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Monitoring of simple linear profiles and change point estimation in the presence of within-profile ARMA autocorrelation

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Abstract –In statistical process control applications, the quality of certain processes or products can be accurately described by either a univariate or multivariate distribution. Nonetheless, in certain instances, the quality of a process or product can be defined by a profile, which represents the relationship between independent and response variables. Numerous studies have examined the monitoring of simple linear profiles that incorporate uncorrelated observations. Nevertheless, in practice, this assumption is seldom met as a result of spatial autocorrelation or time collapse, which can result in unsatisfactory outcomes. In numerous studies, the autocorrelation structure between observations is modeled as a first-order autoregressive ($AR(1)$) model. However, a wide range of autocorrelation between observations might not be modeled by $AR(1)$ models. Therefore, this paper examines a simple linear profile and assumes an autoregressive moving average ($ARMA$) autocorrelation structure between each observation, which is more flexible than AR models. It is assumed that in each profile, random errors follow an $ARMA(p,q)$ model. This article mainly focuses on the Phase II monitoring of simple linear profiles, with a particular emphasis on the estimation of change points, which can lead to substantial reductions in time and cost. This paper aims to estimate the change point for each simple linear profile that possesses an autocorrelation structure of $ARMA(p,q)$. To achieve this, a maximum likelihood estimator is developed. Simulation experiments are conducted to compare Hotelling's T^2 control chart with the proposed control chart. Additionally, the proposed change point estimator is compared to one of the built-in estimators for exponentially weighted moving average (EWMA) control charts. The results demonstrate that the proposed estimator has accurately estimated the change point regardless of the shift size and the $ARMA(p,q)$ coefficients, and it outperforms the built-in control chart estimator in terms of accuracy.

Keywords– *Autocorrelation, Autoregressive moving average process, Change point estimation, $EWMA-3$ control chart, Simple linear profile.*

I. INTRODUCTION

Statistical process control applications typically rely on a univariate or multivariate statistical distribution to describe the quality of a process or product. However, in some cases, the evaluation of quality can involve analyzing the correlation between a response variable and one or multiple explanatory variables. This relationship, which may have a

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linear or nonlinear pattern, is known as a profile. Different types of profiles exist based on the relationship between variables, such as simple linear profiles, multiple linear profiles, polynomial profiles, nonlinear profiles, waveform profiles, spline profiles, and profiles based on generalized linear models. These profiles are utilized for various purposes in both manufacturing and services. Kang and Albin (2000) focused on monitoring a linear process in semiconductor manufacturing. Mahmoud and Woodall (2004) proposed a method using indicator variables in a multiple regression model. To enhance performance, Wang and Tsung (2005) suggested utilizing charting schemes that rely on quantile-quantile (Q-Q) plots and profile monitoring techniques. Some other authors, including Stover and Brill (1998), Woodall et al. (2004), and Woodall (2007), have discussed practical applications of profiles. In recent years, many researchers have focused on profile monitoring, particularly for simple linear profiles, due to their specific use in calibration. Profile monitoring is performed in two phases. The objective of Phase I analysis is to assess the stability of the process and estimate its parameters. Salmasnia et al. (2019) proposed the concurrent use of $EWMA$ and range (R) control charts for profile monitoring. Several other authors, including Kang and Albin (2000), Kim et al. (2003), Mahmoud et al. (2007), Mahmoud and Woodall (2004), and Stover and Brill (1998), have investigated Phase I monitoring of simple linear profiles.

The primary objective of Phase II analysis is to promptly identify any shifts in the process parameters. Many studies have concentrated on Phase II profile monitoring, assuming that the in-control parameters are already established. Atashgar and Adelian (2023) monitored the mean vector of multivariate processes using a wavelet-based model. Haq et al. (2022) suggested four control charts, namely the maximum cumulative ($MaxCUSUM$), maximum Crosier CUSUM, maximum $EWMA$, and maximum double $EWMA$ charts, that utilize individual observations to monitor the Phase II parameters of simple linear profiles. Fallahdizcheh and Wang (2022) introduced a transfer learning framework aimed at extracting the inter-relationship between profiles to enhance monitoring accuracy. Amiri et al. (2022) investigated four different approaches for monitoring binomial regression profiles during Phase II. The T^2 and $MEWMA$ control chart, the likelihood ratio test (LRT) and $LRT/EWMA$ method were among the methods investigated, whose effectiveness was assessed using the ARL metric in both simulation experiments and a numerical example. Kim et al. (2003) proposed the use of three univariate control charts for Phase II monitoring. Additionally, other researchers, such as Gupta et al. (2006), Kang and Albin (2000), Zou et al. (2006), and Saghaei et al. (2009), have examined Phase II monitoring of simple linear profiles.

The studies mentioned above have assumed that the error terms are independently distributed within and between profiles, but in practice, this assumption is frequently violated, particularly when samples are taken at brief intervals. There have been many studies exploring the monitoring of autocorrelated profiles during Phase I and II. Jensen et al. (2008) employed linear mixed models to observe autocorrelated linear profiles in Phase I. Meanwhile, Noorossana et al. (2008) put forward three techniques to monitor simple linear profiles when there is autocorrelation between successive profiles in Phase II. Additionally, Soleimani et al. (2009) focused on autocorrelated simple linear profiles and suggested a solution to remove the impact of first-order autoregressive autocorrelation between observations in each profile. Rahimi et al. (2021) created two control charts for monitoring the mean vector and covariance matrix of autocorrelated multivariate simple linear profiles simultaneously, even when the assumption of independence among observations within each profile is not met. Nadi et al. (2023) investigated how autocorrelations within and between profiles influence the effectiveness of four monitoring techniques for simple linear profiles in Phase II. Kazemzadeh et al. (2015, 2016), Noorossana et al. (2010), Soleimani et al. (2013), Narvand et al. (2013), Koosha and Amiri (2013), Zhang et al. (2014), Khedmati and Niaki (2015), Niaki et al. (2015), Sogandi and Vakilian (2015), Wang and Tamirat (2015), Kamranrad and Amiri (2016), Kazemzadeh et al. (2016b), Tamirat and Wang (2016), Chiang et al. (2017), Maleki et al. (2018), Pini et al. (2018), Taghipour et al. (2017), and Cheng and Yang (2018) are among the authors who have investigated monitoring autocorrelated profiles.

Once the control chart indicates an out-of-control condition, the investigation into assignable causes of variation begins. Although control charts are useful, their drawback lies in the delay between the actual time of a process change, referred to as the change point, and the time at which the chart identifies the change, attributable to the inertia characteristic of the charts. Single-step, drift, isotonic, multiple-step, and sporadic changes are among the various types of changes that can cause a process to deviate from the in-control state. Although control charts are efficient in tracking process changes, they do not offer information about the timing or underlying causes of process variations. As a result, accurately estimating the change point is crucial for process engineers to promptly identify and address the root causes of variation, which can result in enhanced process quality. Accurate change point estimation can result in substantial time and cost savings. Many authors such as Perry and Pignatiello (2010), Ghazanfari et al. (2008), Noorossana and Shadman (2009), Amiri and Allahyari (2012), Asghari Torkamani et al. (2014), Sogandi and Amiri (2014), Shadman et al. (2015), Nie and Du (2017), Shadman et al. (2017), Sogandi and Amiri (2017) and so on studied the change point estimation.

Numerous researchers have examined change point estimation in profile monitoring. Mahmoud et al. (2007) developed a maximum likelihood estimator (MLE) for the change point of simple linear profiles using LRT statistics in Phase I. In this phase, Sharafi et al. (2012, 2013) put forth a MLE technique for identifying real-time step changes and linear trend changes in logistic profiles, respectively. They also presented an MLE-based method for estimating the time of drift and step changes in Poisson profiles. Kazemzadeh et al. (2015) expanded the MLE method to take into account linear disturbances in the parameters of multivariate linear regression profiles. Sogandi and Amiri (2014) introduced estimators for step and drift in Gamma regression profiles. Moreover, Ayoubi et al. (2016) developed a maximum likelihood-based approach to estimate sporadic changes in the mean of multivariate linear profiles during Phase II. Ayoubi and Ebadi (2022) used the maximum likelihood approach to propose the step and linear drift change points estimators for multivariate multi-nominal contingency tables.

There is a limited number of research on estimating change points in autocorrelated profiles. Khedmati et al. (2013) concentrated on first-order autoregressive autocorrelation structures in polynomial profiles. They proposed a solution for minimizing the effect of autocorrelation in Phase II monitoring of autocorrelated polynomial profiles, and they introduced a Generalized Linear Test (GLT)-based control chart for tracking the coefficients of polynomial profiles. To monitor the variance of the error term, they employed an R chart and recommended a likelihood ratio estimator for estimating the change points in the parameters of autocorrelated polynomial profiles. Kazemzadeh et al. (2016b) utilized MLE and clustering methods to estimate the point at which a step change occurs in monitoring autocorrelated linear profiles. To address the issue of autocorrelation between observations in each profile, they applied a transformation. The researchers then evaluated and compared the effectiveness of the proposed estimators using simulation studies. He et al. (2021) presented a Phase I technique to detect and estimate the change point in Poisson profiles that exhibit autocorrelation and have design points that are distributed unevenly or randomly. Other research on autocorrelated profile change point estimation includes Sogandi and Vakilian (2015) and Maleki et al. (2018).

This study addresses the monitoring of simple linear profiles when there is autocorrelation within each profile. In addition, it is assumed that there is no correlation between profiles. Instead of AR models commonly used to model within-profile autocorrelation, this research uses an autoregressive moving average model to express the autocorrelation structure within profiles. Since the $ARMA$ models are more flexible in expressing the autocorrelation structures in most cases compared to AR models, this type of autocorrelation between observations is considered in this research. The proposed approach involves a linear transformation to eliminate the impact of autocorrelation within each profile. This research aims to develop a monitoring scheme for simple linear profiles in the presence of $ARMA$ autocorrelation. Additionally, a maximum likelihood estimator that is based on the joint probability density function of $ARMA(p, q)$ observations within each profile is used to estimate the actual time of the step change. To evaluate its effectiveness, the proposed estimator is compared to one of the built-in change point estimators of the $EWMA$ control chart that was developed by Nishina (1992).

The organization of the rest of the paper is as follows: Formulation of the problem is presented in Section 2. A quick review of $ARMA$ processes is presented in Section 2.1. The autocorrelated simple linear profile modeling and the proposed transformation for eliminating the autocorrelation effect are given in Section 2.2. The $EWMA$ control charts for monitoring purposes are presented in Section 2.3. The Proposed change point estimator is also discussed in section

2.4. The simulation studies and the performance evaluation of the proposed change point estimator are indicated in Section 3. The confidence set estimator of the change point was constructed in Section 4, and its average cardinality and coverage probability were computed. A real case is discussed in Section 5. Finally, conclusions and further studies are made in Section 6.

II. PROBLEM FORMULATION

A. Review of autoregressive moving average processes

As stated in Section 1, the majority of profile monitoring research assumes that observations are independent. Nonetheless, this assumption is frequently violated in practice due to issues such as spatial autocorrelation or time collapse, which may have an adverse impact on the effectiveness of the relevant control charts. To address this issue, researchers have explored the autocorrelation structure between observations, with many studies focusing on an $AR(1)$ model. However, a wide range of autocorrelation between observations might not be modeled by $AR(1)$ models in some cases. As a result, this study uses $ARMA$ models, which are generalized to AR models, to model autocorrelation within observations. The random errors are assumed to follow an $ARMA$ model in each profile. For this purpose, according to Box et al. (2015), $ARMA$ models are briefly reviewed in this Section.

Consider the general stationary and invertible $ARMA(p, q)$ process defined by:

$$\phi(B)\tilde{z}_t = \theta(B)a_t, t = 1, 2, \ldots, n \tag{1}$$

where $\tilde{z}_t = (z_t - \mu)$ and z_t is the value of the process at time t, while μ denotes the process's mean. The random error terms are denoted by a_t, which are normally independently distributed with $E(a_t) = 0$ and $Var(a_t) = \sigma_a^2$. Moreover, the stationary autoregressive operator of order p is denoted by $\phi(B) = (1 - \phi_1 B -\cdots - \phi_p B^p)$, while the invertible moving average operator of order q is denoted by $\theta(B) = (1 - \theta_1 B -\cdots - \theta_q B^q)$. In addition, the backshift operator B is defined such that $B^k \tilde{z}_t = \tilde{z}_{t-k}$. To ensure that the process in equation (1) is stationary and invertible, all roots of $\phi(B) = 0$ and $\theta(B) = 0$ must lie outside the unit circle, respectively.

