

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