Journal of Quality Engineering and Production Optimization

Vol. 8, No. 1, Winter & Spring 2023

http://jqepo.shahed.ac.ir

Research Paper

DOI: 10.22070/jqepo.2023.17599.1256

Designing a Green-Resilient Supply Chain Network for Perishable Products Considering a Pricing Reduction Strategy to Manage Optimal Inventory: A Column Generation-based Approach

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Abstract –This paper aims to design an integrated green supply chain by considering several drivers of resiliency for perishable products. Supply chain resiliency seeks to prevent the chain from moving toward unfavorable conditions and to manage and mitigate disruptions. To minimize the effects of disruption, new strategies are investigated, including horizontal cooperation between distribution centers, emergency inventory reserves in production and distribution centers, and contracts with backup suppliers. Additionally, to avoid resource wastage, the strategy of reducing the selling price of products nearing their expiration date is considered, allowing the model to control the optimal amount of production and inventory. In addition to resiliency, all supply chain members must adopt a green supply chain approach to conduct production and distribution activities in a manner that reduces environmental damage. Therefore, optimal routing is implemented to deliver products at the minimum possible cost and with the least pollution. A column generation decomposition algorithm is developed to improve solution time as the model becomes complex and unsolvable or takes a long time to solve with standard mixed-integer programming solvers. After designing an integrated supply chain for perishable products, we initially solved several test cases to determine the applicability of the column generation method to these issues. Subsequently, we employed the proposed solution method for a specific case study on the PEGAH dairy industry. A sensitivity analysis was conducted on the product's shelf life to examine variations in optimal production and inventory levels. The results indicated that as the product's lifetime decreases, a lower quantity of the product is produced and stored.

Keywords– Supply chain design, Perishable, Price reduction, Resiliency, Column generation.

I. INTRODUCTION

A Supply chain network (SCN) is a facility network that procures raw materials, converts them into products, and transmits them to customers through a distribution network. The supply chain network design deals with various sectors, such as waste management, energy, health systems, and humanitarian and relief logistics, which play an essential role in the better function of the SC. The evaluation of sustainability in the crop supply chain has a significant role in establishing an efficient supply chain and enhancing food safety in developing countries (Mehrbanfar et al., 2020).

Manuscript Received: 17-January-2023 & Revised: 8-March-2023 & Accepted: 20-April-2023
ISSN: 2423-3781

The Food SC is one of the largest industries, often dealing with products that have a limited lifetime and are perishable, which adds to the complexity of this sector. This complexity leads to uncertainty and makes the supply chain more vulnerable (Chobar et al. 2022; Deng et al., 2019). Perishable food must be consumed within its lifetime; Otherwise, it becomes waste and results in a loss of resources. Due to the unique storage and transportation conditions, these products must reach customers immediately. Hence, attention to the transportation system within the supply chain is particularly important. The perishability of products affects production planning, product storage policy, retailer decision-making, and sail policy. Retailers should also focus on the lifetime of the products in their decisions (Noya et al., 2016). If the retailer can provide fresh produce to the customer, it can capture a significant share of the market (Mirmajlesi and Shafaei, 2016). In these conditions, an agile distribution mechanism is essential for delivering products quickly.

Many industries implement a price reduction strategy as they approach the end of their sales period to minimize waste and losses (Leventhal and Breur, 2012). Adjusting the price according to the product specifications, instead of a fixed price throughout the lifetime, can increase sales. As a result, this approach may increase the income and balance the supply and demand according to information such as the inventory levels and price elasticity (Babaeinesami et al. 2022; Adenso-Díaz et al., 2017). Therefore, it is necessary to consider price reductions for products approaching their lifetime to prevent the wastage of resources and achieve greater profitability.

Following the cases mentioned earlier, disruption and the loss of infrastructure resources also lead to economic losses, damaging the company's reputation and harming the environment. Dependencies between nodes are high; thus, a disruption in one supply chain echelon can quickly transfer to related echelons, causing partial or total disruptions in the entire chain. (Wang et al., 2017). Therefore, resiliency is an essential and inevitable goal in designing a resilient perishable supply chain network (RPSCN).

There are various definitions of resilience in the literature. According to Christopher and Lee (2004), resilience is "the ability of a system to return to its original state or move to a new, more desirable state after being disturbed." In their study, Hosseini et al. (2019) categorized the conceptual drivers of a resilient supply chain into resiliency, information sharing, collaboration, visibility, and agility. Different definitions of these drivers can be found in the literature, some of which are summarized in Table 1.

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Table I. DRIVERS OF RESILIENCE IN SC

Drivers	Definition	author
Agility	SC's capability to promptly adapt the network structure to the customer's dynamic needs	Dubey et al. (2018)
Visibility	The location, identity, and status of entities passing through the SC are recorded in timely event messages, along with the scheduling date of these events.	Francis (2008)
Flexibility	As one of the characteristics of SCR, flexibility determines the company's ability to respond to market changes appropriately.	Millar (2015)
Collaboration	Cooperation of several independent companies to achieve common goals	Scholten and Schilder (2015)
Information Sharing	Mitigate the bullwhip effect under demand disturbances	Yang and Fan (2016)

Greenness in the supply chain entails reducing natural resource consumption, minimizing pollution, and enhancing environmental performance throughout the supply chain processes. This article delves into optimizing product delivery routes for minimal lead time while also considering cost-reduction strategies for products nearing the end of their lifetime (Sarkis et al., 2011). By prioritizing customer satisfaction and ensuring prompt responsiveness, these measures not only contribute to a decrease in pollution emissions and fuel consumption but also prevent resource wastage and protect products from vulnerability to corruption.

Governments prioritize the design of strategic supply chain networks considering the potential occurrence of disruptions (Daneshvar et al. 2023; Hamidieh et al., 2018). Resilience in the supply chain is a systemic capability that allows companies and organizations to maintain adequate and sustainable performance in the face of changes and adverse events. This capability includes coping with problems, demonstrating flexibility in the face of changes, ensuring rapid recovery after errors and incidents, and maintaining proper performance under exceptional conditions (Tukamuhabwa et al., 2015). This study considers the horizontal relationships between distribution centers, the design of an integrated supply chain, and the inclusion of backup suppliers as resilience drivers.

The term "perishable supply chain" refers to a chain in which products susceptible to spoilage and decay are carefully managed from production to distribution and sales. This type of supply chain is specifically designed to uphold the quality and freshness of perishable products through specific conditions, including proper temperature and humidity control, swift delivery speed, and the utilization of methods for preserving freshness while ensuring rigorous quality control throughout the entire supply chain (Goli et al., 2020) .This study focuses on perishability in distribution centers, where a reduction in selling price is considered to incentivize sales and prevent spoilage for products with a limited lifetime. In this research, we aim to achieve greater profitability by considering several drivers for the resilience of the PSCN and implementing a strategy for price reduction. By reviewing the existing literature in the RPSCN field, our contributions are as follows:

- This study prevents possible shortages by considering the horizontal and vertical collaboration of the layers, emergency storage in centers, backup suppliers, and multiple resilience drivers. Also, avoiding perishable products and determining the optimal route help the greenness of the supply chain. Therefore, considering sustainability increases the SC's resiliency, which has yet to be comprehensively pointed out in previous studies.
- Minimizing the number of perishable products
- Using the appropriate fuzzy method to face uncertainty
- Developing a decomposition structure for solving the model with an exact column generation method, which can be used for large-scale problems within a reasonable time..
- Conducting a real case study to validate the efficiency of the proposed solution method

The remainder of the paper is organized as follows: Section 2 presents a literature review in the RPSCND field. In Section 3, the mathematical model is shown with its assumptions. The solution method is explained in Section 4. A description of the case study, i.e., Pegah Dairy Co., is presented in Section 5, followed by the results and sensitivity analysis in Section 6. Finally, in Section 7, conclusions and suggestions for future studies and improvement to the mathematical model are presented.

II. LITERATURE REVIEW

This section briefly reviews the most important related sources based on PSCN, resiliency, and the problem-solving methods. Garcia-Herreros et al. (2014) proposed the RSCN model and used a multi-cut Benders decomposition algorithm to solve it. Torabi et al. (2015) submitted a model to enhance the supply chain's resilience, identifying several strategies such as contracting with backup suppliers, implementing suppliers' business continuity plans, and strengthening supplier relationships. Jabbarzadeh et al. (2016) presented a model for designing an RSCN considering supply and demand disturbances and facilities. They used Lagrange relaxation method to solve their proposed model. In their model, disturbances are categorized as complete shutdowns of facilities or reductions in their capacity. Rahimi et al. (2017) introduced an inventory routing problem for perishable products. Their model has three objectives. The first objective is to minimize distribution and inventory costs, the second is to increase customer satisfaction, and the third is to reduce carbon emissions in distribution activities. They used the Non-Dominated Sorting Genetic Algorithm II (NSGA-II) to solve their presented model.

