Journal of Quality Engineering and Production Optimization

Vol. 8, No. 1, Winter & Spring 2023

http://jqepo.shahed.ac.ir

Research Paper

DOI: 10.22070/jqepo.2023.16027.1230

Multivariate Statistical Process Control Using Wavelet Approach

Karim Atashgar [1*], Froogh Adelian [2]

[1] *Department of Industrial Engineering, Malek-Ashtar University of Technology, Tehran, Iran.*

*** Corresponding Author:** *Karim Atashgar (Email: atashgar@iust.ac.ir)*

Abstract – This paper attempts to monitor the mean vector of multivariate processes using a wavelet-based model. In the case of monitoring several related technical specifications, wavelet approach is an attractive contribution to analyze the performance of the multivariate process over time statistically. This advanced approach of signal processing enables more effective process monitoring compared to the traditional methods. The wavelet capability can lead practitioners to a root cause analysis sooner than the traditional schemes when the process shifts to an out-of-control condition. In this paper a new statistic named TMO-WAVE is proposed to analyze the variation of a multivariate process. The capability of the proposed scheme is compared numerically with different methods in this paper. The numerical comparative reports address high capability of the proposed wavelet-based method compared to the models in the literature in terms of average run length (ARL).

Keywords– *Multivariate process, Phase II, Statistical process control, Wavelet.*

I. INTRODUCTION

In the case that a process includes more than one technical specification, one is concerned with monitoring the performance of the process using measurements on a vector of technical specifications at a given time. In this case, the technical specifications are structured by a joint multivariate probability distribution. If the joint statistical structure is ignored, missed data corresponding to the correlation of the technical specifications will be provided. In this missing data process the error-I of the monitoring is increased. Advances in technology have made it possible for practitioners to consider correlated data of a process with the aim of evaluating its stability over time using different signal processing approaches. This type of research is considered attractive research in the literature of statistical process control (SPC) (Cohen and Atoui, 2022).

The work of monitoring statistically multivariate cases originally considered by Hotelling (1947). He proposed T^2 statistic to monitor statistically a process characterized by a vector of technical specification. The Hotelling (1947) method follows the properties of Shewhart-chart type. The Multivariate Shewhart-chart follows the same weakness of the univariate type. When a shewhart chart is used, one ignores the information given by the sequence of samples. This chart type is not sensitive effectively to small and moderate magnitudes of parameter(s) shifts. In the aim of increasing the sensitivity of small and moderate shifts of parameter(s), multivariate cumulative sum (MCUSUM) and multivariate exponentially weighted moving average (MEWMA) control charts proposed by researchers. Properties of these sensitive charts led researchers to develop these method types for monitoring multivariate cases. Woodall and Ncube

Manuscript Received:1-December-2022 & Revised: 17-February-2023 & Accepted: 26-March-2023
ISSN: 2423-3781

(1985), Crosier (1988), Pignatiello and Runger (1990), Ngai and Zhang (2001), Runger and Testik (2004), Haq et al. (2020), and Xie et al. (2021) approached monitoring a multivariate process using MCUSUM method. Furthermore, several researchers such as Lowry et al. (1992), Yumin (1996), Fasso (1999), Runger et al. (1999), Tseng et al. (2002), Yeh et al. (2003), Testik and Borror (2004), Huwang et al. (2019), Gharib et al. (2021) also contributed to develop MEWMA chart.

The importance of the multivariate process monitoring in real cases led several researchers to approach soft computing methods to analyze signals of this process type. Hwarng (2004), Guh (2007), Noorossana et al. (2011), and Atashgar and Noorossana (2012) considered the process monitoring using ANN-based model where the process involves several correlated technical specifications. Furthermore, Guh and Shiue (2008) contributed the same environment focusing on the decision tree method.

Developing different schemes using various approaches in literature addresses the important of the sensitivity issue in monitoring the multivariate environment of SPC. Analyzing signals of a multivariate process is considerably more complicated compared to a univariate process type. Wavelet, as a powerful method in signal processing, can be utilized to overcome the complexities of the multivariate environment. In this research, a wavelet-based model is proposed to statistically monitor the variation of the mean vector of a process in phase-II. In this investigation, signals are processed using the scale and decomposition parameters of a discrete wavelet type to achieve a more sensitive chart for the multivariate environment.

The remainder of this paper is structured as follows: The next section is dedicated to the wavelet literature of SPC. In Section 3, the proposed wavelet-based chart of this paper is discussed, providing clear steps for constructing the proposed scheme. Section 4 provides a numerical analysis of the capability of the proposed chart compared with different charts under various scenarios. Finally, the last section is dedicated to remarks and conclusions.

