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A nonlinear mathematical model for autonomous vehicle routing problem by considering traffic congestion for each route

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Abstract *–Emerging technologies, such as vehicle-to-road infrastructure connectivity via wireless telecommunications systems, in addition to reducing the role of humans in driving activities, can meaningfully improve road performance compared to traditional traffic control systems. Today, automated vehicles (AVs), as an emerging player in modern urban transportation, would significantly influence customer satisfaction. For AVs, the optimal routes must be found by a decision support system. This problem becomes more challenging when a suitable route is concurrently chosen by the majority of vehicles and network congestion occurs. In this research, a mathematical model for seeking the optimal route and scheduling of AVs by considering road traffic is presented. GAMS software is used for solving and analyzing the mathematical model. The results for a sample example show that the optimal routes are successfully obtained for the AVs. Sensitivity analysis reveals that as traffic time increases, so do the cost and service time. This model calls for government agencies to allocate portions of road networks to AVs to regulate vehicle movement and thus increase the output and performance of the network.*

Keywords*– Automated vehicle; Road Traffic; Performance Improvement; Transportation Networks; Routing; Traffic*

I. INTRODUCTION

Modern technologies enable significant reductions in the human limitations associated with transportation, as well as an improvement in the safety and effectiveness of transportation systems (Soltanzadeh et al., 2020). Intelligent utilization of data interchange between infrastructure and vehicles can lower latency and improve road capacity. Among these technologies, automated vehicle (AV) technology is one that has the potential to enhance both road safety and performance (Drexl, 2013). By designating roads for automated control systems, the optimal service capacity of routes is increased (Kesting et al., 2010; Levin & Boyles, 2016; Chen et al., 2016).

Researchers have proposed transforming some AVs into private AV lines to reduce traffic congestion and improve passenger safety (Levin & Boyles, 2016). Chen et al. (2016) proposed a time-dependent model to maximize the use of AV lines in the public network, consisting of AVs and conventional vehicles, to boost AV lines' utilization. Godsmark and Kakkar (2014) assert that AV zones' presence increases AVs' use while optimizing their performance and that coordinating a large fleet of AVs to provide customer service using a demand-based strategy is an appealing operational

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paradigm. The autonomous mobility on demand (AMoD) system can reduce travel costs and produce long-term surplus benefits like higher use of public transit generally, less need for infrastructure supporting urban parking, and decreased pollution (Mitchell et al., 2010; Spieser et al., 2014). The main advantages of AMoD result from vehicle sharing, in which, after providing customer service, each vehicle autonomously navigates to the next customer location or anticipates upcoming customer demand (Pavone et al., 2012; Perez et al., 2010). A Route Optimization System (ROS) aims to reduce costs and other negative social and environmental impacts while optimizing vehicle routes to meet transportation demands. The performance of ROS is influenced by daily operations, varying traffic conditions, shifting restriction rules, road construction, and drivers' growing familiarity with routes and destinations. Kakimoto et al. examined the impact of AV on safety, efficiency, and comfort on a single-lane highway (2018). Kim et al. (2019) examined a future transportation landscape in which AV technologies are fully developed and have supplanted connected vehicles (CVs).

Given the significance of the problem, a mathematical model is proposed to determine the best routes for AVs when more than one vehicle chooses the same route. This mathematical programming model can simulate real-world conditions by simulating traffic and scheduling processes for these AVs along the same route. The proposed model determines the optimal route for AVs by analyzing traffic on each route. The current study is organized as follows: The relevant literature is reviewed in Section 2. Section 3 describes the problem statement. Section 4 describes the mathematical model. Section 5 contains the traffic time sensitivity analysis. Section 6 includes the travel time sensitivity analysis. Section 7 discusses conclusions and future research.

II. LITERATURE REVIEW

The development of autonomous vehicle technology has accelerated recently (Cao et al., 2017). From driver assistance to complete automation, the Society of Automobile Engineers (SAE) International outlines five stages of autonomous driving (SAE, 2021). Many automakers and IT companies worldwide are currently implementing level 4 (high automation) tests after many vehicles have completed level 1 (driver assistance) and level 2 (partial automation) testing (Litman, 2017). Levels of automated operation have been created by the Society of Automotive Engineers (SAE), ranging from zero automation (Level 0) to complete automation (Level 1). (Level 5, also referred to as autonomous or self-driving cars) We are now getting close to the stage where the challenges and potential related to AVs are starting to become more apparent after a time of intense excitement. Although the pros and cons of its use are arguable, almost every significant technology company and automaker invests billions of dollars annually in an effort to get a competitive edge in this industry (Korosec, 2018).

In dynamic (shared autonomous taxi) SAT systems, AVs serve as public transportation. Currently, taxi drivers choose the fastest route based on their knowledge and experience (Yuan et al., 2011; Zheng et al., 2010). Due to the ad hoc nature of actual road networks, customers might not be able to get to their destinations on time (Wu et al., 2012). Customers thus place a high value on the possibility of arriving on time; as a result, this has been regarded as an essential component of urban transportation, particularly for customers with limited time and who face severe penalties for being late. Zhu et al. (2018) investigated the potential benefits of a road pricing scheme. Accurate travel planning information and a solution that may improve customers' dependability on on-time arrival are unquestionably in high demand in dynamic SAT systems. SATs may provide travel flexibility because they can serve several consumers (Liu et al., 2019). Customers call in to seek SATs, which are then automatically assigned to them by a fictional central control system. It has been discovered that SAT systems can reduce taxi fleets and clients' travel expenses (Liu et al., 2018a; Fagnant and Kockelman, 2018). In a study by Burghout et al. (2015), personal vehicles (PVs) were used instead of SATs in Stockholm. Participants in the sharing programs included consumers with the same origin and destination, customers with the same origin but different destinations, and customers with different origins but the same destination. The results show that only 5% of today's private automobiles would be required for passenger transportation. The examined shared taxi systems can be divided into three categories depending on their objectives and criteria for sharing: time-based, cost-based, and distance-based shared systems. The literature has looked at a wide range of travel time statistics, such as a shared taxi service that costs you according to how much time you spend traveling.