Rafie-Majd et al. (2018) presented an inventory-location-routing problem considering stochastic demand for retailers with products of limited lifetime. Their model also allocates vehicles to more than one distribution center. To solve their model, they used the Lagrangian Relaxation method. Jabbarzadeh et al. (2018) proposed a closed-loop RSC by considering lateral transshipment. They used a model to manage disruption risk and minimize the supply chain costs. Rajesh (2019) proposed a fuzzy index to measure the resilience of firms, identifying critical attributes for strategic decision-making. Data were collected at linguistic levels from industry experts and then transformed into a fuzzy triangular membership function. Biuki et al. (2020) proposed an approach to evaluate sustainable suppliers and network design problems, using GA and PSO algorithm to solve their model..

Yavari and Zaker (2020) considered disruptions in supply chains and power networks in their design to minimize total network costs and carbon emissions, employing five risk-reduction strategies in their study. Yavari et al. (2020) analyzed the RSCN based on dynamic pricing and destructive pricing for demand management, modeling location, inventory, and routing in which demand depends on price and product lifetime. Ouhader (2020) proposed a Location-Routing problem utilizing horizontal collaboration in urban freight delivery. The results showed that horizontal cooperation reduces transportation costs and increases partners' performance while addressing environmental constraints.

Arabsheybani and Khasmeh (2023) developed an integrated SCND with two objectives: maximizing total profits and total resiliency scores. They applied the proposed model to the spice and flavor industries. Keshmiry Zadeh et al. (2021) developed a mathematical model for designing a green closed-loop supply chain under disruptions, using the ε-constraint method for small-scale problems and developing a non-dominated sorting genetic algorithm for larger issues. Other studies such as Daroudi et al. (2021), presented a SCND model for perishable products to minimize the cost of the entire chain, product delivery time, and pollution from supply chain activities. Hemmati et al. (2022) proposed an agile perishable supply chain with two objective functions in two phases. In the first stage, the suppliers are evaluated with network data envelopment analysis, and the evaluation indicators are extracted.

In the second phase, selected resilient supplier are integrated into the supply chain. In their proposed model, demand for products is considered a fuzzy parameter. Their proposed inventory-routing problem is solved by the column generation method. Aloui et al. (2022) presented an RSCN to deal with demand fluctuations, whose objective functions aimed at minimizing costs, CO_2 emissions, energy consumption, and maximizing employment. To solve their model, they used the Lagrange relaxation algorithm. The results showed that cooperation between suppliers could lead to cost savings and reduction of CO_2 missions. Bahrampour et al. (2022) introduced a scenario-based multi-objective fuzzy model for sustainable closed-loop supply chains (SCLSC), employing Multiple Objective Particle Swarm Optimization (MOPSO) and NSGA-II algorithms to solve it, with parameters optimized using the Taguchi method. The model was evaluated using robust criteria and tested in diverse problem dimensions implemented in the Chipboard Pooya Company in Iran. Mohammadi Jozani et al. (2022) developed a supply chain model integrating production and distribution activities, incorporating a multi-period routing problem. The model considers uncertain parameters and incorporates a two-stage stochastic optimization approach. NSGA_II, MOPSO, and a hybrid algorithm were employed to solve the model.

Hashemi-Amiri et al. (2023) developed a mixed-integer programming (MIP) model for the production-distribution-routing problem in a sustainable agricultural product supply chain network. Their objective functions aim to minimize economic costs and environmental impacts while maximizing social impacts. Several meta-heuristic algorithms were applied, and two hybrid algorithms were developed to solve the complex and NP-hard nature of problem. The efficiency of the algorithms was assessed through various test instances, comparisons, and sensitivity analyses. Goodarzian et al. (2023) proposed a bi-objective optimization model for a three-echelon perishable food supply chain (PFSC) with multiple products, addressing supplier selection, production scheduling, and vehicle routing. The model aims to mitigate demand and supply uncertainties and optimize network costs and supplier reliability. A distributionally robust modeling paradigm is used to handle uncertain demand, and a chance-constrained approach ensures that retailer demands and vehicle capacities are met with high probability. The model is reformulated as a mixed-integer linear program using duality and linearization techniques. The weighted goal programming approach is applied to address the multi-objectiveness of the model. Shi et al. (2023) focused on analyzing a sustainable three-echelon supply chain structure, specifically from the retailer's perspective, for perishable products. This study aims to maximize profit by determining the retailer's optimal cycle time, selling price, and inventory period. Additionally, it conducts a comparative analysis of four different payment methods: upstream ACC, advance, cash, credit, and downstream partial credit payments. Foroozesh et al. (2023) focused on the challenge of designing efficient and resilient Agri-Food Supply Chains (AFSCs) in emerging countries while considering perishability constraints. Their study proposes a methodology that combines optimization and simulation techniques to identify critical aspects affecting the resilience of AFSCs. The framework is evaluated using a Colombian coffee supply chain, demonstrating its effectiveness in measuring, predicting, comparing, and improving the resilience of supply chains.

Blood platelets, which are perishable products with a five-day shelf life, often result in wastage due to limited storage time. Shortages can also occur during emergencies like wars and pandemics. To address this, a resilient and sustainable supply chain model was proposed by Shokouhifar and Goli (2023), considering transshipment and costs (economic, social, and environmental). The model incorporates lateral transshipment between hospitals to mitigate shortages and disruptions. The implementation of a metaheuristic approach reduces financial costs, shortages, and wastage by 3.61%, 30.1%, and 18.8%, respectively.

Based on the literature and summarized findings in Table II, some contributions are noted as follows:

First, in the RSCND field, the proposed model can tackle several types of disruptions by considering more resilience drivers than previous studies, enhancing the overall resilience of the supply chain. Second, the proposed RSCND for perishable products (PRSCND) used in the dairy industry is considered as a resilient-green supply chain. As seen in columns 2 to 5 of Table II, this study has focused on resilience and sustainability, that have received less attention in previous studies. Third, the Column Generation method was developed to solve the problem and reach optimal solutions. The Column Generation method is a decomposition approach used to solve models that are not solvable by standard solvers. Fourth, an integrated multi-product and multi-period MIP model with a four-echelon structure is designed, employing the column generation method to obtain optimal solutions in a reasonable time. Fifth, The proposed model includes a price reduction strategy for products nearing the end of their lifetime, aimed at preventing perishability, increasing profitability, attracting more customers, and reducing resource waste. With this strategy, the company seeks optimization in inventory management and production levels. The objective is to improve profitability and minimize product vulnerability to perishability. Previous research did not consider these features, but this study focuses on them. Using the proposed model makes optimizing inventory storage and production levels possible, resulting in increased profits and reduced vulnerability to perishability. This strategy helps optimize resource utilization while making products available to meet customer needs.

Implementing this strategy allows the company to benefit from the following advantages:

Increased profitability: By optimizing inventory storage and production levels, the company can reduce costs and increase its profits.

Reduced perishability risk: By lowering the prices of products at the end of their lifetime, the likelihood of perishability and resource waste is reduced.

Customer attraction: By offering products at affordable prices in the final stages of their lifetime, the company can attract new customers and expand its market.

Resource utilization optimization: The company can efficiently use resources and reduce waste by optimizing inventory storage and production levels.

Overall, the price reduction strategy for products nearing the end of their lifetime can help the company improve its profitability, reduce product perishability, and meet customer needs.

Table II. Literature review of perishable SCND

Article	Characteristic				Decision Variables				Number of echelons	Uncertainty				Solution Method		DISCOUNT		Case study
	Sustainability			Resiliency	Location	Transportation	Routing	Inventory		Probability	Fuzzy	Stochastic	Robust	Exact	Metaheuristic	Purchase	Sales	
	En.	So.	Ec.															
Garcia-Herreros et al. (2014)				✓	✓	✓		✓	3			✓		✓				Dairy
Torabi et al. (2015)				✓				✓	1		✓	✓			✓			Frozen Food
Jabbarzadeh et al. (2016)				✓	✓	✓			2			✓	✓	✓				Oil
Rahimi et al., (2017)	✓		✓			✓	✓		1						✓			-
Rafie-Majd et al. (2018)					✓	✓	✓	✓	3			✓		✓				-
Jabbarzadeh et al. (2018)				✓	✓	✓			4			✓	✓	✓				Glass
Rajesh (2019)				✓					4		✓							Electronics
Biuki et al. (2020)	✓	✓	✓		✓	✓	✓	✓	4		✓				✓	✓		-
Yavari and Zaker (2020)	✓		✓	✓	✓	✓		✓	4	✓		✓		✓				Dairy
Yavari et al. (2020)				✓	✓		✓	✓	3			✓			✓		✓	-
Ouhader (2020)	✓		✓		✓		✓		2			✓		✓				-
Arabsheybani and Arshadi Khasmeh (2023)				✓		✓		✓	3		✓		✓	✓				Flavor
Daroudi et al., (2021)					✓		✓	✓	3		✓				✓			-
Keshmiry Zadeh et al. (2021)	✓		✓	✓	✓	✓			4						✓			-
Hemmati et al. (2022)	✓		✓	✓	✓	✓	✓	✓	4		✓			✓				Dairy
Bahrampour et al. (2022)	✓	✓	✓		✓		✓		4			✓			✓			Chipboard
Mohammadi Jozani et al. (2022)					✓	✓	✓	✓	3			✓			✓			Pharmaceutical
Aloui et al. (2022)	✓		✓		✓	✓	✓	✓	2					✓				-
Goodarzian et al. (2023)	✓	✓	✓				✓		3						✓			-
Hashemi-Amiri et al. (2023)							✓		3	✓			✓	✓				Poultry
Shi et al. (2023)	✓	✓	✓					✓	3					✓		✓		-
Foroozesh et al. (2023)	✓	✓	✓	✓					3		✓		✓	✓				Food, dairy, and drink
Hasan-Zadeh (2023)			✓			✓			2					✓				-
Shokouhifar and Goli (2023)	✓	✓	✓	✓		✓		✓	3						✓			Blood platelets
This study	✓		✓	✓	✓	✓	✓	✓	4		✓			✓			✓	Dairy

III. PROBLEM DESCRIPTION AND FORMULATION

This study aims to develop a green-resilient model for perishable products that can be resilient against all types of disruptions and have the least waste of resources. The proposed model includes four echelons: suppliers, factories (F), distribution centers (DC), and customers, with several members at each echelon, as shown in Figure 1.