II. LITERATURE OF WAVELET IN SPC

The Wavelet transform is an advanced approach compared to the Fourier transform. Fourier provides the signal representation in the frequency domain, while the wavelet represents the signal in both the time and frequency domains simultaneously. This capability allows one to efficiently access localized information related to the signal. However, when looking at a signal using the Fourier transform, it is impossible to determine the exact time when a specific event took place. (For more details about the wavelet, see, for example, Fleet (2019) or Gao and Yan (2010)). There have been several advances in the theory and application of wavelet as an effective pre-processing tool. However, the literature indicates that less attention has been given to propose a wavelet-based scheme for statistical process monitoring.

The use of the wavelet in control chart of SPC can be classified into two branches: 1) Scheme that is sensitive to the mean change, and 2) Scheme that is sensitive to the variance change. In this type of research, a few studies have focused on monitoring the coefficients of the wavelet. Another classification can be implemented based on the application environment. In this classification, the use of the wavelet in SPC is categorized into three groups: 1) univariate process, 2) multivariate process, and 3) profile environment. The profile environment refers to the functional relationship between the technical specifications of a process or product (For more details, readers can refer to Atashgar and Asghari (2017) and Atashgar and Abbasi (2021)). Some authors have also explored the combination of the wavelet and support vector machine. Additionally, the combination of wavelet and artificial neural network (ANN) also has been approached in the literature. The literature on the use of the wavelet in SPC is summarized in Table 1. The first column of the table provides the references of the literature, and the following three columns show the environments in which the models can be practically used. As shown in Table 1, most of the wavelet-based methods have been developed for the univariate condition. The "Sensitivity" column of Table 1 indicates the parameter whose variation is monitored by the model addressed in the reference. The two last columns of Table 1 show whether the proposed models have been combined with ANN or machine learning. This paper focuses on the statistical monitoring of the mean vector of multivariate processes using the wavelet transform method in cases where the process follows a multivariate normal distribution and the parameters have been effectively estimated in phase-I of the statistical process control.

TABLE I. SUMMARIZED LITERATURE OF THE WAVELET IN SPC

Reference	Environment Type			Sensitivity		Wavelet Combined with Neural Network	Wavelet Combined with Machine Learning
	Univariate	Profile	Image	Mean	Variance		
Al-Assaf (2004)	*			*		*	
Jeong et al. (2006)	*			*			
Wang and Kuo (2007)	*			*		*	
Chen et al. (2007)	*			*		*	
Cahng & Yadama, (2010)		*		*			
Ranaee & Ebrahimzadeh (2011)	*			*			*
Lee et al. (2012)		*		*			
Du et al. (2013)	*			*			*
Jeong et al. (2013)		*		*			
Nikoo & Noorossana (2013)		*		*			
Wang et al. (2015)	*			*			
Cohen et al. (2015)	*			*			
Stewarda,& Rigdon (2015)	*			*			
McGinnity et al.(2015)		*		*			
Cohen et al. (2016a)	*			*			
Cohen et al. (2016b)	*			*	*		
Yin and Hou, (2016)	*			*			*
Chicken et al. (2009)		*		*			
Mansouri et al.(2018)	*			*			
Harrou et al. (2018a)	*			*			
Harrou et al.(2018b)	*			*			
Harrou et al.(2019)	*			*			
Alamelu Manghai & Jegadeeshwaran (2019)	*			*			*
Li et al.(2019)	*			*			
Piri et al. (2021)		*		*			
Koosh et al. (2022)			*	*			

III. THE PROPOSED WAVELET-BASED SCHEME

Assume $\boldsymbol{X}_1, \boldsymbol{X}_2, \boldsymbol{X}_3, \dots, \boldsymbol{X}_\tau, \boldsymbol{X}_{\tau+1}, \dots, \boldsymbol{X}_T$ are observed independent vectors in a multivariate case, where the vector $\boldsymbol{X}$ follows a known distribution with a joint probability function $f(X,\Theta)$, where Θ indicates the parameters of the known distribution. Let the vector $\boldsymbol{X}$ includes p related random variables of a product or process. In this case, the vector has a $p \times 1$ dimension, so that the j^{th} random variable is the j^{th} quality specification of the product or process. Suppose that the process works statistically in-control condition up through the change point, i.e., time τ , but after the change point an assignable cause(s) affects the mean vector of the distribution and then the process shifts to an out-of-control condition. In this condition, the process remains in the out-of-control until at a later time T the shift of the process is identified by a chart. The distance between the times τ and T is referred to as the sensitivity of the chart. The closer the change point (i.e., τ) is to the signal time (i.e., T), the more sensitive the method is considered.