Equation (1) for the $ARMA(p, q)$ process implies that $E(z_t) = \mu$. By multiplying equation (1) with $\tilde{z}_{t-k}$ and taking the expectation, one can derive the autocovariance function of the $\tilde{z}_t$ process. The resulting lag k autocovariance is denoted by γ_k. Then, according to Box et al. (2015), the autocovariance function is given by:

$$\gamma_k = \phi_1\gamma_{k-1} + \cdots + \phi_p\gamma_{k-p} - \sigma_a^2(\theta_k\psi_0 + \theta_{k+1}\psi_1 + \cdots + \theta_q\psi_{q-k}) \tag{2}$$

where $\frac{\theta(B)}{\phi(B)} = \psi(B) = 1 + \sum_{j=1}^{\infty}\psi_j B^j$, with the convention that $\theta_0 = -1$:

$$\tilde{z}_t = \psi(B)a_t \tag{3}$$

$$\psi(B) = \phi^{-1}(B)\theta(B) \tag{4}$$

$$\psi(B) = 1 + \psi_1 B^1 + \psi_2 B^2 + \cdots \tag{5}$$

In time series literature, there are two different representations of linear processes. The first is shown by a linear combination of random error terms. Through this representation, equation 1 can be rewritten as equation (3), in which the weights $\psi_j; j = 0,1,2, \ldots$, corresponding to the operator of infinite order moving average.

In the second representation, $\tilde{z}_t$ is shown as a linear combination of previous process values plus the current error term. Using this representation, equation (1) can be rewritten as:

$$\pi(B)\tilde{z}_t = a_t \tag{6}$$

$$\pi(B) = \phi(B)\theta^{-1}(B) \tag{7}$$

$$\pi(B) = 1 - \pi_1 B^1 - \pi_2 B^2 - \cdots \tag{8}$$

where the $\pi_j; j = 0,1,2,...$, weights corresponding to the infinite order autoregressive operator. It can be concluded from equations (3) and (6) that $\pi(B)\psi(B) = 1$.

For $ARMA$ processes, the weights of π_j's and ψ_j's are available in Box et al. (2015). In this study, π weights are used to remove autocorrelation within profiles. For an $ARMA(p,q)$ model, they are given below:

$$\begin{cases} \pi_j = \theta_1\pi_{j-1} + \theta_2\pi_{j-2} + \cdots + \theta_q\pi_{j-q} + \phi_j \;\; ; \;\; j > 0 \,; for\; j > p \;\; \phi_j = 0 \\ \pi_0 = -1 \\ \pi_j = 0 \;\; ; \;\; j < 0 \end{cases} \tag{9}$$

According to equation (6), it can be concluded that the value of independent random shocks is obtained as follows:

$$a_t = \tilde{z}_t - \pi_1\tilde{z}_{t-1} - \pi_2\tilde{z}_{t-2} - \cdots = \sum_{j=0}^{\infty} \pi_j \tilde{z}_{t-j} \tag{10}$$

Using equation (10), a series of infinite terms is needed to calculate the value of independent shocks. However, since the underlying time series is invertible, it is evident that the $\pi_j; j = 1,2,...$, weights vanish. Thus, a finite number of these weights can be used, and the approximate values of random shocks can be calculated as follows:

$$a_t \approx \sum_{j=0}^{M} \pi_j \tilde{z}_{t-j} \tag{11}$$

where this study uses the hyperparameter M to perform a transformation to remove the autocorrelation effect from observations. This paper uses a graphical method to determine the value of M. For example, consider an $ARMA(1,1)$ model with parameters $\phi = 0.8$ and $\theta = 0.5$. According to equation (9), π_j may be obtained as follows:

$$\begin{cases} \pi_1 = \theta_1\pi_0 + \phi_1 = (0.5 \times -1) + 0.8 = 0.3 \\ \pi_2 = \theta_1\pi_1 = (0.5 \times 0.3) = 0.15 \\ \pi_3 = \theta_1\pi_2 = (0.5 \times 0.15) = 0.075 \\ \pi_4 = \theta_1\pi_3 = (0.5 \times 0.075) = 0.0375 \\ \pi_5 = \theta_1\pi_4 = (0.5 \times 0.0375) = 0.0187 \\ \pi_6 = \theta_1\pi_5 = (0.5 \times 0.0187) = 0.0093 \\ \vdots \end{cases}$$

π_j weights against lags for this example are shown in Figure 1:

According to Figure 1, π_j weights vanish as j tends to infinity, so the last lag before $\pi_j; j = 1,2,...$, becomes approximately zero, is considered the value of the M hyperparameter. Here, it is appropriate to consider $M = 6$.

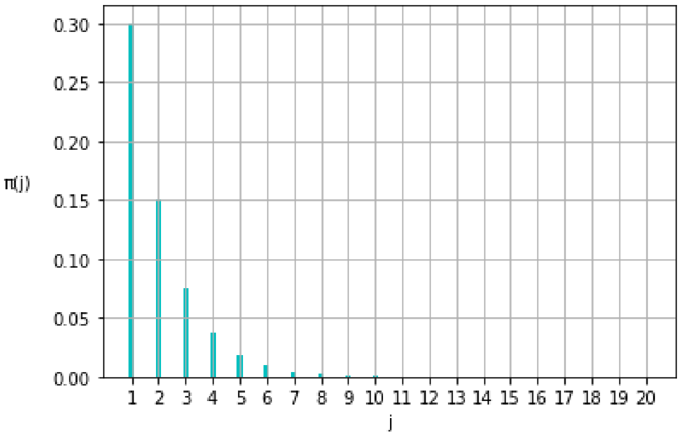

Figure 1. π weights against lags for ARMA (1,1) model

B. Autocorrelated simple linear profile and the proposed method of removing autocorrelation

In numerous studies concerning the monitoring of simple linear profiles, it is assumed that there is no autocorrelation among the observations within each profile. Nonetheless, in practical applications, this assumption is seldom observed due to spatial autocorrelation or time collapse, which can result in poor outcomes for the related control charts. This study assumes that when the process is in-control, the relationship between the response variable and independent variable for the i^{th}; $i = 1,2, \dots, n$, observation in the $j^{th}; j = 1,2, \dots$, sample can be expressed as follows:

$$y_{ij} = \mathrm{A}_{0j} + \mathrm{A}_{1j} x_i + \varepsilon_{ij} \tag{12}$$

$$\varepsilon_{ij} = \phi_1 \varepsilon_{(i-1)j} + \dots + \phi_p \varepsilon_{(i-p)j} + a_{ij} - \theta_1 a_{(i-1)j} - \dots - \theta_q a_{(i-q)j} \tag{13}$$

in which $\varepsilon_{ij}'s$ follow the $ARMA(p,q)$ model.

In equation (13) $a_{ij}'s$ are normally independently distributed with $E(a_{ij}) = 0$, and $Var(a_{ij}) = \sigma_a^2$.

Let B denote the backshift operator such that $B^k \varepsilon_{ij} = \varepsilon_{(i-k)j}$. Using this operator, equation (13) can be written as follows:

$$(1 - \phi_1 B - \cdots - \phi_p B^p)\varepsilon_{ij} = (1 - \theta_1 B - \cdots - \theta_q B^q) a_{ij} \tag{14}$$

In equation (14), $\phi(B) = 1 - \phi_1 B - \cdots - \phi_p B^p$, and $\theta(B) = 1 - \theta_1 B - \cdots - \theta_q B^q$, are the autoregressive and moving average operators, respectively. It is required that all roots of $\phi(B) = 0$, and $\theta(B) = 0$, lie outside of the unit circle for an *ARMA* process to be stationary and invertible, respectively.

As an example, for $ARMA(1,1)$ model, $\phi(B) = (1 - \phi B)$, and $\theta(B) = (1 - \theta B)$, are the autoregressive and moving average operators, respectively. Thus, for this model $\pi(B) = \frac{(1-\phi B)}{(1-\theta B)}$, using equation (6), the relationship between autocorrelated error terms, ε_{ij}, and independent error terms a_{ij} is obtained as follows:

$$\pi(B)\varepsilon_{ij} = a_{ij} \tag{15}$$

In equation (12), it is assumed that the x-values remain constant and consistent across all profiles. The present article focuses on Phase II analysis, where the in-control values of parameters A_{00}, A_{10} and σ_{a0}^2 are considered to be known. The model described in equation (12) features an $ARMA(p,q)$ autocorrelation structure within each profile but not between profiles.

It can be shown that the relationship between the error terms in the autoregressive moving average structure results in autocorrelation among the observations within each profile. For this purpose, one may multiply both sides of equation (12) by $\phi(B) = 1 - \phi_1 B - \cdots - \phi_p B^p$, which gives the following result:

$$\phi(B)y_{ij} = \phi(B)\mathrm{A}_{0j} + \mathrm{A}_{1j}\phi(B)x_i + \phi(B)\varepsilon_{ij} \tag{16}$$

According to equation (14), equation (16) may be rewritten as follows,

$$y_{ij} = (\phi_1 B + \cdots + \phi_p B^p)y_{ij} + \phi(B)\mathrm{A}_{0j} + \mathrm{A}_{1j}\phi(B)x_i + \theta(B)a_{ij} \tag{17}$$

Based on equation (17), it can be inferred that there is an autocorrelation among the response values within each profile. To facilitate monitoring, it is advisable to eliminate the autocorrelation between observations. In this research, a linear transformation is used to make the observations uncorrelated. According to equation (15), using linear operator $\pi(B)$ one may transform ε_{ij}'s to a_{ij}'s which are uncorrelated. Thus, to remove autocorrelation between observations, it is proposed to multiply both sides of equation (12) by $\pi(B)$ operator. This operator is an infinitive order polynomial in *B*. However, it can be shown that if the $ARMA$ model is invertible, π_j weights vanish as *j* tends to infinity, according to Box et al. (2015). Thus, instead of using an infinite order polynomial $\pi(B)$, which is shown in equation (8), one may use a truncated polynomial $\tilde{\pi}(B)$ as follows:

$$\tilde{\pi}(B) = 1 - \pi_1 B^1 - \pi_2 B^2 - \cdots - \pi_M B^M \tag{18}$$

in which the hyperparameter *M* is a sufficiently large number where $\pi_j \approx 0$, for $j > M$. Since this paper primarily focuses on phase II analysis, it is assumed that in-control values of profile parameters such as ϕ and θ are available from Phase I to obtain the π weights according to equation (9). The value of M can also be obtained from the analysis of π weights.

Therefore, to remove autocorrelation between observations, both sides of equation (12) are multiplied by the linear operator $\tilde{\pi}(B)$ which gives the following results:

$$\tilde{\pi}(B)y_{ij} = \tilde{\pi}(B)\mathrm{A}_{0j} + \mathrm{A}_{1j}(\tilde{\pi}(B)x_i) + \tilde{\pi}(B)\varepsilon_{ij} \tag{19}$$

After simplifying equation (19), the result is as follows:

$$y'_{ij} = A'_{0j} + \mathrm{A}_{1j}x'_i + a_{ij} \tag{20}$$

where $y'_{ij} = \tilde{\pi}(B)y_{ij}$, $A'_{0j} = \tilde{\pi}(B)\mathrm{A}_{0j}$, $x'_i = \tilde{\pi}(B)x_i$, and $a_{ij} = \pi(B)\varepsilon_{ij} \approx \tilde{\pi}(B)\varepsilon_{ij}$. Note that:

$$y'_{ij} = y_{ij} - \pi_1 y_{(i-1)j} - \cdots - \pi_M y_{(i-M)j}; i > M ; j = 1,2, \dots \quad (21)$$

$$A'_{0j} = A_{0j}(1 - \pi_1 - \cdots - \pi_M) \quad (22)$$

$$x'_i = x_i - \pi_1 x_{i-1} - \cdots - \pi_M x_{i-M} ; i > M ; j = 1,2, \dots \quad (23)$$

As a result of transforming the variables y'_{ij}'s and x'_i's, a linear profile model with independent error terms can be established, as presented in equation (20). Subsequently, the profile parameters can be estimated using the ordinary least squares (OLS) approach.

Samples are taken sequentially from the process to monitor the process in Phase II. Transformed variables are obtained using equations (21) – (23). For each profile, the least squared estimators of regression parameters, A'_{0j} and A_{1j} are then computed. These estimators are used to construct test statistics for monitoring the process in Phase II. The next Section addresses the construction of control charts for monitoring the process.

C. EWMA-3 control charts to monitor simple linear profiles in Phase II

In this study, the $EWMA-3$ control chart, which was initially proposed by Kim et al. (2003), is used to monitor the profile parameters following the transformation suggested in the preceding section. By reducing the impact of autocorrelation between observations, this transformation enables straightforward monitoring of the profile parameters during Phase II. Several studies have demonstrated that the $EWMA-3$ approach exhibits superior performance compared to other methods, such as T^2 and $EWMA/R$, in detecting shifts in the individual parameters (see Kim et al. (2003) and Soleimani et al. (2009) for more details). The $EWMA-3$ control charts employ three distinct test statistics to monitor the slope, intercept, and variance of the error term separately. Since these charts monitor the process parameters separately, the root causes of the process being out-of-control can be determined more precisely after generating an out-of-control signal on these charts. Thus, the shifts in these parameters may be detected more rapidly. This technique centers the x' variable by subtracting its mean, such that the resulting mean of the centered x' values is zero. The objective of this transformation is to attain independence between the estimators of the intercept and slope. This enables the use of distinct control charts to monitor each parameter, making it easier to interpret any out-of-control signals. The transformed model can be formulated as follows:

$$y'_{ij} = \beta_{0j} + \beta_{1j} x''_i + a_{ij} \quad (24)$$

where $\beta_{0j} = A'_{0j} + A_{1j}\bar{x}'$, $\beta_{1j} = A_{1j}$, $x''_i = x'_i - \bar{x}'$, $\bar{x}' = \frac{\sum_{i=M+1}^{n} x'_i}{n-(M+1)}$.

