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Robust Phase I monitoring of Poisson Regression Profiles in Multistage Processes

Fatemeh Sogandi [1*]

[1] *Department of Industrial Engineering, Faculty of Engineering, University of Torbat Heydrieh, Mashhad, Iran*

*** Corresponding Author:** Fatemeh Sogandi *(Email: f.sogandi@torbath.ac.ir)*

Abstract *–The challenge of monitoring and controlling processes where the response variable follows a Poisson distribution, rather than a normal distribution, is addressed in this paper. The significant impact of outliers on model estimators, which subsequently affects the performance of control charts, is also highlighted. To overcome these challenges, the use of robust estimators for monitoring profiles and enhancing control chart efficiency is proposed. Furthermore, as industries increasingly adopt multistage processes in manufacturing instead of a single stage, monitoring these processes becomes essential. Therefore, two robust control charts, namely the* $T^2_{I_{u_{js}}}$ *and* $T^2_{R_{u_{js}}}$ *control charts, designed for Poisson regression profiles in multistage processes, are proposed. The efficiency of the proposed control chart is assessed using the signal probability criterion and its performance is compared to that of a classic control chart in Phase I. Extensive simulation studies are conducted to appraise the performance of these monitoring schemes under different shifts and stages, considering contamination scenarios as well. Based on the simulation results, it is found that the control chart using robust estimators provides better performance compared to the classic control chart, demonstrating the effectiveness of the proposed method. Additionally, a real example is presented to further evaluate the performance of this approach.*

Keywords– Poisson regression profiles, Multistage processes, Phase I, Robust control chart.

I. INTRODUCTION

Profile monitoring is an emerging research field in statistical process monitoring. It is also applied in many applications, as noted by He et al. (2019) and Prabhu and Runger (1997). In this respect, Jones et al. (2021) provided a manual for professionals about monitoring profiles using parametric, nonparametric, and semiparametric methods. As the first review paper, Woodall (2007) surveyed significant contributions up to 2007. Moreover, Maleki et al. (2018) presented comprehensive and classified papers in this area from 2008 up to 2018. Afterwards, many researhere studied diffrent types of linear profiles, such as Chang and Chen (2020), Yao et al. (2020), Yeganeh et al. (2023), Haq et al. (2022), Ghashghaei and Amiri (2017), and Haq (2022). With incomplete samples within profiles, Fallahdizcheh and Wang (2022) provided a transfer learning framework that extracts inter-relationships between profiles by modeling them as multiple output Gaussian processes, and a designed covariance structure was given to increase the computational load in optimality of the multi-output Gaussian process parameters. Ahmadi Karavigh and Amiri (2022) and Yeganeh et al. (2021) utilized different run rules to control the linear profiles. Saeed et al. (2018) designed a memory-based structure that uses progressive means to efficiently monitor linear profile parameters simultaneously. Ghashghaei et al. (2019) and Sabahno and Amiri (2023) proposed some novel monitoring schemes to control both the

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mean vector and covariance matrix of multivariate multiple linear profiles at the same time. Khalili and Noorossana (2022) took into account the correlation of autocorrelated multivariate multiple linear profiles using a multivariate linear mixed model. Also, there are several index works on monitoring schemes for non-parametric profiles, for instance, Abbasi et al. (2022), Zhou and Qiu (2022), Nassar and Abdel-Salam, (2021), and Nasiri et al. (2022). Recently, some works proposed profile monitoring schemes for different types of profiles using artificial neural networks and machine learning approaches, for instance, Yeganeh and Shadman (2021) and Yeganeh et al. (2022a, 2022b). Ding et al. (2023) surveyed a model for a profile that takes into account various factors using the Gaussian process. This model was used to predict what the profile would be like. Based on the differences between the actual and predicted profiles, two control charts were created. More recently, Li and Tsai (2023) developed control charts for profile monitoring of within-profile correlations using the dispersion process model. In Phase I for non-linear profiles, Nie et al. (2021) proposed a new monitoring scheme based on a modified Hausdorff distance algorithm and clustering analysis.

Numerous profiles have response variables that belong to exponential family distributions. To model such profiles, generalized linear models (GLMs) are utilized, and they are commonly referred to as GLM-based profiles. As a pioneering work, Yeh et al. (2009) utilized logistic regression models for binary profiles. In this field, Mohammadzadeh et al. (2021) suggested a monitoring scheme for logistic profiles using a variable sample interval approach. Also, Maleki et al. (2022) conducted a Phase II monitoring scheme for logistic regression profiles using a parameter estimation method. After that, Cheema et al. (2023) applied monitoring of the logistic regression profiles to deal with the challenge of COVID-19 epidemiology. Mammadova and Ozkale (2021) designed some monitoring scheme using the ridge deviance residuals for Poisson and COM–Poisson regression profiles under a multicollinearity problem. Then, Mammadova and Ozkale (2023) conducted a study where they compared deviance and ridge deviance residual-based control charts to monitor Poisson regression profiles under the multicollinearity problem. For the profile monitoring of Poisson distribution, a wavelet approach was utilized by Piri et al. (2021). Amiri et al. (2015) examined binomial and Poisson response profiles in Phase I monitoring using methods including the F method, Hotelling T^2 statistic, and the Likelihood Ratio Test (LRT). Shadman et al. (2015) continued this research by utilizing a change point method to monitor the same profiles. Sogandi and Amiri (2017) suggested a monotonic change point estimation method for GLM-based profiles in Phase II monitoring. Furthermore, Qi et al. (2016) extended a control chart to monitor GLM-based profiles using weighted LRTs. To find additional examples of GLM-based profiles, refer to Koosha and Amiri (2013), Amiri et al. (2018), Maleki et al. (2017), and others.

Usually, the quality characteristic at a stage in the process is affected by the quality characteristic of previous stages. However, ignoring the impact of previous stages on the quality of a product in a multistage process can result in inaccurate monitoring results and make it challenging to determine which stage caused the alarm. Specific multistage process monitoring methods, such as the regression-adjusted control chart, the cause-selecting control chart, and linear State-Space Models (SSMs), were extended to address this issue. Examples of researchers who have used these methods include Tsung et al. (2008) and Asadzadeh et al. (2008). Some monitoring schemes adopt multistage processes in a linear SSM based on engineering knowledge. For example, Sogandi et al. (2019, 2021) proposed a Bernoulli SSM to monitor multistage healthcare processes during Phase I and II, respectively. Regarding profile monitoring in multistage processes, most studies focus on simple linear profiles, such as those examined by Khedmati and Niaki (2016a, 2016b). For monitoring general linear profiles, Khedmati and Niaki (2017) applied a T^2 chart and a control chart using a likelihood-ratio test for a multistage piston manufacturing line. They also proposed a monitoring procedure for general linear profiles with autocorrelation between profiles. Recently, Derakhshani et al. (2020, 2021) provided control charts for Poisson regression profiles and Bernoulli regression profiles in multistage processes, respectively. Bahrami et al. (2021) used a Multivariate Exponentially Weighted Moving Average (MEWMA) for monitoring multivariate simple linear profiles in multistage processes.