To provide a new wavelet-based statistic to plot on a proposed control chart, one should initially focus on the weighted wavelet coefficient. In the case of investigating changes of the mean vector of a multivariate process, the weighted coefficient is used in this vector of the process. The statistic that is plotted in the proposed control chart is based on weighted wavelets coefficients, which are provided through the Discrete Wavelets Transform. Selecting a suitable wavelet for a given application is valuable. The selected type affects the results of the wavelet transform and then influences the performance of the method. In this research, the coefficients are produced using Daubechies db2 wavelet family. The db2 wavelet is known as a discrete wavelet transform. The following equation (1) introduces the wavelet function:

$$\psi_{j,k}(t) = 2^{j/2}\boldsymbol{\psi}(2^j t - k) \tag{1}$$

where j, k, and $\boldsymbol{\psi}$ are the scale parameter, translation parameter, and mother wavelet, respectively. Scaling a wavelet means stretching or compressing it, and translating a wavelet simply means delaying or hastening its onset. In this equation, the t parameter addresses the time of the function. To design the proposed wavelet-based scheme for the multivariate environment, the following steps are considered in this research:

Step 1) Normal standard: In the first step, the vector of the observations is transformed to the normal standard distribution. In this case all variables follow the normal distribution with a mean of 0 and standard deviation of 1. This measure helps to clearly define the distributions of the wavelet coefficients. In this case, the process is considered in control when the mean and variance of the variables are zero and one, respectively.

Step 2) Wavelet transform: The multi-resolution of the wavelet transform provides a framework for signal analysis. In this step, to utilize this property, the wavelet coefficients are calculated using the following equations (2) and (3). Assuming that the quality specification vector of the multivariate process in each observation is defined as $\boldsymbol{X} = \begin{pmatrix} x_1 & x_2 & x_3 & \dots & x_p \end{pmatrix}^T$, two types of coefficients (i.e., approximation and detail) for each x_i $(i = 1, 2, \dots, p)$ are obtained. These wavelet coefficients include frequency of signal information.

$$a_j(k) = \sum_{i=0}^{l} h(i)\, a_{j-1}[2k - i] \tag{2}$$

$$d_j(k) = \sum_{i=0}^{l} g(i)\, a_{j-1}[2k - i] \tag{3}$$

where $a_j(k)$ denotes the approximation coefficient and $d_j(k)$ indicates the detail coefficient corresponding to the wavelet. In these equations, $a_0 = x_i$ is the original signal of x_i $(i = 1,2, \dots, p)$, j denotes to decomposition scale, $k \in Z$, l indicates the length of the filter, h (low-pass) is scaling, and g (high-pass) is the wavelet filter.

Step 3) Window size: In the wavelet transform approach to project data into a wavelet base, a window analysis is used. The size of the window addresses the observations that can be used with the *db2* wavelet. In this paper, the size of the window is set to 8. This size is the smallest value that can be used for db2 case in which the discrete transform type is approached (Cohen et al. (2016a)). When the window size is set to 8, it means that one gets 8 wavelet coefficients in each window observation.

Step 4) Performing random vector: To evaluate the performance of the multivariate process, a vector based on the wavelet analysis is performed. This vector has a $p \times 1$ dimension, such that the j^{th} random variable corresponds to the j^{th} quality specification. In other words, the number of dimensions are equal to the number of technical specifications of the multivariate process. The member values of the vector are obtained from the wavelet coefficients. Without loss of generality, if each technical specification follows standard normal distribution (i.e.$x_i \sim N(0,1); i = 1,2,\dots,p$), when a shift equal to δ value manifests itself to the mean of the distribution (i.e. μ_0 shifts to $\mu_1 = \mu_0 + \delta\sigma$) it can be shown that $a_j(k) \sim N(2^{j/2}\delta, 1)$ and $d_j(k) \sim N(0,1)$. As mentioned before, the wavelet coefficients are provided through the Daubechies db2 wavelet, which is an orthonormal wavelet. The sensitivity analysis reported by Cohen et al. (2016a) indicates that the first detail coefficient (d_1) is more sensitive than d_1, d_2, d_3, d_4, and approximation coefficients are more sensitive compared to detail coefficients. Finally, it was concluded that each *O* is better to construct based on a_1, a_2, a_3, a_4, d_1 wavelet coefficients. The $\boldsymbol{O}$ vector statistic of this paper is defined as follows:

$$\boldsymbol{O}_i = \begin{bmatrix} O_1 \\ O_2 \\ \dots \\ O_p \end{bmatrix} \tag{4}$$

So that;

$$
\begin{aligned}
O_1 &= w_1 * a_{1,i} + w_2 * a_{2,i} + w_3 * a_{3,i} + w_4 * a_{4,i} + w_5 * d_{1,i} \\
O_2 &= w_1 * a_{1,i} + w_2 * a_{2,i} + w_3 * a_{3,i} + w_4 * a_{4,i} + w_5 * \mathrm{d}_{1,\mathrm{i}} \\
&\dots\dots \\
O_p &= w_1 * a_{1,i} + w_2 * a_{2,i} + w_3 * a_{3,i} + w_4 * a_{4,i} + w_5 * d_{1,i}
\end{aligned} \tag{5}
$$

where *i* addresses the *TMO* index, which is calculated in the step 5, and furthermore:

$\sum_{j=1}^{5} w_j = 1$, $0 \le w_j \leqq 1 \; for\; j = 1,2,3, and\; 4$, $-1 \le w_j \leqq 0 \; for\; j = 5$.