SAT simulators were developed by Fagnant and Kockelman (2018) for users with a range of origins and destinations. When deciding whether a ride can be shared, the total travel time and increase in remaining travel time for riders, the total travel time for a new passenger, the likelihood that the new passenger will be picked up in the next five minutes, and the total travel time for the two passengers are taken into account. Their research suggested that sharing could cut down on users' overall service time (including waiting time) and costs. According to Lioris et al. (2016), a potential client should be turned away if the time required to serve them is longer than the permitted time. A previous study found that it is better to give an empty or public SAT to the customer who comes to the customer's location first when both are available (Tao, 2007).

The combined latency of all SAT system users was decreased by Alonso-Mora et al. (2017). According to Kruege et al. (2016), customers pay attention to wait times. By lowering overall routing costs, Cordeau (2006) used a trip cost-based shared taxi system to address the shared taxi issue. To increase overall profit, Hosni et al. (2014) proposed a shared taxi system.

Miller and How (2017) observed a ride-sharing system with independent automobiles where the cost of all applicants is maximized based on applicant location prediction. According to Ma et al. (2015), a trip request submitted using a smartphone app should be assigned to the taxi that minimizes the increase in travel distance caused by the request while meeting the arrival time, capacity, and financial restrictions of both the new and existing clients.

In order to reduce the overall distance traveled by all customers in a shared taxi, Lokhandwala and Cai (2018) designed a ride-sharing taxi system that combines traditional and autonomous vehicles. They also took into account the permitted trip time for each consumer in their analysis. Every customer's authorized trip time was taken into account in their study.

The transportation system today is becoming more and more complicated, which presents challenges for planners and politicians. They must maintain efficient traffic flows in populated areas and offer sufficient service in remote areas. The fleet of vehicles must minimize energy use and have as little of an environmental impact as possible, yet the overall increase in trips necessitates constant system capacity augmentation. Self-driving taxi services have only lately begun to operate all over the world, making autonomous vehicles a reality. Future predictions predict that intelligent and networked cars will significantly boost the effective road capacity (Fortune, 2016; Navya, 2016; Abdullah, 2016).

The requirement for parking in cities would be drastically reduced by a fleet of self-driving taxis, freeing up additional lanes to expand capacity or improve the aesthetics, sustainability, and livability of urban areas (Tientrakool et al., 2011; Friedrich, 2015). On the other side, more vehicles may be seen on the roads if AVs become so cozy, economical, and accessible that aggregation means like buses or trains become redundant (Skinner and Bidwell, 2015).

The use of AVs may have a number of benefits, including increased network capacity and a decrease in accidents. The results show that the average travel time increased by 50%, while parking lots that can be used again range from 14.6% to 32.27%. Due to the lack of affordable parking options in the congested downtown area, drivers spend 8% to 74% of their route time looking for parking spaces. Due to the inefficiency of the walking distance between the parking lot site and the users' destination, privately owned human-driven vehicles (HVs) restrict the number of potential parking spots close to their destinations. Despite the low parking fee, the farther an automobile owner must walk, the less likely it is that they will choose this parking lot. A major concern for the transportation system is that traffic congestion could get worse as a result of induced trips made by AVs to far-off parking lots. Previous research primarily focused on AV parking methods, morning–evening come and go trips using a monocentric model (Zhang et al., 2019a) (the bottleneck-constrained highway model assumed only one origin and one destination), and multiclass traffic with HVs and AVs (Levin and Boyles, 2016).

The quick development and mass production of autonomous vehicles (AVs) has the potential to change the way people travel in terms of mobility, safety, and travel habits. Shared autonomous cars (Mao et al., 2020; Kang and Levin, 2021; Kruege et al., 2016), cooperative adaptive cruise control (Wang et al., 2020; Lai et al., 2020; Gong and Du, 2018), policy and safety measures for AV/HV mixed flow (Li et al., 2020, 2021; Gong and Du, 2018; Dresner and Stone, 2008; Wu et al., 2019; Kang et al., 2021). The introduction of self-driving cars has the potential to increase traffic capacity, encourage car sharing, and provide convenience to the elderly or disabled who are unable to drive on their own, motivating industry and university researchers to evaluate AVs.

However, there are also drawbacks to adopting AVs, such as the fact that AVs may initially lower capacity if AV owners choose comfortable acceleration speeds and that zero-occupant vehicles may increase vehicle hours traveled (Kang and Miller, 2018). Transportation planners must consider the possibility of traffic congestion brought on by induced AV excursions in search of inexpensive parking spots as more people use AVs. A parking reservation approach with confined transport modes and parking lots was investigated by Wu et al. (2021). They anticipated that if commuters arrived late, the parking lot reservation would expire or that they would have to pay additional fees to extend the reservation.