In these references, model parameters at each stage are often estimated by methods that work appropriately in the absence of outliers. However, in many applications, there may exist some outliers due to different reasons, such as measurement error. If traditional parameter estimation techniques are utilized when there are outliers, the resulting estimates of the model parameters would be inaccurate, which in turn would lead to suboptimal performance of the control charts. To address these challenges, several control charts have been designed using robust estimators for process monitoring. In the field of robust methods for profile monitoring, Zi et al. (2012) proposed a robust distribution-free method for monitoring linear profiles based on rank-based regression. In 2014, Ebadi and Shahriari (2014) proposed applying two effective techniques, namely Huber and Bi-square M-estimators, for determining the parameters of uncomplicated linear profiles in Phase I. Kamranrad and Amiri (2016) extended a robust monitoring scheme for auto-correlated simple linear profiles in Phase II, while Shahriari et al. (2016) used nonparametric approachs for robust parameter estimation of complicated profiles in Phase I. Shahriari and Ahmadi (2017) proposed a robust method for estimating complicated profiles using wavelets, and Ahmadi et al. (2018) suggested a robust wavelet-based profile monitoring. Furthermore, Hakimi et al. (2017) employed classical method and robust methods that relied on two M-estimators for determining logistic regression profile parameters. Kordestani et al. (2020) utilized three robust M, S, and MM - estimators to monitor multivariate simple linear profiles. After that, Moheghi et al. (2021) surveyed robust estimation to monitor GLM-based profiles in persense of outliers. More recently, Salam (2022) has suggested a novel robust multivariate CUSUM control chart known as a semi-parametric technique for conducting Phase II profile monitoring based on linear mixed models.

Several works have focused on multistage process monitoring. However, few works have concentrated on robust estimators in multistage processes. For example, Asadzadeh and Aghaie (2009) suggested a robust cause-selecting chart using Huber's M-estimator in Phase I parameter estimation. In other words, they utilized a suitable weighting function to decrease the impacts of outlying points concerning the data center. In a two-stage process, Hassanvand et al. (2019) used two robust M-estimators, Huber's and bi-square, to estimate model parameters in a simple linear profile and decrease the effect of outliers. Khedmati and Niaki (2022) proposed two robust estimators for robust parameter estimation of simple linear profiles in multistage processes with contamination in historical data in Phase I monitoring.

Despite numerous studies on GLM-based profile monitoring and robust estimation separately, no research has focused specifically on robust estimation for GLM-based profiles in multistage processes. Given the application of the Poisson distribution in various fields such as finance, biology, and engineering, this paper proposes Phase I monitoring Poisson regression profiles using a robust estimation method in multistage processes. In Phase I monitoring, which has been less addressed, a comparison is made between the estimators acquired from MLE and the robust method. Furthermore, comprehensive simulation results are provided to show the performance of the control chart under different contamination, changes, and stages. This research is organized as follows. In Section 2, the problem modeling, assumption, and classic parameter estimation are discussed. The parameter estimation of Poisson regression profiles using a robust estimator is introduced in Section 3. The next section designs the proposed robust control charts for Poisson regression profile monitoring in multistage processes in Phase I. Section 5 is dedicated to the extensive simulation studies and a numerical example to evaluate the performance of the proposed control charts. To evaluate the performance of the method, a real example is also presented in Section 6. Conclusions are made in the last section.

II. POISSON REGRESSION PROFILES IN MULTISTAGE PROCESSES

This section outlines two main topics: the underlying modeling and assumptions addressed in this study and the classic parameter estimator for Poisson regression profiles in multistage processes during Phase I.

A. Underlying modeling and assumptions

The underlying process consists of multiple stages, and quality of stages is determined by a Poisson regression profile. This implies that a Poisson regression model is utilized to represent the connection between the response variable and independent variables. Assume that *m* samples, with the size *n* are gathered at the stage *s* of a multistage process. The values of the independent variables in every profile are predetermined and definite. Additionally, it is assumed that there is a cascade effect between the stages. Also, (x_{ijs}, y_{ijs}) denotes observation $i = 1,\ 2,\ ...,\ n$ for sample or profile $j = 1,\ 2,\ ...,\ m$ and stage $s = 1,\ 2,\ ...,\ S$. Hence, regarding the link function of GLM, which links the average of the response variable to the explanatory variables, a Poisson regression profile in a multistage process can be represented as follows:

$$\begin{aligned}
\lambda_{ij1} &= \exp(\mathbf{x}_{ij1}^T \boldsymbol{\beta}_{j1}) \\
\lambda_{ijs} &= \exp(\mathbf{x}_{ijs}^T \boldsymbol{\beta}_{js}) + \Phi \lambda_{ij(s-1)} \\
\lambda_{ijs} &= \Phi^{s-1} \lambda_{ij1} + \sum_{r=2}^{s} \Phi^{s-r} \exp(\mathbf{x}_{ijr}^T \boldsymbol{\beta}_{jr}),
\end{aligned} \tag{1}$$

in which $\mathbf{x}_{ijs} = \left(x_{1ijs}, x_{2ijs}, ..., x_{qijs}\right)$ and $\boldsymbol{\beta}_{js} = \left(\beta_{0js}, \beta_{1js}, ..., \beta_{qjs}\right)$ shows independent variables and the vector of parameter for j^{th} sample in stage s, respectively. Note that Φ is the autocorrelation coefficient of the process. These design structures for modeling are partly similar to those presented by Derakhshani et al. (2020). Since this research copes with the Phase I monitoring, the in-control process model is unknown. Moreover, $E\left(y_{ijs}\right) = \lambda_{ijs}$ is the average of the response variable for the observation i in the j^{th} sample at stage s. To decrease or remove the cascade property between different stages, the U statistic is applied. Based on this statistic in the first stage and stage s for the j^{th} sample:

$$\begin{aligned}
\mathbf{u}_{j1} &= \hat{\boldsymbol{\beta}}_{j1} \\
\mathbf{u}_{js} &= \hat{\boldsymbol{\beta}}_{js} - \sum_{s(s-1)} \sum_{(s-1)(s-1)}^{-1} \hat{\boldsymbol{\beta}}_{j(s-1)},
\end{aligned} \tag{2}$$

where $\hat{\boldsymbol{\beta}}_{js}$ denotes the estimate of the model parameters of sample j in stage s. Besides, $\sum_{(s-1)(s-1)}$ shows the variance-covariance matrix of $\hat{\boldsymbol{\beta}}_{j(s-1)}$, and $\sum_{s(s-1)}$ represents the covariance matrix between $\hat{\boldsymbol{\beta}}_{js}$ and $\hat{\boldsymbol{\beta}}_{j(s-1)}$. According to Jearkpaporn et al. (2007), applying U statistic causes the current stage is not impacted by the previous stages. After that, some researchers like Khedmati and Niaki (2017) also used U statistic for the profile monitoring. It is worth mentioning that the $U_j s$ are independent of each other. In this regard, the average vector and variance matrix of the U statistic in both the initial and s^{th} stages can be described by Eq. (3).

$$\begin{aligned}
\boldsymbol{\mu}_{u_{j1}} &= \boldsymbol{\mu}_{\hat{\boldsymbol{\beta}}_{j1}} \\
\boldsymbol{\mu}_{u_{js}} &= \boldsymbol{\mu}_{\hat{\boldsymbol{\beta}}_{js}} - \sum_{s(s-1)} \sum_{(s-1)(s-1)}^{-1} \boldsymbol{\mu}_{\hat{\boldsymbol{\beta}}_{j(s-1)}} \\
\sum_{u_{j1}} &= \sum_{11} \\
\sum_{u_{js}} &= \sum_{ss} - \sum_{s(s-1)} \sum_{(s-1)(s-1)}^{-1} \sum_{(s-1)s}.
\end{aligned} \tag{3}$$

B. Classic parameter estimation

For the sake of simplicity, the independent variable is assumed to have fixed values for the stages of the responses. Hence, $\mathbf{x}_{ijs} = \mathbf{x}_i$ for all profiles and stages. Consider a scenario with a set of n observations and q independent variables that denoted by $\mathbf{x}_i = \left(x_{1i}, x_{2i}, \ldots, x_{qi}\right)$. The quality characteristics for each of these independent variables, y_{ijs}, are distributed as a Poisson distribution with the parameter λ_{ijs}. One of the most popular link functions for the exponential family of distributions is the Logit link function, as shown in Eq. (4).