The *w* values are obtained using the local search algorithm addressed by Cohen et al. (2016a). It is clear that the *O* statistic is developed based on weighted wavelet coefficients. It should be mentioned that based on the reproductive property of normal distribution, each variable O_i follows a normal distribution. In this case, the random vector $\boldsymbol{O}_i$ follows a *p*-variate normal distribution.

Step 5) Variance- covariance matrix: As described before, in a multivariate process, the technical specifications (i.e. $x_i; i = 1,2,\dots,p$) are related together. $\boldsymbol{O}_i$ vector has a variance-covariance matrix, where $\boldsymbol{O} = \left(O_1, O_2, \dots, O_p\right)^T$ is the random vector obtained in Step 4. Since $\boldsymbol{O}$ follows a *p*-variate normal distribution, one can simply find the variance-covariance matrix $\boldsymbol{S_o}$ (Equation 6).

$$\boldsymbol{S_o} = \begin{pmatrix} \sigma_{O_1}^2 & \cdots & \sigma_{O_1 O_p} \\ \vdots & \ddots & \vdots \\ \sigma_{O_p O_1} & \cdots & \sigma_{O_p}^2 \end{pmatrix} \tag{6}$$

Step 6) TMO-WAVELET statistic: In this research, to monitor a multivariate process, a statistic based on the T^2 Hotelling (1947) is developed. The proposed statistic, which is named TMO-WAVELET provides an effective condition to signal an out-of-control condition, when the multivariate process (modeled by wavelet coefficients in Step 4) is affected by an assignable cause(s). The proposed TMO-WAVELET statistic is as below:

$$TMO_i = \text{n}\ (\bar{\boldsymbol{O}}_i - \bar{\bar{\boldsymbol{O}}})'\ \boldsymbol{S}_o^{-1}\ (\bar{\boldsymbol{O}}_i - \bar{\bar{\boldsymbol{O}}}) \qquad (7)$$

where n is the window size value, $\bar{\bar{\boldsymbol{O}}}$ indicates the mean vector of the $\boldsymbol{O}$ samples in the case that the process works under in-control condition, $\bar{\boldsymbol{O}}_i$ addresses the mean of the vector of the i^{th} observation, and $\boldsymbol{S}_o^{-1}$is the inverse of the variance-covariance matrix obtained in Step 5.

Step 7) In-control ARL: Before a quality engineer uses the proposed scheme, the scheme should be adjusted for a selected in-control ARL leading to determination of the upper control limit. This is performed using Mont Carlo simulation. In this paper, 10,000 iterations are used to obtain the control limit of the chart based on the error-I (i.e., a specified α).

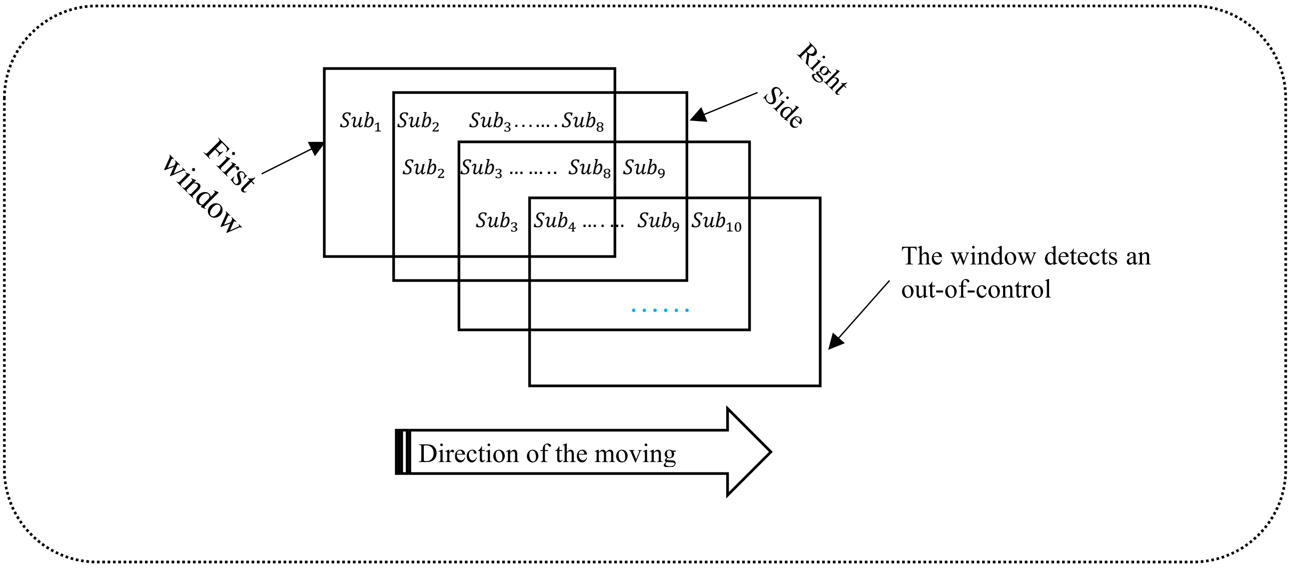

Fig 1. Moving window concept

Step 8) Moving window: In this research, the moving window method is employed to identify the out-of-control condition of the process. With this approach, as the window moves from the right side (When the process is in-control), new observations enter the window, while old observations leave this window from the left side, one by one. The concept of moving window is illustrated in Figure 1 (where Sub indicates the subgroup). When the proposed scheme signals an out-of-control condition, it implies that a shift corresponding to the observation of the mean vector has entered to the window. The moving window has effectively been utilized in previous studies to detect an out-of-control condition, such as Noorossana et al. (2011) and Atashgar & Noorossana (2012).