The least squared estimator of the intercept for the j^{th} sample "$b_{0(j)}$" is used in the $EWMA$ statistics as follows:

$$EWMA_I(j) = \lambda b_{0(j)} + (1-\lambda) EWMA_I(j-1) \quad (25)$$

where $0 < \lambda \leq 1$, λ is a smoothing parameter, and $EWMA_I(0) = \beta_{00} = A'_{00} + A_{10}\bar{x}'$. Equation (26) establishes the upper and lower control limits for the control chart. If the calculated $EWMA_I$ statistics are within these control limits, the profile's intercept is considered to be in statistical control.

$$(LCL_I, UCL_I) = \beta_{00} \pm L_I \sigma_{a0} \sqrt{\lambda / (2-\lambda)(n-1)} \quad (26)$$

where $L_I > 0$ is chosen to give a specified in-control average run length (ARL).

The least squared estimator of the slope for the j^{th} sample "$b_{1(j)}$" is used to define the $EWMA$ statistics as follows:

$$EWMA_s(j) = \lambda b_{1(j)} + (1-\lambda)EWMA_s(j-1) \tag{27}$$

where again $0 < \lambda \leq 1$ is a smoothing constant and $EWMA_s(0) = \beta_{10} = A_{10}$. Equation (28) provides the lower and upper control limits for this control chart. If the $EWMA_s$ statistics fall within the control limits, then the slope of the profile is deemed to be in statistical control.

$$(LCL_s, UCL_s) = \beta_{10} \pm L_s \sigma_{a0} \sqrt{\lambda \Big/ (2-\lambda) \sum_{i=M+1}^{n} x_i''^2} \tag{28}$$

where $L_s > 0$ is chosen to give a specified in-control ARL.

To monitor the variance of the error term "σ_a^2", a third $EWMA$ control chart is employed. This control chart utilizes the estimator of the error term variance, which is based on the mean squared error of residuals (MSE_j). The $EWMA$ statistics for this control chart are calculated as follows:

$$EWMA_E(j) = \max\left(\lambda\left(MSE_j - 1\right) + (1-\lambda)EWMA_E(j-1), 0\right) \tag{29}$$

$$e_{ij} = y'_{ij} - b_{0(j)} - b_{1(j)} x''_i \tag{30}$$

$$MSE_j = \frac{\sum_{i=M+1}^{n} {e_{ij}}^2}{n-(M+1)} \tag{31}$$

where $0 < \lambda \leq 1$ is a smoothing constant and $EWMA_E(0) = 0$. The upper control limit of this control chart is given in equation (32).

$$UCL_E = L_E \sqrt{\lambda var\left(MSE_j\right) \Big/ (2-\lambda)} \tag{32}$$

where $L_E > 0$ is chosen to give a specified in-control ARL, and $var\left(MSE_j\right) = \frac{2\sigma_a^4}{n-1}$.

Following the creation of three control limits outlined in equations (26), (28), and (32), the test statistics defined in equations (25), (27), and (29) are calculated for each sample taken during Phase II, and they are compared to the corresponding control chart. If the test statistics fall outside the corresponding control limit, the process is deemed to be out of control. In such cases, it is necessary to estimate the change point. The forthcoming section introduces the proposed approach for estimating the change point.

D. Proposed change point estimator

As long as the test statistics remain within the control limits, the process is considered to be in statistical control with known parameters. However, if a change occurs at an unknown time τ, the process shifts to an out-of-control state with unknown parameters. This study assumes that any alteration to the profile parameters takes the form of a step change, and the affected parameter remains at the adjusted level until the underlying causes are identified and remediated. Under this model, for the profiles $j = 1, 2, \ldots, \tau$, the process is considered to be in-control, and the parameters β_{00}, β_{10}, and σ_{a0}^2 are known. While for profiles $j = \tau + 1,\ \tau + 2, \ldots, T$, the parameters change to out-of-control values β'_0, β'_1, and σ'^2_a; where T is the time the first test statistic falls out of the corresponding control limits.

Note that at time T, some test statistics may fall inside the corresponding control limits. Thus, the corresponding parameters are in control in this situation. Suppose that, for instance, at time T, the control chart for the intercept indicates that the process is out of control. While the other two control charts do not signal out-of-control conditions in slope and error term variance. Thus, it is concluded that at time T, a change in the intercept has occurred while other parameters have not changed.

Transformed observations are used to compute the maximum likelihood estimators of τ. In what follows, the worst case is considered. In essence, the assumption is that all three control charts indicate out-of-control conditions at time T. However, if some control charts at time T do not reveal out-of-control conditions, there is no need to estimate the corresponding parameter..

To determine the maximum likelihood estimator of the change point, it is required to obtain the likelihood function of y'_{ij}'s. Since a_{ij}'s are normally and independently distributed, it can be concluded that the $y'_{ij}; j = 1,2, \ldots, \tau$, are also normally and independently distributed with mean $\beta_{00} + \beta_{10}x''_i$, and variance of σ_{a0}^2. Therefore, the probability density function of $y'_{ij}; j = 1,2, \ldots, \tau$, is as follows:

$$f(y'_{ij}) = \frac{1}{\sigma_{a0}\sqrt{2\pi}} e^{-\frac{(y'_{ij} - (\beta_{00} + \beta_{10}x''_i))^2}{2\sigma_{a0}^2}} \quad ; i = M+1, M+2, \ldots, n; j = 1,2, \ldots, \tau \tag{33}$$

After change point τ, y'_{ij}'s are normally independently distributed with mean $\beta'_0 + \beta'_1 x''_i$, and variance σ'^2_a. The probability distribution function of $y'_{ij}; j = \tau + 1, \tau + 2, \ldots, T$, will be calculated as follows:

$$f(y'_{ij}) = \frac{1}{\sigma'_a\sqrt{2\pi}} e^{-\frac{(y'_{ij} - (\beta'_0 + \beta'_1 x''_i))^2}{2\sigma'^2_a}} \quad ; i = M+1, M+2, \ldots, n; j = \tau+1, \tau+2, \ldots, T \tag{34}$$

If it is assumed that a change takes place at time τ, the probability distribution function of the transformed observations can be expressed as follows:

$$\begin{aligned} L(\acute{\beta}_0, \acute{\beta}_1, \acute{\sigma}_a^2, \tau\,;\, y'_{ij}) &= \prod_{j=1}^{T} \prod_{i=M+1}^{n} f(y'_{ij}) \\ &= \frac{1}{(2\pi\sigma_{a0}^2)^{\frac{\tau(n-M)}{2}}} e^{-\frac{1}{2\sigma_{a0}^2}\sum_{j=1}^{\tau}\sum_{i=M+1}^{n}(y'_{ij} - (\beta_{00} + \beta_{10}x''_i))^2} \\ &\times \frac{1}{(2\pi\sigma'^2_a)^{\frac{(T-\tau)(n-M)}{2}}} e^{-\frac{1}{2\sigma'^2_a}\sum_{j=\tau+1}^{T}\sum_{i=M+1}^{n}(y'_{ij} - (\beta'_0 + \beta'_1 x''_i))^2} \end{aligned} \tag{35}$$

Considering the logarithm of equation (35), the following can be obtained:

$$\begin{aligned} \ln\left(L(\beta'_0, \beta'_1, \sigma'^2_a, \tau\,;\, y'_{ij})\right) &= -\frac{\tau(n-M)}{2}\ln(2\pi\sigma_{a0}^2) - \frac{1}{2\sigma_{a0}^2}\sum_{j=1}^{\tau}\sum_{i=M+1}^{n}(y'_{ij} - (\beta_{00} + \beta_{10}x''_i))^2 \\ &- \frac{(T-\tau)(n-M)}{2}\ln(2\pi\sigma'^2_a) - \frac{1}{2\sigma'^2_a}\sum_{j=\tau+1}^{T}\sum_{i=M+1}^{n}(y'_{ij} - (\beta'_0 + \beta'_1 x''_i))^2 \end{aligned} \tag{36}$$

Equation (36) involves a log-likelihood function that requires the estimation of four parameters: $\beta'_0, \beta'_1, \sigma'^2_a$, and τ. Initially, the maximum likelihood estimators for β'_0, β'_1 and σ'^2_a are determined based on a fixed value for the change point, which falls within the range $0 \le t < T$. It is important to note that the estimation of τ is also required for the log-likelihood function stated in equation (36), making a total of four unknown parameters that need to be estimated. To determine these estimators, the partial derivative of equation (36) concerning β'_0, β'_1 and σ'^2_a are computed and set to zero as follows:

$$\frac{\partial \ln L\left(\beta'_0, \beta'_1, \sigma'^2_a, \tau\, ;\, y'_{ij}\right)}{\partial \beta'_0} = 0 \tag{37}$$

$$\frac{\partial \ln L\left(\beta'_0, \beta'_1, \sigma'^2_a, \tau\, ;\, y'_{ij}\right)}{\partial\ \beta'_1} = 0 \tag{38}$$

$$\frac{\partial \ln L\left(\beta'_0, \beta'_1, \sigma'^2_a, \tau\, ;\, y'_{ij}\right)}{\partial \sigma'^2_a} = 0 \tag{39}$$

Solving equations (37) – (39), the estimators of the parameters β'_0, β'_1 and σ'^2_a can be obtained as follows:

$$\hat{\beta}'_1(t) = \frac{\sum_{j=\tau+1}^{T}\sum_{i=M+1}^{n}(y'_{ij} - \bar{y}')\ (x''_i - \bar{x}'')}{\sum_{i=M+1}^{n}(x''_i - \bar{x}'')^2} = \frac{(S_{x''y'})_{t,T}}{(S_{x''x''})_{t,T}} \tag{40}$$

$$\hat{\beta}'_0(t) = \frac{\sum_{i=M+1}^{n} y'_{ij}}{n - (M+1)} = (\bar{y}')_{t,T}\quad ;\quad \tau + 1 \le j < T \tag{41}$$

$$\hat{\sigma}'^2_a = \frac{\sum_{j=\tau+1}^{T}\sum_{i=M+1}^{n}(y'_{ij} - (\hat{\beta}'_{0(t)} + \hat{\beta}'_{1(t)}x''_i))^2}{(T-\tau)(n-M)} \tag{42}$$

where the value of $(.)_{t,T}$ is determined using the profiles ranging from t to T. Subsequently, these estimated parameters are employed in equation (36) to derive the maximum likelihood estimate of the change point.

$$\hat{\tau} = \arg\max_{\tau} \left\{ -\frac{\tau(n-M)}{2}\ln(2\pi\sigma^2_{a0}) - \frac{1}{2\sigma^2_{a0}}\sum_{j=1}^{\tau}\sum_{i=M+1}^{n}(y'_{ij} - (\beta_{00} + \beta_{10}x''_i))^2 - \frac{(T-\tau)(n-M)}{2}\ln(2\pi\hat{\sigma}'^2_a) - \frac{(T-\tau)(n-M)}{2} \right\}\ ;\ \ 0 \le t < T \tag{43}$$

The succeeding paragraphs provide a summary of the proposed method:

At first, the value of the hyperparameter M, which depends on the $ARMA(1,1)$ coefficients, is estimated. The second step involves implementing the suggested transformation on the observations, known parameters, and x values. After that, the underlying profile is monitored using the $EWMA - 3$ control chart. A signal from a control chart denotes that the process is not in-control anymore. Upon receiving the signal, the investigation into the sources of variation begins. The proposed change point estimator is ultimately utilized to obtain a precise estimation of the change point, which is helpful in identifying the underlying causes of the change. The steps of the proposed method to monitor simple linear profiles are shown in Figure 2:

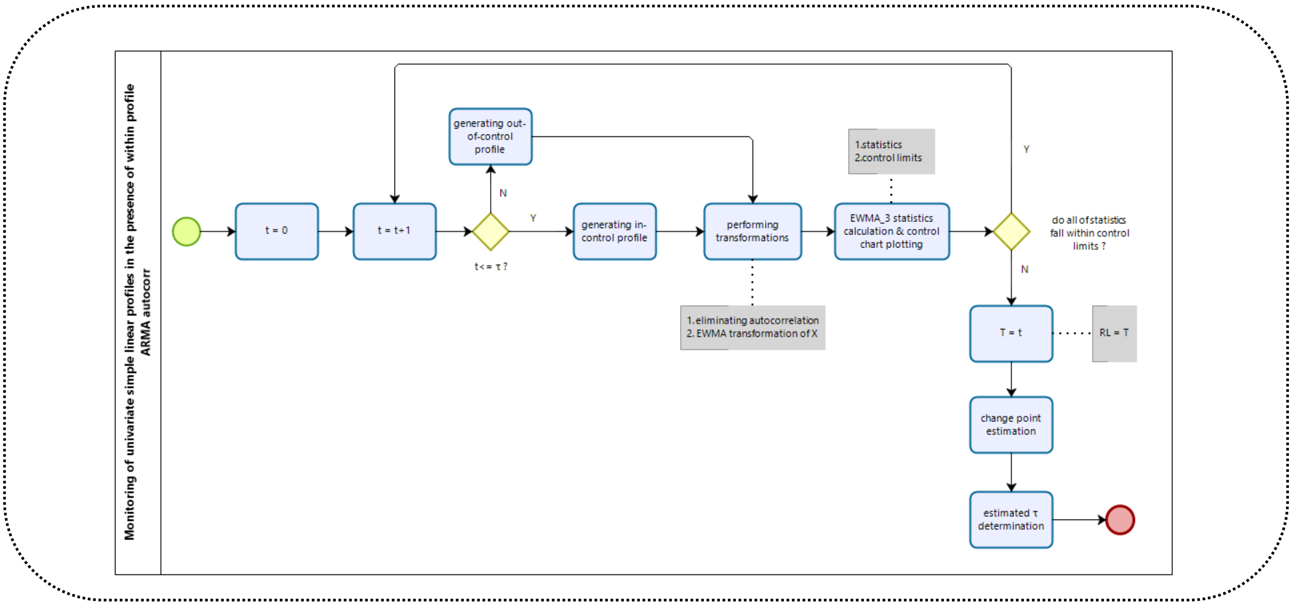

Figure 2. Proposed method flowchart

III. SIMULATION STUDIES

In this Section, first, the performance of $EWMA-3$ control chart is compared with T^2 control chart in terms of average run length criterion. Considering equation (12), the T_j^2 statistics are written as follows:

$$T_j^2 = ((\widehat{A_{0j}}, \widehat{A_{1j}}) - (\mathrm{A}_{00}, \mathrm{A}_{10}))S^{-1}((\widehat{A_{0j}}, \widehat{A_{1j}}) - (\mathrm{A}_{00}, \mathrm{A}_{10}))^T \tag{44}$$

where $\widehat{A_{0j}}$ and $\widehat{A_{1j}}$ are the least squared estimators of the intercept and slope for the j^{th} sample, respectively, $S = \begin{pmatrix} \sigma_\varepsilon^2(\frac{1}{n-1} + \frac{\bar{x}^2}{S_{xx}}) & -\sigma_\varepsilon^2 \frac{\bar{x}}{S_{xx}} \\ -\sigma_\varepsilon^2 \frac{\bar{x}}{S_{xx}} & \frac{\sigma_\varepsilon^2}{S_{xx}} \end{pmatrix}$, T stands for transpose operator, and $S_{xx} = \sum_{i=1}^{n}(x_i - \bar{x})^2$.

Under in-control conditions, the T_j^2 statistic follows a chi-squared distribution with two degrees of freedom. As a result, the upper control limit for the T_j^2 control chart is $UCL = \chi_{2,\alpha}^2$, where $\chi_{2,\alpha}^2$ represents the $(1-\alpha)100$ percentile of the chi-square distribution with two degrees of freedom.

As a part of this study, the effectiveness of the suggested change point estimator is evaluated by comparing it with the built-in change point estimator of the $EWMA$ control chart proposed by Nishina (1992). Nishina (1992) proposed an estimator that utilizes signals from an $EWMA$ control chart to detect the change point in processes monitored by the control chart. This methodology is employed on the control chart that generates an out-of-control signal, as the process is monitored using three distinct $EWMA$ control charts. Therefore, if the $EWMA_I$ control chart generates an out-of-control signal, the estimated change point indicated by $\hat{\tau}_{EWMA}$ is given by equation (45):

$$\hat{\tau}_{EWMA} = \begin{cases} \max\{j: EWMA_I(j) \le \beta_{00}\} & if \quad EWMA_I(T) > UCL_I \\ \max\{j: EWMA_I(j) \ge \beta_{00}\} & if \quad EWMA_I(T) < LCL_I \end{cases} \tag{45}$$

If the $EWMA_S$ control chart generates an out-of-control signal, the change point is estimated by:

$$\hat{\tau}_{EWMA} = \begin{cases} \max\{j: EWMA_S(j) \le \beta_{10}\} & if \quad EWMA_S(T) > UCL_S \\ \max\{j: EWMA_S(j) \ge \beta_{10}\} & if \quad EWMA_S(T) < LCL_S \end{cases} \tag{46}$$

Finally, if the $EWMA_E$ control chart generates an out-of-control signal, the estimated change point can be obtained by:

$$\hat{\tau}_{EWMA} = \max\{j: EWMA_E(j) \leq \ln \sigma_{a0}^2\} \tag{47}$$

This paper has used Python to perform all programming tasks in the simulation experiments that compare the performance of the proposed change point estimator with the built-in $EWMA$ estimator.

The effectiveness of the suggested method for monitoring simple linear profiles is assessed through Monte Carlo simulation. The simulation studies consider a simple linear profile with $ARMA(1,1)$ error terms. Additionally, a comparison is made between the proposed estimator and the estimators obtained by the built-in change point estimator of the $EWMA$ control chart. The simulation studies employ the following model:

$$y_{ij} = 3 + 2x_i + \varepsilon_{ij} \tag{48}$$

$$\varepsilon_{ij} = \phi\varepsilon_{(i-1)j} + a_{ij} - \theta a_{(i-1)j} \tag{49}$$

where a_{ij}s are normally independently distributed with mean zero and variance one, and the explanatory variable is set equal to $2, 4, 6, \dots, 50$, which is fixed from profile to profile.

To achieve an overall in-control ARL of approximately 200 under $ARMA(1,1)$ parameters with ϕ values of 0.2, 0.5, and 0.8 and θ values of 0.2, 0.5, and 0.8, the $EWMA - 3$ control chart employs a smoothing parameter of 0.2, along with parameter values of L_I, L_S, L_E set to 3.014, 3.012, 3.870, respectively.

In simulation studies, the change point value of $\tau = 10$ is considered. Hence, the first ten profiles are generated from a simple linear profile with $ARMA(1,1)$ error terms and known parameters ($\beta_{00} = 3,\ \beta_{10} = 2,\ \sigma_{a0}^2 = 1$) for the in-control state. Starting from the 11th profile, the observations are generated randomly from an out-of-control process that features a step shift in parameters, as $(3 + \delta\_1), (2 + \delta\_2)$, and $(1 + \delta\_3)$, where δ_1 indicates the magnitude of both increasing and decreasing step changes in intercept, and it varies from -2 to -0.2 and from 0.2 to 2 with a 0.2 increment. δ_2 indicates the magnitude of both increasing and decreasing step changes in slope, which varies from -0.25 to -0.025 and from 0.025 to 0.25 with a 0.025 increment. Moreover, δ_3 indicates the magnitude of the increasing step change in error term variance, which varies from 0.4 to 2 with a 0.4 increment. In this case, it is assumed that only one parameter changes at a time.

The proposed transformation removes autocorrelation between observations by estimating M using a graphical method. Using simulation studies, the value of 10 is considered for M, which gives a proper transformation for all levels of $ARMA(1,1)$ coefficients. Then, the test statistic is computed for each profile, and the results are plotted on both the $EWMA - 3$ and T^2 control charts to compare their performance. The generation of profiles continues until the EWMA-3 control chart indicates that the process is out of control. At this stage, the observation generation ceases, and the suggested change point estimator is computed. This procedure is repeated 10,000 times, and the average run length, mean, and mean squared errors of the change point estimations obtained from all iterations are calculated. ARL is calculated using the following formula:

$$ARL = \frac{\sum_{i=1}^{m} RL}{m} \quad ; \quad m = 10{,}000 \tag{50}$$

where $RL = T$ is the signal time.

Table 1 and Table 2 contain $ARL(EWMA)$, $ARL(T^2)$, mean, and mean squared errors (MSE) of the change point estimates under a step change in the intercept from β_{00} to $\beta_0' = \beta_{00} + \delta_1$ for the proposed change point estimator ($\hat{\tau}_0$) and the built-in change point estimator ($\hat{\tau}_1$) under different $ARMA(1,1)$ coefficients. Note that the same control charts are used for monitoring profiles with the proposed change point estimator and the built-in change point estimator. Since ARLs are also the same, only one is given in the following tables.

Table 1. Mean and MSE of the two change point estimators for the increasing step shifts in the intercept based on different $ARMA(1,1)$ coefficients

	$\phi = 0.2, \theta = 0.2$						$\phi = 0.2, \theta = 0.5$						$\phi = 0.2, \theta = 0.8$					
δ_1	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE ($\hat{\tau}_1$)	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)
0.2	23.902	59.673	11.403	26.537	10.158	34.456	14.613	44.304	10.589	6.78	8.676	19.904	13.141	49.175	10.988	1.521	8.293	20.517
0.4	14.819	45.814	10.765	3.480	8.509	19.591	13.120	38.583	10.783	1.979	8.344	19.761	11.906	38.311	10.377	1.413	8.27	20.372
0.6	13.249	36.243	10.759	1.515	8.404	19.483	12.325	32.147	10.614	1.762	8.302	19.663	11.510	32.156	9.968	1.380	8.266	20.287
0.8	12.617	27.926	10.704	1.075	8.313	19.452	11.968	25.401	10.293	1.423	8.344	19.506	11.484	29.629	9.951	1.336	8.242	20.173
1	12.271	21.554	10.568	0.965	8.333	19.329	11.711	20.972	10.150	1.414	8.316	19.438	11.423	28.231	9.958	1.284	8.276	20.122
1.2	12.035	18.324	10.391	0.958	8.388	19.033	11.627	18.314	10.057	1.397	8.347	19.195	11.527	25.323	9.966	1.249	8.228	20.034
1.4	11.680	17.977	10.191	0.883	8.411	18.995	11.612	16.829	10.018	1.289	8.320	19.072	11.495	23.134	9.955	1.217	8.258	20.012
1.6	11.728	15.641	10.057	0.875	8.300	18.887	11.596	14.204	9.987	1.218	8.284	19.047	11.470	18.283	9.950	1.173	8.277	20.007
1.8	11.621	15.256	10.002	0.831	8.351	18.568	11.617	13.716	9.934	1.197	8.333	19.032	11.512	14.347	9.931	1.118	8.304	19.962
2	11.612	14.842	9.998	0.784	8.364	18.234	11.631	13.062	9.915	1.182	8.368	19.013	11.488	12.110	9.908	1.091	8.308	19.943
	$\phi = 0.5, \theta = 0.2$						$\phi = 0.5, \theta = 0.5$						$\phi = 0.5, \theta = 0.8$					
δ_1	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE ($\hat{\tau}_1$)	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE ($\hat{\tau}_1$)	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE ($\hat{\tau}_1$)
0.2	46.284	78.732	14.602	124.993	26.083	1048.281	23.934	57.393	11.291	26.037	10.122	34.464	14.862	42.894	10.860	2.196	8.416	20.109
0.4	19.493	62.652	10.656	14.152	9.070	21.658	14.935	47.925	10.711	3.308	8.541	19.733	12.581	38.123	10.743	1.752	8.368	19.362
0.6	15.261	48.397	10.757	4.494	8.583	19.756	13.252	45.860	10.816	1.657	8.374	19.712	11.969	33.879	10.342	1.524	8.355	19.397
0.8	13.582	38.817	10.736	2.190	8.394	19.552	12.614	32.663	10.747	1.314	8.359	19.547	11.632	31.499	10.009	1.381	8.255	20.398
1	13.163	28.601	10.740	1.071	8.440	19.487	12.292	26.569	10.538	1.271	8.324	19.464	11.580	26.960	9.958	1.342	8.304	19.996
1.2	12.717	24.174	10.662	1.063	8.351	19.418	12.041	24.287	10.373	1.152	8.306	19.451	11.571	24.802	9.924	1.270	8.306	19.830
1.4	12.445	22.914	10.679	0.926	8.307	19.354	11.832	23.946	10.191	1.079	8.304	19.390	11.569	23.065	9.960	1.223	8.268	20.146
1.6	12.314	17.273	10.578	0.876	8.395	18.316	11.675	17.591	10.071	1.066	8.245	19.317	11.554	19.302	9.952	1.217	8.298	20.078
1.8	12.135	15.138	10.438	0.834	8.347	18.289	11.653	16.963	9.961	1.028	8.371	19.248	11.532	15.407	9.962	1.203	8.298	19.850
2	12.011	13.665	10.320	0.801	8.307	18.175	11.590	13.793	9.915	1.008	8.326	19.123	11.524	12.164	9.946	1.172	8.356	19.408
	$\phi = 0.8, \theta = 0.2$						$\phi = 0.8, \theta = 0.5$						$\phi = 0.8, \theta = 0.8$					
δ_1	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE ($\hat{\tau}_1$)	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE ($\hat{\tau}_1$)
0.2	167.602	188.304	25.225	454.151	154.404	46056.651	82.079	107.245	20.031	258.944	65.475	7927.153	23.778	48.094	11.700	25.227	10.037	35.742
0.4	66.019	168.985	16.792	190.370	47.093	3951.625	28.941	88.183	12.071	52.100	12.711	108.616	14.912	38.093	10.726	3.598	8.541	19.461
0.6	34.545	107.084	12.520	94.490	16.039	271.849	19.337	63.859	10.414	16.202	9.100	21.302	13.250	32.958	10.763	1.398	8.339	20.219
0.8	23.810	84.276	10.716	22.987	10.180	34.607	16.107	58.873	9.903	11.264	8.721	19.291	13.613	31.213	10.685	1.247	8.372	19.369
1	19.323	67.888	10.754	12.783	9.069	21.415	14.717	47.877	9.854	10.832	8.539	19.345	12.261	27.315	10.572	1.159	8.317	19.723
1.2	17.139	44.308	10.160	11.360	8.745	19.912	13.849	31.319	9.619	10.082	8.392	20.144	12.072	25.743	10.358	1.115	8.397	18.895
1.4	15.754	22.509	10.052	9.945	8.606	19.553	13.341	20.617	9.738	8.780	8.414	19.504	11.820	22.537	10.139	1.037	8.324	19.687
1.6	14.907	18.499	9.943	9.215	8.577	19.343	12.946	19.189	9.772	8.763	8.362	19.804	11.741	17.931	10.035	1.022	8.360	19.203
1.8	14.281	14.468	9.788	9.093	8.419	20.203	12.734	15.828	9.736	8.736	8.348	19.603	11.709	16.796	9.991	0.991	8.411	18.801
2	13.916	14.051	9.860	7.990	8.480	19.278	12.550	13.064	9.548	8.413	8.349	19.498	11.644	13.846	9.955	0.958	8.350	19.251