$$g(\lambda_{ijs}) = \mathbf{x}_i^T \boldsymbol{\beta}_{js}. \tag{4}$$

According to the link function, the mean of y_{ijs}s for Poisson regression profiles is as follows:

$$\lambda_{ijs} = \exp\left(\mathbf{x}_i^T \boldsymbol{\beta}_{js}\right), \tag{5}$$

in which $\boldsymbol{\beta}_{js}$ is estimated using maximum likelihood method for each profile and stage. Based on this approach, the likelihood function for response variables is given in Eq. (6) for each profile and stage.

$$L\left(\lambda_{js}, y_{js}\right) = \prod_{i=1}^{n} \frac{e^{-\lambda_{ijs}} \lambda_{ijs}^{y_{ijs}}}{y_{ijs}!}. \tag{6}$$

After taking the logarithm of $L\left(\lambda_{js}, y_{js}\right)$, Eq. (7) is obtained.

$$L\left(\lambda_{js}, y_{js}\right) = \sum_{i=1}^{n} \ln\ \mathrm{e}^{-\lambda_{ijs}} + \sum_{i=1}^{n} \ln\ \lambda_{ijs}^{\ y_{ijs}} + \sum_{i=1}^{n} \ln\ \left(\frac{1}{y_{ijs}!}\right). \tag{7}$$

Link function can be substituted in Eq. (7).

$$L\left(\lambda_{js}, y_{js}\right) = -\sum_{i=1}^{n} \mathrm{e}^{\mathbf{x}_i^T \boldsymbol{\beta}_{js}} + \sum_{i=1}^{n} y_{ijs} \mathbf{x}_i^T \boldsymbol{\beta}_{js} - \sum_{i=1}^{n} \ln\ \left(y_{ijs}!\right). \tag{8}$$

Eq. (9) is obtained by taking the derivative concerning $\boldsymbol{\beta}_{js}$ for values of *j* and *s*.

$$\frac{\partial L(\lambda, y)}{\partial \boldsymbol{\beta}_{js}} = \mathbf{x}^T \left(\mathbf{y}_{js} - \boldsymbol{\mu}_{js}\right), \tag{9}$$

in which $\boldsymbol{\mu}_{js} = E\left(\mathbf{y}_{js}\right) = \left(\lambda_{1js}, \lambda_{2js}, \ldots, \lambda_{njs}\right)^T$. According to Yeh et al. (2009), the MLE of parameters follows a *q*-variate Normal distribution asymptotically, $\mathbf{N}\left(\boldsymbol{\beta}_{js}, \left(\mathbf{X}_s^T \mathbf{W}_{js} \mathbf{X}_s\right)^{-1}\right)$ where $\mathbf{W}_{js} = diag\left(\lambda_{1js}, \lambda_{2js}, \ldots, \lambda_{njs}\right)$ and $\mathbf{X} = \left(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n\right)^T$. So far, this classic estimation method has been applied in many works such as Amiri et al. (2015). However, none of them have considered the impact of outlier data in Phase I monitoring of multistage processes.

III. ROBUST ESTIMATION OF POISSON REGRESSION PROFILES

As previously stated, historical data often contains outliers that can indicate special causes for production conditions that are out of control. To decrease the impact of outliers on parameter estimation, robust estimators are utilized. In situations where only a small number of contaminants exist in data, two methods can be employed: (a) removing the sample from the historical data set, but this may result in the loss of valuable information, or (b) keeping the sample, but this may lead to inaccurate estimates for model parameters. However, robust estimators can overcome these drawbacks as they are designed to be insensitive to contamination. This results in partly accurate estimates of parameters, regardless of whether the data is free of outliers or contaminated with them. One approach recommended by Huber and Ronchetti (2012) to decrease the impact of outliers is to utilize a limited function, known as the ρ function, in the estimation process. This approach led to the development of M-estimator which is a type of robust estimator that generalize the MLE. By assigning proper weights to the observations based on an appropriate function, outliers are allocated smaller weights with outliers. This decreases the impact of outliers on the estimators. MLEs for GLM parameters are highly sensitive to contamination. To address this issue, Cantoni and Ronchetti (2001) proposed robust estimators for GLM by extending the quasi-likelihood estimators. In this method, the quasi-likelihood estimator for parameters for each profile and stage can be obtained by solving a system of equations.

$$\sum_{i=1}^{n} \boldsymbol{\Psi}(y_{ijs}, \eta_{ijs}) = \mathbf{0}, \tag{10}$$

in which η_{ijs} is equal to λ_{ijs}^{-1}, and $\boldsymbol{\Psi}(y_{ijs}, \eta_{ijs}) = \upsilon(y_{ijs}, \eta_{ijs})\omega(x_{ijs})\eta_{ijs}{}' - \alpha(\boldsymbol{\beta}_{js})$ and $\alpha(\boldsymbol{\beta}_{js}) = \mathrm{E}\left[\upsilon(y_{ijs}, \eta_{ijs})\right]$ with the expectation taken with respect to the conditional distribution of $y|\mathbf{x}$, $\upsilon(.,.)$ are weight functions. For sample j in stage s, it is considered that $\upsilon(y_{ijs}, \eta_{ijs}) = \Psi_c\left(r_{ijs}\right)$ in which $r_{ijs} = \dfrac{y_{ijs} - \eta_{ijs}}{V^{1/2}\left(\eta_{ijs}\right)}$ denote the Pearson residuals and Ψ_c show the Huber function given in

$$\Psi_c(r) = \begin{cases} r & |r| \le c \\ c\,\mathrm{sign}(r) & |r| > c. \end{cases} \tag{11}$$

Assume that $k_1 = \left\lfloor \eta_{ijs} - cV^{1/2}\left(\eta_{ijs}\right) \right\rfloor$ and $k_2 = \left\lfloor \eta_{ijs} + cV^{1/2}\left(\eta_{ijs}\right) \right\rfloor$. Regarding to $\mathrm{E}\left[y_{ijs}\right] = V\left(\eta_{ijs}\right) = \eta_{ijs}$ in the Poisson regression profile,

$$\mathrm{E}\left[\Psi_c\left(\frac{y_{ijs} - \eta_{ijs}}{V^{1/2}\left(\eta_{ijs}\right)}\right)\right] = c\left(\mathrm{P}\left(\mathrm{Y}_{ijs} \ge k_2 + 1\right) - \mathrm{P}\left(\mathrm{Y}_{ijs} \le k_1\right)\right) + \frac{\eta_{ijs}}{V^{1/2}\left(\eta_{ijs}\right)}\left[\mathrm{P}\left(\mathrm{Y}_{ijs} = k_1\right) - \mathrm{P}\left(\mathrm{Y}_{ijs} = k_2\right)\right]. \tag{12}$$

Hence, the proposed approach by Cantoni and Ronchetti (2001) is applied for robust parameter estimation. R software is utilized to conduct the MLE and robust estimation for parameters of Poisson regression profiles. In other words, “stats” and “robustbase” packages are applied to obtain MLE and robust estimators for each stages, respectively. To calculate the parameters of Poisson regression profiles by MLE, the stats package's glm function is employed alongside the robustbase package's glmrob function to calculate robust estimates of the parameters of Poisson regression profiles. Besides, the option Mqle is applied based on the function glmrob. Mqle fits a generalized linear model using Huber type robust estimators. Note that the option weights.on.x= "robCov" makes sense if all explanatory variables are continuous. Moreover, a auxiliary function is applied as user interface for glmrob fitting when method Mqle is applied.

IV. PROPOSED CONTROL CHART

Multistage processes involve several dependent stages where each stage's output serves as the input for the subsequent stage. Yeh et al. (2009) proposed five T^2 control charts for monitoring logistic profiles. After that, Amiri et al. (2009) used two of the best T^2 control charts proposed by Yeh et al. (2009) which are $T^2_{I_{u_{js}}}$ and $T^2_{R_{u_{js}}}$ control charts. Hence, in this study these control charts based on U statistic are used to monitor the Poisson regression profiles in multistage processes. On this subject, Figure 1 shows a flowchart of the $T^2_{I_{u_{js}}}$ robust charts for Poisson regression profiles in multistage processes.