IV. PERFORMANCE EVALUATION

To assess the capability of the proposed model and test the sensitivity of its performance, the proposed model is compared with three well-known traditional schemes in this section (i.e., T^2, MEWMA, and MCUSUM).

A. T^2 Control chart

Let the studied case follow a joint normal probability distribution identified by $\boldsymbol{\mu}$ = (0, 0) and $\Sigma=\begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$ parameters. Assume that the process works under in-control condition, and the first 98 data of observations are produced from an in-control bivariate normal distribution. Suppose at the time of observation 99, due to a special cause, the mean vector of the process departs to an out-of-control condition. In this analysis, the environment is simulated for

10,000 sets of observations. Furthermore, to provide a comprehensive comparative analysis, three different scenarios are considered: 1) Only one of the members of the mean vector affecting an assignable cause shift to a new mean value, 2) Both members of the vector with the same magnitude shift to an out-of-control condition, and 3) Both members of $\boldsymbol{\mu}$ vector depart to an out-of-control condition with different shift values.

To identify the out-of-control condition, the proposed scheme of this paper, as described in Section 3, is used. The probability of the false signal is set to α=0.0027, i.e., ARL_{In}=370 using a simulation process. In this simulated process, the upper control limit (UCL) is obtained to define the in-control condition of the bivariate environment.

Table 2 shows the comparative results for the described three scenarios in term of out-of-control ARL. This ARL is used as a performance indicator of control charts. In other words, the capability of these three charts is evaluated by the out-of-control ARL. A smaller value for ARL indicates a higher sensitivity of the control chart. As shown in these reports, the proposed wavelet-based model is more powerful compared to the capability of the T^2 approach.

TABLE II. COMPARATIVE REPORT FOR T²

Shift value (δ)	Scenario 1		Scenario 2		Scenario 3	
	T^2 chart	Proposed chart	T^2 chart	Proposed chart	T^2 chart	Proposed chart
1	16.23	2.34	13.87	6.37	16.10	3.37
1.5	12.94	2.99	11.32	3.56	12.24	2.99
2	13.99	3.56	6.69	2.77	10.54	2.99
2.5	9.89	3.23	7.85	2.78	9.36	2.98
3	7.53	2.98	7.98	2.86	7.9	2.97

Figures 2-4 depict signals of Scenarios 1-3 in the case of δ=3, respectively. As illustrated in the figures, up to observation 98, both schemes' signals exhibit a similar behavior; however, the proposed model demonstrates a noticeably superior sensitivity compared to the T^2 chart.