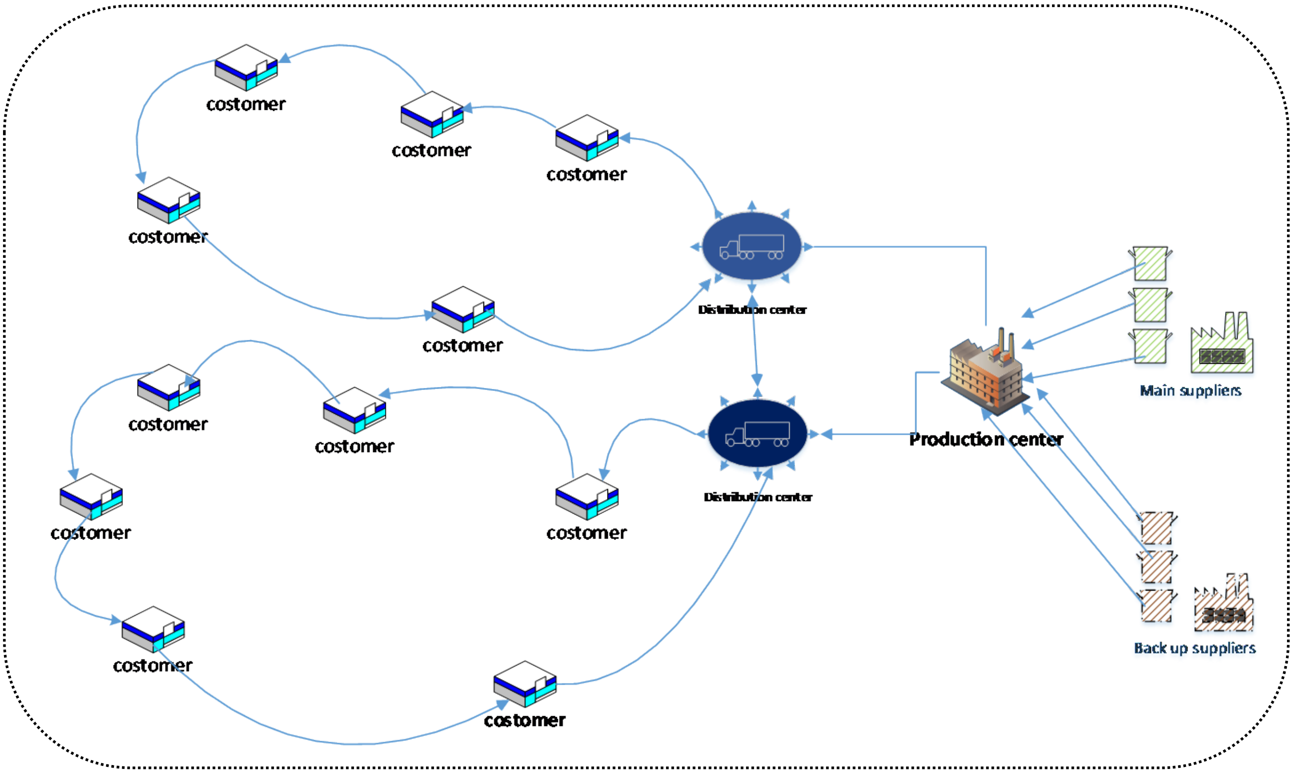

Fig. 1. Structure of proposed PSCN

Products are manufactured in factories using raw materials supplied by first-class members. The products received from the factories are sent to the DCs to be delivered to the customers, and the customer's demand is met.

The assumptions of the problem are as follows:

1. The SCND model is multi-period, and multi-product.
2. The inventory of DCs is considered zero at the beginning of each period.
3. Products have a limited lifetime; once they expire, they must be discarded.
4. Discount strategy is considered for products that are approaching their lifetime.
5. The routing may differ in each period; vehicles are not homogeneous and have different costs.
6. The customers' demand, which is uncertain and represented as a fuzzy number, must be met for all customers.
7. The products delivered to the DCs are at the beginning of their lifetime.
8. There is a horizontal collaboration between distributors.

In this research, considering the above assumptions, we aim to enhance the supply chain's resilience, increase the chain's income (reducing costs), reduce resource wastage, and determine optimal product distribution routes. To increase the resilience of the supply of raw materials, backup suppliers are used if the main suppliers cannot supply raw materials. Additionally, to improve the agility of the distribution, we promote horizontal cooperation between distributors to prevent shortages at the DCs. Also, emergency inventory is considered in factories and DCs.

A. Notations

The symbols used for the mathematical formulation are listed in Tables III to V.

Table III. Symbols used in the model and their description

Symbol	Description
b, b=1,2,…,B	Backup suppliers
i, i=1,2,…,I	Distribution centers
$n, n' \in N = (I \cup J)$	DCs and customers.
j, j=1,2,..J	Customers
r, r=1,2,…,R	Raw materials
t, t`=1,2,…,T	Period of the planning horizon
v, v=1,2,…,V	Vehicles
k, k=1,2,…,K	Main suppliers
f, f=1,2,…,F	Factories
p, p=1,2,…,P	Products

Table IV. Parameters used in the model and their description

Parameter	Description
Pr_{ipt}	The unit price of product *p* at period *t* by DC *i*
λ_{ipt}	The product sales discount factor of product *p* at period *t* by DC *i*
$C_{nn'vt}$	The unit cost of transmission between nodes n, n' by vehicle *v* in the period *t*
$CY_{ii'pt}$	The unit cost of transmission between DCs i, i' in the period *t*
$CIT_{ipt't}$	Perishability cost of products received at the time t' but not sold until period *t* and have reached the end of their lifetime
$CI_{ipt't}$	The cost of maintaining product inventory in DCs received in the period t'
CS'_{fipt}	The cost of production processes and transmission of product *p* from factory *f* to DC *i* in each period *t*
CIF_{fpt}	The unit cost of maintaining product *p* at the factory *f* in period *t*
CS_{kfrt}	The unit cost of procurement, supply, and transmission of raw materials from main supplier *k* to factory *f* in period *t*.
CBS_{bfrt}	The unit cost procurement, supply, and transmission of raw materials from backup supplier *b* to factory *f* in each period
D_{jpt}	Estimated demand of node *j* for products *p* in each period *t*
$capF_{fp}$	The maximum production capacity of factory *f* for product *p*
cap_{fp}	The maximum storage capacity of factory *f* for product *p*
$capI_{ip}$	The maximum storage capacity of DC *i* for product *p*
$capk_{krt}$	The capacity of materials *r* for the main supplier in the period *t*.
lp	The lifetimes of the products
p_t	A period that includes a price reduction
M	A big number

Table V. The variables used and their description

variables	Description
$I_{ipt't}$	The amount of inventory of products in DCs that have been shipped in the period t'
S_{fipt}	The amount of product *p* transmitted from factory *f* to DC *i* in each period *t*
$W_{ii'pt}$	The amount of horizontal transfer of product *p* at the distribution echelon
SS_{kfrt}	The amount of raw material received by factory *f* from main suppliers *k*
BS_{bfrt}	The material amount received by factory *f* from backup suppliers *b* in each period *t*
IF_{fpt}	The amount of emergency inventory product *p* in the factory *f* at the end of each period *t*
Y_{njvt}	1if vehicle *v* travels from one customer or DC to another customer or DC; 0 otherwise
Z_{ijt}	1 if the DC i meets node j; 0 otherwise
YS_{frt}	1 if the backup suppliers are required; 0 otherwise
YYS_{bfrt}	1 if the backup supplier *b* sends raw material *r* to the factory *f*; 0 otherwise

B. Objective and related constraints

The proposed RSCND model for perishable products has two objectives. Objective F1 maximizes the revenue of the SC, considering the costs of supplying, producing, and distributing the product, as well as the transformation cost between layers and the costs of inventory maintenance. In the first objective, the cost of perishability is considered to prevent waste. A discount strategy is applied to products that are approaching the end of their lifetime to reduce perishability. The terms related to revenues and supply chain costs are formulated in Tables VI to VII.