A liner aisle with a bi-directional through road was used by Su and Wang (2021) in order to observe a spatial parking planning design that had both AVs and HVs. Another track for the balance analysis of selecting parking spots is the static traffic assignment model. An approach was introduced by Jiang et al. (2014) to design the balance analysis on parking spot choice for electric automobiles with a time restriction. Although they examined two types of cars, the demand for each was indicated, implying that no mode alternative was considered. Zhang et al. (2019) proposed a variational inequality-based approach. In some studies, mode split and route selection were combined into a single mathematical model. Two methods for integrating modal split and equilibrium assignment models, as well as elastic demand modeling techniques, were presented by Abdulaal and LeBlanc in 1979.

Numerous ways in which AVs may be advantageous to passengers have been thoroughly investigated as a hot issue. Because AVs may self-direct to a cheap parking facility, the resulting journeys can make the traffic situation worse. There is a lot of potential for AVs and driver assistance technology to reduce accidents and increase network capacity. Cooperative driving by AVs can lower vehicle emissions, but when AVs self-navigate to far-off parking lots, they run the risk of raising emissions. Decision-makers must choose between using less parking space in crowded core areas and lengthening routes when AVs and HVs coexist.

The potential impact of self-driving technology on urban mobility has generated both enthusiasm and worry. Approximately 30 car manufacturers or IT companies have licenses and are already testing their AVs in actual environments. There are several ways in which we can make use of transformational technology (Litman, 2017).

According to some analysts, this technology will be a key part of the sharing economy since it will make it possible for shared AVs (SAVs), a new door-to-door transportation option, to exist (Krueger et al., 2016). According to current expressed preference survey results, privately owned AVs (PAVs) will soon outnumber SAVs in the market, according to other studies (Zhang & Guhathakurta, 2021). Whatever the form of AV deployment, this disruptive technology will radically change how people travel, causing a change in how cities are developed.

AVs are expected to significantly improve the safety and efficiency of existing roads and transportation systems. Although it will take years for the use of AV technology to spread, recent advancements indicate that we are rapidly approaching its use. As of October 2016, Google's AVs had traveled over 2 million miles on public roads. In Singapore, Nutonomy Software recently launched the world's first self-driving taxi. Multiple automobile manufacturers, including Volvo and Audi, are currently developing and testing prototypes of AVs. In the United States, government agencies in states such as Nevada, Florida, California, Michigan, and Washington, D.C., have yet to modify various policies and procedures in order to utilize and improve the application of this technology.

Extensive research papers on AVs have recently been published, making it necessary to have a broad perspective in order to synthesize the existing knowledge base. There have been attempts to bring together pertinent studies (Gkartzonikas and Gkritza, 2019; Becker and Axhausen, 2017; Miller and How, 2017; Soteropoulos et al., 2018). Becker et al. (2017) evaluate studies on autonomous car adoption up to 2016. Gkartzonikas et al. (2019) categorize survey research on AVs based on the study's aims. BMW Group has teamed up with Intel and the Mobileye Team to develop

AVs for ride-sharing by 2021 (BMW Group, 2021). In 2021, Ford aims to launch its AVs in a ride-hailing or ride-sharing service (The Ford Company, 2021). O'Kane (2018) says that by 2021, the Volkswagen Group and Hyundai will work with Aurora Innovations to launch a self-driving on-demand service. Redistributing cars, according to Vosooghi et al. (2019), has a substantial impact on service performance, such as modal share and fleet utilization. Dandl et al. (2019) in their research work on the influence of geographical and temporal aggregation of demand forecasts used for vehicle redistribution. Gurumurthy et al. (2019) created a model of a system that included AVs with flexible ride-sharing options and congestion pricing during peak hours. Simoni et al. (2019) investigated four distinct congestion pricing and tolling techniques, which are divided into two categories: classic and advanced. The International Transport Forum (2018) examines the concerns of road safety and security connected with autonomous cars, as well as solutions for addressing them. Vosooghi et al. (2019) find that the AV service can lower the number of automobiles in the network by at most 1.7 percent when considering dynamic demand and a multimodal network. In Table I, we summarized the studies that are similar to the present article.

Table I. Components of AV modeling and indicative references

#	*Authors*	*Booking Type*		*Traffic assignment*		*Problem*	
		on demand	*Reservation based*	*Fix Travel Time*	*simulation system*	*Objective function*	*Multi-objective*
1	Alonso-Mora et al. (2017)	×				Min-Time	
2	Lamotte et al. (2017)		×			Max-profit	
3	Fagnant and Kockelman (2018)		×		×		
4	Gurumurthy and Kockelman (2018)		×		×		
5	Hörl, S (2017)	×			×		
6	Hyland and Mahmassani (2018)		×			Min-Time	
7	Levin (2017)		×		×		
8	Lokhandwala and Cai (2018)		×		×		
9	Mahmassani (2018)		×		×		
10	Ma et al. (2017)		×			Min-cost	
11	Dandl et al. (2021)	×			×		
12	Pimenta et al. (2017)		×			Min number of stops	
13	Chen et al. (2016)					Min-Time	
14	Oke et al., (2020)	×			×		
15	Boesch et al. (2016)				×		
16	Dia and Javanshour (2017)				×		
17	Oh et al.(2020)	×			×		
18	Zhou et al.(2021)				×		
19	Alam and Habib (2018)				×		
20	Dandl et al. (2019)				×		
21	Vosooghi et al. (2019)				×		
22	Liu et al. (2019)			×	×		
23	Bracy et al. (2019)	×			×		
24	Zhang and Guhathakurta (2021)	×			×		
25	Mousavi et al. (2021)				×		

26	Kang et al. (2022)	×				Min-Time	
27	This research	×		×		Min-Time Min-Cost	×

The literature has begun to pay attention to the potential for AVs to improve traffic flow. While much of the literature on CVs and AVs uses micro-simulation, this study provides bi-objective mathematical modeling to investigate the effects of traffic on a city network with dynamic user equilibrium. This research helps in the creation of a road traffic assignment. In addition, the suggested model solves using the Lagrangian relaxation approach in medium and large cases.