Table 2. Mean and MSE of the two change point estimators for the decreasing step shifts in the intercept based on different $ARMA(1,1)$ coefficients

	$\phi=0.2,\theta=0.2$						$\phi=0.2,\theta=0.5$						$\phi=0.2,\theta=0.8$					
δ_1	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE ($\hat{\tau}_1$)	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)
-2	11.536	14.840	9.943	0.793	8.357	19.130	11.564	13.065	9.965	1.179	8.361	19.198	11.481	12.118	9.976	1.092	8.318	19.957
-1.8	11.587	15.257	10.010	0.827	8.369	19.496	11.575	13.719	9.928	1.192	8.337	19.445	11.526	14.336	9.957	1.116	8.331	19.949
-1.6	11.609	15.643	10.047	0.872	8.317	19.865	11.580	14.196	9.913	1.216	8.289	20.162	11.472	18.278	9.946	1.177	8.272	20.735
-1.4	11.712	17.981	10.182	0.880	8.398	18.963	11.628	16.821	9.937	1.291	8.316	19.710	11.533	23.139	9.959	1.215	8.257	20.469
-1.2	12.137	18.327	10.375	0.962	8.403	19.105	11.624	18.318	9.951	1.392	8.351	19.197	11.520	25.331	9.957	1.242	8.239	20.758
-1	12.478	21.555	10.570	0.961	8.348	19.524	11.711	20.970	10.067	1.411	8.319	19.627	11.487	28.231	9.951	1.280	8.258	20.118
-0.8	12.548	27.930	10.721	1.072	8.316	19.869	11.967	25.406	10.285	1.425	8.348	19.513	11.494	29.637	9.949	1.339	8.234	20.679
-0.6	13.343	36.254	10.759	1.517	8.414	19.487	12.338	32.144	10.610	1.765	8.296	19.659	11.522	32.170	9.964	1.374	8.260	20.281
-0.4	14.685	45.799	10.768	3.474	8.509	19.593	13.121	38.576	10.779	1.982	8.342	19.767	11.893	38.304	10.397	1.415	8.293	20.361
-0.2	23.745	59.687	11.399	26.524	10.218	34.451	14.609	44.314	10.581	6.765	8.671	19.909	13.121	49.184	10.997	1.526	8.217	20.520
	$\phi=0.5,\theta=0.2$						$\phi=0.5,\theta=0.5$						$\phi=0.5,\theta=0.8$					
δ_1	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE ($\hat{\tau}_1$)	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE ($\hat{\tau}_1$)	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)
-2	12.019	13.619	10.328	0.811	8.314	19.914	11.582	13.736	9.927	1.012	8.376	19.714	11.537	12.156	9.940	1.171	8.347	19.428
-1.8	12.145	15.156	10.419	0.832	8.350	19.274	11.654	16.917	9.953	1.023	8.363	19.237	11.558	15.430	9.967	1.206	8.263	19.851
-1.6	12.318	17.289	10.563	0.879	8.336	18.939	11.672	17.527	10.119	1.068	8.249	20.459	11.562	19.363	9.950	1.219	8.291	20.067
-1.4	12.441	22.932	10.671	0.924	8.317	19.829	11.861	23.946	10.187	1.074	8.304	19.834	11.567	23.052	9.961	1.221	8.279	20.149
-1.2	12.725	24.178	10.654	1.065	8.351	19.621	12.047	24.280	10.369	1.150	8.301	19.871	11.557	24.836	9.929	1.263	8.300	19.871
-1	13.169	28.619	10.749	1.076	8.439	19.087	12.261	26.552	10.547	1.272	8.327	19.632	11.582	26.969	9.951	1.304	8.317	19.928
-0.8	13.579	38.862	10.763	2.187	8.397	19.993	12.619	32.641	10.739	1.316	8.355	19.574	11.630	31.491	10.090	1.389	8.257	20.392
-0.6	15.263	48.337	10.747	4.493	8.576	19.662	13.250	45.893	10.811	1.658	8.317	19.737	11.961	33.886	10.343	1.526	8.347	19.397
-0.4	19.419	62.662	10.663	14.129	9.081	21.650	14.937	47.971	10.719	3.307	8.537	19.561	12.571	38.137	10.744	1.750	8.369	19.323
-0.2	46.223	78.740	14.619	124.983	26.053	1048.271	23.925	57.399	11.281	26.029	10.302	34.448	14.855	42.863	10.861	2.204	8.413	20.111
	$\phi=0.8,\theta=0.2$						$\phi=0.8,\theta=0.5$						$\phi=0.8,\theta=0.8$					
δ_1	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE ($\hat{\tau}_1$)	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)
-2	13.935	14.043	9.872	7.986	8.468	19.265	12.536	13.061	9.587	8.416	8.364	19.490	11.654	13.839	9.934	0.955	8.374	19.239
-1.8	14.287	14.451	9.737	9.095	8.412	20.214	12.719	15.837	9.729	8.737	8.334	19.623	11.732	16.784	9.984	0.997	8.402	18.824
-1.6	14.917	18.502	9.990	9.211	8.585	19.375	12.957	19.197	9.763	8.764	8.369	19.896	11.745	17.930	10.052	1.019	8.376	19.215
-1.4	15.769	22.513	10.053	9.947	8.619	19.547	13.340	20.623	9.751	8.789	8.413	19.547	11.831	22.535	10.130	1.038	8.362	19.674
-1.2	17.124	44.318	10.119	11.358	8.752	19.918	13.834	31.336	9.682	10.083	8.384	20.140	12.069	25.749	10.351	1.114	8.371	18.884
-1	19.312	67.857	10.731	12.789	9.079	21.475	14.721	47.870	9.837	10.831	8.567	19.336	12.260	27.327	10.594	1.152	8.319	19.737
-0.8	23.729	84.262	10.709	22.987	10.103	34.639	16.098	58.842	9.905	11.268	8.720	19.275	13.672	31.238	10.636	1.249	8.364	19.360
-0.6	34.594	107.041	12.534	94.490	16.041	271.832	19.341	63.841	10.497	16.218	9.109	21.328	13.743	32.942	10.749	1.397	8.347	20.227
-0.4	66.036	168.994	16.784	190.312	47.090	3951.619	28.930	88.174	12.064	52.163	12.717	108.660	14.911	38.086	10.719	3.592	8.540	19.458
-0.2	167.671	188.317	25.231	454.152	154.414	46056.612	82.062	107.250	20.027	258.947	65.439	7927.137	23.767	48.067	11.708	25.231	10.025	35.732

Tables 1 and 2 demonstrate that the $EWMA-3$ control chart performs better than the T^2 control chart when considering the ARL criterion. The results reveal that both estimators produce satisfactory mean values for both increasing and decreasing shifts. However, the proposed change point estimator delivers more precise estimates than the built-in estimator of the $EWMA$ control chart for nearly all shift magnitudes and $ARMA(1,1)$ coefficients. Furthermore, the estimated change points for small shifts are closer to the actual change point. Figure 3 showcases the results presented in Table 1.

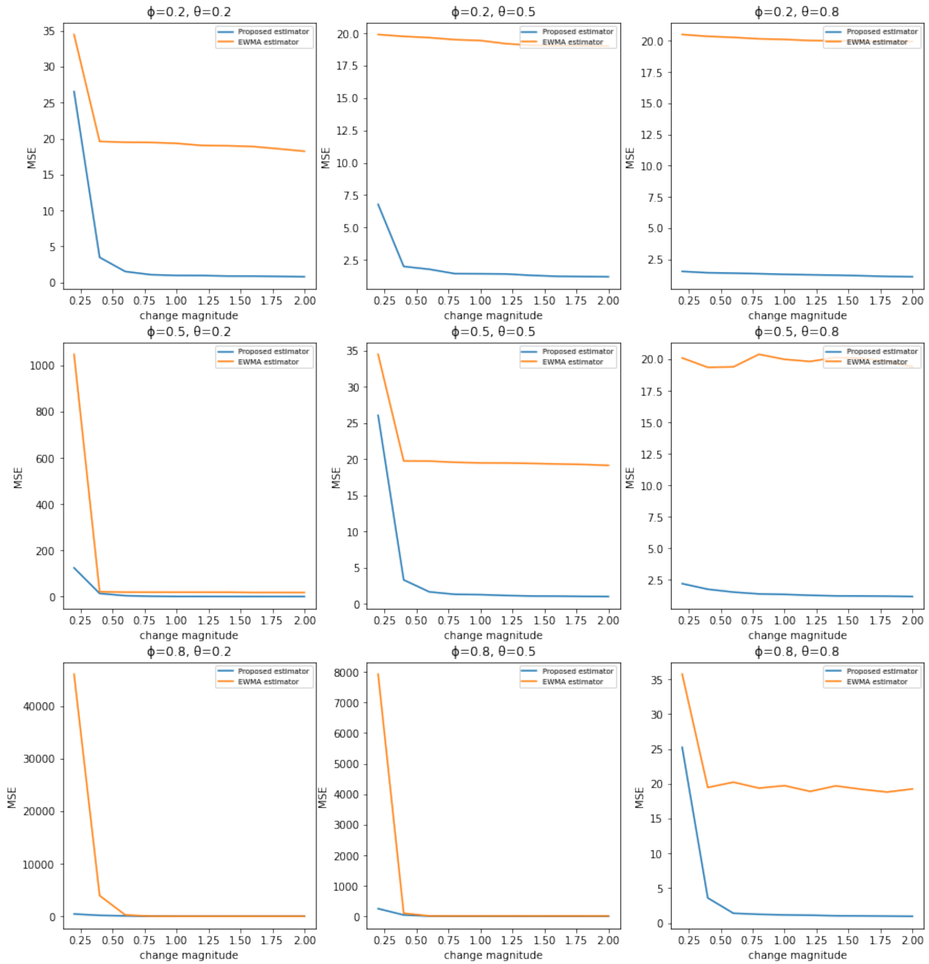

Figure 3. MSE of two estimators for different change magnitudes in the intercept parameter.

Based on Figure 3, for all shift magnitudes and $ARMA(1,1)$ coefficients, the proposed estimator of the change point has lower MSE or, in other words, it is less scattered than the built-in estimator of $EWMA$ control chart.

Table 3. Mean and MSE of the two change point estimators for the increasing step shifts in the slope based on different autocorrelation coefficients.