Table VI. the revenues of the supply chain

Notation	Formulation	Description
A	$\sum_{i\in I}\sum_{p\in P}\sum_{t'\in T}\sum_{\substack{t\in T;\\ t\le t'+lt-p_t-1}} pr_{ipt}\cdot\left(I_{ipt'(t-1)}-I_{ipt't}\right)$	The amount of profit from the sale at the original price
B	$\sum_{i\in I}\sum_{t\in T}\sum_{p\in P} pr_{ipt}\left(\left(\sum_{f} S_{fipt}-\sum_{\substack{i'\\ i'\neq i}} W_{ii'pt}+\sum_{\substack{i'\\ i'\neq i}} W_{i'ipt}\right)-I_{iptt}\right)$	The amount of profit from the sale at the original price
C	$\sum_{i\in I}\sum_{p\in P}\sum_{t,t'\in T}\sum_{\substack{t\in T\\ t'+lt-p_t\le t\\ t\le t'+lt-2}} pr_{ipt}\left(1-\lambda_{ipt}\right).\left(I_{ipt'(t-1)}-I_{ipt't}\right)$	The sale of the product at the reduced price

Terms A and B represent the sale of the product at the approved price. Terms C offers the sale of the product at the discount price, which is described as follows:

When the product approaches its lifetime, a discount price is applied to prevent perishability. To reach this goal, Figure 2 is presented. In Figure 2, L_t represents the product's lifetime, p_t is the time the product approaches lifetime, and the discount price is applied. Price reduction is expressed as expression $(1\text{-}\boldsymbol{\lambda})Pr_t$.

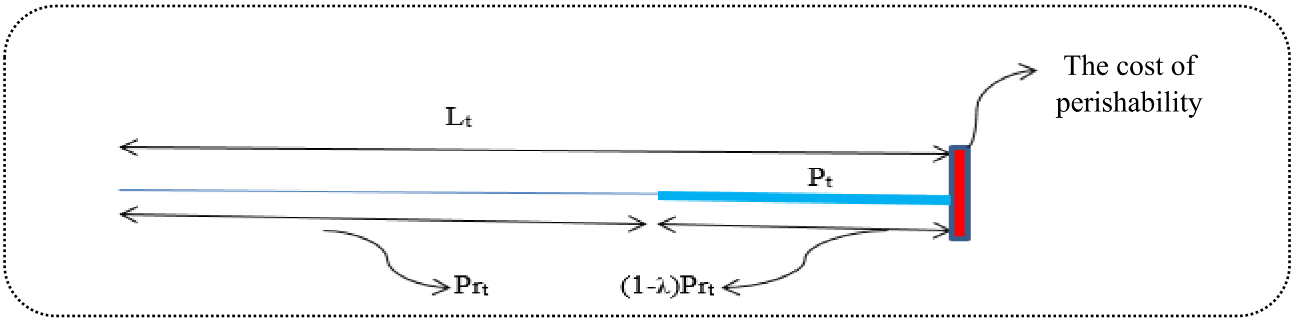

Fig. 2. Apply discount strategy in product sales

As shown in Figure 2, the price Pr_t is applied at $L_t - P_t$, and the price $(1-\lambda)\mathrm{Pr}_t$ is at the time P_t, which λ is the price reduction rate. During this period of price reduction, the model tries to first adjust the amount of production and inventory. Secondly, no product remains until the end of the period because the products are thrown away at the end of their lifetime, which prevents income from sales and incurs the cost of perishability. The lifetime of each product starts from the moment they arrive at the DCs. With the specified lifetime and the time of sending the product (i.e. t') along with the current time (i.e., t), the selling price of the products is determined with the original price and the reduced price, and based on that, the balance is adjusted in the inventory and production level. Figure 3 shows the time allocation of sales prices during the product's lifetime.

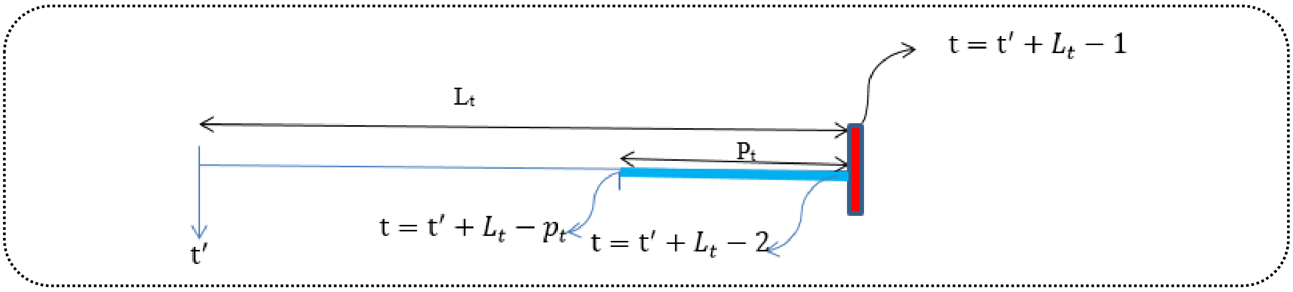

Fig. 3. The start times of the discounted price

The total cost of the SC includes the following items:

Table VII. the costs of the supply chain

Notation	Formulation	Description
D	$\sum_{n'\in N}\sum_{n\in N}\sum_{v\in V}\sum_{t\in T} C_{nn'vt}\, Y_{nn'vt}$	The costs of transportation of the product from DCs to the customers
E	$\sum_{i\in I}\sum_{i'\in I}\sum_{p\in P}\sum_{t\in T} CY_{ii'pt}\, W_{ii'pt}$	The costs of transportation between DCs
F	$\sum_{i\in I}\sum_{p\in P}\sum_{t'\in T}\sum_{\substack{t\in T\\ t=t'+lt-1}} CIT_{ipt't}\, I_{ipt't}$	The costs of perishability
G	$\sum_{i\in I}\sum_{p\in P}\sum_{t'}\sum_{t<t'+lt} CI_{ipt't}\, I_{ipt't}$	The costs of maintaining inventory in DCs

Continue Table VII. the costs of the supply chain

Notation	Formulation	Description
H	$\sum_{p\in P}\sum_{i'\in I}\sum_{i\in I}\sum_{t\in T} CS'_{fipt}\, S_{fipt}$	The costs of transportation of the product p from the factory f to the DCs
M	$\sum_{f\in F}\sum_{p\in P}\sum_{t\in T} CIF_{fpt}\, IF_{fpt}$	The costs of maintaining inventory in the factories
N	$\sum_{k\in K}\sum_{f\in F}\sum_{r\in R}\sum_{t\in T} CS_{kfrt}\, SS_{kfrt}$	The costs of transmission and procurement and purchase of materials from main suppliers to factories
O	$\sum_{b\in B}\sum_{f\in F}\sum_{r\in R}\sum_{t\in T} CBS_{bfrt} BS_{bfrt}$	The costs of transmission and procurement and purchase of materials from backup suppliers to factories

Based on the above formulation, the first objective function is as follows.

$$\text{Max}\, F1 = (A + B + C) - (D + E + F + G + H + M + N + O)$$

C. constraints Vehicle routing constraint

$$\sum_{i\in I} Z_{ijt} = 1 \qquad \forall j \in J, \qquad \forall t \in T \tag{1}$$

$$\sum_{\text{n}\in\text{N}}\sum_{v e V} Y_{njvt} = \sum_{i\in I} Z_{ijt} \quad \forall j \in J,\ \forall t \in T \tag{2}$$

$$\sum_{j\in J}\sum_{v\in V} Y_{ijvt} = \sum_{i\in I} Z_{ijt} \quad \forall i \in I, \quad \forall t \in T \tag{3}$$

$$\sum_{n'} Y_{nn'vt} = \sum_{n'} Y_{n'nvt} \quad \forall n, n' \in (I \cup J), \qquad \forall\, v \in V, \qquad \forall\, t \in T \tag{4}$$

$$\sum_{n\in N} Y_{invt} + \sum_{n\in N} Y_{njvt} \ \le 1 + Z_{ijt} \qquad \forall i \in I, \quad \forall\, j \in J, \qquad \forall\, v \in V, \qquad \forall\, t \in T \tag{5}$$

$$U_{n'vt} \ge U_{jvt} + |N| - M(1 - Y_{jn'vt}) \quad \forall n' \in N,\ \forall\, v \in V,\ \forall\, t \in T, \forall\, j \in J \tag{6}$$

Constraint (1) determines the relationship between customers and DCs. Constraints (2) and (3) determine the vehicle's routes to the customers in each period. Constraint (4) guarantees that the vehicle entered and exited each node is the same. Constraints (5) and (6) consider the movement path from the DCs to the customers without creating a sub-tour.

