Power Control and Scheduling For Low SNR Region in the Uplink of Two Cell Networks

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Abstract- In this paper we investigate the sub-channel assignment and power control to maximize the total sum rate in the uplink of two-cell network. It is assumed that there are some sub-channels in each cell which should be allocated among some users. Also, each user is subjected to a power constraint. The underlying problem is a non-convex mixed integer non-linear optimization problem which does not have a trivial solution. To solve the problem, having fixed the consumed power of each user, and assuming low co-channel interference region, the sub-channel allocation problem is reformulated into a more mathematically tractable problem which is shown it can be tackled through the so-called Hungarian algorithm. Then, the consumed power of each user is reformulated as a quadratic fractional problem which can be numerically derived. Numerical results demonstrate the superiority of the proposed method in low SNR region as compared to existing works addressed in the literature.

Index Terms- Sub-Channel assignment, Power control, Mixed Integer Non Linear Problem, Hungarian algorithm

I. INTRODUCTION

Resource allocation is considered as a major challenge in wireless communication networks. This is due to the limited available bandwidth, and total power, while there is a non-stop demand for emerging communication services. It is widely recognized that Orthogonal Frequency Division Multiple Access (OFDMA) can effectively divide the available bandwidth into orthogonal sub-channels to be allocated among active users. OFDMA, on the other hand, can be employed in multi-user multi-path dispersive channel as it can effectively divide a frequency selective fading channel into some narrowband flat fading channels [1]. Noting this, OFDMA technique has been widely adopted in broadband wireless communications over the last decade, due to its flexibility in resource allocation.

It is worth mentioning that in an OFDMA system, the intra-cell interference is simply avoided due

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to the orthogonality among sub-channels [2]-[7].

The problem of assigning sub-channels and allocating power to users in an OFDMA system has attracted many attentions in recent years. Moreover, the resource allocation problem in the uplink is more challenging than that of the downlink as the uplink interference is mostly affected by neighbouring co-channel users[7]-[8]. This is due to the fact that the overall SNR of users throughout the uplink channel is lower than that of the downlink due to the limited battery life of mobile users. Thus, more efforts are devoted to proper scheduling and power control mechanisms in the uplink.

In this regard, a plethora of works are devoted to explore effective ways of assigning sub-channels as well as allocating power to optimize a performance function [3]-[11]. For instance, the author of [3] investigates the joint sub-channel assignment and power control mechanism in terms of maximizing the sum rate in an uplink OFDMA network. This problem is non-convex mixed integer non-linear problem which can be solved by adding a penalty term to the objective function and relax the integer variables can be converted into a difference of two concave function (DC) problem. It is worth mentioning that the sub-optimal problem derived in [3] is too complex and the authors do not provide the condition under which the proposed method approaches the optimal solution.

The author of [4] determines the resource allocation in multi-cell OFDMA networks in order to jointly optimize the energy efficiency and spectral efficiency performance which allocate the sub-channel and power iteratively. This method, however, suffers from poor performance as compared to [3].

In [5] the joint sub-channel assignment and power control problems in a cellular network with the objective of enhancing the quality of-service is studied. Accordingly, this problem is tackled in two steps. First, it attempts to assign the sub-channels assuming all users make use of an equal power. Then, the power of each user is optimized for the assigned channels. Again, this problem has a poor performance as compared to [3].

This motivated us to pursue addressing this issue in a two-cell network through proposing a novel approach as follows. First, it is demonstrated that the joint sub-channel scheduling and power control mechanism is a highly non-convex mixed integer non-linear problem with a computationaly infeasible solution for a large network. As a result, we took some simple steps to simplify the problem through relying on low to mid SNR regime when tackling the sub-channel scheduling, where it is demonstrated that the sub-channel assignment can be thought as a so-called assignment problem with efficient solution named as Hungarian method [13]. Second, to get some further improvment in mid SNR regime, it is demonstrated that the power control mechanism can be effectively done for the assigned sub-channels through using a fractional programming approach based on the so-called Dinkelbach Algorithm. Simulation results confirm the effectivness of the proposed method in low to mid SNR regions as compared to exsiting known methods addresssed in the literature.

Also, it is worth mentioning that the current study is not limitted to a two-cell network and it can be extended to the cases in which the network planning is done in a way that each cell receives interference from some co-channel cells, however, one of the co-channel cells are the most influenced one. In this case, it is advisble that the proposed approach can be tackled on every two dominant co-channel pairs, individually. The second implication of the studied model is in the cases that the network operator takes the fractional frequency reuse factor of unity, meaning the whole cells make use of a single frequency band. However, again there is just one neighboring cell with the most interfering power due to the close-distance to the intended cell. For example, one can think of a Micro or Pico cell inside an original cell with the same frequency band, making a two-cell pair with the most influenced interferer on each other.

The rest of the paper is organized as follows.The system model of an OFDMA in the uplink direction is introduced in section II. in section III optimization problem is formulated to a mathematically tractable form. in section IV the sub-channel assignment based on the Hungarian method is investigated. The power control for the chosen sub-channels are determined in section V. Section VI presents the simulation results, and section VII concludes the paper.