III. PROBLEM DESCRIPTION

In recent years, technology advancements, particularly in the digital realm, have prompted changes to traffic regulations intended to increase safety. The traffic monitoring system, which is still in use today, has several disadvantages, including the inability to communicate information to other drivers and processing delays. Recent research has also focused on newer ideas, such as the wireless network.

Because freeways and highways are critical components of the transportation infrastructure, it is essential to manage traffic in a way that minimizes delays. New traffic control technology can aid in this regard. Both are effective in enhancing the capacity of the flow and disaster management. The new concept of automobile communication systems and traffic infrastructure enables the management of freeways and highways in which vehicles can communicate with each other and with infrastructure. Before they are released to the public, vehicle communication systems must be evaluated for their impact on vehicle traffic, their interaction with one another, and the traffic infrastructure. People who cannot or will not drive are expected to behave safely as the use of independent vehicles grow rapidly in popularity. The following framework serves as the basis for formulating the research problem. Initially, AVs were assigned to a portion of a transportation network. In this situation, only AVs are permitted to use these communication paths (links). They also follow predetermined routes. In addition, the departure point and shortest route to each region are determined after entering AVs to various areas. If multiple AVs are available, the shortest route is used to determine the optimal network for each AV. This strategy increases the use of autonomous vehicles by improving the starting and end positions. Additionally, if there are insufficient routes, multiple vehicles may be assigned to a single route.

In this proposed system, only the customer is involved in requesting a ride, allocating the trip, arranging the arrival, and routing the vehicle via phone. The driver is solely responsible for following the computer's instructions. Due to the rapid growth of autonomous vehicles, customers will eventually be able to order an AV through their smartphone or the internet and ride it to their destination alone. Examining shared vehicle systems has been a fundamental strategy for improving conventional vehicle systems in terms of customer convenience and reduction of traffic congestion. Studies on this methodology have recently garnered considerable interest. Systems for dynamically shared autonomous vehicles are seen as a practical means of enhancing travel flexibility. Since AVs lack human drivers, they require accurate traffic data to create suitable routes; on-time arrival is a critical service characteristic in AV systems. In this study, the reliable route concept and obtained travel time data were used to support path discovery for AVs to increase the probability of on-time arrival, and the potential benefits were examined.

IV. MATHEMATICAL FORMULATION

A. Assumptions

1. AVs can travel on designated roadways.

2. Each AV has a starting and terminating station.

3. Uncertainty is not taken into account and all parameters are assumed to be constant.

4. All AVs are considered equal.

5. Each AV is limited to passing via the stations to which it has been assigned.

The goal of this research's mathematical model is to propose a model for the autonomous vehicle routing problem by taking into account road traffic for each route. The suggested model includes the following necessary information:

B. Sets, Parameters and Variables

Sets

I	Set of all AVs; $i, i' \in I$
J	Set of all station; $j, j' \in J$
K	Set of all routes; $k, k' \in K$
$\mathbf{J_i}$	Set of stations that a vehicle can pass through (each AV can only pass through assigned stations)
$\mathbf{A_i}$	First station for AV (each AV's first station is specified.)
$\mathbf{z_i}$	Last station for AV (each AV's The end station is specified.)

Parameters

$\boldsymbol{PT_{ijk}}$	The amount of time it takes to go from station $i \in I$ to station $j \in J$ using route k.
$\boldsymbol{ST_{ijk}}$	The traffic time for the route k, with the purpose of arriving at station $j \in J$ by the starting station $i \in I$.
$\boldsymbol{TD_i}$	Cost of traveling by $i \in I$

Decision Variables

$\boldsymbol{C_{ijk}}$	The time it takes to travel via route k from station $i \in I$ to station $j \in J$
$\boldsymbol{Y_{ijk}}$	binary variable, if the $i \in I$ uses the route k to arrive at the station j, it will be equal to one; otherwise zero
$\boldsymbol{X_{ii'jk}}$	binary variable, if two AVs want to travel the same route, it represents that $i' \in I$ arrives at the station j after $i \in I$ while using the route k, it will be equal to one; otherwise zero

C. Proposed Mathematical Model

The objectives of the proposed mathematical model are to minimize overall transportation costs and completion times while taking into account traffic conditions along each route. The following demonstrates how the proposed mathematical programming model is presented:

$$Max \sum_{K \in k_{iz_i}} C_{iz_ik} \tag{1}$$

In this case, the objective function (1) minimizes $Max \sum_{K \in k_{iz_i}} C_{iz_ik}$, which is used to minimize the traveling time.

The objective function (1) is linearized using a decision variable named T. In this case, the whole expression in the Min function is equal to this decision variable:

$$\sum_{K \in k_{iz_i}} C_{iz_ik} = T \tag{2}$$

Where $Min\ (T)$ is the objective function (1), and constraint (15) maximizes the whole expression.

Since costs play a significant role in realistic situations, it is essential to consider the cost objective function. This objective function is presented in equation (3).