D. Inventory and flow constraints

Inventory constraints and related constraints are given based on constraints 7 to 13.

$$I_{iptt} + \sum_{\substack{t' \in T \\ t' < t}} I_{ipt't} = \sum_{f \in F} S_{fipt} + \sum_{\substack{t' \in T \\ t' < t}} I_{ipt'(t-1)} - \sum_{j \in J} Z_{ijt}\, D_{jpt} - \sum_{\substack{i' \in I \\ i' \neq i}} W_{ii'pt} + \sum_{\substack{i' \in I \\ i' \neq i}} W_{i'ipt} \quad \forall i \in I,\ \forall t \in T,\quad \forall p \in P \tag{7}$$

$$I_{iptt} \leq \sum_{f \in F} S_{fipt} + \sum_{\substack{i' \in I \\ i' \neq i}} W_{i'ipt} - \sum_{\substack{i' \in I \\ i' \neq i}} W_{ii'pt} \qquad \forall i \in I,\ \forall t \in T,\quad \forall p \in P \tag{8}$$

$$I_{ipt't'} \geq I_{ipt't} \qquad \forall i \in I,\ \forall p \in P,\ \forall t, t' \in T,\ t' < t \tag{9}$$

$$I_{ipt't} \geq I_{ipt'(t+1)} \qquad \forall i \in I,\ \forall p \in P,\ \forall t, t' \in T,\ t' < t \tag{10}$$

$$I_{ipt't} = 0 \qquad \forall i \in I,\ \forall p \in P,\quad \forall t, t' \in T, t \geq t' + lt_p \tag{11}$$

$$I_{iptt} - \sum_{\substack{i,i' \in I \\ i \neq i'}} W_{ii'pt} + \sum_{\substack{i,i' \in I \\ i \neq i'}} W_{i'ipt} \leq M. \sum_{j \in J} Z_{ijt} \qquad \forall i \in I, p \in P, t \in T \tag{12}$$

$$IF_{fpt} = \sum_{r \in R}\sum_{p \in P} \alpha_{rp}\, SS_{kfrt} + \sum_{r \in R}\sum_{b \in B} \alpha_{rp} BS_{bfrt} + IF_{fp(t-1)} - \sum_{i \in I} \mathbf{S}_{fipt} \quad \forall f \in F, \forall p \in P, \forall t \in T \tag{13}$$

Constraint (7) guarantee the adjustment of inventories in the DCs in each period of the products. Constraint (8) specifies the inventory level available in the period *t*. Constraints (9) and (10) state that the inventory level before transmitting must be greater than the inventory level after transmission. Constraint (11) indicate the inventory of the products after lifetimes must be zero. Constraint (12) ensures transmission between centers if customers are assigned to those centers. Constraint (13) specifies the balance of the inventory levels for factory products in each period.

E. Facility capacity and related constraints

The capacity of centers in each layer of the proposed SC and related constraints are given based on relations 14 to 22.

$$\sum_{i \in I} S_{fipt} \leq capF_{fp} \qquad \forall f \in F,\quad \forall p \in P,\quad \forall t \in T \tag{14}$$

$$IF_{fpt} \leq cap_{fp} \qquad \forall f \in F,\quad \forall p \in P,\quad \forall t \in T \tag{15}$$

$$\sum_{t' \in T} I_{ipt't} \leq capI_{ip} \quad \forall i \in I,\quad p \in P,\quad t \in T \tag{16}$$

$$YS_{frt} * M \geq -\left(\sum_{k \in K}\sum_{p \in P} \alpha_{rp} SS_{kfrt} + \sum_{p \in P} IF_{fp(t-1)}\right) + \sum_{i \in I}\sum_{p \in P} S_{fipt} \quad \forall f \in f, \forall r \in R, \forall t \in T \tag{17}$$

$$(YS_{frt} - 1) * M \leq -\left(\sum_{k \in K}\sum_{p \in P} \alpha_{rp} SS_{kfrt} + \sum_{p \in P} IF_{fp(t-1)}\right) + \sum_{i \in I}\sum_{p \in P} S_{fipt} \quad \forall f \in f, \forall r \in R, \forall t \in T \tag{18}$$

$$\sum_{b \in B} YYS_{bfrt} \geq \mathrm{YS}_{\mathrm{frt}} \qquad \forall \mathrm{f} \in \mathrm{F}, \forall \mathrm{r} \in \mathrm{R}, \forall \mathrm{t} \in \mathrm{T} \tag{19}$$

$$\sum_{b \in B} YYS_{bfrt} \leq \mathrm{YS}_{\mathrm{frt}} * M \qquad \forall \mathrm{f} \in \mathrm{F}, \forall \mathrm{r} \in \mathrm{R}, \forall \mathrm{t} \in \mathrm{T} \tag{20}$$

$$\sum_{f \in F} BS_{bfrt} \leq capb_{brt} * \sum_{f \in F} YYS_{bfrt} \qquad \forall b \in B, \forall r \in R, \forall t \in T \tag{21}$$

$$\sum_{f \in F} SS_{kfrt} \leq capk_{krt} \qquad \forall r \in R, \forall t \in T, \forall k \in K \tag{22}$$

Constraint (14) specifies the capacity of factories that can send to DCs. Constraint (15) refers to inventory storage capacity in the factories. Constraint (16) takes into account the capacity of the DCs. Constraints (17) to (22) examine the conditions for using backup suppliers and determine the capacities of the main and backup suppliers. Constraint (23) defines the model's non-negative variables and Constraint (24) determines the binary variables in the model.

$$I_{ipt't}, S_{fipt}, W_{ii'pt}, SS_{kfrt}, BS_{bfrt}, IF_{fpt} \geq 0$$
$$\forall i \in I, \forall j \in J, \forall f \in F, \forall b \in B, \forall r \in R, \forall p \in P, \forall v \in V, \forall t, t' \in T, k \in K \tag{23}$$

$$Y_{njvt}, \mathrm{YS}_{\mathrm{frt}}, Z_{ijt}, YYS_{bfrt} \in \{0 \text{ or } 1\}, \forall n \in N, \forall j \in J, \forall t \in T, \forall i \in I, \forall v \in V, \forall b \in B, \forall f \in F, \forall r \in R \tag{24}$$

IV. THE COUNTERPART MODEL OF UNCERTAINTY

Generally, the source of data uncertainty stems from fuzziness and randomness (Torabi et al., 2015). Randomness arises from the random nature of data when an operation is repeated multiple times. The probability distribution of this data is estimated with the presence of sufficient historical data. A stochastic programming method is used to deal with this type of uncertainty. Most supply chain planning researchers consider the uncertainties in the SC using probability distributions, which usually use historical data. Since historical statistical data can be unreliable or may not always be available; therefore probabilistic models may not be the best choice. However, epistemic uncertainty has faced the nature of ambiguity in the data, which is related to the lack of knowledge about detecting their probability distribution. To cope with epistemic uncertainty, it is common to use probabilistic programming (Sabouhi et al., 2018). This product is new to the supply chain under study, so sufficient historical data is unavailable. Thus, to approach reality, we consider the demand for the product as a fuzzy number. Data collection for the case study initially involved interviews with the sales manager, production manager, and CEO about similar products, factors affecting demand, the amount of production of current products and the amount of raw materials purchased, transportation methods, etc. Then, a questionnaire was designed, and ten relevant experts provided their responses to these questions. Additionally, visits were made to the sales and production locations to observe the processes, and based on the collected reliable information, modeling, and problem-solving were carried out. In this study, the parameter of demand for the product, given that it is new and lacks historical data, has been considered based on predictions and opinions of experts and sellers. The demand for the product has been estimated using a fuzzy triangular numerical approach, considering the worst, average, and best values. This method can estimate the product's demand as fuzzy numbers, considering differences and various opinions. This method is commonly used in demand forecasting for new products due to the lack of historical data. Constraint (7) encompass the demand parameter, which we will attempt to de-fuzzify. For this purpose, we rewrite Constraint (7) as Constraint (25), which includes the fuzzy parameter of demand, i.e. $\widetilde{D_{jpt}}$.

Subsequently, a questionnaire was designed, and responses were collected from ten relevant experts. Additionally, visits were made to the sales and production locations to observe the processes firsthand. Based on the reliable information gathered, modeling and problem-solving were conducted.

In this study, the demand parameter for the product, given that it is new and lacks historical data, has been estimated based on predictions and opinions from experts and sellers. The demand for the product has been estimated using a fuzzy triangular numerical approach, considering the worst, average, and best values. This method effectively captures the demand as fuzzy numbers, accommodating differences in opinions. It is commonly used in demand forecasting for new products due to the absence of historical data.