II. SYSTEM MODEL

In this paper, we consider a two-cell network composing of I users per cell trying to send their signals to the base station in the uplink channel. Also, there are N sub-channels to be used by each user pair such that the signal arising from a co-channel user in one cell is treated as an interference in the other cell. The set of users in the j^{th} cell is represented by $I_j = \{1,2,...,\mathrm{I}\}$ where $J = \{1,2\}$, $j \in J$.

The network structure is depicted in Fig. 1, where the dotted lines represent inter-cell interference paths. Moreover, the uplink transmission from the user i in the j^{th} cell to the base station j goes through a Rayleigh flat fading channel denoted by $g^{n}_{i(j)j}$, where the channel strength is denoted by

$h^{n}_{i(j)j} = \left|g^{n}_{i(j)j}\right|^2$

p^{n}_{ij} and x^{n}_{ij} , respectively, represent the transmit power and a zero/one indicator showing if the n^{th} sub-channel is assigned to the i^{th} user residing in cell j where $N = \{1,2,...,\mathrm{N}\}$.

Lets us we define that $\mathbf{x} = [\mathbf{x}_{11},...,\mathbf{x}_{|I_j|j}]^T$ and $\mathbf{p} = [\mathbf{p}_{11},...,\mathbf{p}_{|I_j|j}]^T$ as a vector of $\mathbf{x}_{ij} = [x_{ij}^{1},...,x_{ij}^{n}]$ and $\mathbf{p}_{ij} = [p_{ij}^{1},...,p_{ij}^{n}]$.

It is worth mentioning that channel state information is globally known at the base stations.

The data rate of the user i located in the cell j using the n^{th} sub-channel according to the Shannon capacity formula can be mathematically written as follows:

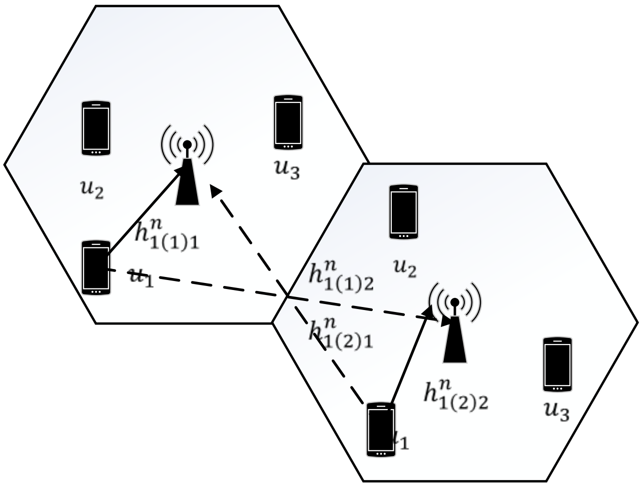

Fig. 1. Structure of the considered network

$$R_{ij}^{n} = \log_2\left(1+\frac{p_{ij}^{n}h_{i(j)j}^{n}}{\sum_{j'\in J, j'\neq j}\sum_{k\in I_{j'}} x_{kj'}^{n}p_{kj'}^{n}h_{k(j')j}^{n}+\sigma^2}\right) \tag{1}$$

Where $h_{i(j)j}^{n}$ indicates the channel from the i^{th} user in the j^{th} cell to its corresponding the j^{th} BTS and $h_{k(j')j}^{n}$ indicates the interference channel from the k^{th} user in the j' cell to the j^{th} BTS. Also, σ^2 is the noise power. Moreover, in the studied network, the set of users taking the same sub-channel are named as user pairs, where each user pair consists of one user in each cell, so its cardinality is equal to the number of cells. For the studied model, since the cell number is two, each two users having the same sub-channel from different cells make a user pair. Moreover, since different user pairs make use of different orthogonal sub-channels, the user pairs do not get interference from other pairs and their interference is just depend on user(s) inside the pair.

III. PROBLEM FORMULATION

The aim of this section is to find sub-channel assignment and optimal power allocation such that the sum-rate of the network is maximized. Mathematically speaking, the optimization problem is written as (2).

In (2), C_1 represents the power constraint of each user. C_2 indicates transmit powers take positive values. In addition, C_3 shows that a single sub-channel should be allocated to each user and C_4 indicates that the indicator x_{ij}^{n} takes zero/one values.

The problem of (2) is non-convex in general. We formulated the problem into a more mathematically

$$
\begin{aligned}
&\max_{\mathbf{p},\mathbf{x}} \sum_{j=1}^{2} \sum_{i\in I_j} \sum_{n\in N} \log_2\left(1+\frac{x_{ij}^n p_{ij}^n h_{i(j)j}^n}{\sum_{j'\in J, j'\neq j} \sum_{k\in I_{j'}} x_{kj'}^n p_{kj'}^n h_{k(j')j}^n + \sigma^2}\right) \\
&s.t. \quad c_1 : \sum_{n\in N} x_{ij}^n p_{ij}^n \le p_{\max} \qquad \forall j \in J, \forall i \in I_j \\
&\qquad\; c_2 : p_{ij}^n \ge 0 \qquad \forall j \in J, \forall i \in I_j \\
&\qquad\; c_3 : \sum_{n\in N} x_{ij}^n = 1 \qquad \forall j \in J, \forall i \in I_j \\
&\qquad\; c_4 : x_{ij}^n \in \{0,1\} \qquad \forall\, n \in N,\ \forall j \in J, \forall i \in I_j
\end{aligned} \tag{2}
$$

tractable form. It is noteworthy that since x_{ij}^n is a binary variable we can write*:*

$$
x_{ij}^n R_{ij}^n = \log_2\left(1+\frac{x_{ij}^n p_{ij}^n h_{i(j)j}^n}{\sum_{j'\in J, j'\neq j} \sum_{k\in I_{j'}} x_{kj'}^n p_{kj'}^n h_{k(j')j}^n + \sigma^2}\right) \tag{3}
$$