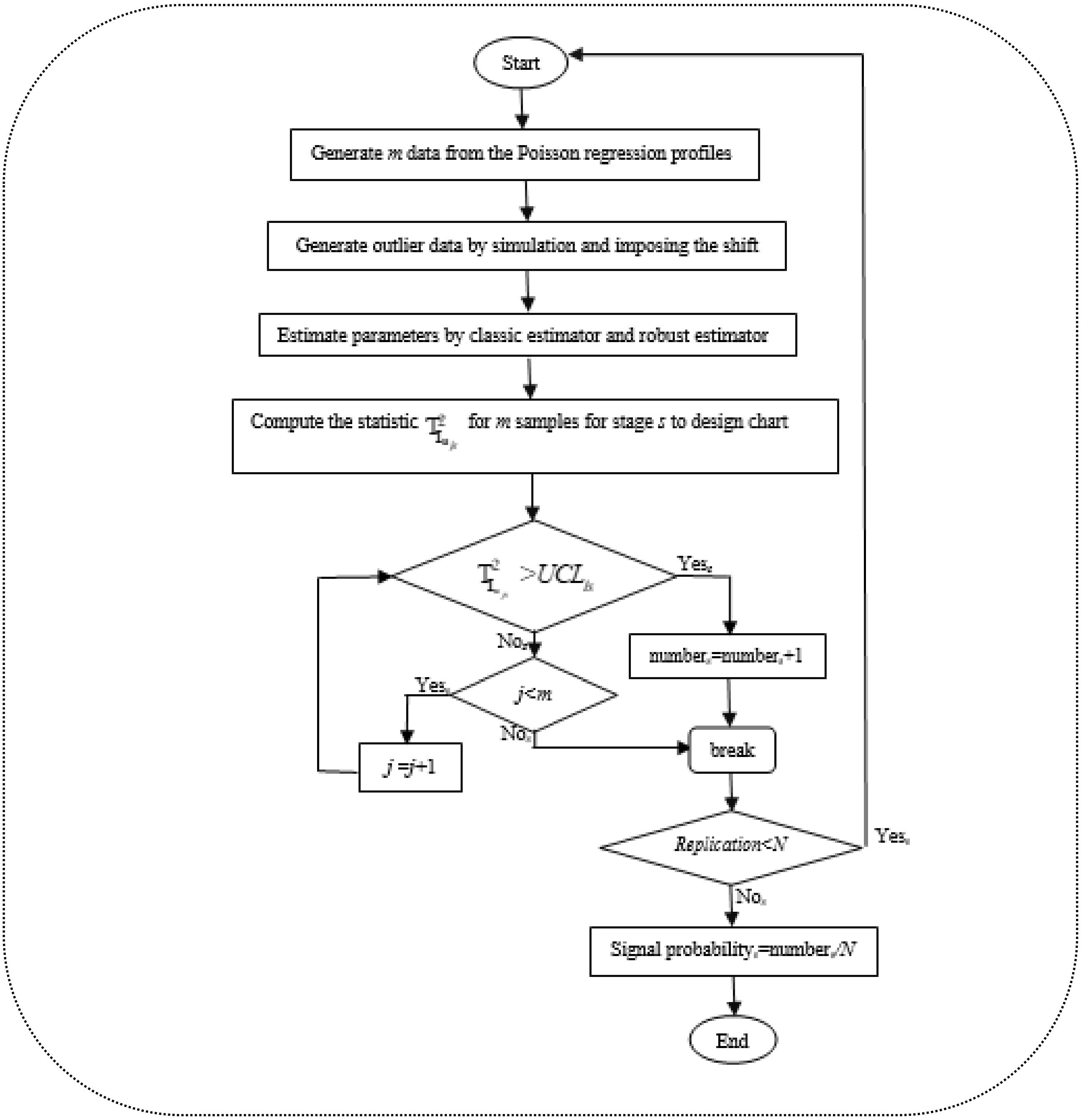

Figure 1. A schematic algorithm for proposed Phase I scheme of Poisson regression profiles in a s-satge process.

A. $T^2_{I_{u_{js}}}$ Control chart

Hauck et al. (1999) proposed the *U* statistic to address the nature od the multistage process monitoring. They showed the *U* control chart is helpful when the magnitude of the shift is large. In this developed control chart, at first, the *U* statistic of the *j*th profile is computed using Eq. (2), then the modified $T^2_{I_{u_{js}}}$ statistic which is T^2 based on the sample average and intra-profile pooling is computed by Eq. (14):

$$T^2_{I_{u_{js}}} = \left(\hat{\mathbf{u}}_{js} - \bar{\mathbf{u}}_s\right)^T \mathbf{S}^{-1}_{I_{us}} \left(\hat{\mathbf{u}}_{js} - \bar{\mathbf{u}}_s\right), \qquad j = 1,2,...,m, \;\; s = 1,2,...,S, \tag{13}$$

where $\mathbf{S}_{I_{us}} = \frac{1}{m}\sum_{j=1}^{m} \text{var}(\hat{\mathbf{u}}_{js}),\ \bar{\mathbf{u}}_s = \frac{1}{m}\sum_{j=1}^{m}\hat{\mathbf{u}}_{js}$, and the Upper Control Limit $\left(UCL_{Is}\right)$ of $T^2_{I_{u_{js}}}$ control chart in stage s^{th} is achieved to meet a given overall signal probability.

B. $T^2_{R_{u_{js}}}$ Control chart

Based on Yeh et al. (2009), T^2 can be developed based on on the sample average and moving ranges for Poisson multistage processes. In this respect, $T^2_{R_{u_{js}}}$ statistic can be achieved by Eq. (14) for each stage and sample.

$$T^2_{R_{u_{js}}} = \left(\hat{\mathbf{u}}_{js} - \bar{\mathbf{u}}_s\right)^T \mathbf{S}^{-1}_{R_{us}} \left(\hat{\mathbf{u}}_{js} - \bar{\mathbf{u}}_s\right), \qquad j = 1,2,...,m, \;\; s = 1,2,...,S, \tag{14}$$

where $\mathbf{S}_{R_{us}} = \frac{1}{2(m-1)}\sum_{j=1}^{m}(\hat{\mathbf{u}}_{j+1,s} - \mathbf{u}_{j,s})(\hat{\mathbf{u}}_{j+1,s} - \mathbf{u}_{j,s})^T$ for each stage and sample.

V. PERFORMANCE OF THE PROPOSED METHOD

This section examines the implementation of the proposed control charts through some simulation studies. The response variable of each stage is assessed using the Poisson regression profile, implying that the response variable of the initial stage influences that of the subsequent stage. Let the following Poisson regression profiles for both stages of a two-stage process for i=1, 2, ..., n and j=1, 2, ..., m:

$$\lambda_{ij1} = \exp(\beta_{0j1} + \beta_{1j1}x_{ij1})$$
$$\lambda_{ij2} = \exp(\beta_{0j2} + \beta_{1j1}x_{ij2}) + \Phi\lambda_{ij1},$$

To appraise the performance of the control charts, a numerical example is surveyed. consider a 2-stage process for generating a product in which quality characteristics of two stages are Poisson regression profiles. Φ is equal to 0.5. It is assumed that there is only one independent variable in each stage of the process, and each stage has nine levels of independent variables. 30 Poisson profiles with mean λ_{ij1} and λ_{ij2} are generated for in-control and out-of-control profiles where $\beta_{01} = 3,\ \beta_{11} = 4,\ \beta_{02} = 2,$ and $\beta_{12} = 1$ for all profiles. Moreover, values of independent varaibles are considered as $\mathbf{x}_{j1} = \left(0.1, 0.2, ..., 0.9\right)$ and $\mathbf{x}_{j2} = \left(1, 2, ..., 9\right)$ for the first and second stages, respectively. A percentage of the simulated data is contaminated by shifting the parameters of the Poisson regression profile as $\beta_{out} = \beta_{in} + \delta \times \sigma_{\beta_{in}}$. Then, to appraise the performance of the control charts, the UCLs should be set such that a given probability of Type I error is calculated when there are no outliers in the process,. For classic and robust estimation, the

UCL$_{I1}$ of the $T^2_{I_{u_{js}}}$ chart is set equal to 16.77 and 16.22, respectively to obtain $\alpha = 0.05$ for the first stage. Similarly, in the second stage, the UCL$_{I2}$ is set equal to 20.58 and 18.97, for classic and robust estimation, respectively. Furthermore, to achieve a target signal probability, the UCL$_{R1}$ for the classic estimation method is determined as 19.3 and 17.45 for the first stage. Likewise, for the second stage, the UCL$_{R2}$ values for classic and robust estimation are set to 25.77 and 22.64 respectively to ensure the desired level of control.