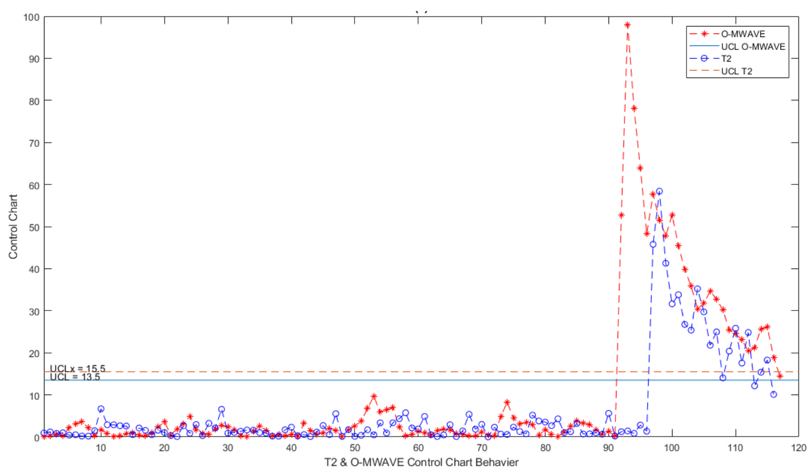

Fig 2. Comparative signals for Scenario 1

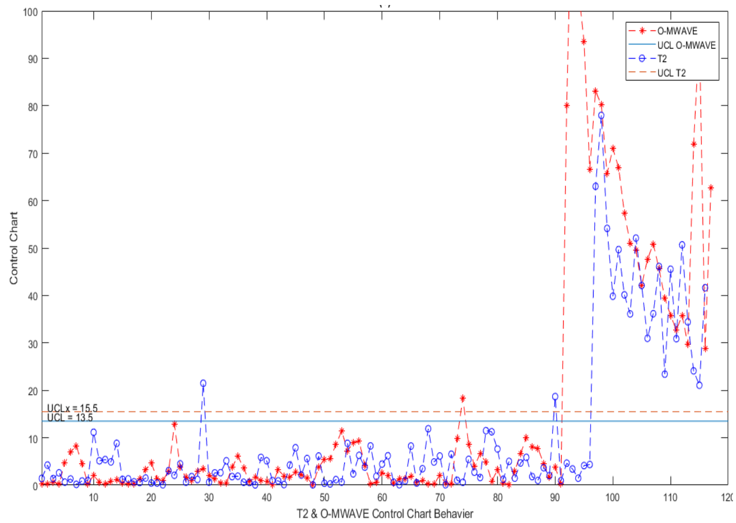

Fig 3. Comparative signals for Scenario 2

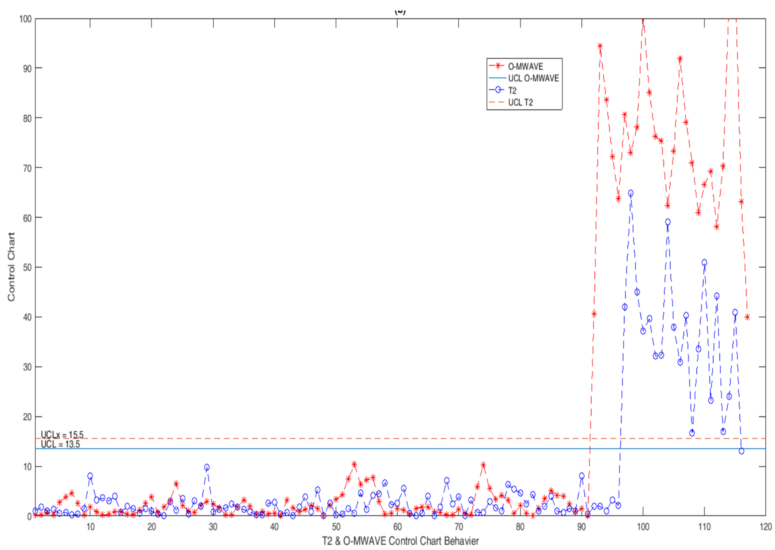

Fig 4. Comparative signals for Scenario 3

Table 3 reports the capability of the proposed scheme (TMO-WAVE) compared to MEWMA scheme (for different *r* cases) and MCUSUM based on the term of ARL. As shown in Table 3, the performance of the proposed model is superior compared to MEWMA and CUSUM charts.

TABLE III. COMPARATIVE REPORT FOR MEWMA AND TMO-WAVE

Shift value (δ)	MEWMA *r* value				MCUSUM	TMO-WAVE
	0.8	0.6	0.4	0.2		
0	200	200	200	200	200	200
1	28.10	19.30	13.2	10.10	9.35	3.98
1.5	10.30	7.24	5.74	5.50	5.94	2.87
2	4.57	3.86	3.54	3.80	4.20	2.76
2.5	2.57	2.53	2.55	2.91	3.26	1.98
3	1.91	1.88	2.04	2.42	2.78	1.97

Tables 2 and 3 indicate that the ARL values of the proposed method of this paper are smaller compared to the three schemes. The result addresses that TMO-WAVE is more sensitive when a special cause affects the multivariate process.

V. CONCLUSION

Monitoring several correlated technical specifications of a process is more complex compared to a univariate case. In this research, a new scheme was proposed based on the wavelet approach to monitor a process with multiple technical specifications. In this research it is assumed that the technical specifications are provided in a correlated environment. The proposed scheme, named TMO-WAVE, utilizes the multi-resolution of the wavelet transform. The evaluation of the proposed model's performance indicated an acceptable capability for the approach. Comparative analysis in this research suggests that the capability of TMO-WAVE is superior to three well-known schemes in the literature. This research highlights the potential wavelet approach, as an advanced signal processing technique, to effectively control complex multivariate processes statistically. Statistically monitoring multistage processes with cascade properties using the wavelet approach represents a promising direction for further research . This advanced approach is capable of supporting the advanced purposes of industrial information and Industry 4.0.

REFERENCES

Alamelu Manghai, T., & Jegadeeshwaran, R. (2019). Vibration based brake health monitoring using wavelet features: A machine learning approach. *Journal of Vibration and Control*, 25(18),2534–2550.

Al-Assaf, Y. (2004). Recognition of control chart patterns using multi-resolution wavelets analysis and neural networks. *Computer & Industrial Engineering*, 47 (17-29), 17-29.

Atashgar, K., & Abbasi, L. (2021). A new model to monitor very small effects of a polynomial profile. *International Journal of Quality & Reliability Management*, 38(4), 1023-143.

Atashgar, K., & Zargarabadi, O. A. (2017). Monitoring multivariate profile data in plastic parts manufacturing industries: An intelligently data processing. *Journal of Industrial Information Integration*, *8*, 38-48.