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$$MIN\ C = \sum_i TD_i * \left(\sum_{K,J} C_{ijk}\right) \tag{3}$$

$$Min \quad (T) \tag{4}$$

$$\sum_{\substack{i\in I\\ i\neq i1}} \sum_{i'\in I} X_{ii'jk} \leq 1 \qquad \forall\, j \in J_i\,, k \in k_{ij} \tag{5}$$

$$\sum_{k\in K} Y_{ijk} = 1 \qquad \forall i \in I\,, j \in J_i \tag{6}$$

$$\sum_{k\in K} Y_{ijk} = 1 \qquad \forall i \in I\,, j \in J_i \tag{7}$$

$$C_{ijk} \geq \sum_{k'\in k_{ij'}} C_{ij'k'} + ST_{ijk} + PT_{ijk} - M * \left(1 - Y_{ijk}\right) \qquad \forall i \in I, j' \in J_i, j \neq A_i, j' = j - 1, k \in k_{ij} \tag{8}$$

$$C_{ijk} \geq Y_{ijk} * \left(ST_{ijk} + PT_{ijk}\right) \qquad \forall i \in I, \forall j \in A_i, \forall k \in k_{ij} \tag{9}$$

$$X_{ii'jk} + X_{i'ijk} \leq 1 \qquad \forall i, i' \in I, i < i', j \in J_i \cap J_{I'}, k \in k_{ij} \cap k_{i'j} \tag{10}$$

$$2 * X_{ii'jk} \leq Y_{ijk} + Y_{i'jk} \qquad \forall i, i' \in I, i < i', j \in J_i \cap J_{I'}, k \in k_{ij} \cap k_{i'j} \tag{11}$$

$$Y_{ijk} + Y_{i'jk} \leq X_{i'ijk} + X_{ii'jk} + 1 \qquad \forall i, i' \in I, i < i', j \in J_i \cap J_{I'}, k \in k_{ij} \cap k_{i'j} \tag{12}$$

$$C_{ijk} \geq C_{i'jk} + ST_{ijk} + PT_{ijk} - M * X_{i'ijk} - 2M + MY_{ijk} + MY_{i'jk} \qquad \forall k \in K\colon N_k > 1, j = j_k, i, i' \in I_k\,, i < i' \tag{13}$$

$$C_{i'jk} \geq C_{ijk} + ST_{i1jk} + PT_{i1jk} - M * X_{ii'jk} - 2M + MY_{ijk} + MY_{i'jk} \qquad \forall k \in K\colon N_k > 1, j = j_k, i, i' \in I_k\,, i < i' \tag{14}$$

$$T \geq \sum_{k\in k_{iz_i}} C_{iz_ik} \qquad \forall i \in I \tag{15}$$

$$C_{ijk} \geq 0 \tag{16}$$

$$T \geq 0 \tag{17}$$

$$X_{ii'jk:binary} \tag{18}$$

$$y_{ijk}\colon binary \tag{19}$$

Constraint 4 minimizes the time it takes to reach the last station. Constraint (5) maintains that each AV is selected once for each route to reach the destination. In constraint (6), each AV must traverse a route to reach its destination. Constraint (7) ensures that if a route is chosen for transportation, its gate becomes open. Constraint (8) represents that the transportation time in each stage includes traveling time at the previous station, the time for the presence of a vehicle in traffic, and the traveling time to reach the destination. This constraint is applied to all stages of navigation, except for the first one. Also, constraint (9) expresses that the traveling time in the first stage includes the traveling time on each route and the traffic time on the route. Constraints (10)–(12) impose restrictions on the two vehicles' route selection because they must adhere to the traffic queue. Constraint (12) guarantees that if the $i' \in I$ departs from route k before the $i \in I$, the $i \in I$ has a longer transport end time. Constraint (13) enforces that if the $i \in$ AV moves ahead of $i' \in I$ from route k, the $i' \in I$ has a longer transport end time. Also, constraints (15)-(17) are model linearization constraints. Besides, constraints (18) and (19) specify the types of variables. The following specifications are applied to the schematic representation of the research model .Table II gives the time that an AV reaches destination j via route k.

D. Solution approach: weighted sum

The weighted sum method integrates the objective functions by allocating appropriate weights. The weights are chosen by decision-makers ($w1$ and $w2$). The weights of objectives may also be determined using some methods, such as AHP.

It is important to consider that $w1, w2 \geq 0$ and $w1 + w2 = 1$ The equation is found in Eq. (20). Since the model has two objectives, the weighted sum method (WSM) is used to combine the two objectives into one objective.

$$MIN\ final = w1 \times \sum_i TD_i * \left(\sum_{K,J} C_{ijk}\right) + w2 \times (T) \tag{20}$$

$s.t$

$Eqs. (5) - (19)$

E. Validation of the mathematical model

A numerical experiment is considered, the details of which are presented in Table II, to validate the mathematical model. The CPLEX solver is used to solve the model, and the results are presented in Table III and Fig. (1).

Table II. Specification of an instance

I*: *Set of all AVs	$\{AV1, AV2, AV3, AV4, AV5\}$
K*: *Set of all routes	$\{Aq1, Aq2, Aq3, Aq4, Aq5,, Aq6, Aq7, Aq8, Aq9\}$
S*: *Set of all stations	$\{s1, s2, s3\}$
$\boldsymbol{PT_{ijk}}$	$round(uniform(1,3))$
$\boldsymbol{ST_{ijk}}$	$round(uniform(1,3))$
$\boldsymbol{TD_i}$	$round(uniform(25,50))$

Table III. Presenting the travel time for each AV at each stage

AV and S	***K***								
	Aq1	***Aq2***	***Aq3***	***Aq4***	***Aq5***	***Aq6***	***Aq7***	***Aq8***	***Aq9***
$AV1.s1$		6							
$AV1.s2$						9			
$AV2.s1$		2							
$AV2.s2$						4			
$AV3.s1$			5						
$AV3.s2$				9					
$AV4.s1$	3								
$AV4.s2$					6				
$AV5.s1$			2						
$AV5.s2$				5					
$AV1.s3$									12
$AV2.s3$							7		
$AV3.s3$								13	
$AV4.s3$									9
$AV5.s3$							10		

The travel time for each AV at each stage can be seen in Table III. The proposed model is solved with the specifications in Table II. Certain routes receive more traffic than others; for example, the Aq2 and Aq6 routes are used by AV1 and AV2. For instance, the route Aq1, Aq5, and Aq8 have been used only once, indicating a route of low importance, whereas the routes Aq3, Aq4, Aq2, Aq7, Aq9, and Aq6 are significant. Fig. (1) provides the optimal routes for each AV.