To assess robust and classical estimates, various levels of contamination percentages are taken into account using global outliers. The mean and standard deviation of estimates in the Poisson regression profile are then calculated. To simulate global outliers, (*p*) percent of outliers in the data of all profile and the remaining (100-*p*) percent using the pre-specified Poisson regression profile are generated. However, in local outlying, outliers are generated in only some profiles. This distinction is clarified by stating that for local contamination, *q* profiles of all profiles are generated from the contaminated distribution, while the rest are from a clean distribution. Consequently, localized disturbances affect all data in *q* profiles. In the global outlying in the second stage, consider 20 Poisson profiles. Each profile consists of 20 values for explanatory variables, resulting in a total of 400 observations. If a 5 percent contamination in the data is assumed, one observation in all profiles should be replaced with outliers.

The accuracy and standard deviation of the estimators in the presence of outliers are shown in Table 1 based on the simulation studies where Std denotes the standard deviation of estimates in the Poisson regression profile. Hence, the performance of classical and robust estimators under contamination in parameters for the first stage is given in Table 1. To apprise the performance of the two monitoring schemes, its signal probability is calculated after estimating the regression coefficients. It is worth mentioning that shifts are imposed to define the global contamination under different *p*. Shifts represent sustained changes in the process, the outliers are temporary and sporadic deviations from normal behavior. These sporadic deviations are made from different contamination percentages and different shifts in simulation runs. In this respect, the signal probabilities are shown under shifts of different magnitudes in the presence of outliers in parameters of Poisson regression profile for the first stage in Table 2. Similarly, Table 3 presents the signal probabilities for different shift magnitudes in the first stage for $T^2_{R_{u_{js}}}$ control chart, considering the presence of contamination in its parameters. It is worth mentioning that the other shifts and stages have a similar trend, but they are omitted for the sake of conciseness.

The simulation results presented in Table 1 indicate that two estimators produce almost identical results in the absence of contamination. However, the proposed robust estimation method outperforms the classical estimator when contamination is present. This means that the robust estimator gives more appropriate estimates of the model parameters rather than the estimator achieved by the classic approach regardless of outlier and shift magnitudes. Also, Table 2 indicates that if the parameters are contaminated, using a robust control chart will result in significantly better performance than a classical monitoring schemes. In addition, the presence of contamination increases signal probabilities. Furthermore, both estimators exhibit larger signal probability values as the shift size increases.

Comparing Tables 2 and 3 shows that $T^2_{I_{u_{js}}}$ chart performs better than $T^2_{R_{u_{js}}}$ chart in detecting different shifts and contamination.Therfore, the simulations are continued using the $T^2_{I_{u_{js}}}$ control chart to evaluate the robust control chart against the classic one for Phase I of Poisson regression profiles.

Similarly, for the second stage, the performance of classical and robust estimations and the corresponding signal probabilities can be achieved under different contamination. In this respect, the performance of $T^2_{I_{u_{js}}}$ chart is shown for the second stage in Figure 2. The analysis of simulation results is similar to the first stage. Other simulation studies with various shifts, stages, σ and p values also support these results, but they are not included here for brevity. This figure illustrates that the robust estimator performs well in the presence of global outliers, as it effectively minimizes their influence.

Table 1. Performance evaluation of the estimators under contamination in parameters of the Poisson regression profile for the first stage

Parameter		β_{01}					β_{11}		
Method		***Classic***		***Robust***		***Classic***		***Robust***	
p	δ	**Mean**	**Std**	**Mean**	**Std**	**Mean**	**Std**	**Mean**	**Std**
5	**0**	3.1113	1.086	3.1016	0.92	4.1463	1.296	4.1026	0.908
	0.3	3.3043	1.091	3.1096	0.921	4.3393	1.301	4.1106	0.909
	0.6	3.3553	1.119	3.1276	0.923	4.3903	1.329	4.1286	0.911
	0.9	3.4213	1.133	3.1336	0.924	4.4563	1.343	4.1346	0.912
	1.2	3.4843	1.144	3.1826	0.935	4.5193	1.354	4.1836	0.923
	1.5	3.6193	1.152	3.2396	0.946	4.6543	1.362	4.2406	0.934
10	**0**	3.2063	1.108	3.0766	0.919	4.2413	1.318	4.0776	0.907
	0.3	3.4473	1.114	3.1086	0.92	4.4823	1.324	4.1096	0.908
	0.6	3.5033	1.122	3.1406	0.925	4.5383	1.332	4.1416	0.913
	0.9	3.5823	1.127	3.2296	0.932	4.6173	1.337	4.2306	0.92
	1.2	3.7583	1.137	3.3086	0.952	4.7933	1.347	4.3096	0.94
	1.5	3.7763	1.175	3.3696	0.968	4.8113	1.385	4.3706	0.956
15	**0**	3.2403	1.086	3.0646	0.916	4.2753	1.296	4.0656	0.904
	0.3	3.4553	1.1	3.1566	0.926	4.4903	1.31	4.1576	0.914
	0.6	3.5703	1.107	3.2746	0.934	4.6053	1.317	4.2756	0.922
	0.9	3.6143	1.125	3.3336	0.962	4.6493	1.335	4.3346	0.95
	1.2	3.9773	1.135	3.3816	0.964	5.0123	1.345	4.3826	0.952
	1.5	4.1453	1.228	3.5306	0.982	5.1803	1.438	4.5316	0.97
20	**0**	3.2543	1.125	3.0786	0.929	4.2893	1.335	4.0796	0.917
	0.3	3.4433	1.126	3.1336	0.936	4.4783	1.336	4.1346	0.924
	0.6	3.6723	1.135	3.2956	0.94	4.7073	1.345	4.2966	0.928
	0.9	3.9383	1.145	3.4326	0.956	4.9733	1.355	4.4336	0.944
	1.2	4.0343	1.15	3.6806	0.961	5.0693	1.36	4.6816	0.949
	1.5	4.3253	1.19	3.8636	0.966	5.3603	1.4	4.8646	0.954

Table 2. Signal probability of $T^2_{I_{u_{js}}}$ control chart in the presence of contamination in parameters of the Poisson regression profile for the stage 1

Parameter		β_{02}		β_{12}	
Method		Classic	Robust	Classic	Robust
c	Shift				
5	0	0.0062	0.0066	0.0072	0.0078
	0.3	0.3292	0.6176	0.3302	0.6188
	0.6	0.4622	0.6806	0.4632	0.6818
	0.9	0.5642	0.7426	0.5652	0.7438
	1.2	0.6682	0.7726	0.6692	0.7738
	1.5	0.7552	0.8106	0.7562	0.8118
10	0	0.0062	0.0066	0.0072	0.0078
	0.3	0.4452	0.7346	0.4462	0.7358
	0.6	0.6442	0.8356	0.6452	0.8368
	0.9	0.8212	0.9176	0.8222	0.9188
	1.2	0.8902	0.9626	0.8912	0.9638
	1.5	0.9612	0.9886	0.9622	0.9898
15	0	0.0062	0.0066	0.0072	0.0078
	0.3	0.5672	0.7886	0.5682	0.7898
	0.6	0.8032	0.9266	0.8042	0.9278
	0.9	0.9322	0.9826	0.9332	0.9838
	1.2	0.9832	0.9786	0.9842	0.9798
	1.5	0.9932	0.9996	0.9942	1.0008
20	0	0.0062	0.0066	0.0072	0.0078
	0.3	0.6302	0.6576	0.6412	0.7188
	0.6	0.8162	0.8203	0.8172	0.8378
	0.9	0.9492	0.9497	0.9382	0.9488
	1.2	0.9892	0.9906	0.9889	0.9918
	1.5	0.9642	0.9926	0.9987	0.9993

Table 3. Signal probability of $T^2_{R_{u_{js}}}$ Control chart in the presence of contamination in parameters of the Poisson regression profile for the stage 1.