	$\phi = 0.2, \theta = 0.2$						$\phi = 0.2, \theta = 0.5$						$\phi = 0.2, \theta = 0.8$					
δ_2	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)
0.025	12.615	27.149	10.731	1.683	8.554	20.06	12.053	28.752	10.368	1.763	8.361	19.819	11.531	27.111	9.936	1.449	8.265	20.169
0.050	11.779	23.108	10.042	1.656	8.361	19.257	11.611	23.524	9.949	1.527	8.331	19.36	11.516	20.543	9.960	1.396	8.355	19.487
0.075	11.613	18.619	9.924	1.642	8.290	19.707	11.625	19.642	9.939	1.481	8.347	19.32	11.519	18.405	9.949	1.332	8.276	20.147
0.100	11.611	16.737	9.929	1.632	8.347	19.374	11.612	16.659	9.909	1.472	8.322	19.583	11.494	17.453	9.973	1.337	8.210	20.882
0.125	11.603	15.632	9.907	1.591	8.329	19.531	11.584	15.704	9.934	1.469	8.338	19.501	11.481	15.938	9.960	1.278	8.263	20.434
0.150	11.609	15.034	9.925	1.558	8.235	20.504	11.625	15.299	9.935	1.454	8.334	19.603	11.535	14.821	9.974	1.272	8.338	19.567
0.175	11.623	14.234	9.917	1.482	8.317	19.64	11.636	14.356	9.93	1.446	8.391	18.935	11.484	14.168	9.954	1.257	8.209	20.853
0.200	11.631	13.607	9.914	1.337	8.339	19.453	11.619	13.665	9.936	1.411	8.28	20.018	11.489	13.968	9.989	1.161	8.181	21.026
0.225	11.595	13.187	9.954	1.229	8.254	20.241	11.637	13.597	9.932	1.364	8.342	19.485	11.450	13.344	9.945	1.136	8.182	21.153
0.250	11.634	12.243	9.923	0.744	8.371	19.287	11.611	12.634	9.926	1.158	8.349	19.474	11.513	12.116	9.947	1.048	8.233	20.575
	$\phi = 0.5, \theta = 0.2$						$\phi = 0.5, \theta = 0.5$						$\phi = 0.5, \theta = 0.8$					
δ_2	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)
0.025	13.776	29.383	10.778	1.791	8.995	20.132	12.635	28.008	10.716	1.742	8.586	19.335	11.773	29.475	10.148	1.639	8.324	19.989
0.050	12.234	21.758	10.543	1.739	8.431	19.835	11.726	21.709	10.036	1.635	8.328	19.823	11.562	22.254	9.940	1.492	8.235	20.614
0.075	11.748	19.515	10.035	1.535	8.301	19.905	11.621	18.963	9.933	1.529	8.301	19.933	11.571	19.056	9.919	1.471	8.356	19.331
0.100	11.614	16.816	9.960	1.513	8.231	20.513	11.593	16.350	9.939	1.447	8.275	19.897	11.619	16.013	9.963	1.470	8.335	19.642
0.125	11.603	15.908	9.922	1.498	8.302	19.828	11.594	15.933	9.943	1.439	8.346	19.577	11.565	15.861	9.960	1.387	8.280	19.992
0.150	11.642	15.076	9.932	1.382	8.341	19.436	11.612	14.523	9.921	1.412	8.281	19.901	11.536	15.293	9.948	1.353	8.249	20.493
0.175	11.581	14.340	9.941	1.314	8.318	19.784	11.629	14.476	9.940	1.353	8.326	19.626	11.594	14.407	9.928	1.276	8.282	20.046
0.200	11.627	13.878	9.929	1.281	8.370	19.066	11.594	13.197	9.896	1.341	8.372	19.170	11.597	13.637	9.964	1.237	8.296	19.899
0.225	11.595	13.702	9.952	1.279	8.356	19.348	11.617	13.215	9.963	1.231	8.288	19.926	11.582	13.192	9.937	1.184	8.333	19.544
0.250	11.614	12.652	9.954	0.941	8.370	19.092	11.581	12.607	9.937	0.811	8.299	19.887	11.561	12.740	9.927	1.091	8.228	20.579
	$\phi = 0.8, \theta = 0.2$						$\phi = 0.8, \theta = 0.5$						$\phi = 0.8, \theta = 0.8$					
δ_2	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)
0.025	19.823	34.151	10.450	12.612	13.392	57.795	15.225	33.761	10.621	3.976	9.861	21.353	12.654	29.828	10.686	1.755	8.558	19.574
0.050	13.992	29.404	10.690	1.884	9.309	19.443	12.683	26.189	10.781	1.681	8.631	19.558	11.754	20.318	10.061	1.581	8.314	19.840
0.075	12.824	23.690	10.730	1.541	8.736	19.146	12.092	20.101	10.355	1.679	8.456	19.287	11.596	19.467	9.935	1.536	8.312	19.680
0.100	12.365	19.524	10.577	1.533	8.498	19.775	11.761	19.594	10.082	1.633	8.325	19.825	11.597	17.240	9.905	1.511	8.314	19.694
0.125	12.049	15.652	10.349	1.435	8.356	20.085	11.635	16.349	9.970	1.514	8.331	19.631	11.601	15.774	9.925	1.482	8.283	20.107
0.150	11.814	15.335	10.119	1.361	8.345	19.707	11.627	15.011	9.908	1.459	8.286	19.982	11.576	15.372	9.956	1.437	8.214	20.728
0.175	11.681	14.379	9.969	1.342	8.320	19.807	11.612	14.081	9.943	1.382	8.334	19.468	11.603	14.071	9.930	1.426	8.301	19.750
0.200	11.602	13.204	9.960	0.981	8.300	19.929	11.621	13.203	9.911	1.357	8.328	19.451	11.631	13.708	9.928	1.379	8.371	19.142
0.225	11.643	13.328	9.919	0.979	8.428	18.676	11.639	13.853	9.903	1.313	8.323	19.667	11.614	13.102	9.922	1.287	8.298	19.897
0.250	11.605	12.797	9.936	0.883	8.314	19.524	11.573	12.101	9.936	0.994	8.314	19.611	11.609	12.926	9.937	1.114	8.379	19.032

Table 4. Mean and MSE of the two change point estimators for the decreasing step shifts in the slope based on different autocorrelation coefficients.

	$\phi = 0.2, \theta = 0.2$						$\phi = 0.2, \theta = 0.5$						$\phi = 0.2, \theta = 0.8$					
δ_2	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)
-0.250	11.601	15.049	9.913	1.661	8.228	20.491	11.614	12.647	9.935	1.157	8.351	19.469	11.509	12.114	9.952	1.054	8.234	20.569
-0.225	11.605	13.197	9.967	1.21	8.26	20.236	11.623	13.573	9.931	1.364	8.339	19.491	11.458	13.340	9.946	1.137	8.180	21.152
-0.200	11.609	18.611	9.937	1.482	8.298	19.717	11.619	13.651	9.935	1.409	8.279	20.021	11.483	13.972	9.993	1.146	8.179	21.031
-0.175	11.613	15.626	9.916	1.65	8.322	19.526	11.627	14.363	9.937	1.446	8.382	18.943	11.474	14.161	9.947	1.259	8.212	20.850
-0.150	11.621	16.741	9.915	1.589	8.362	19.388	11.630	15.279	9.930	1.459	8.347	19.598	11.537	14.827	9.975	1.271	8.335	19.572
-0.125	11.621	13.613	9.908	1.618	8.353	19.443	11.588	15.720	9.940	1.465	8.327	19.507	11.491	15.934	9.963	1.281	8.267	20.431
-0.100	11.623	12.261	9.916	1.586	8.357	19.296	11.623	16.664	9.939	1.475	8.329	19.571	11.488	17.945	9.978	1.337	8.214	20.886
-0.075	11.627	14.22	9.91	1.674	8.328	19.63	11.620	19.643	9.939	1.482	8.340	19.319	11.522	18.405	9.941	1.359	8.272	20.145
-0.050	11.762	23.117	10.057	1.332	8.358	19.261	11.619	23.524	9.947	1.524	8.327	19.360	11.515	20.547	9.965	1.391	8.351	19.489
-0.025	12.620	27.134	10.733	0.734	8.54	20.077	12.061	28.750	10.368	1.701	8.370	19.815	11.538	27.106	9.930	1.451	8.268	20.171
	$\phi = 0.5, \theta = 0.2$						$\phi = 0.5, \theta = 0.5$						$\phi = 0.5, \theta = 0.8$					
δ_2	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)
-0.250	11.618	12.655	9.956	0.944	8.372	19.095	11.584	12.601	9.941	0.805	8.301	19.888	11.557	12.734	9.925	1.093	8.231	20.582
-0.225	11.588	13.707	9.953	1.279	8.350	19.352	11.595	13.190	9.892	1.228	8.370	19.170	11.588	13.187	9.931	1.187	8.330	19.540
-0.200	11.627	13.876	9.930	1.283	8.374	19.061	11.622	13.213	9.958	1.342	8.289	19.923	11.593	13.633	9.964	1.234	8.287	19.902
-0.175	11.579	14.341	9.945	1.311	8.319	19.787	11.621	14.479	9.941	1.358	8.321	19.627	11.589	14.415	9.923	1.281	8.285	20.054
-0.150	11.637	15.079	9.937	1.388	8.344	19.433	11.619	14.519	9.920	1.403	8.283	19.918	11.537	15.290	9.947	1.357	8.250	20.487
-0.125	11.601	15.911	9.925	1.495	8.303	19.831	11.597	15.931	9.942	1.444	8.349	19.579	11.556	15.869	9.962	1.399	8.287	19.994
-0.100	11.608	16.818	9.962	1.515	8.230	20.515	11.604	16.350	9.943	1.444	8.277	19.891	11.610	16.013	9.959	1.469	8.334	19.641
-0.075	11.744	19.513	10.034	1.534	8.301	19.896	11.629	18.959	9.932	1.525	8.297	19.930	11.571	19.051	9.920	1.473	8.361	19.327
-0.050	12.232	21.761	10.548	1.737	8.439	19.837	11.727	21.711	10.035	1.628	8.320	19.823	11.569	22.257	9.937	1.498	8.243	20.619
-0.025	13.770	29.381	10.772	1.788	8.994	20.136	12.631	28.018	10.719	1.744	8.581	19.341	11.773	29.472	10.145	1.645	8.320	19.993
	$\phi = 0.8, \theta = 0.2$						$\phi = 0.8, \theta = 0.5$						$\phi = 0.8, \theta = 0.8$					
δ_2	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)
-0.250	11.611	12.778	9.934	0.879	8.317	19.526	11.572	12.096	9.941	0.989	8.308	19.609	11.612	12.929	9.942	1.103	8.382	19.037
-0.225	11.608	13.203	9.962	0.972	8.321	19.927	11.628	13.210	9.917	1.31	8.332	19.450	11.609	13.107	9.918	1.281	8.289	19.891
-0.200	11.652	13.325	9.922	0.98	8.432	18.679	11.641	13.858	9.914	1.359	8.325	19.661	11.637	13.715	9.931	1.37	8.373	19.137
-0.175	11.676	14.382	9.971	1.346	8.320	19.812	11.616	14.083	9.948	1.388	8.334	19.467	11.598	14.067	9.930	1.426	8.307	19.790
-0.150	11.819	15.339	10.120	1.365	8.347	19.703	11.620	15.018	9.914	1.463	8.275	19.983	11.571	15.371	9.952	1.438	8.217	20.723
-0.125	12.047	15.650	10.349	1.427	8.349	20.089	11.631	16.357	9.963	1.502	8.333	19.630	11.607	15.779	9.979	1.484	8.291	20.116
-0.100	12.371	19.527	10.574	1.535	8.492	19.782	11.769	19.593	10.091	1.632	8.331	19.835	11.587	17.243	9.911	1.508	8.310	19.699
-0.075	12.823	23.691	10.734	1.539	8.733	19.152	12.083	20.112	10.365	1.683	8.451	19.281	11.607	19.465	9.936	1.53	8.307	19.672
-0.050	13.995	29.415	10.610	1.88	9.311	19.440	12.695	26.192	10.775	1.685	8.628	19.559	11.790	20.319	10.057	1.58	8.313	19.840
-0.025	19.821	34.146	10.451	12.615	13.379	57.799	15.231	33.768	10.619	3.981	9.858	21.347	12.653	29.833	10.676	1.749	8.561	19.571

Tables 3 and 4 present the mean and mean squared errors of the estimated change points when encountering a step change in the slope parameter from β_{10} to $\beta_1' = \beta_{10} + \delta_2$. Based on these findings, the $EWMA-3$ control chart performs better than the T^2 control chart concerning the ARL criterion, and the proposed change point estimator delivers precise estimates for almost all shift values and $ARMA(1,1)$ coefficients. Notably, the proposed estimator offers more accurate results than both ARL and $\hat{\tau}_1$. Figure 4 illustrates the results presented in Table 3.