It should be noted that as x_{ij}^n takes zero/one values, hence, both sides of (3) becomes zero when x_{ij}^n is zero. Similarly, referring to the definition of R_{ij}^n in (1), equation (3) holds for the case of $x_{ij}^n = 1$. One can readily verify that the optimization problem in (2) involves some continuous variables p_{ij}^n and integer variables x_{ij}^n, hence, it is not convex in general [12]. Thus, it does not yield to a trivial solution. However, having fixed the transmit power at its maximum allowable value, and considering low SNR region, it is shown that using some approximations, the original problem can be converted to an assignment problem leading to the best sub-channel selection. Then, the optimal powers associated with selected sub-channels are numerically derived in terms of maximizing the sum-rate of the network.

IV. Sub-Channel Assignment

This section tends to determine the sub-channel assignment for each user. It is assumed that all users use all of their available power and operate at low SNR region. Thus, for a two-cell network, the following approximation (4) can be used to simplify the rate of i^{th} user in the j^{th} cell at n^{th} sub-channel.

In this case noting that when SINR is much smaller than one, the log function can be approximated as $\log_2(1+x) \cong x$. Where it is assumed that the noise power is greater than that of the interference. As a result, one can use equation (4) as an approximated achievable rate of the i^{th} user in the j^{th} cell for

$$R_{ij}^{n} = \log_2\left(1+\frac{p_{\max} h_{i(j)j}^{n}}{p_{\max} h_{k(j')j}^{n} + \sigma^2}\right) \cong \frac{p_{\max} h_{i(j)j}^{n}}{p_{\max} h_{k(j')j}^{n} + \sigma^2} = \frac{h_{i(j)j}^{n}}{h_{k(j')j}^{n} + \frac{\sigma^2}{p_{\max}}} \cong p_{\max}\frac{h_{i(j)j}^{n}}{\sigma^2} \quad (4)$$

$j \in \{1,2\}\, j' \in \{3-j\}$

The n^{th} sub-channel. As a result, the optimization problem can be reformulated as,

$$\begin{aligned}
&\max_{\mathbf{x}} \sum_{j=1}^{2}\sum_{i\in I}\sum_{n\in N} x_{ij}^{n}\left(\frac{p_{\max} h_{i(j)j}^{n}}{\sigma^2}\right) \\
&\text{s.t} \quad c_3 : \sum_{n\in N} x_{ij}^{n} = 1 \qquad \forall j \in J, \forall i \in I_j \\
&\qquad c_4 : x_{ij}^{n} \in \{0,1\} \qquad \forall\, n \in N,\ \forall j \in J, \forall i \in I_j
\end{aligned} \quad (5)$$

This problem can be tackled through the so called assignment problem. The assignment problem is one of the fundamental combinatorial optimization problems. It consists of finding a maximum weight matching (or minimum weight perfect matching) in a weighted bipartite graph and can be solved by using the Hungarian algorithm [13]. Suppose that we have N sub-channels to be assigned to N users on a one to one basis. Also, the cost of assigning sub-channels to users are known. It is desirable to find the optimal assignment minimizing the total cost. Let's c_{ij}^{n} be the cost of assigning the n^{th} sub-channel to the i^{th} user. We define the $n \times n$ cost matrix C associated with the assignment problem such that the element of the i^{th} row and the j^{th} column, i.e, c_{ij}^{n} is set to,

$$c_{ij}^{n} = \left(\frac{p_{\max} h_{i(j)j}^{n}}{\sigma^2}\right) \quad (6)$$

The Hungarian method attempts to find the best sub-channel in the following maximization problem

$$\begin{aligned}
&\max_{\mathbf{x}} \sum_{j=1}^{2}\sum_{i\in I_j}\sum_{n\in N} x_{ij}^{n}\left(C_{ij}{}^{n}\right) \\
&\text{s.t} \quad c_3 : \sum_{n\in N} x_{ij}^{n} = 1 \qquad \forall j \in J, \forall i \in I_j \\
&\qquad c_4 : x_{ij}^{n} \in \{0,1\} \qquad \forall\, n \in N,\ \forall j \in J, \forall i \in I_j
\end{aligned} \quad (7)$$

It should be noted that the assignment problem can be extended to a more general case of having N sub-channels and I users through without imposing any constraint on the size of sub-channels and users. It should be noted that the number of users (I) and sub-channels (N) is not restricted to be the same in the assignment problem. For the case of I<N, one can add N-I virtual users with the corresponding –Inf (a relatively large negative value) values for the added users, where I out of N sub-channels would be assigned to current I users. Then for the remaining unassigned N-I sub-channels, one can run another optimization problem as follows. To set the number of unassigned sub-channels (N-I) equal to that of current N users, one can add I virtual sub channels with corresponding -Inf cost values for current N users to make a square cost matrix again. Then, solve the new assignment problem to choose the best users for the remaining sub-channels. For the case of N < I, one can add N-I virtual sub-channels with the corresponding -Inf cost values for the current users to make a square cost matrix and then solve the assignment problem. Again, the actual sub-channels are assigned to the best N users.