Parameter		β_{02}		β_{12}	
Method		Classic	Robust	Classic	Robust
c	Shift				
5	**0**	0.005382	0.005938	0.007152	0.006898
	0.3	0.299717	0.526112	0.318607	0.566127
	0.6	0.457859	0.650728	0.419204	0.612899
	0.9	0.537416	0.655087	0.52712	0.685242
	1.2	0.587808	0.710061	0.574043	0.763463
	1.5	0.72537	0.759828	0.754441	0.739903
10	**0**	-0.06141	-0.04293	-0.07247	-0.07683
	0.3	0.348009	0.64304	0.375265	0.695514
	0.6	0.573632	0.740365	0.621762	0.807427
	0.9	0.766421	0.894157	0.808605	0.903565
	1.2	0.790286	0.899102	0.880754	0.864666
	1.5	0.938439	0.895893	0.905886	0.97246
15	**0**	-0.06736	-0.00423	-0.02719	-0.00121
	0.3	0.560327	0.712762	0.485365	0.730947
	0.6	0.7104	0.902175	0.736071	0.864095
	0.9	0.868675	0.975204	0.871188	0.921442
	1.2	0.889049	0.944412	0.97402	0.922164
	1.5	0.935776	0.962756	0.963324	0.995466
20	**0**	-0.05201	-0.04885	-0.01923	-0.09165
	0.3	0.554232	0.55886	0.573638	0.675367
	0.6	0.794193	0.764036	0.792158	0.77717
	0.9	0.8693	0.885169	0.901146	0.861101
	1.2	0.985417	0.9247	0.974382	0.989269
	1.5	0.916831	0.958037	0.913956	0.970182

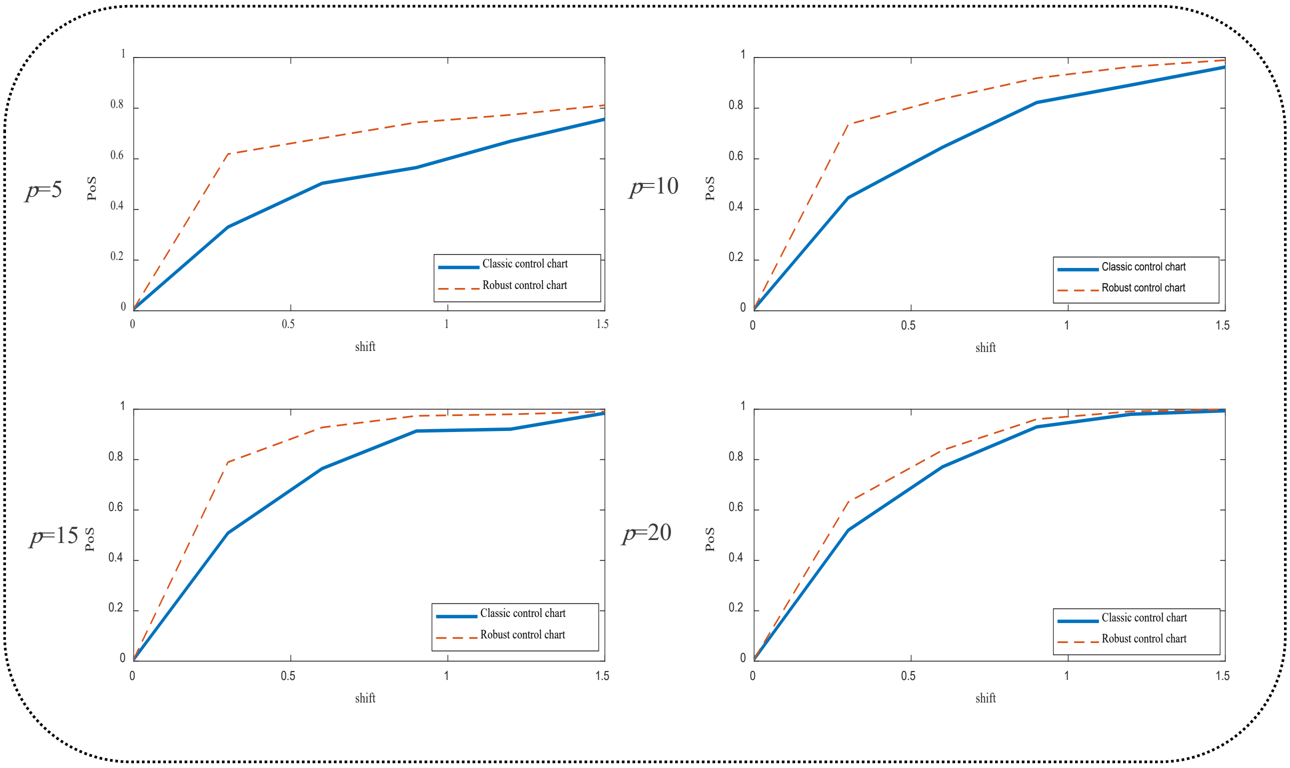

Figure 2. Signal probability of $T^2_{I_{u_{js}}}$ chart for different shifts in β_{02} of the Poisson regression profile for the second stage under p percent contaminated data

Generally, the simulation results indicate that classical and robust estimation methods are pretty similar when applied to uncontaminated data. However, the robust estimator is more effective at reducing the influence of outliers on estimated parameter means. This means that the robust estimator produces values that are closer to the expected parameters than the classical estimator when there are shifts or outlier observations. Additionally, a comparison of the standard deviation between the two methods shows that the robust estimation method outperforms the classical estimator in the presence of outliers. However, note that without contamination, the classical approach for estimating standard deviation may perform slightly better than the robust estimator.

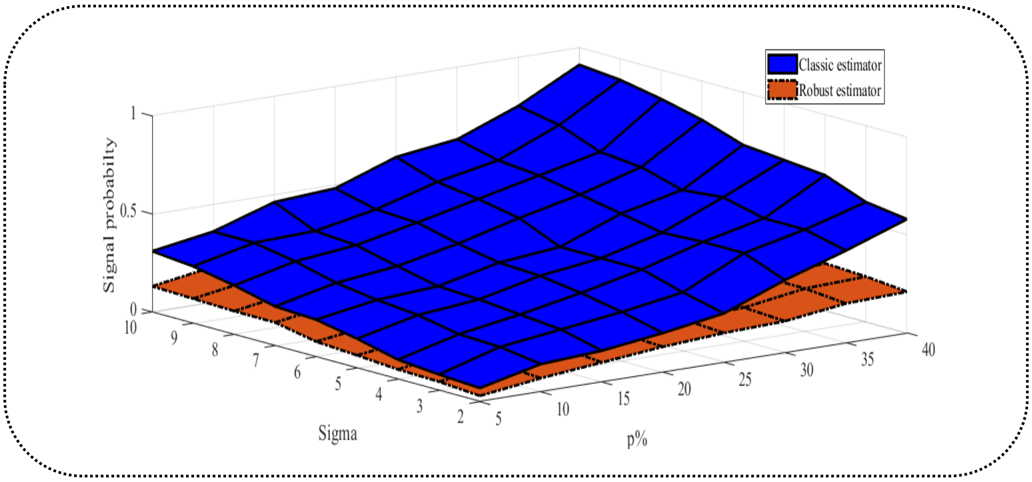

Figure 3. Performance of $T^2_{I_{u_{js}}}$ chart for *p*% outliers in error terms distribution in the second stage.