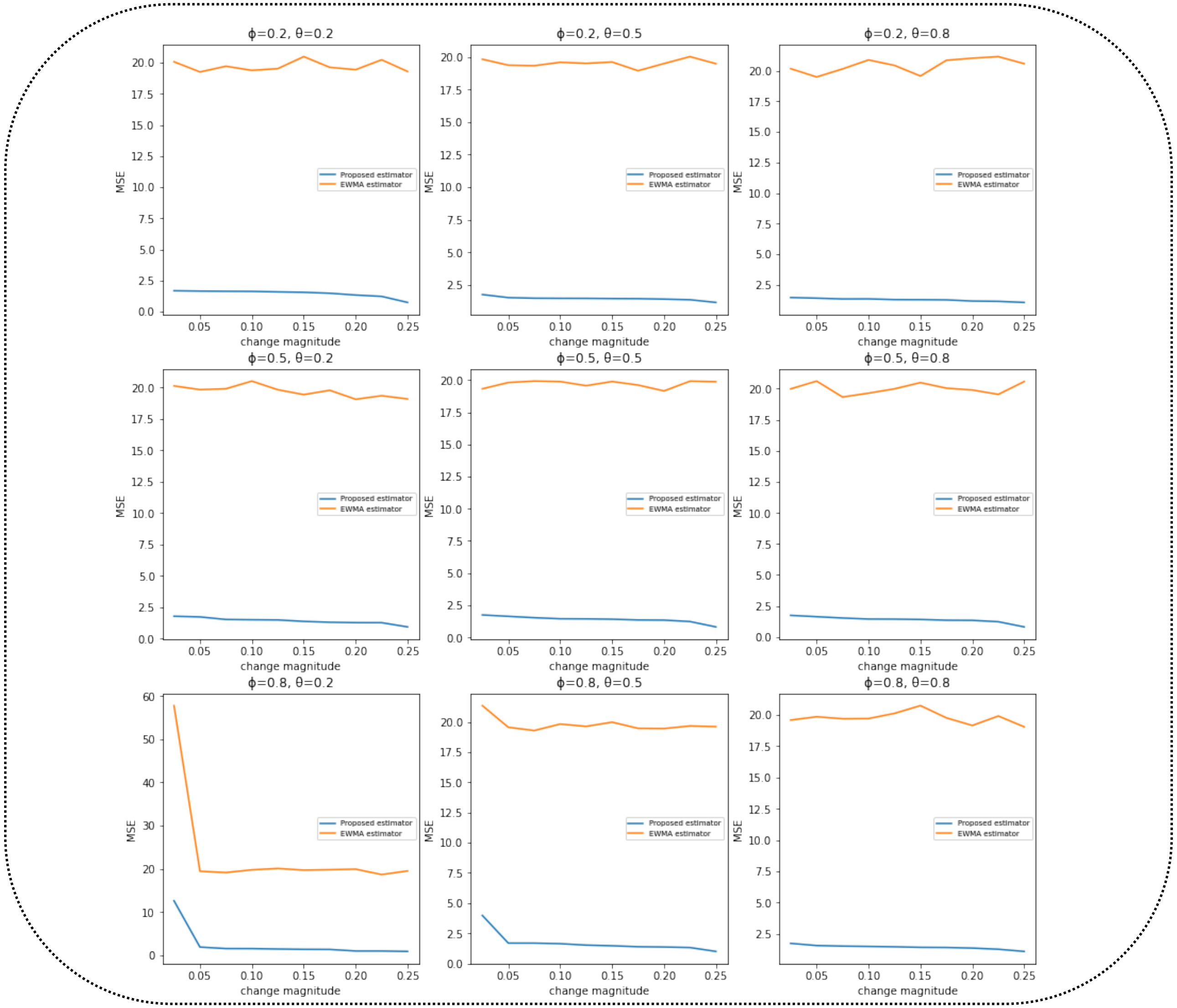

Figure 4. MSE of two estimators for different change magnitudes in slope parameter.

According to Figure 4, again, for all shift magnitudes and $ARMA$ (1, 1) coefficients, the proposed change point estimates have lower MSE than the built-in estimator of $EWMA$ control chart.

Table 5. Mean and MSE of the two change point estimators for the increasing step shifts in the error term variance based on different autocorrelation coefficients

	$\phi = 0.2, \theta = 0.2$						$\phi = 0.2, \theta = 0.5$						$\phi = 0.2, \theta = 0.8$					
δ_3	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)
0.4	20.082	30.292	18.353	88.379	16.552	84.544	20.060	32.226	18.886	93.666	16.568	88.865	19.024	32.489	18.169	76.790	15.586	73.058
0.8	15.617	26.628	12.062	6.102	12.373	23.964	15.680	27.152	12.066	6.190	12.530	24.187	15.467	23.982	11.886	4.448	12.281	24.519
1.2	14.539	18.725	10.874	0.805	11.521	18.403	14.585	18.507	10.875	0.788	11.517	18.735	14.574	17.310	10.949	0.844	11.530	19.887
1.6	13.896	14.771	10.781	0.676	10.936	16.975	13.937	15.128	10.810	0.476	11.005	16.780	13.955	16.391	10.810	0.723	11.077	17.929
2	13.380	14.598	10.669	0.589	10.460	16.540	13.474	14.256	10.690	0.358	10.627	15.560	13.522	14.412	10.763	0.540	10.646	17.733
	$\phi = 0.5, \theta = 0.2$						$\phi = 0.5, \theta = 0.5$						$\phi = 0.5, \theta = 0.8$					
δ_3	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)
0.4	20.135	32.469	18.783	93.422	16.695	89.719	20.147	30.037	18.710	91.343	16.585	89.989	19.906	33.998	18.617	88.868	16.464	87.047
0.8	15.610	24.602	12.038	5.604	12.417	24.462	15.652	21.205	11.926	5.169	12.535	24.387	15.675	28.627	12.008	5.035	12.539	25.083
1.2	14.597	18.083	10.900	0.758	11.534	18.587	14.542	19.034	10.931	0.935	11.501	18.814	14.613	19.785	10.944	0.685	11.652	19.415
1.6	13.949	15.160	10.752	0.712	11.013	16.569	13.876	15.852	10.749	0.814	10.908	17.435	14.068	16.772	10.827	0.496	11.149	17.099
2	13.412	14.913	10.717	0.532	10.539	15.857	13.454	14.87	10.687	0.789	10.581	15.559	13.608	14.006	10.747	0.413	10.728	16.518
	$\phi = 0.8, \theta = 0.2$						$\phi = 0.8, \theta = 0.5$						$\phi = 0.8, \theta = 0.8$					
δ_3	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)	ARL (EWMA)	ARL(T^2)	E($\hat{\tau}_0$)	MSE($\hat{\tau}_0$)	E($\hat{\tau}_1$)	MSE($\hat{\tau}_1$)
0.4	20.007	33.804	18.716	95.509	16.494	88.935	19.922	33.464	18.591	94.351	16.378	87.943	20.127	32.087	18.513	90.840	16.597	89.282
0.8	15.646	27.335	12.061	5.927	12.485	24.452	15.564	24.997	11.991	5.448	12.445	24.121	15.463	24.095	12.075	6.033	12.507	23.921
1.2	14.568	17.646	10.864	0.837	11.566	18.360	14.543	18.492	10.889	0.594	11.529	18.096	14.510	18.478	10.861	0.842	11.464	19.069
1.6	13.927	16.413	10.740	0.693	10.943	17.707	13.868	16.704	10.781	0.572	10.925	16.782	13.890	15.587	10.752	0.701	10.978	16.810
2	13.426	14.552	10.701	0.666	10.542	15.998	13.356	14.723	10.659	0.476	10.511	15.883	13.446	14.298	10.701	0.558	10.507	16.111

Table 5 displays the means and mean squared errors of the change point estimations when encountering a step change in the error term variance from σ_{a0}^2 to $\sigma'^2_a = \sigma_{a0}^2 + \delta_3$. Once again, the results indicate that the $EWMA - 3$ control chart outperforms the T^2 control chart in terms of the ARL criterion, and the proposed change point estimator $\hat{\tau}_0$ delivers more accurate estimates than $\hat{\tau}_1$ for nearly all shift magnitudes and autocorrelation coefficients. Figure 5 presents the results provided in Table 5.

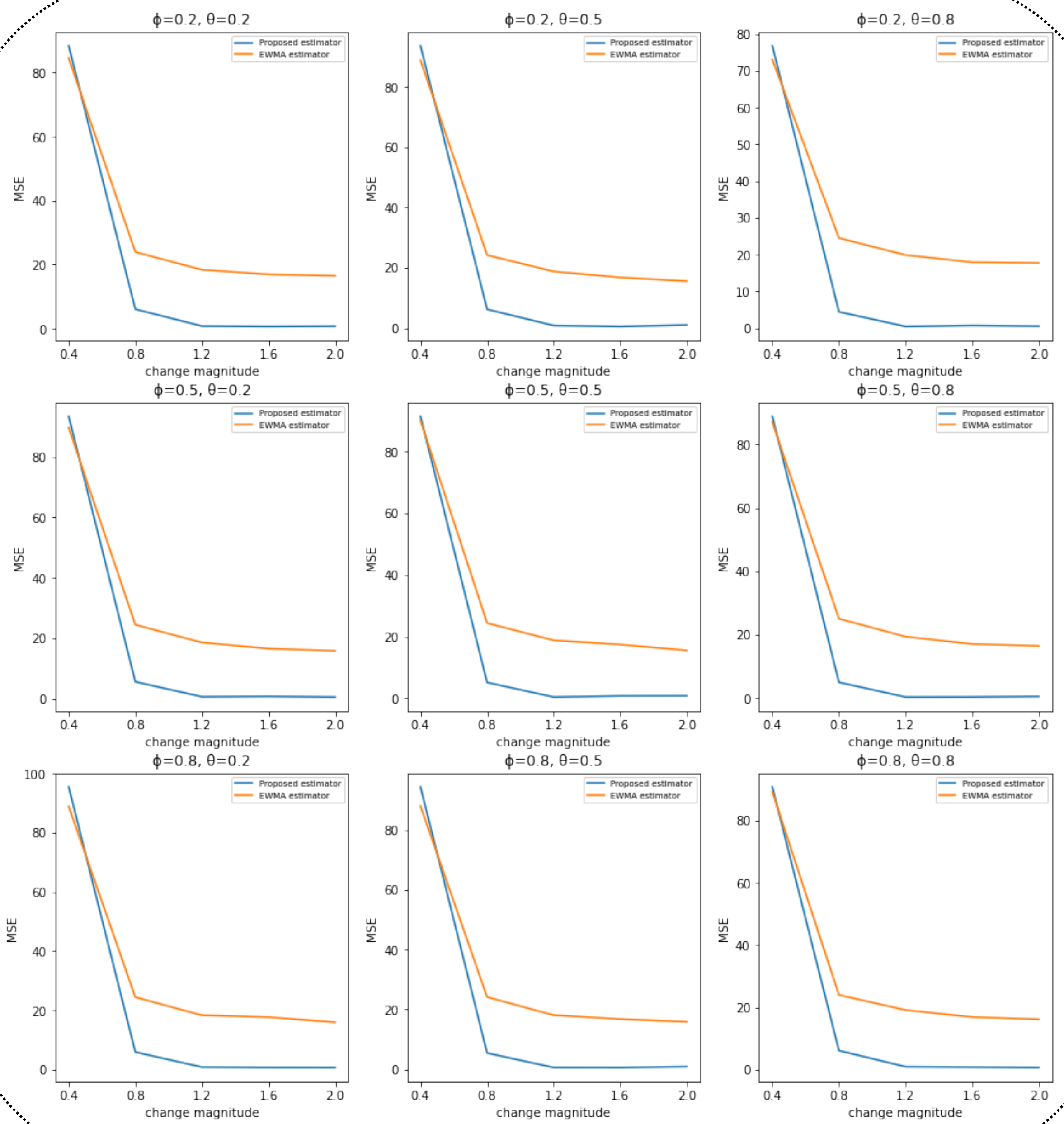

Figure 5. MSE of two estimators for different change magnitudes in error variance.

Finally, Figure 5 demonstrates that for the majority of shift magnitudes and $ARMA(1,1)$ coefficients, the proposed change point estimator yields lower mean squared errors compared to the built-in estimator of the $EWMA$ control chart.

To summarize, the proposed change point estimator for a step change in the parameters of a simple linear profile with $ARMA(1, 1)$ error terms deliver sufficiently precise estimates of the change point, irrespective of the shift magnitude and $ARMA(1,1)$ coefficients. Furthermore, the simulation study results suggest that $\hat{\tau}_0$ performs better than $\hat{\tau}_1$ for nearly all shift values, with $\hat{\tau}_1$ frequently underestimating the actual change point.

IV. CARDINALITY AND COVERAGE PERCENTAGE OF CONFIDENCE SET ESTIMATOR

This section develops a confidence set for the change point in the process. This set offers process engineers a set of potential change points to start their investigation into a specific cause. With this set, they can identify a range of possible change points that encompass the actual process change point with a certain degree of confidence. Box and Cox (1964) suggest constructing a confidence set estimator of the parameter by utilizing the likelihood function. By employing this technique, a confidence set can be derived in the following format:

$$CS = \{t: \ln L(t) > \ln L(\hat{\tau}) - D\} \tag{51}$$

The confidence set is constructed based on the maximum of the log-likelihood function, $\ln L(\hat{\tau})$, calculated across all feasible change points t. If the log-likelihood function value at t, $\ln L(t)$, surpasses the maximum log-likelihood function minus a reference value D, then t is considered as a part of the confidence set. The number of points included in the set is known as the cardinality. The coverage probability is estimated by dividing the cardinality of the confidence set by the number of samples taken until the control chart issues an alarm.