V. POWER CONTROL STRATEGY

In the previous section, each sub-channel is assigned to one user pair. In the studied network, each user in a cell has the same sub-channel as another user in the neighboring cell. These two users with the same sub-channel are named as a user pair. Different user pairs have different sub-channels and their transmitted signals do not interferer on each other. Therefore, each user pair has no effect on other pairs. Thus, the total network throughput maximization problem can be simplified by the throughput maximization on each individual pair. Typically, the sum rate maximization problem for the first user pair can be written as:

$$\max_{p_{11}^n, p_{12}^n} \log_2\left(1+\frac{p_{11}^n h_{1(1)1}^n}{p_{12}^n h_{1(2)1}^n+\sigma^2}\right)+\log_2\left(1+\frac{p_{12}^n h_{1(2)2}^n}{p_{11}^n h_{1(1)2}^n+\sigma^2}\right)$$
$$s.t. \quad C_1 : p_{11}^n \le p_{max} \;, p_{12}^n \le p_{max}$$
$$C_2 : p_{11}^n \ge 0,\; p_{12}^n \ge 0 \qquad (8)$$

To simplify, some variables are changed using the definitions $a = h_{1(1)1}^n, b = h_{1(2)1}^n, c = h_{1(2)2}^n, d = h_{1(1)2}^n$ Then, the problem (8) is changed to,

$$\max_{p_{11}^n, p_{12}^n} \log_2\left(1+\frac{p_{11}^n a}{p_{12}^n b+\sigma^2}\right)+\log_2\left(1+\frac{p_{12}^n c}{p_{11}^n d+\sigma^2}\right)$$
$$s.t. \quad C_1 : p_{11}^n \le p_{max} \;, p_{12}^n \le p_{max}$$
$$C_2 : p_{11}^n \ge 0,\; p_{12}^n \ge 0 \qquad (9)$$

Noting $\log(1+a).\log(1+b) = \log(1+a+b+ab)$ thus, the optimization problem in (9) can be simplified to,

$$\max_{p_{11}^n, p_{12}^n} \left(\frac{p_{11}^n a}{p_{12}^n b + \sigma^2} \right) + \left(\frac{p_{12}^n c}{p_{11}^n d + \sigma^2} \right) + \left(\frac{p_{11}^n p_{12}^n ac}{(p_{11}^n d + \sigma^2)(p_{12}^n b + \sigma^2)} \right) \tag{10}$$

$$s.t. \quad C_1 : p_{11}^n \le p_{max} \ , p_{12}^n \le p_{max}$$

$$C_2 : p_{11}^n \ge 0, \ p_{12}^n \ge 0$$

Which can be reformulated as:

$$\max_{p_{11}^n, p_{12}^n} \left(\frac{ad\left(p_{11}^n\right)^2 + a\sigma^2 p_{11}^n + bc\left(p_{12}^n\right)^2 + c\sigma^2 p_{12}^n + acp_{11}^n p_{12}^n}{bdp_{12}^n p_{11}^n + d\sigma^2 p_{11}^n + b\sigma^2 p_{12}^n + \sigma^4} \right) \tag{11}$$

$$s.t. \quad C_1 : p_{11}^n \le p_{max} \ , p_{12}^n \le p_{max}$$

$$C_2 : p_{11}^n \ge 0, \ p_{12}^n \ge 0$$

To optimize the power of each user pair, we make use of the following lemma:

Lemma1: let's consider a nonlinear fractional programming problem:

$$\max_x \left(\frac{h(x)}{g(x)} \right) \tag{12}$$

$$s.t. \quad x \in X \subset R^n$$

The objective function of (12) is a fraction of two convex function called fractional problem. To address the optimal solution, we define the following function:

$$F(x;\lambda) = h(x) - \lambda g(x), \lambda > 0 \tag{13}$$

Where $x \in X, \lambda \succ 0$

Let,

$$x(\lambda) = \arg\max_x F(x;\lambda) \tag{14}$$

Also let

$$\Pi(\lambda) = \max_{x \in R^n \, F(x,\lambda) = F(x(\lambda),\lambda)} F(x;\lambda) \tag{15}$$

If there exist $\lambda^* \ge 0$ **for which** $\Pi(\lambda^*) = 0$ **then** $x \equiv x(\lambda^*)$ **is the optimal solution of (12).**

Proof: see [14]