To survey the impact of contamination on error term variances, a proportion of (1-p) percent of ε_{ij} values follow a Normal distribution with N(0, σ^2), while p percent of the residuals are distributed as a different Normal distribution. This means that a model can be created which assumes that all observations are from a N(0,1) distribution, but in reality, the disturbances may have different probabilities of being drawn from either a N(0,1) or N(0,9) distribution. Figure 3 displays the signal probabilities when there are different shifts and contamination in the variance of error terms in the second stage.

Figure 3 shows that the robust method outperforms the classical approach when shift magnitudes and outlier percentages increase. The classical estimator of parameters is not significantly affected by low contamination in variance of the ε_{ij}'s, however increasing the contaminated error terms variance results in significant differences from the actual value. Additionally, Figure 3 shows that the performance of robust method increases the contamination percentage.

A. A numerical example

This section examines the practical implementation of the proposed methods through a numerical example involving a 2-stage process. The assessment of each stage's quality in the process is conducted using the Poisson regression profile. With the assumptions outlined in Section 4, the profiles at each stage of the process are descibed as follows:

$$\lambda_{ij1} = \exp(3 + 2x_{ij1})$$

$$\lambda_{ij2} = \exp(4 + x_{ij2}) + 0.5\,\lambda_{ij1},\ i = 1,\ 2,\ 3,\ \ldots, 9.$$

The explanatory variables are shown as

$$\mathbf{x}_{j1} = (0.1, 0.2, \ldots., 0.9),\ \mathbf{x}_{j2} = (1, 2, \ldots, 9)$$

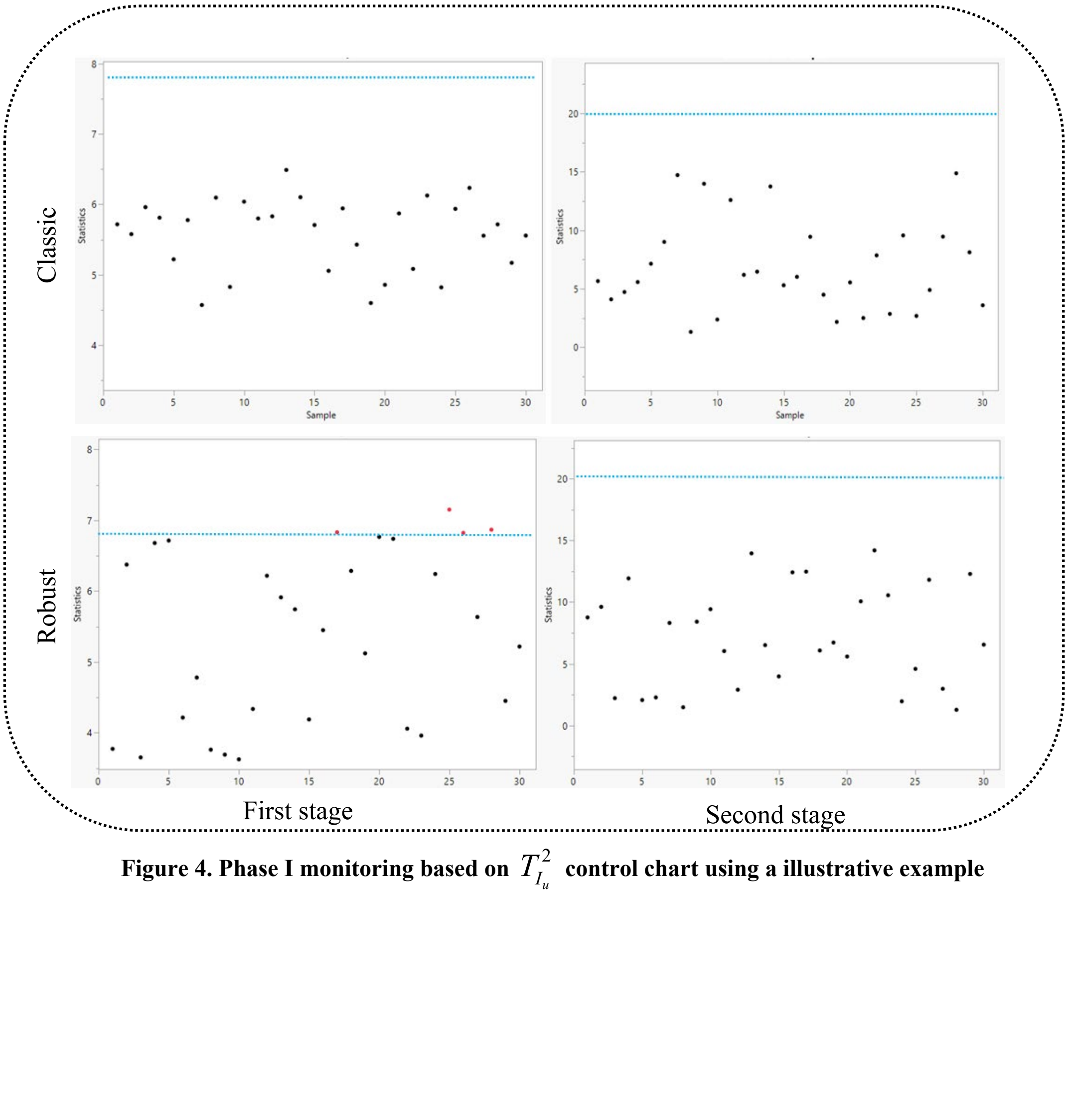

Figure 4. Phase I monitoring based on $T^2_{I_u}$ control chart using a illustrative example

10% of global outliers is considered by imposing changes in the intercept parameter of the Poisson regression profile at the first stage. The profile parameters are estimated using both the classic and robust parameter estimation method. From these estimates, the in-control values of the model parameters are computed for each stage. By employing simulation and considering a Type I error of 5%, the UCL$_{IS}$ of the $T^2_{I_{u_{js}}}$ control chart, the better approach, is calculated. Upon analyzing the classical estimates, it is evident that outliers have an influence on the estimated parameters. To assess the process stability and identify any abnormal profiles, the proposed robust approach is utilized too. The $T^2_{I_{u_{js}}}$ statistics for the control chart, computed for the samples using equation (13), are determined with both the classic and robust estimators. The results of these calculations are plotted in Figure 4. Upon examining the figure, the robust chart indicate some out-of-control signal based on the the red points for the first stage. In contrast, the control chart based on the classic estimator does not show signals for both stages.

VI. A REAL CASE

Here, a data set from Derakhshani et al. (2020) is used to show the applicability of the proposed robust monitoring scheme. This section presents a tangible example of how automobile glasses are produced, using a multistage process depcited in Figure 4. The first stage involves cutting glass beads into pieces with a machine that may leave scratches. The link between cutting speed and the number of scratches is modeled using Poisson regression considering the initial conditions. Also, in the second stage, heat lines are printed onto the glass using a machine. If the print speed exceeds the allowed tolerance, it may create defects in the glass. The link between print speed and the number of defects is modeled using Poisson regression. Moreover, the number of defects in the stage 2 is influenced by the number of scratches in the previous stage. To monitor a two-stage process, a dataset is gathered during the period, comprising 20 profiles with five observations each, where the levels of cutting speed are 500, 550, 600, 650, and 700 rpm, and the levels of printing speed are 10, 11, ..., 14 meter per second. The number of scratches and defects is recorded at each level. Using the classic and robust parameter estimation method, the profile parameters are estimated. Hence, the in-control parameters at each stage are computed based on the classic and robust estimated parameters from each profile.