Constraints (7) encompass the demand parameter, which we will attempt to de-fuzzify. To achieve this, we rewrite constraints (7) as constraints (25), which include the fuzzy demand parameter, i.e., D*D*.

$$I_{iptt} + \sum_{t'<t} I_{ipt't} = \sum_{f} S_{fipt} + \sum_{t'<t} I_{ipt'(t-1)} - \sum_{j} ZZ_{ijt}\, \widetilde{D_{jpt}} - \sum_{\substack{i' \\ i'\neq i}} W_{ii'p(t-1)} + \sum_{\substack{i' \\ i'\neq i}} W_{i'ipt}\ \forall i \in I \forall t \in T \forall p \in P; \tag{25}$$

The following variable is interred in Constraints (25) to (28) to deal with the fuzzy parameter properly.

$$I_{iptt} + \sum_{t'<t} I_{ipt't} = \sum_{f} S_{fipt} + \sum_{t'<t} I_{ipt'(t-1)} - \sum_{\substack{i'\neq i \\ i,i'\in I}} W_{ii'p(t-1)} + \sum_{\substack{i'\neq i \\ i,i'\in I}} W_{i'ipt} - \sum_{j} Dz_{ijpt}$$
$$\forall\, t \in T, \forall p \in P, \forall\, i, i' \in I; \tag{26}$$

$$Dz_{ijpt} - M\left(1 - ZZ_{ijt}\right) \leq \widetilde{D_{jpt}}\ \forall i \in I \quad \forall\, t \in T \quad \forall \mathrm{j} \in \mathrm{J} \forall p \in P \tag{27}$$

$$Dz_{ijpt} + M\left(1 - ZZ_{ijt}\right) \geq \widetilde{D_{jpt}} \qquad \forall\, t \in T \quad \forall i \in I \quad \forall \mathrm{j} \in \mathrm{J} \tag{28}$$

$$\sum_{P} Dz_{ijpt} \leq MZZ_{ijt}\ \forall i \in I \quad \forall\, t \in T \quad \forall \mathrm{j} \in \mathrm{J} \forall p \in P \tag{29}$$

By transforming Constraint (25), we reach Constraints (26) to (29), where only the right side is in the form of a fuzzy number. When there is a fuzzy parameter on the right side of the constraint, it will be easier to deal with it. Various methods have been proposed to address fuzzy parameters on the right side of the fuzzy limit. In this study, we will employ the method developed by Delgado et al. (1989) to encounter the uncertainty of the fuzzy parameters, which will be detailed in the following section.

The model is called asymmetric if several constraints in the mathematical model have a fuzzy source. In the proposed SCND model, Constraints (27) and (28) incorporate fuzzy parameters. To address this uncertainty, we have applied the Verdegay fuzzy method.

Suppose the asymmetric fuzzy mathematical model in the form of (30) and the parameter $\breve{b_j} \in \left[b_j, b_j + p\right]$

$$\begin{aligned} &\max z = cx \\ &(A_1x)_i \leq b_i \quad i = 1,2,\dots,n \\ &(A_2x)_j \leq \breve{b_j} \quad j = 1,2,\dots,m \\ &\qquad\quad x \geq 0 \end{aligned} \tag{30}$$

Model (30) can be rewritten as model (31) by considering $\breve{b_i} \in [b_i, b_i + \theta p_i]$, $\theta \in [0,1]$

$$\begin{aligned}
&\max z = cx \\
&s.t.\ (A_1x)_i \le b_i \qquad i = 1,2,\dots,n \\
&\quad (A_2x)_j \le b_j + \theta p_j \quad j = 1,2,\dots,m \\
&\quad x \ge 0
\end{aligned} \tag{31}$$

If $\theta = 1 - \alpha$, the model (31) becomes model (32).

$$\begin{aligned}
&\max z = cx \\
&\quad (A_1x)_i \le b_i \qquad i = 1,2,\dots,n \\
&\quad (A_2x)_j \le b_j + (1-\alpha)p_j \qquad j = 1,2,\dots,m \\
&\qquad x \ge 0\ ,\ \alpha \in [0,1]
\end{aligned} \tag{32}$$

In general, model (32) can be written as model (33), and the membership degree of fuzzy constraint will be in the form of (34).

$$\begin{aligned}
&\max \qquad z = cx \\
&\quad s.t.\quad (A_1x)_i \le b_i \qquad i = 1,2,\dots,n \\
&\qquad x_j \in X_\alpha \quad j = 1,2,\dots,m \\
&X_\alpha = \{x \mid \mu_j(x) \ge \alpha\} \\
&\quad x \ge 0\ ,\ \alpha \in [0,1]
\end{aligned} \tag{33}$$

$$\mu_j(x) = \begin{cases} 1 & (A_2x) < b_j \\ 1 - \left[(A_2x)_j - b_j\right]/p_j & b_j \le (A_2x)_j < b_j + p_j \\ 0 & (A_2x)_j > b_j + p_j \end{cases} \tag{34}$$

the final asymmetric fuzzy model form as the model (35):

$$\begin{aligned}
&\max \qquad z = cx \\
&\quad s.t.\quad (A_1x)_i \le b_i \qquad i = 1,2,\dots,n \\
&\qquad\quad (A_2x)_j \le b_j + p_j(1-\alpha) \qquad j = 1,2,\dots,m
\end{aligned} \tag{35}$$

Thus by applying the fuzzy Delgado method, Constraints (29) and (30) become the form (36) and (37).

$$\forall p \in P Dz_{ijpt} + M\left(1 - ZZ_{ijt}\right) \ge D_{jpt} - (1-\alpha)D_{jpt} \qquad \forall\, t \in T\ \forall i \in I\quad \forall \mathrm{j} \in \mathrm{J} \tag{36}$$

$$Dz_{ijpt} - M\left(1 - ZZ_{ijt}\right) \le D_{jpt} + (1+\alpha)D_{jpt} \qquad \forall \mathrm{j} \in \mathrm{J}, \forall i \in I\ , \forall p \in P, \forall\, t \in T \tag{37}$$

V. PROPOSED SOLUTION METHOD

The proposed PSCN model cannot be solved directly using a MIP solver on a large scale within an acceptable time. We must apply a decomposition method to solve the model in the multiple sub-problems. Thus, we propose a branch-and-price approach, which is an exact optimization approach, to solve the PSCN model.

In this approach, all the model variables are not applied at first, and these variables are gradually added to the model until the optimal solution is reached.So, the solution can be reached in less time without examining all the variables and by examining a smaller number of these variables. Consequently, a Restricted Master Problem (RMP) is developed that encompasses several variables from the OP. The variables placed in RMP constitute a feasible solution for the OP. When solving a subproblem optimization problem, the following variables are identified and iteratively added to the RMP, which may improve the existing solution. There are a small number of variables of the OP problem in RMP, and to check the rest of the variables in the OP, we use the dual coefficients of the corresponding constraints from the pricing sub-problem. Therefore, by examining the reduced costs of these variables, we take steps to improve the value of the objective function. Repetitions are carried out until no improvement is achieved (in the problem of minimizing the non-negative values of all reduced costs, this indicates that no further improvement in the problem are possible).

A. Developing Master Problem (MP)

Suppose we have the following OP model, where the constraint set Bx≥d is difficult to satisfy.

$$Min\ \ c^t x \tag{38}$$

$$subject\ to: \quad Ax \geq b \tag{39}$$

$$Bx \geq d \tag{40}$$

$$x \geq 0 \tag{41}$$

Let's be a polyhedron formed by vectors x that satisfy the hard constraints, considering the non-negative variables x.

$$S = \{x \geq 0 | \mathrm{B}x \geq d\} \tag{42}$$

That x can be rewritten as equation (42) according to the Representation Theorem.

$$x = \sum_{q \in Q} p_q\, y_q + \sum_{r \in R} p_r\, y_r \qquad , \ \sum_{q \in Q} y_q = 1, \qquad y \in R_+^{|Q|+|R|} \tag{43}$$

According to the representation theorem, every point of the feasible region can be expressed as a convex combination of extreme points, along with a non-negative linear combination of extreme rays. This way, the Representation Theorem transforms the MP into relations (43) to (46).

$$MP: \qquad \min \sum_{q \in Q} c_q\, y_q + \sum_{r \in R} c_r\, y_r \tag{44}$$

$$subject\ to: \quad \sum_{q \in Q} a_q\, y_q + \sum_{r \in R} a_r\, y_r \geq b \tag{45}$$

$$\sum_{q \in Q} y_q = 1 \tag{46}$$

$$y_q \geq 0 \tag{47}$$

B. The Restricted Master Problem (RMP)

The number of variables in the MP is | Q + R| where the MP will have a larger size. The number of variables of MP is typically less than OP, but the size of the MP problem will also be considerable. Solving a larger problem using standard solvers will take a lot of time, and sometimes it is impossible to solve them. Therefore, RMP, which includes a smaller number of variables ($|Q'| \leq |Q|\ , |R'| \leq |R|$) is introduced.

$$RMP \coloneqq \min \sum_{q \in Q'} c_q y_q + \sum_{r \in R'} c_r y_r \tag{48}$$

$$st\colon \sum_{q\in Q'} a_q\, y_q + \sum_{r\in R'} a_r\, y_r \geq b \tag{49}$$

$$\sum_{q\in Q'} y_q = 1 \tag{50}$$

$$y \geq 0 \tag{51}$$

C. The Pricing Sub –problem (SP)

Corresponding to the Constraints (39) and (40), the dual multipliers $m_1$and m_2, are considered. Negative values of the reduced cost in the minimization problems indicate that adding the relevant variable to the solution will improve the objective function. These candidate variables must also be feasible in OP to enter the basic solution. Therefore, the SP of pricing can be expressed as follows.

$$SP\colon \qquad \min\,(C^T - m_1^T A).x - m_2 \tag{52}$$

$$s.t. \quad Bx \geq d \tag{53}$$

$$x \geq 0 \tag{54}$$

If all reduced cost values are non-negative, no further improvement will occur, and the current extreme point (or extreme rays) constitutes the optimal value. If some of the reduced cost values are negative, we can expect an improvement in the optimal value by adding extreme points or rays, and if SP is unbounded below, the solution of SP is an extreme ray, which must then be added to the RMP.