According to the lemma 1 and referring to (11), we define $F(p,\lambda)$ as follows:

$$
\begin{aligned}
&F(\mathbf{p},\lambda) = \max\{f(\mathbf{p}) - \lambda g(\mathbf{p}) \mid \mathbf{p} \in \mathbf{P}^2\} \\
&\mathbf{P}^2 = [0 \quad p_{\max}]*[0 \quad p_{\max}] \\
&s.t. \quad \mathbf{p} \in \mathbf{P}^2 \subset R^n, \mathbf{a_i}^T \mathbf{p} \le p_{\max}, -(\mathbf{a_i}^T \mathbf{p}) \le 0, a_1 = [10]^T\ a_2 = [01]^T, i = \{1,2\} \\
&(p_1 \le p_{\max}, p_2 \le p_{\max}, -p_1 \le 0, -p_2 \le 0)
\end{aligned}
\tag{16}
$$

We define $f(\mathbf{p})$ and $g(\mathbf{p})$ where*:*

$$f(\mathbf{p}) = ad\left(p_{11}^n\right)^2 + a\sigma^2 p_{11}^n + bc\left(p_{12}^n\right)^2 + c\sigma^2 p_{12}^n + acp_{11}^n p_{12}^n \tag{17}$$

$$g(\mathbf{p}) = bdp_{12}^n p_{11}^n + d\sigma^2 p_{11}^n + b\sigma^2 p_{12}^n + \sigma^4 \tag{18}$$

The problem in (11) is a standard fractional problem which can be solved by the Dinkelbach algorithm [16] in polynomial time. The algorithm summarized in proposition 1.

Proposition1 (optimality). $F(\lambda^*) = \{f(\mathbf{p}) - \lambda^* g(\mathbf{p}) \mid \mathbf{p} \in \mathbf{P}^2\} = 0$ **if and only if** $\lambda^* = f(\mathbf{p}^*)/g(\mathbf{p}^*) = \max\{f(\mathbf{p})/g(\mathbf{p}) \mid \mathbf{p} \in \mathbf{P}^2\}$

Algorithm 1 Dinkelbach Algorithm

1. initialize $\lambda_1 = 0$ and $\mathbf{p}_1$ as a feasible power vector.
2. set error tolerance $\delta << 1$ and iteration index $n = 1$
3. repeat
4. $\mathbf{p}_n = \arg\max_{\mathbf{p}\in\mathbf{P}^2}\{f(\mathbf{p}) - \lambda_n g(\mathbf{p})\}$
5. $\lambda_{n+1} = f(\mathbf{p}_n)/g(\mathbf{p}_n)$
6. $n = n + 1$
7. until $f(\mathbf{p}_{n-1}) - \lambda_n g(\mathbf{p}_{n-1}) \le \delta$ (convergence check)

Based on Lemma1, the optimal power allocation can be obtained.

To solve the step 4 of the this algorithm, we note that the objective function associated with the optimization problem of (11) can be reformulated as follows,

$$\max_{\mathbf{p}} \frac{\mathbf{p}^T \mathbf{M}\mathbf{p}+\mathbf{c}^T \mathbf{p}+\alpha}{\mathbf{p}^T \mathbf{Q}\mathbf{p}+\mathbf{d}^T \mathbf{p}+\beta}$$

$$\mathbf{P}^2 = [0 \quad p_{\max}]*[0 \quad p_{\max}]$$

$$s.t. \quad \mathbf{p} \in \mathbf{P}^2 \subset R^n, \mathbf{a_i}^T \mathbf{p} \le p_{\max}, -(\mathbf{a_i}^T \mathbf{p}) \le 0, a_1 = [10]^T a_2 = [01]^T, i = \{1,2\} \tag{19}$$

$$(p_1 \le p_{\max}, p_2 \le p_{\max}, -p_1 \le 0, -p_2 \le 0)$$

Where $\mathbf{M} = \begin{bmatrix} ad & ac \\ 0 & bc \end{bmatrix}, \mathbf{Q} = \begin{bmatrix} 0 & bd \\ 0 & 0 \end{bmatrix}, \mathbf{c} = \begin{bmatrix} a\sigma^2 & c\sigma^2 \end{bmatrix}^T, \mathbf{d} = \begin{bmatrix} d\sigma^2 & b\sigma^2 \end{bmatrix}^T$

In this case, the step 4 of the preposition becomes,

$$F(\mathbf{p},\lambda) = \max\left\{\mathbf{p}^T \mathbf{M}\mathbf{p}+c^T \mathbf{p}+\alpha - \lambda_n(\mathbf{p}^T \mathbf{Q}\mathbf{p} + d^T \mathbf{p} + \beta) \mid \mathbf{p} \in \mathbf{P}^2\right\}$$

$$= \mathbf{p}^T(\mathbf{M} - \lambda_n \mathbf{Q})\mathbf{p} + (\mathbf{c} - \lambda_n \mathbf{d})^T \mathbf{p} + \alpha - \lambda_n \beta \tag{20}$$

$$s.t. \quad \mathbf{p} \in \mathbf{P}^2 \subset R^n, \mathbf{a_i}^T \mathbf{p} \le p_{\max}, -(\mathbf{a_i}^T \mathbf{p}) \le 0, a_1 = [10]^T a_2 = [01]^T, i = \{1,2\}$$

$$(p_1 \le p_{\max}, p_2 \le p_{\max}, -p_1 \le 0, -p_2 \le 0)$$

One can readily observe that $\mathbf{M} - \lambda\mathbf{Q}$ is positive semi definite matrix since the eigen values of the matrix $\mathbf{H}_n = \mathbf{M} - \lambda_n \mathbf{Q} = \begin{bmatrix} ad & ac - \lambda_n bd \\ 0 & bc \end{bmatrix}$ are $v_1 = ad, v_2 = bc$. It should be noted that the eigen values of matrix $\mathbf{H}_n = \mathbf{M} - \lambda_n \mathbf{Q}$ are independent of λ_n. Moreover, based on [17], if $\mathbf{H}_n = \mathbf{M} - \lambda_n \mathbf{Q}$ is a positive semi definite matrix, the optimal value of $F(\mathbf{p},\lambda)$ can be obtained from the following steps,