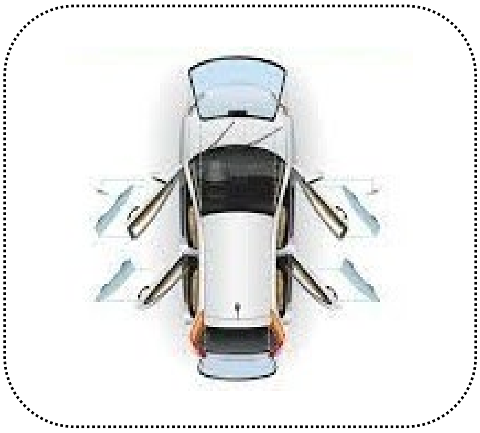

Figure 5. Schematic of the real case for automotive glass manufacturing process

The average of the y_{ijs}s at each level of observations is computed using $\lambda_{is} = \frac{\sum_{j=1}^{20} y_{ijs}}{20}$ in which 20 is the number of samples in Phase I monitoring. After that, the correlation between the average of the y_{ijs}s at each level of observations is considered as an estimate of Φ which is equal to 0.2. Then, having these estimates and using simulation and considering the overall Type I error equal to 5%, the UCL of the control chart is computed. Moreover, the classic and robust estimations are given in Table 3.

TABLE 3. The estimates of the case using classical and the robust estimators

Outlier percentages	Method	β_{01}	β_{11}	β_{02}	β_{12}
0	Classic	0.7934	-0.0003	1.0560	0.0023
	Robust	0.7933	-.00002	1.0572	0.0021
20	Classic	0.9101	-0.0012	1.0029	0.0068
	Robust	0.7893	-.00018	1.0607	0.0018

To evaluate the stability of the process and identify any unusual profiles, T_U^2 can be employed based on classic and robust approach. The T_U^2 statistics for the control chart are calculated for the samples by equation (14) with both the classic and robust estimators. These results are then plotted in Figure 6. By examining this figure, it becomes apparent that the robust control charts indicated an out-of-control signal based on the last two and one data points for the stages 1 and 2, respectively. In contrast, the classic estimator-based control chart showed that two and six sample as signal for the first and second stages, respectively. Any data points that fall outside these limits are considered signals of process variation and should be investigated further to identify the source of the variation to meet a stable process. Generally, this real case shows that the proposed classic and robust control charts perform well, and the robust control chart performs better than the classic one.

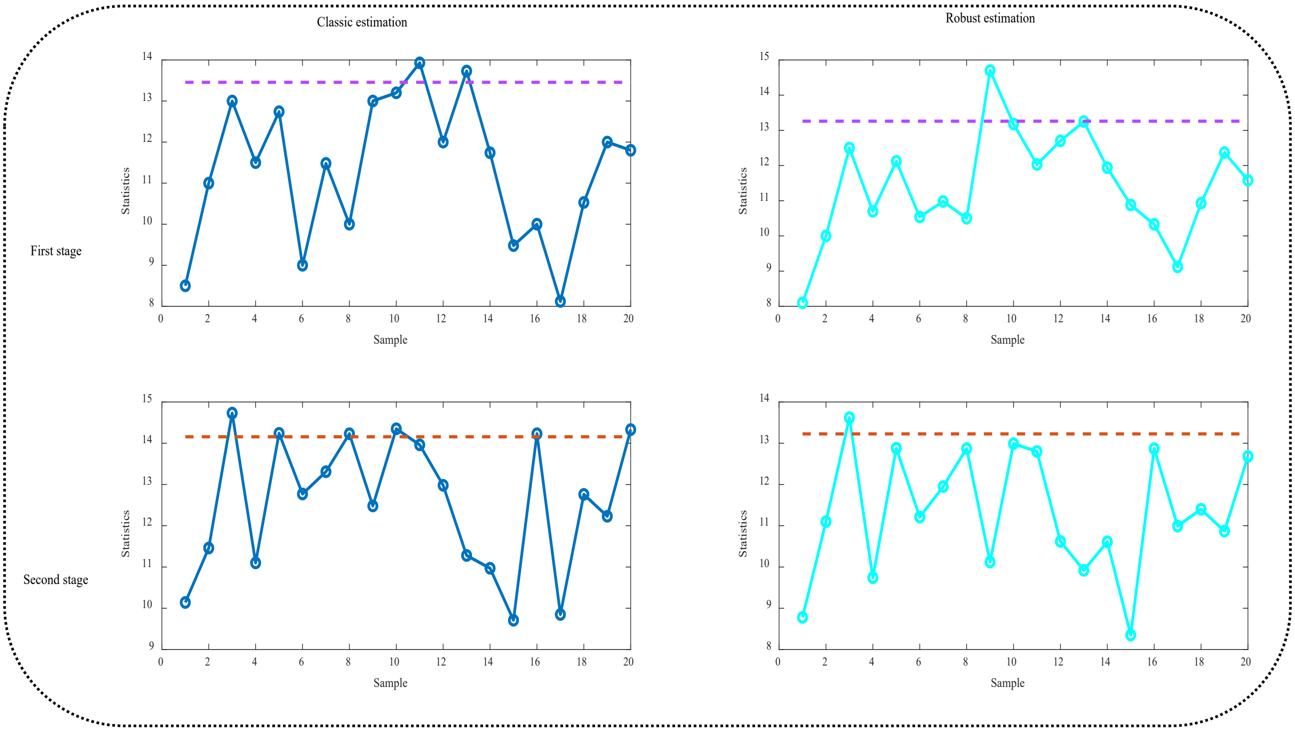

Figure 6. Phase I monitoring based on $T_{I_u}^2$ control chart with classical and robust estimators in two-stage process of real case

VI. CONCLUSION AND FURTHER STUDIES

In many applications of profile monitoring, the response variable follows a Poisson distribution. Furthermore, when data is contaminated, classical estimation methods may not perform well. To address this issue, two new approaches , namely $T^2_{I_{u_{js}}}$ and $T^2_{R_{u_{js}}}$ control charts were proposed for Phase I monitoring Poisson regression profiles. First, the performance of these control charts was compared to find the best method for monitoring Poisson profiles under various shifts and contamination. The results indicated that $T^2_{I_{u_{js}}}$ control chart performs better than $T^2_{R_{u_{js}}}$ chart, hence, the simulations were continued using the control chart $T^2_{I_{u_{js}}}$ to evaluate the robust control chart against the classic one. The classic and robust estimators were evaluated through extensive simulation studies with and without outliers, and the results indicated that the robust estimator outperformed the classic one in all scenarios. Then, the $T^2_{I_{u_{js}}}$ control chart's performance with the classical and robust estimates was compared under different shifts with and without outliers, and it was found that using the robust method to estimate regression parameters improved the performance of the proposed $T^2_{I_{u_{js}}}$ control chart under different shifts. Overall, the study demonstrated that the robust approach is more effective than the classic method in handling outliers and improving the performance of the monitoring scheme. In addition, the signal probability was computed for a numerical example and a real case.

While the proposed method showed promising results in handling outliers and improving the performance of the monitoring scheme, the applicability of the method to other types of profiles may require further investigation. Additionally, the proposed method assumes independent observations, and considering autocorrelation within or between profiles at each stage in multistage processes could be an area for future research.

For future recommendations, the other Phase I monitoring like F and LRT method could be developed for these profiles. Also, new robust estimators such as MM, S, Fast τ and so on, in the literature could be applied to demonstrate greater efficiency. Also, expanding this research to monitor other distributions of exponential families could be considered a fruitful area for work. Additionally, it could survive the estimation error and measurement error on this monitoring scheme.

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