Atashgar, K., & Noorossana, R. (2012). Diagnosing the source(s) of a monotonic change in the process mean vector. *International Journal of Advanced Manufacturing Technology*, 60, 1175–1183.

Cahng, S.l., & Yadama, S. (2010). Statistical process control for monitoring non-linear profiles using wavelet filtering and B-Spline approximation. *International Journal of Production Research*, 48 (4), 1049-1068.

Chen, Z., Lu, S. & Lam, S. (2007). A hybrid system for SPC concurrent pattern recognition. *Advanced Engineering Informatics* 21(3,: 303–310.

Chicken, E., Pignatiello Jr, J. J., & Simpson, J. R. (2009). Statistical process monitoring of nonlinear profiles using wavelets. *Journal of Quality Technology*, *41*(2), 198-212.

Cohen, A., & Atoui, M.A.A. (2022). On Wavelet-based Statistical Process Monitoring. *Transactions of the Institute of Measurement and Control*, 44 (3), 525-538.

Cohen, A., Tiplica, T., & Kobi, A. (2015). Statistical process control for AR(1) or non-gaussian processes using wavelets coefficients. *Journal of Physics: Conference Series*. 659 012043.

Cohen, A., Tiplica, T., & Kobi, A. (2016a). Owave control chart for monitoring the process mean. *Control Engineering Practice*, 54, 223–230.

Cohen, A., Tiplica, T., & Kobi, A. (2016b). Design of experiments and statistical process control using wavelets analysis. *Control Engineering Practice,* 49, 129–138

Crosier, R.B. (1988). Multivariate Generalization of Cumulative Sum Quality Control Schemes. *Technometrics*. 30(3), 291-302.

Du, S., Huang, D., & Lv, J. (2013). Recognition of concurrent control chart patterns using wavelet transform decomposition and multiclass support vector machines. *Computers and Industrial Engineering,* 66(4), 683–695.

Fasso, A. (1999). One-sided MEWMA control charts. *Communications in Statistics -Theory and Methods*. 28(2), 381-401.

Fleet, P.J.V. (2019). *Discrete Wavelet Transformations: An Elementary Approach with Applications*, New York, John Wiley & Sons, Inc.

Gao, R.X., & Yan, R. (2010). Wavelets: *Theory and Applications for Manufacturing*. New York: Springer

Gharib, A., Amiri, A., & Jalilibal, Z. (2021). Designing a multivariate exponentially weighted moving average control chart with measurement errors. *Journal of Quality Engineering and Production Optimization*, 5(1), 215-232.

Guh, R.S. (2007). On-line Identification and Quantification of Mean Shifts in Bivariate Processes using a Neural Network-based Approach. *Quality and Reliability Engineering International*. 23(3), 367-385.

Guh, R.S., & Shiue, Y.R. (2008). An effective application of decision tree learning for on-line detection of mean shift in multivariate control charts. *Computer and Industrial Engineering*. 55(2), 475-493.

Haq, A., Munir, T., & Shah, B.A. (2020). Dual multivariate CUSUM charts with auxiliary information for process mean. Quality and Reliability Engineering International. 36(3), 861-875.

Harrou, F., Taghezouit, B., & Sun, Y. (2018a). A robust monitoring technique for fault detection in grid-connected PV plants. In: 2018 7th International Conference on Renewable Energy Research and Applications (ICRERA), Istanbul, Turkey, 594–598.

Harrou, F., Khaldi, B., Sun, Y., & Cherif, F. (2018b). Statistical detection of faults in swarm robots under noisy conditions. In: 2018 6th International Conference on Control Engineering & Information Technology (CEIT), Istanbul, Turkey, 1–6. IEEE.

Harrou, F., Taghezouit, B., & Sun, Y. (2019). Robust and flexible strat-egy for fault detection in grid-connected photovoltaic systems. *Energy Conversion and Management*. 180, 1153–1166.

Hotelling, H. (1947) *Multivariate quality control-Illustrated by the air testing of sample bombsights. Techniques of Statistical Analysis*, Eisenhart, C., Hastay, M.W., Wallis, W.A. (Eds). McGraw-Hill: New York.

Huwang, L., Lin, L.W., & Yu, C.T. (2019). A spatial rank–based multivariate EWMA chart for monitoring process shape matrices. *Quality and Reliability Engineering International*, 35(6), 1716-1734.

Hwarng, H.B. (2004). Detecting process mean shift in the presence of autocorrelation: a neural network based monitoring scheme. *International Journal of Production Research*, 42(3),573-595.

Jeong, M.K., Lu, J.C., Huo, X., Vidakovic, B., & Chen, D. (2006). Wavelet-based data reduction techniques for process fault detection. *Technometrics,* 48(1): 26–40.

Jeong, Y.S., Kim, B. & Ko, Y.D. (2013). Exponentially weighted moving average-based procedure with adaptive thresholding for monitoring nonlinear profiles: Monitoring of plasma etch process in semiconductor manufacturing. *Expert Systems with Applications,* 40(14), 5688–5693.