Step 4-1: Let's define $n = 1$ as the first iteration number and set $\lambda_1 = 0$. Also, let's define $\mathbf{H}_n = \mathbf{M} - \lambda_n \mathbf{Q}$, $\mathbf{c}_n = \mathbf{c} - \lambda_n \mathbf{d}$.

4-2 Solve the optimization problem of (20) as follows,

$$F(\mathbf{p},\lambda) = \max\{\mathbf{p}^\mathbf{T}\mathbf{H_n}\mathbf{p} + (\mathbf{c}_n)^T \mathbf{p}\}$$

$$s.t. \quad \mathbf{p} \in \mathbf{P}^2 \subset R^n, \mathbf{a_i}^T \mathbf{p} \le p_{\max}, -(\mathbf{a_i}^T \mathbf{p}) \le 0, a_1 = [10]^T a_2 = [01]^T, i = \{1,2\} \tag{21}$$

$$(p_1 \le p_{\max}, p_2 \le p_{\max}, -p_1 \le 0, -p_2 \le 0)$$

To this end, the Lagrangian form of (21) becomes,

$$L(\mathbf{p}, v_1, v_2, v_3, v_4) = \{\mathbf{p}^\mathbf{T}\mathbf{H}_n\mathbf{p} + (\mathbf{c}_n)^T \mathbf{p} - v_1(\mathbf{a_1}^T \mathbf{p} - p_{\max}) - v_2(\mathbf{a_2}^T \mathbf{p} - p_{\max}) \tag{22}$$

$$+ v_3(\mathbf{a_1}^\mathbf{T}\mathbf{p}) + v_4(\mathbf{a}_2^\mathbf{T}\mathbf{p})\}$$

Taking the gradient with respect to $\mathbf{p}$, results in,

$$2\mathbf{H}_n\mathbf{p} + \mathbf{c}_n - v_1\mathbf{a_1} - v_2\mathbf{a_2} + v_3\mathbf{a}_1 + v_4\mathbf{a}_2 = 0 \tag{23}$$

Yielding the following solution,

$$\mathbf{p} = \frac{1}{2}\mathbf{H}_n^{-1}(v_1\mathbf{a_1} + v_2\mathbf{a_2} - \mathbf{c}_n - v_3\mathbf{a}_1 - v_4\mathbf{a}_2) \tag{24}$$

Considering the slackness condition, there are four possible choices as follows, Case 1: The constraints are met with equality and occurred at extremal points, i.e., $\mathbf{a}_1^T\mathbf{p} = 0$ or $p_{\max}$ and $\mathbf{a}_2^T\mathbf{p} = 0$ or $p_{\max}$. In this case, the optimum vector $\mathbf{p}$ can be among one of the four possible choices of $\mathbf{p} = [0,0]$, $[0, p_{\max}]$, $[p_{\max}, 0]$ or $[p_{\max}, p_{\max}]$.

Case 2: In this case, we assume that $\mathbf{a}_1^T\mathbf{p} = 0$ or $p_{\max}$ and $0 < \mathbf{a}_2^T\mathbf{p} < p_{\max}$. Therefore, noting the slackness conditions, one can conclude that $v_2 = v_4 = 0$, where depending on the value of $p_1 = 0, p_{\max}$, one of the following equations are satisfied, yielding the value of p_2.

$$\mathbf{p} = \begin{bmatrix} p_{\max} \\ p_2 \end{bmatrix} = \frac{1}{2}\mathbf{H}_n^{-1}(v_1\mathbf{a}_1 - \mathbf{c}_n) \text{ or } \mathbf{p} = \begin{bmatrix} 0 \\ p_2 \end{bmatrix} = \frac{1}{2}\mathbf{H}_n^{-1}(-v_3\mathbf{a}_1 - \mathbf{c}_n) \quad (25)$$

Case 3: It happens when p_2 gives one of the extremal points of $p_2 = 0, p_{\max}$. By the same token as is stated in Step2, p_1 can be extracted form the following equations,

$$\mathbf{p} = \begin{bmatrix} p_1 \\ p_{\max} \end{bmatrix} = \frac{1}{2}\mathbf{H}_n^{-1}(v_2\mathbf{a}_2 - \mathbf{c}_n) \text{ or } \mathbf{p} = \begin{bmatrix} p_1 \\ 0 \end{bmatrix} = \frac{1}{2}\mathbf{H}_n^{-1}(-v_4\mathbf{a}_2 - \mathbf{c}_n) \quad (26)$$

Case 4: Finally, considering both constraints do not satisfied with equality, i.e., $0 < \mathbf{a}_1^T\mathbf{p} < p_{\max}$ and $0 < \mathbf{a}_2^T\mathbf{p} < p_{\max}$, according to the slackness condition we have $v_1 = v_2 = v_3 = v_4 = 0$, using (24), the optimal value of $\mathbf{p}$ is computed as follows,

$$\mathbf{p} = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \frac{-1}{2}\mathbf{H}_n^{-1}(\mathbf{c}_n). \quad (27)$$

Where the vector $\mathbf{p}$ from the above cases which maximizes (21) would be the optimal solution. Dinkelbach algorithm proceeds until satisfying the convergence condition of step 7.