This section uses different values for *D*, shift in intercept, and shift in slope to compute the cardinality and coverage probability of the confidence set estimator of the process change point. The value of D varies between 1 and 5 with an increment of 1. δ_1 indicates the magnitude of a shift in intercept, ranging from 0.2 to 4 with an increment of 0.2. δ_2 indicates the magnitude of a shift in slope, and it varies from 0.02 to 2 with an increment of 0.05.

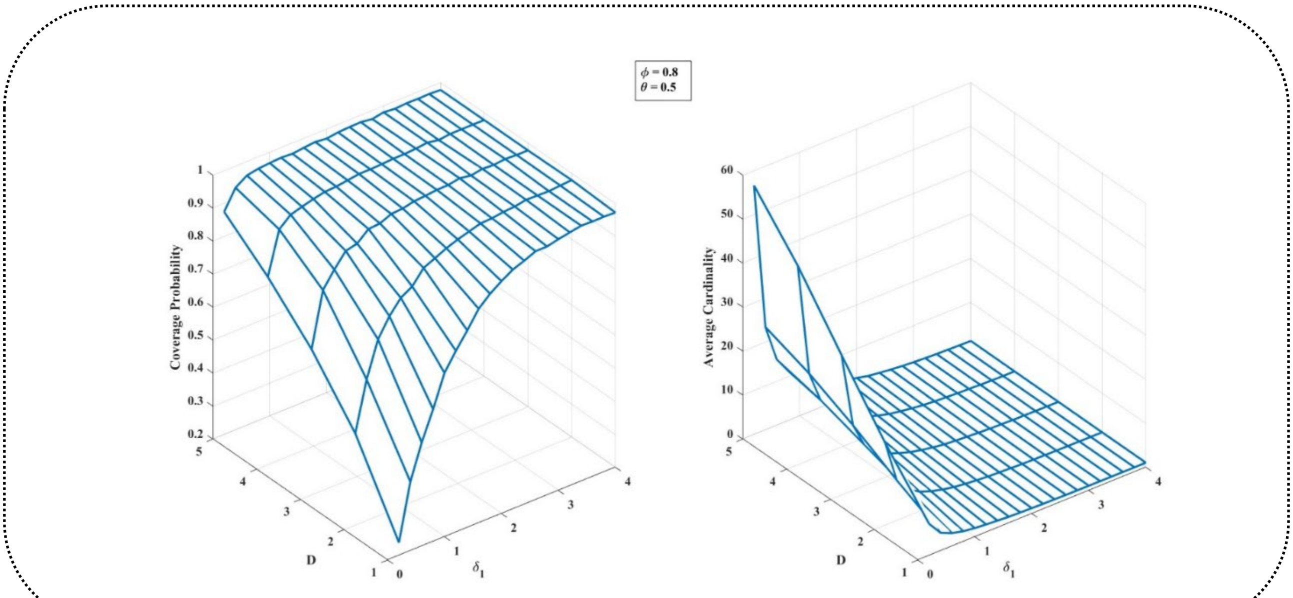

Figure 6. The coverage probability and average cardinality for the proposed confidence set estimator under increasing step shifts in the intercept parameter

Simulation studies were conducted to compute the average cardinality and coverage probability of the confidence set estimator. In each simulation run, ten random samples were generated from the in-control process shown by equations (48) and (49) with $\phi = 0.8$ and $\theta = 0.5$. From sample 11 onwards, profiles were generated from the out-of-control process with an intercept equal to $(3 + \delta_1)$ or slope equal to $(2 + \delta_2)$. The *EWMA*-3 control charts were used to monitor this process. After any of these control charts signals that the process is out of control, the change point was estimated using the proposed change point estimator. Then, the confidence set estimator of the change point was computed using equation (51). For each value of D and δ_1 or δ_2 , the average cardinality and the coverage probability of the confidence set were computed over 10000 simulation runs.

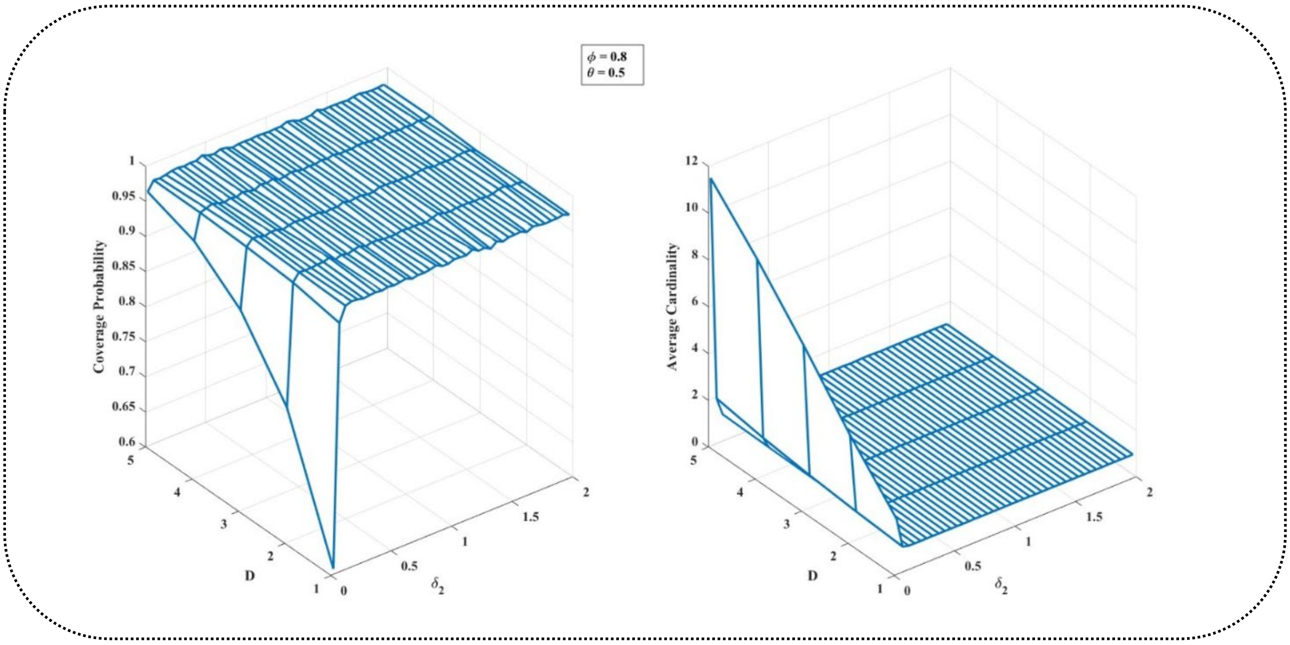

Figure 7. The coverage probability and average cardinality for the proposed confidence set estimator under increasing step shifts in the slope parameter

The average cardinality and coverage probability of the confidence set estimator for various values of δ_1 and D are depicted in Figure 6, while Figure 7 shows the average cardinality and coverage probability for different values of δ_2 and D. For instance, when δ_1 is set to 1 and D to 4, the expected cardinality of the resulting confidence set estimator is approximately 10. Moreover, in 0.85 percent of cases, this confidence set includes the actual change point.

V. CASE STUDY

In this section, a case study is presented to demonstrate the practical performance of the proposed change point estimation technique. In this regard, a study has been conducted on the relationship between the weight of newborn babies, the response variable, and their age (months), the independent variable. This weight of newborn babies is an essential criterion for measuring their health, and it is also crucial for specialists. Hence, it is vital to monitor the weight profile of babies to understand their health status. The data set of a health center in one of the big cities (the name of the city and the health center are not mentioned to preserve the information) was used as a case study.

In phase I of profile monitoring, the weight profiles of 50 newborn female babies were recorded. The profile parameters, including the intercept and slope, were estimated based on these data. Also, the residual analysis showed that an $ARMA$ (1,1) model was appropriate to express the autocorrelation between error terms.

To verify whether the proposed approach can detect changes in the weight profile of babies or not, the weight profiles of 19 newborn babies were used in phase II. The data were divided into two groups of male and female babies. The first seven profiles were for female babies, while the others were for male babies.

The $EWMA$-3 test statistics corresponding to the 19 weight profiles were computed. The test statistics and the corresponding control limits are shown in Figure 8.

Figure 8 illustrates the $EWMA - 3$ control chart for the intercept signals at sample 15, with the estimated change point determined using the proposed change point estimation method. The estimated change point was 7. It is the point where the last weight profile of female babies was plotted on the control charts.

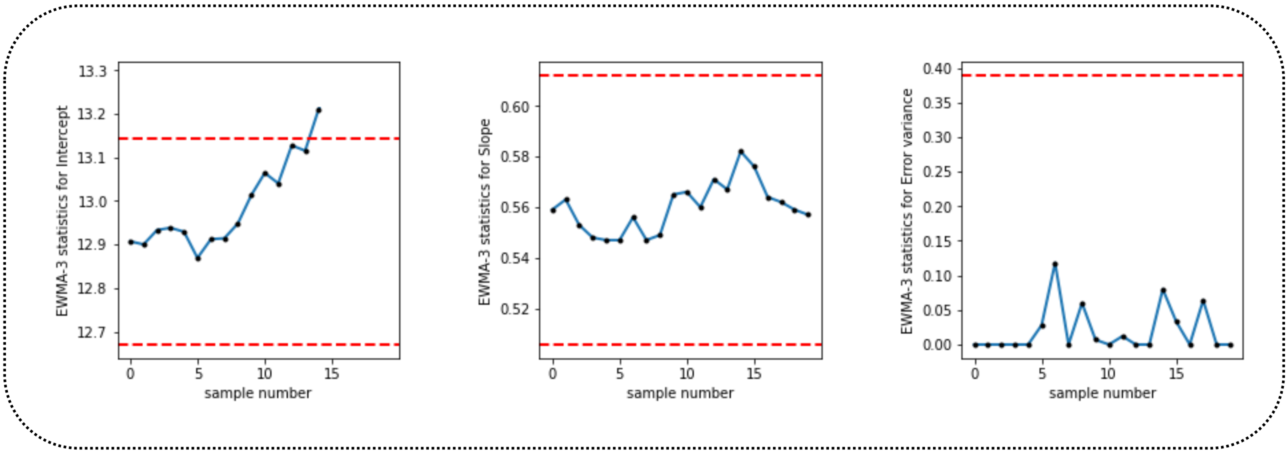

Figure 8. The EWMA-3 control charts for weight profiles of newborn babies in phase II

As expected, the estimated change point computed by the proposed method indicates that a change has occurred in the intercept parameter at sample 7, from which the test statistics of the weight profile of male babies were plotted.

According to the results, the intercept of the weight profile of male babies is different from the intercept of the weight profile of female babies. It means that at a certain age, the average weight of male babies is higher than that of female babies.

VI. CONCLUSIONS AND FUTURE RESEARCH

This paper aims to monitor simple linear profiles in the presence of within-profile autocorrelation while assuming no correlation between profiles. Instead of relying on commonly used AR models to model within-profile autocorrelation, this research employs an autoregressive moving average model, $ARMA(p,q)$, to capture the autocorrelation structure between observations within each profile. This is why, in most instances, $ARMA$ models are more versatile in representing autocorrelation structures than AR models. Assuming an autoregressive moving average model, $ARMA(p,q)$, for the autocorrelation structure between observations within each profile, the proposed transformation eliminates the autocorrelation effect and yields a simple linear profile model with uncorrelated error terms. The intercept, slope, and error term variance of the profile are monitored individually using an $EWMA-3$ control chart until a signal is detected. For evaluating its effectiveness, the performance of the $EWMA-3$ control chart is compared to that of the T^2 control chart based on the ARL criterion. Additionally, a maximum likelihood estimator is proposed to estimate the change point. Simulation was conducted to assess the performance of the proposed estimator, and the results were compared to those of the built-in change point estimator of the $EWMA$ chart. The findings indicate that, in terms of the ARL criterion, the $EWMA-3$ control chart outperforms the T^2 control chart for both increasing and decreasing shifts. Additionally, the results showed that the proposed estimator performs better than the built-in change point estimator of the $EWMA$ chart in accurately estimating the change point, regardless of the magnitude of the shift and the $ARMA(1,1)$ coefficients. The study also analyzed the cardinality and coverage percentage of a confidence set estimator under step shifts. Finally, a real case was presented to demonstrate the application of the proposed estimator.

The proposed method can be extended by considering a change in autocorrelation coefficients and evaluating its effect on the change point estimator performance. In addition, the impact of the smoothing parameter can be investigated in further studies.

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