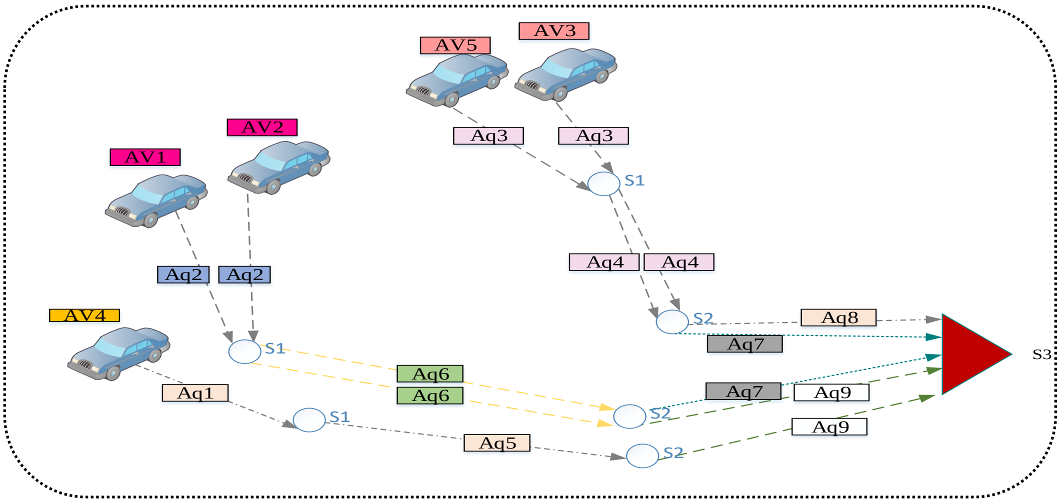

Fig 1. The routes of AVs to reach the destination j by using route k

Optimal routes for each AV are shown in Fig. (1). It depicts the routes and stations that each AV uses to avoid traffic on its trip. Vehicles 3 and 5 use the Aq3 route to S1, then the Aq4 route to S2, then the Aq8 route to the final station (S3), while vehicle 5 takes the Aq4 route to S2, then the Aq7 route to the final station (S3).

For analyzing the sensitivity, we consider the parameters TD_i, PT_{ijk} by increasing 1 unit in each iteration, their effect on the objective function is measured. In this analysis, the model is executed 10 times and each time one unit is added to the previous value. The results of each parameter are presented in Fig. (2).

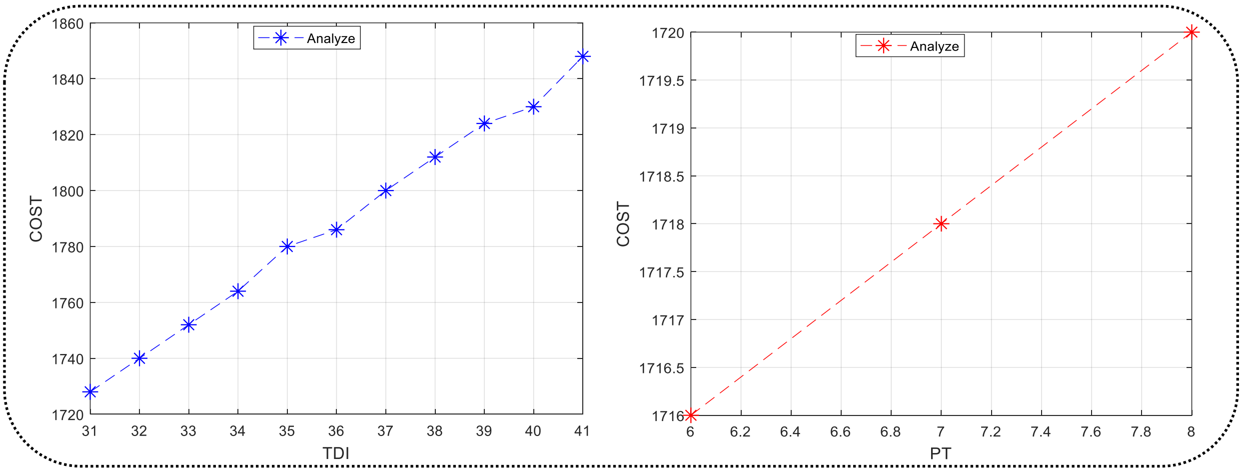

Fig 2. Analyzing the sensitivity by considering the parameters TD_i, PT_{ijk}

Results after 10 iterations can be seen in Fig. (2), by increasing 1 unit of TD_i, PT_{ijk} in each iteration, the cost increases as expected. In addition to sensitivity analysis, the mathematical model can be validated. Also, by increasing 1 unit of TD_i in each iteration the cost increases.

V. CONCLUSION

The sensitivity of the parameters must be investigated and appropriate scheduling policies to improve the scheduling under study. The traffic time was evaluated with a tolerance of ±25%.