VI. SIMULATION RESULTS

This section aims at comparing the performance of the proposed sub-channel assignment and power control mechanism in the uplink of a two-cell network to that of proposed in the literature including what is reported in [3]. We assume that the number of sub-channels as well as the number of users are three. The wireless channel has Rayleigh fading distribution.We assume that the direct and cross channel gain values are independent Gaussian random variables and are composed of a path loss and a Gaussian variable representing a Rayleigh flat fading environment. In this case, denoting $h_{i(j)k}^n$ as the channel gain value from the i^{th} user of the j^{th} cell to the k^{th} BTS in the n^{th} sub-channel, it is taken from $h_{i(j)k}^n = \phi^n d^{-\alpha}_{i(j)k}$ where, $d_{i(j)k}$ is the distance between the i^{th} user of the j^{th} cell to the k^{th} BTS and ϕ^n is a Gaussian distribution with unit power. In this case, the power of each channel gain can be computed as $E\left[\left|h^{n}_{i(j)k}\right|^{2}\right] = d^{-2\alpha}_{i(j)k}$ where, $d_{i(j)k}$ is the distance between the i^{th} user of the

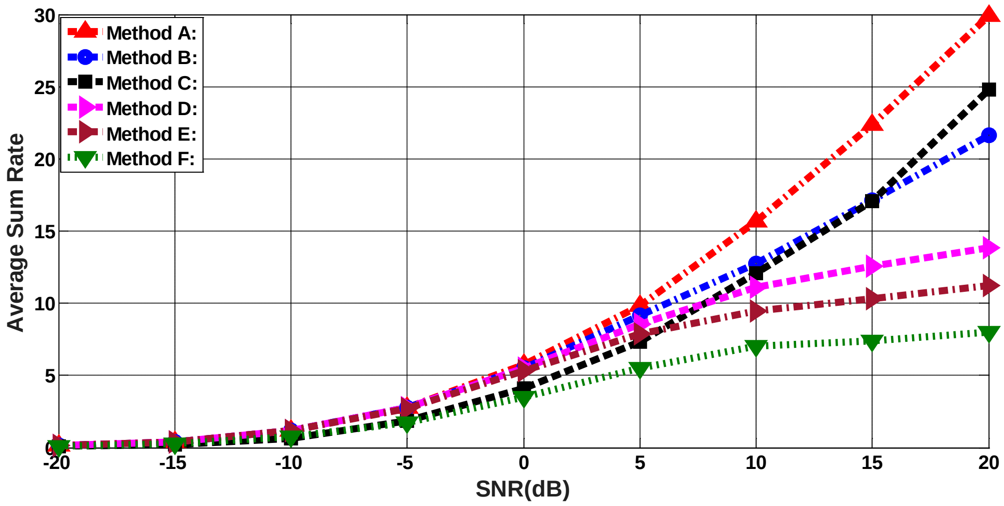

Fig. 2. Comparison of results for the average sum rate

j^{th} cell to the k^{th} BTS. Throughout the simulation, the distance of each user from the corresponding base station is set to 100m, while the distance to the neighboring co-channel base station is set to 500m. and the normalized noise power is set to one.

We use Monte Carlo simulation in which the maximization problem is solved for many realizations of the channel and we represent the average results.

Fig. 2 illustrates the average of sum rate versus SNR for different methods. Method A represents the optimal solution for which channel assignment is performed using exhaustive search and the optimal power control is done according to our proposed optimal power allocation policy. Method B depicts the proposed method for sub-channel assignment based on the Hungarian method and the proposed optimal power control. Method C indicates that sub-channel assignment and power control is done according to the iterative method addressed in [3] through the use of DC-programming approach. the Hungarian method with full power assumption to the simulation results confirming our assertion regarding the advantage of the proposed method in mid SNR region. Moreover, the sub-channel assignment based on exhaustive search method when incorporating full power is also added to the figures, showing the advantage of power control mechanism for SNR values greater than zero. Finally last curve depict random selection strategy with full power control.

Although sub-channel assignment at the high SNR region is not optimal, the results indicate that it yields favorable result even at high SNR region. Moreover, the power allocation policy is optimal. As observed in Fig. 2, at low SNR region, our proposed method outperforms the existing DC-programming and achieves higher data rates as we expected. Also, the average sum-rate of the proposed method coincides that of the exhaustive search with much lower complexity. Also, Method E depicts the Hungarian method with full power assumption confirming our assertion regarding the advantage of the proposed method in mid SNR region. Moreover, Method D illustrates the performance of sub-channel assignment based on exhaustive search method when incorporating full power, showing the advantage of power control mechanism for SNR values greater than one (0dB). Finally, Method E represents the random selection strategy with full power control.