D. Column Generation Method

To solve the proposed PRSCN model, first, the initial solution is provided. The initial solution is considered the initial pattern. The developed patterns will be used to generate the column for adding the decision variables in the following steps. To create a pattern, the following index is modified.

$$\boldsymbol{c_{nvt(pt_n)}} = \boldsymbol{a_{nvt(pt_n)}} * \sum_{\boldsymbol{j}} \boldsymbol{c_{njvt}} \qquad \forall \boldsymbol{n} \in (\boldsymbol{I} \cup \boldsymbol{j}\,) \tag{55}$$

The $\boldsymbol{pt_n}$ parameter is a created pattern representing the possible paths and is related to the variable *y*.

The set of $\boldsymbol{pt_n}$ represents the nth pattern. With this definition, the MP is rewritten as below:

$$z_{MP} = \sum_{pz\in PZ} \sum_{v\in V} YZ_{pt,v} * \sum_{neI\cup J} \sum_{t\in T} a_{nvt(pt_n)} * c_{nvt(pt_n)} \tag{56}$$

$$\sum_{\mathrm{pt}\in \mathrm{PT}} \sum_{\mathrm{t}\in \mathrm{T}} \sum_{\mathrm{v}\in \mathrm{V}} \mathrm{a}_{\mathrm{nvt(pt_n)}}.\mathrm{YZ}_{\mathrm{pt,v}} = 1 \qquad \forall \mathrm{n} \in (\mathrm{I} \cup \mathrm{j}\,) \tag{57}$$

With the above transformation, the SP will take the form of the equation (57) to (60)

$$z_{sub} = \sum_{n' \in (I \cup j)} \sum_{v \in V} \sum_{j \in J} \sum_{t \in T} (C_{n'jvt} - m_j Y_{njvt}) \tag{58}$$

$$z = z_{sub} + E + F + G + H + M + N + O - (A + B + C) \tag{59}$$

s.t. (1) to (25) (60)

And (36) to (37)

Figure 4 shows the flowchart of the column generation algorithm.

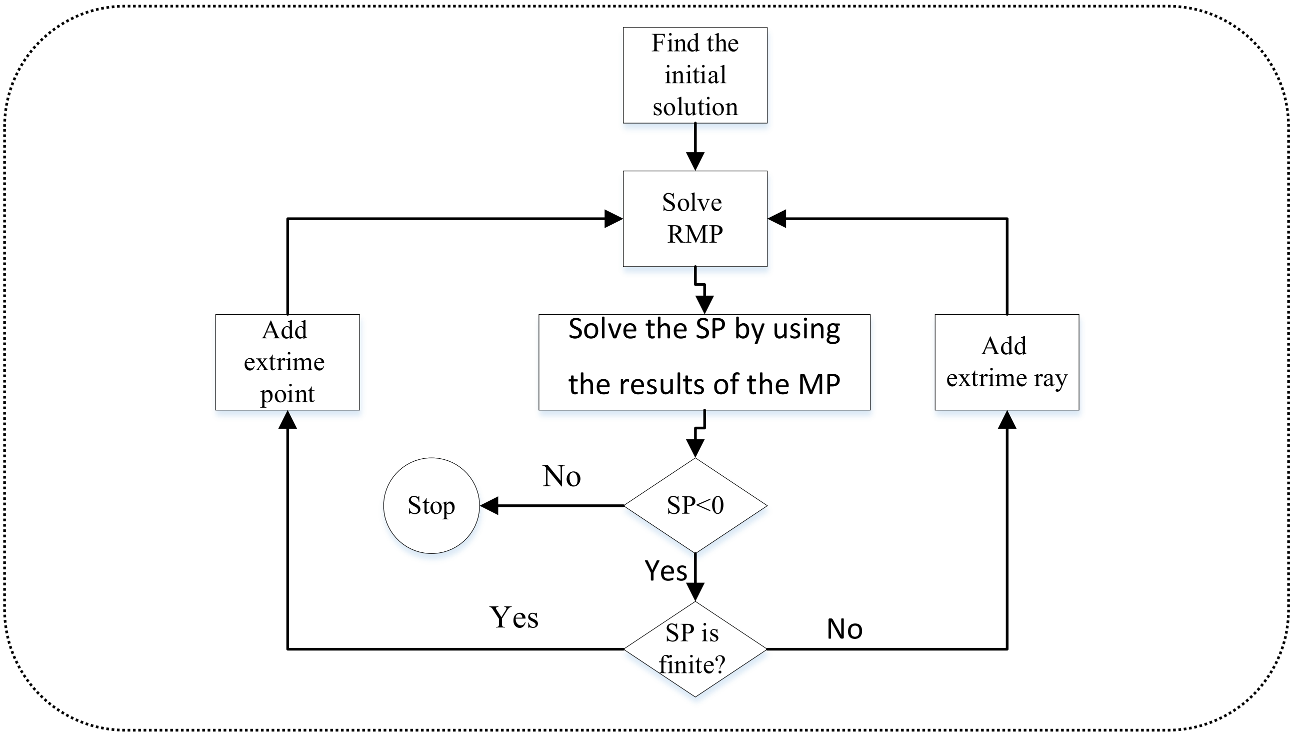

Fig. 4. Column generation algorithm

VI. CASE STUDY

In this part, the case study is explained.

A. CASE DESCRIPTION

The problem of this paper was motivated by the PEGAH dairy industry SC of IRAN, which produces a variety of dairy product. The SC management intends to add a new low-fat and high-fat probiotic yogurt product. For this purpose, the company aims to test the actual sales volume and the profit or loss associated with these products in some regions over the course of one year. This will help them assess profitability and justify proceeding to mass production. Additionally, the company plans to design an RPSCN and apply anti-corruption strategies within the supply chain, due to the perishability of its products. It is also essential to establish optimal routing measures to prevent the waste of fuel resources, reduce environmental pollution, and timely deliver products at the minimum possible cost. In the first stage, the company contracts with 12 main suppliers on an annual basis, according to the evaluations of the actual suppliers. To prevent disruptions in the milk supply, it also contracts with four other suppliers as backup suppliers so that if needed, they can also supply their orders.

After receiving the milk from the suppliers, the factory first sends the milk to the quality assurance unit for microbial and chemical monitoring. After refining, the milk is transferred into the graded tanks. The Probiotic Yogurt Factory follows the seven processes outlined in Table VIII.

Table VIII. The process of factory

stage	Stage name	description
1	Regulation and composition of milk	• The milk used in probiotic yogurt production should be controlled for the number of bacteria and their metabolites. • For the best effect of probiotics, it is necessary to have a high bacteriological quality milk • Milk powder is added to increase whey protein content to create a desirable texture.
2	Pasteurization of milk	• To pasteurize the milk mixture is heated at 203°F (95°C) for 10 minutes.
3	Homogenization	• Homogenization is done to create a stable texture with suitable viscosity. • Pressure (2000 to 2500 psi) produces probiotic yogurt.
4	Cooling the milk	• The milk is cooled to a temperature of 108°F (42 °C) until the yogurt reaches the appropriate growth temperature for primary culture.
5	Adding starters and probiotic bacteria	• Bacterial cultures are mixed with chilled milk. • The milk temperature is kept at 42 degrees until its pH reaches 4.5. This causes fermentation and allows the yogurt to have a soft jelly shape and a fermented flavor
6	Cooling probiotic yogurt	• To stop the fermentation process, the yogurt temperature is adjusted to 7 degrees
7	Packaging	• The obtained probiotic yogurt is pumped from the tank and poured into packaging containers

The products are transferred to the temporary warehouse of the company, and after one day, they are transferred to two DCs equipped with proper cooling systems. In the DCs, the distribution is done based on the demand of the customers and the price announced to the customers. In these centers, a discount policy is applied for products approaching their expiration date to ensure they are sold before perishing.

To increase the agility of the distribution system, there is horizontal cooperation among distributors so that there is no shortage in the DCs. Also, emergency inventory is considered in production facilities and DCs.

B. Computational results

The problem was solved with a computer system with specifications of 20 Gig RAM and a12-core CPU. Five test problems have been solved, as detailed in Table IX. In Table X, for test problems 1 to 4, the solution times show improvement, and for problem 5, which could not be solved using a standard MIP solver, theproposed method achieved a solution in 10900 seconds. Furthermore, the case study, which could not be solved by the standard MIP solver, was successfully addressed using the column generate method in 18063 seconds.

Table IX. specifications of the problems

Problem	Sets								
	$\|I\|$	$\|J\|$	$\|V\|$	$\|P\|$	$\|T\|$	$\|F\|$	$\|K\|$	$\|B\|$	$\|R\|$
1	3	4	3	3	3	2	2	2	2
2	3	7	4	4	5	3	3	3	3
3	4	8	4	5	6	4	4	3	3
4	4	10	4	7	5	3	3	3	3
5	4	15	5	8	5	3	3	3	3
Case study	2	40	5	2	5	1	12	4	1

Table X. Compare of time solution

Problem	Column Generation	MIP Solver
1	2.122(s)	5.44
2	8.689	9.922
3	134.420	197.24
4	533	762
5	10900	-
Case Study	18063	-

The routing results of the case study are shown in Figure 5.