Koosh, M., Noorossana, R., & Ahmadi, O. (2022). Two-dimensional wavelet based statistical monitoring of image data. *Quality & Reliability Engineering International.* 38(7), 3797-3815.

Lee, J., Hur, Y., Kim, S.H, & Wilson, J.R. (2012). Monitoring nonlinear profiles using a wavelet-based distribution-free CUSUM chart. *Journal of Production Research*, 50(22), 6574–6594.

Li, J., Jeske, D.R., Zhou, Y., & Zhang, X. (2019). A wavelet-based non-parametric CUSUM control chart for autocorrelated processes with applications to network surveillance. *Quality and Reliability Engineering International,* 35(2), 644–658.

Lowry, C.A., Woodall, W.H., Champ, C.W., & Rigdon, S.E. (1992). A multivariate Exponential Weighted Moving Average control chart. *Technometrics*. 34(1), 46-53.

Mansouri, M., Al-Khazraji, A., Hajji, M., Faouzi Harkat, M., Nounou, H., & Numan, M. (2018). Wavelet optimized EWMA for fault detection and application to photovoltaic systems. *Solar Energy,* 167, 125–136.

McGinnity, K., Chicken, E., & Pignatiello, J.J. (2015). Nonparametric changepoint estimation for sequential nonlinear profile monitoring. *Quality and Reliability Engineering International*, 31(1), 57–73.

Ngai, H.M., & Zhang, J. (2001). Multivariate cumulative sum control charts based on projection pursuit. *Statistica Sinica*. 11(3), 747-766.

Nikoo, M., & Noorossana, R. (2013). Phase ii monitoring of nonlinear profile variance using wavelet. *Quality and Reliability Engineering International,* 29(7), 1081–1089.

Noorossana, R., Atashgar, K., & Saghaie, A. (2011). An integrated supervised learning solution for monitoring process mean vector. *International Journal of Advanced Manufacturing Technology*, 56, 755–765

Pignatiello, J.J., & Runger, G.C. (1990). Comparisons of multivariate CUSUM charts. *Journal of Quality Technology,* 22(3), 173-186.

Piri, S., Abdel-salam, G., & Boone, E.L. (2021). A wavelet approach for profile monitoring of Poisson distribution with application. Communication in Statistics- Simulation and Computation, 50(2), 525-536.

Ranaee, V., & Ebrahimzadeh, A. (2011). Control chart pattern recognition using a novel hybrid intelligent method. *Applied Soft Computing* 11(2), 2676–2686.

Runger, G.C., Keats, J.B., Montgomery, D.C., & Scranton, R.D. (1999). Improving the performance of a multivariate exponentially weighted moving average control chart. *Quality and Reliability Engineering International*. 15(3), 161-166.

Runger, G.C., & Testik, M.C. (2004). Multivariate extensions to cumulative sum control charts. *Quality and Reliability Engineering International,* 20(6), 587-606.

Testik, M.C., & Borror, C.M. (2004). Design strategies for the multivariate exponentially weighted moving average control chart. *Quality and Reliability Engineering International*, 20(6), 571- 577.

Stewarda, R.M., & Rigdon, S.E. (2015). The Efficiency of a Wavelet Transformation Approach to Prospective Monitoring, *Quality and Reliability Engineering International*, 31(3), 489-500.

Tseng, S., Chou, R., & Lee, S. (2002). A study on a multivariate EWMA controller. *IIE Transactions*, 34(6), 541-549.

Wang, C.H., & Kuo, W. (2007). Identification of control chart patterns using wavelet filtering and robust fuzzy clustering. *Journal of Intelligent Manufacturing,* 18(3), 343–350.

Wang, H., Kim, S.H., Huo, X., Hur, Y., & Wilson, J. (2015). Monitoring nonlinear profiles adaptively with a wavelet-based distribution-free CUSUM chart. *International Journal of Production Research*, 53(15), 4648–4667.

Woodall, W.H., & Ncube, M.M. (1985) Multivariate CUSUM quality- control procedures. *Technimetrics,* 27(3), 285-292

Xie, F., Sun, J., Castagliola, P., Hu, X., & Tang, A. (2021) . A multivariate CUSUM control chart for monitoring Gumbel's bivariate exponential data. *Quality & Reliability Engineering International*, 37(1), 10-33.

Yeh, A.B., Lin, D.K.J., Zhou, H., & Venkataramani, C. (2003). A multivariate exponentially weighted moving average control chart for monitoring process variability. *Journal of Applied Statistics*, 30(5), 507-536.

Yin, Z., & Hou, J. (2016). Recent advances on SVM based fault diagnosis and process monitoring in complicated industrial processes. *Neurocomputing,* 174, 643–650.

Yumin, L. (1996). An improvement for MEWMA in multivariate process control. *Computers and Industrial Engineering,* 31(3-4), 779-781.