Although sub-channel assignment at the high SNR region is not optimal, the results indicate that it yields favorable result even at high SNR region. Moreover, the power allocation policy is optimal. As observed in Fig. 2, at low SNR region, our proposed method outperforms the existing DC-programming and achieves higher data rates as we expected. Also, the average sum-rate of the proposed method coincides that of the exhaustive search with much lower complexity.

VII. CONCLUSION

This paper proposes a joint uplink sub-channel assignment and power control mechanism which is close-to-optimal at low SNR region. To this end, the optimization problem is divided in two steps. First, the sub-channel assignment is selected according to the Hungarian method and then the power of each user is devised through solving a quadratic fractional optimization problem. Numerical results indicate that at low to mid SNR region, the proposed method outperformed existing methods addressed in the literature.

REFERENCES

[1] D. Tse and P. Viswanath, Fundamentals of wireless communications. Cambridge University Press, 2005

[2] D. W. K. Ng, E. S. Lo and R. Schober, “Energy-Efficient Resource Allocation for Secure OFDMA Systems,” *IEEE Trans. Vehicular Technology*, vol. 61, no. 6, pp. 2572-2585, July 2012.

[3] B. Khamidehi, A. Rahmati and M. Sabbaghian, “Joint Sub-Channel Assignment and Power Allocation in Heterogeneous Networks: An Efficient Optimization Method,” *IEEE Communications Letters*, vol. 20, no. 12, pp. 2490-2493, Dec. 2016.

[4] W. Jing, Z. Lu, X. Wen, Z. Hu and S. Yang, “Flexible Resource Allocation for Joint Optimization of Energy and Spectral Efficiency in OFDMA Multi-Cell Networks,” *IEEE Communications Letters*, vol. 19, no. 3, pp. 451-454, March 2015.

[5] D. T. Ngo, S. Khakurel and T. Le-Ngoc, “Joint Sub-channel Assignment and Power Allocation for OFDMA Femtocell Networks,” *IEEE Trans. Wireless Communications*, vol. 13, no. 1, pp. 342-355, January 2014.

[6] S. Jangsher and V. O. K. Li, “Resource Allocation in Moving Small Cell Network,” *IEEE Trans. Wireless Communications*, vol. 15, no. 7, pp. 4559-4570, July 2016.

[7] K. Shen and W. Yu, “A coordinated uplink scheduling and power control algorithm for multi-cell networks,” *49th Asilomar Conference on Signals, Systems and Computers*, Pacific Grove, CA, 2015, pp. 1305-1309.

[8] K. Shen and W. Yu, “Load and interference aware joint cell association and user scheduling in uplink cellular networks,” *IEEE 17th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, Edinburgh, 2016, pp. 1-5.

[9] Y. Sun, D. W. K. Ng, Z. Ding and R. Schober, “Optimal Joint Power and Subcarrier Allocation for MC-NOMA Systems,” *IEEE Global Communications Conference (GLOBECOM),* Washington, DC, 2016, pp.1-6.

[10] A. Khalili, S. Akhlaghi, M. Mirzaee, “Asymptotic Close To Optimal Joint Resource Allocation and Power Control in the Uplink of Two-cell Networks,” arXiv:1711.07913.2017

[11] T. Wang and L. Vandendorpe, “Iterative Resource Allocation for Maximizing Weighted Sum Min-Rate in Downlink Cellular OFDMA Systems,” *IEEE Trans. Signal Processing*, vol. 59, no. 1, pp. 223-234, Jan. 2011.

[12] S. Boyd and L.Vandenberghe, Convex Optimization. Cambridge university press, 2004.

[13] H.W. Kuhn, On the origin of the Hungarian Method, History of mathematical programming; a collection of personal reminiscences (J.K. Lenstra, A.H.G. Rinnooy Kan, and A. Schrijver, eds.), North Holland, Amsterdam, 1991, pp. 77-81.

[14] M. Mirzaee; S. Akhlaghi; G. Aghaei, “Asymptotic Close to Optimal Resource Allocation in Centralized Multi-band Wireless Networks,” *Journal of Communication Engineering*, vol. 2, no. 4, pp. 404-409, Summer and Autumn 2013.

[15] J. Gotoh, and H. Konno, “Maximization of the ratio of two convex quadratic functions over a polytope,” *Computational Optimization and Applications*, vol. 20, no. 1, pp. 43-60, Oct. 2001.

[16] M. Mirzaee, S. Akhlaghi, “Secrecy Capacity of Two-Hop Relay Assisted Wiretap Channels,” *Wireless Personal Communications,* vol. 94, no. 4, pp. 2901-2923, June 2017.

[17] W. Dinkelbach, “On Nonlinear Fractional Programming,” Management Science, vol. 13, no. 7, pp. 492-498, March 1967.

[18] R.G. Ródenas M. L. López D. Verastegui, “Extensions of Dinkelbach's algorithm for solving non-linear fractional programming problems,” *Operations Research & Decision Theory*, vol.7, no.1, pp. 33-70, June 1999