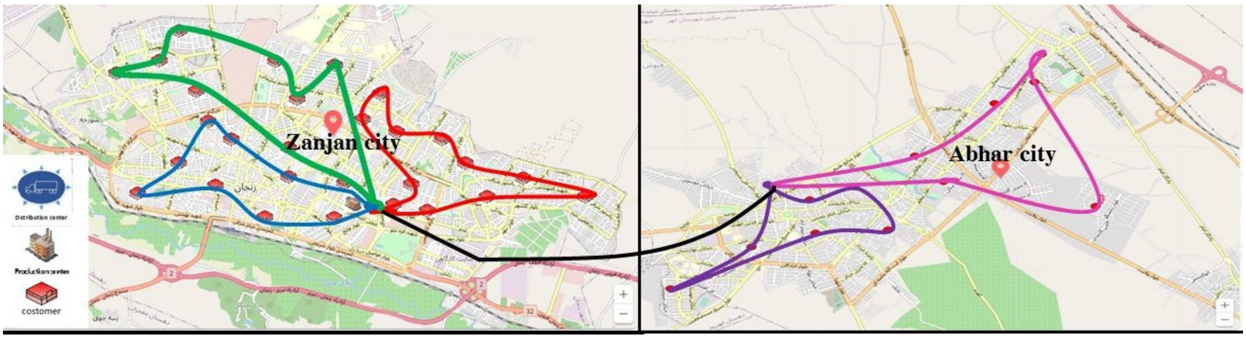

Fig. 5. Vehicle Routing Results

The effects of change in the value of α on the profitability of the SC are exhibited in Figure 6, which decreases with the increase of uncertainty of profitability and vice versa.

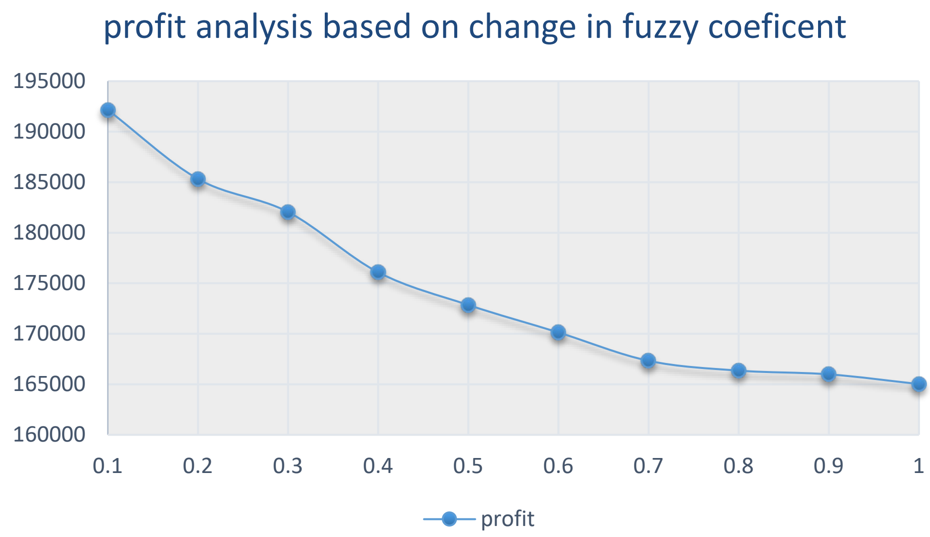

Fig. 6. The total supply chain cost versus demand changes.

Here, the one-dimensional method is used to analyze the sensitivities. Since the proposed model includes different types of costs, without a doubt, changing their values will lead to changes in the model's decisions.

The changes in the capacity of the first factory for one of the products are checked in table XI.

Table XI. change in the capacity of the factory

Parameter	Changes rate	s_{fipt}	F
$capF_{f_1 p_2}$	+10%	$S_{f_1 i_1 p_2 2} = 8.3$	$176.085*10^6$
	0%	$S_{f_1 i_1 p_2 2} = 8.3$	$176.085*10^6$
	-10%	$S_{f_1 i_1 p_2 2} = 7.2$	$170.670*10^6$
	-20%	$S_{f_1 i_1 p_2 2} = 6.8$	$162.452*10^6$

Since the model attempts to optimize inventory based on perishability, increasing the factory's capacity does not improve profitability. Reducing its capacity will decrease its profitability, decreasing the optimal production, sales, and profit. Considering that the model tries to reduce the perishability of the product according to the product's lifetime, we investigate the changes in product lifetime on the profitability of the first objective functions. Table XII examines the case study by considering the changes of -2 and +3 units over five periods.

Table XII. change in lifetime

Lifetime	The rate of changes	F1
LT_p	-2	$157.213*10^6$
	0	$176.085*10^6$
	+3	$198.565*10^6$

In the case of five periods of lifetimes, the total inventory for each product in each period is shown in Table XIII.

With an increase of the lifetime by three units—one of the key factors in storing products in the warehouse, as it extends the durability of the products—along with considerations of production capacity and inventory capacity, we anticipate an increase in inventories. Since the objective function is to maximize profit, a significant portion of this profit derives from inventory levels. Additionally, with the extended shelf life of the products, perishability decreases. This effect is expected to contribute to an increase in storage within the model. The significance of this relationship is illustrated in the outputs presented in Table XIV.

Table XIII. The amount of inventory in the DSs of the periods

Period / $\sum_{p \in P} I_{fipt}$	1	2	3	4	5
i1	4.5	13	22.25	14.25	11
i2	6	12.521	7.6	25.5	12.3
Total inventory in the entire system	10.5	25.521	29.85	39.75	23.3

Now, with an increase of the lifetime by three units, which is one of the main factors of storing products in the warehouse (because the durability of the products will increase), along with considerations of production capacity and inventory , we are expected to see an increase in the inventories. Because the objective function is to maximize profit, a significant share of it is made up of inventories. On the other hand, with the increase in shelf life, the perishability has decreased. As a result, this effect is expected to lead to an increase in storage in the model. According to the outputs in Table XIV, this importance has been determined.

Table XIV. The amount of inventory changes in the DCs by +3 in the lifetime

Period / $\sum_{p\in P} I_{fipt}$	1	2	3	4	5
i1	4.5	8	19.35	22.85	26.5
i2	6	18.2	21	32.4	20.8
Total inventory in the entire system	10.5	26.2	38.33	55.25	47.3

By reducing the lifetime of products by two units of time, the perishability of products increases, which leads to a reduction in storage, as seen in Table XV.

Table XV. The amount of inventory changes in the DCs by -2 in the lifetime

Period / $\sum_{p\in P} I_{fipt}$	1	2	3	4	5
i1	4.5	0	8.5	7.2	6.5
i2	0	15.42	0	14.8	8.6
Total inventory in the entire system	4.5	15.42	8.5	22	15.1

The model tries to store inventory according to the product lifetime. It also tries to make the perishability of the product as low as possible so that resources are not lost, and the supply chain operates sustainably.

The lack of raw materials stops the production line in the dairy industry. Therefore, the proposed model seeks to prevent product shortages in addition to the continuous supply of raw materials at other levels and distribute products at the lowest possible cost through optimal routing. As a government company, the organization under study offers the lowest prices in the market,attracting a large customer base and providing a competitive advantage.What is essential in this supply chain is to pay attention to the product's perishability and manage the product's distribution, production, and inventory so that this work can be done at the lowest cost.

Due to the uncertainty in demand, the proposed model incorporates agility by considering the horizontal relationship among the distributors and having an emergency inventory to the product shortage at the demand points and has an immediate response to the demand satisfaction. It is also sensitive to the lack of raw materials at the initial levels of the chain. Thus, the proposed supply chain network acts resiliently.

VII. CONCLUSION

In this paper, a novel model for the design of an integrated green network for perishable products under uncertainty is proposed. In addition to the resiliency of the supply chain, environmental considerations are also taken into account. Resilience has been a prominent concept in recent years. This research focused on providing new and combined strategies to improve the resilience of the supply chain for perishable products. Different strategies were identified, leading to the design of a new supply chain model. The objective function seeks to minimize the transportation cost and reduce perishability with optimal inventory control. To avoid the waste of resources and gain more profit for the products that are approaching their lifetimes, the sales strategy with price reduction is considered, which causes the optimal control of inventory and production, and it has been paid less attention in previous research. Since the proposed problem is NP-Hard, a column generation decomposition algorithm was presented to solve optimally, improving the problem-solving time. The proposed method solved five test problems, significantly reducing the solution time, and applied to a real case in the PEGAH company. It is suggested to consider reverse supply chain and scenario-based stochastic planning for future studies. Also, it is recommended to develop a proposed solution method using cutting and price accelerators. Furthermore, solving larger-scale problems, combining this approach with heuristic methods, while considering uncertain delivery times and costs, and utilizing uncertainty handling techniques from optimistic and pessimistic perspectives can be regarded as future research directions.

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