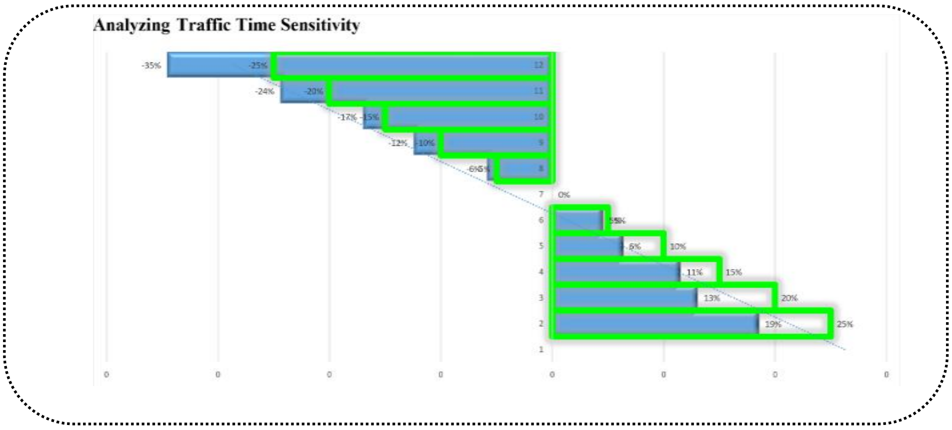

Fig 3. Analyzing traffic time sensitivity

As can be seen in Fig. (3), after a 5% increase in traffic time, the objective function value was increased by 5% due to the increased penalty of delay. On the other hand, a 10% increase in traffic time results in a 6% increase in the objective function value, and when traffic time is increased to 25%, 19% of the objective function's optimum value can fall outside the optimum range. As a result, managers of the studied organizations are advised to plan traffic management so that the time required to complete the task is less than the minimum value, which requires studying the operation of and developing traffic management systems. On the other hand, by reducing traffic time, the overall system's objective function is reduced. It indicates that the mathematical model is highly susceptible to traffic time, such that if traffic time is reduced to 25% of its current value, the objective function value decreases by 35%, indicating the parameter's effect on the mathematical model.

Optimal Cost Changes (%)	19	13	11	6	5	0	-6	-12	-17	-24	-35
Parameter changes (%)	25	20	15	10	5	0	-5	-10	-15	-20	-25

VI. ANALYZING TRAVELING TIME SENSITIVITY

Another factor affecting the model's sensitivity analysis is the travel time.

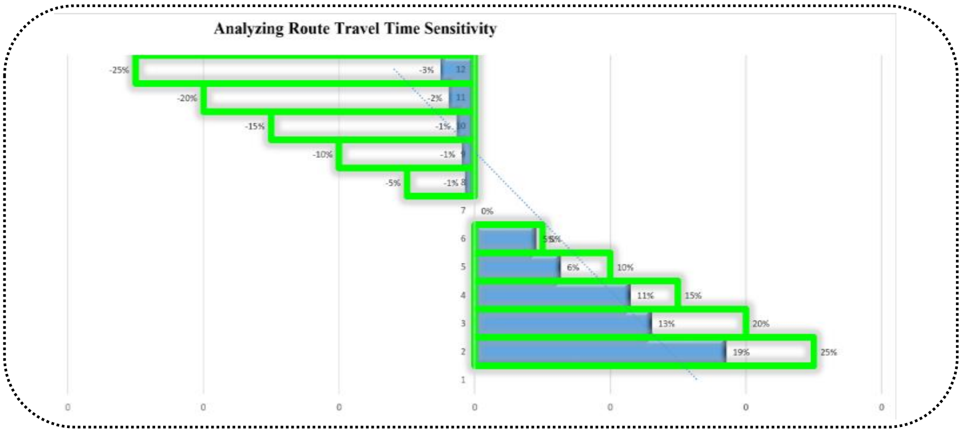

Fig 4. Analyzing route travel time sensitivity

As can be seen in Fig. (4), 5% increase in the travel time results in a 9% deviation from the objective function's optimal value. As is well known, travel time includes numerous effective parameters, It was observed that a 25% increase in travel time resulted in an 18% increase in the objective function value.

Optimal cost changes (%)	19	13	11	6	5	0	-1	-1	-1	-2	-3
Parameter changes (%)	25	20	15	10	5	0	-5	-10	-15	-20	-25

VII. SOLVING THE MATHEMATICAL MODEL USING SPLITTING ALGORITHM OF LAGRANGIAN RELAXATION

The Lagrange relaxation (LR) algorithm is one of the useful techniques for solving complex problems, proposed in 1970. This technique relaxes (removes) complex constraints (constraints that increase complexity; their elimination can simplify the problem) from the original problem and transfers them to the objective function. This relaxation makes the problem simpler (more straightforward) than the original. In the minimization (maximization) problem, the optimal value of the relaxed problem is the lower bound (upper bound) for the main problem. The approach was developed based on Lagrange's theorem. Geoffrion (2010) studies the principles and concepts of LR in detail. Preliminary studies of the LR algorithm are reviewed, and the concepts and principles of this algorithm are reviewed by Fisher (2004). Due to its good performance (Guignard, 2003), the LR algorithm has been used by many researchers in various fields of integer programming. Such an incremental trend is evident in many areas of research (for example, the multi-level location-inventory problem (Fu and Diabat, 2015), the integrated quay crane assignment and scheduling problem (Fu and Diabat, 2015), the vehicle routing problem (VRP) (Imai et al., 2007), the center and orbit network design problem poles and lunar orbits, and source and branch (Alkaabneh et al., 2019). This study uses the LR algorithm too, because it has performed well in solving complex supply chain optimization problems (Hamdan and Diabat, 2020). In view of the fact that the standard form of this algorithm has its drawbacks, in addition to its original form, a modified approach is developed here based on Alkaabneh et al. (2019) in which the standard LR algorithm is improved by updating its multipliers following the violated constraints. In this algorithm, the relaxed constraint is converted to the objective function by the Lagrange multiplier u. Finding the optimal Lagrange multipliers is one of the important issues in this algorithm. In this regard, one of the common approaches that finds the best Lagrange multiplier through an iterative procedure is the subgradient optimization (SO) algorithm. The main problem in SO is determining the step size to ensure that the algorithm converges to the optimal solution. The LR and SO algorithms are combined to overcome these requirements. The pseudo-code for this combination is as follows:

Step 0. Identify the constraints that need to be relaxed.

Step 1. Set the initial Lagrange multiplier u^0 to zero, $t = 1$, and set the initial value θ.

Step 2. Transfer the relaxed constraints to the objective function using Lagrange multipliers.

Step 3. Find a feasible solution to the main problem and set the value of its objective function as the upper bound (UB).

Step 4. Set the initial lower bound (LB) to a small value ($\mathrm{LB}^* = -\infty$).

Step 5. Follow these steps until the stopping criteria are met.

Step 5-1. Solve the relaxed problem and get a new lower bound (LB).

Step 5-2. If $\mathrm{LB} > \mathrm{LB}^*$, then set $\mathrm{LB}^* \longleftarrow \mathrm{LB}$.

Step 5-3. Update the Lagrange multipliers corresponding to the violated constraints.

Step 6. Report the lower bound of the problem.

A. The Proposed LR Algorithm

The LR algorithm used in this study considers the constraints on calculating the travel time and the time to reach the last node. In this mathematical model, constraints (8), (9), (13), and (14) refer to these constraints. It is worth noting that the selected constraints are to examine (test) their properties in the main problem for relaxation, rewritten as follows:

$$-C_{ijk} + \sum_{k' \in k_{ij'}} C_{ij'k'} - M * (1 - Y_{ijk}) \le -ST_{ijk} - PT_{ijk} \qquad \forall i \in I, j' \in J_i, j \ne A_i, j' = j-1, k \in k_{ij} \tag{20}$$

$$-C_{ijk} \le -Y_{ijk} * (ST_{ijk} + PT_{ijk}) \qquad \forall i \in I, \forall j \in A_i, \forall k \in k_{ij} \tag{21}$$

$$-C_{ijk} + C_{i'jk} - M * X_{i'ijk} - 2M + MY_{ijk} + MY_{i'jk} \le -ST_{ijk} - PT_{ijk} \quad \forall k \in K: N_k > 1, j = j_k, i, i' \in I_k, i < i' \tag{22}$$

$$-C_{i'jk} + C_{ijk} - M * X_{ii'jk} - 2M + MY_{ijk} + MY_{i'jk} \le -ST_{i1jk} - PT_{i1jk} \quad \forall k \in K: N_k > 1, j = j_k, i, i' \in I_k, < i' \tag{23}$$

Since the model is of the minimization type, the result of the proposed algorithm is a lower bound for the main problem. For this purpose, the constraints (8), (9), (13), and (14) are removed from the solution space and the equations (24-27) are added to the objective function 20 to obtain a relaxed problem.

$$\sum_{k \in k_{ij}} \sum_{i \in I} \sum_{\substack{j' \in J_i \\ j' = j-1}} \sum_{j \ne A_i} UL_{kj}^{ij'} \times \left[\left[-C_{ijk} + \sum_{k' \in k_{ij'}} C_{ij'k'} - M * (1 - Y_{ijk})\right] - (-ST_{ijk} - PT_{ijk})\right] \tag{24}$$

$$\sum_{k \in k_{ij}} \sum_{i \in I} \sum_{j \ne A_i} VL_{kj}^{i} \times \left[[-C_{ijk}] - \left(-Y_{ijk} * (ST_{ijk} + PT_{ijk})\right)\right] \tag{25}$$

$$\sum_{k \in K: N_k > 1} \sum_{\substack{i, i' \in I_k \\ i < i'}} \sum_{j = j_k} RL_{kj}^{ii'} \times \left[[-C_{ijk} + C_{i'jk} - M * X_{i'ijk} - 2M + MY_{ijk} + MY_{i'jk}] - (-ST_{ijk} - PT_{ijk})\right] \tag{26}$$

$$\sum_{k \in K: N_k > 1} \sum_{\substack{i, i' \in I_k \\ i < i'}} \sum_{j = j_k} SL_{kj}^{ii'} \times \left[[-C_{i'jk} + C_{ijk} - M * X_{ii'jk} - 2M + MY_{ijk} + MY_{i'jk}] - (-ST_{i1jk} - PT_{i1jk})\right. \tag{27}$$

In equations (24)-(27) $UL_{kj}^{ij'}$, VL_{kj}^{i}, $RL_{kj}^{ii'}$, and $SL_{kj}^{ii'}$ are Lagrange multipliers for their corresponding constraints.

B. Updating Lagrange Multipliers

The quality of the solution obtained from the LR algorithm is sometimes unsatisfactory due to the type I zigzag behavior of the pure SO method. Alkaabneh et al. (2019) used the modified SO method to improve the Lagrange multipliers to prevent this behavior. In this study, Lagrange multipliers are updated as follows according to Alkaabneh et al. (2019).

To implement the proposed LR algorithm, the Lagrange multipliers are updated in each iteration based on Equations (28) - (44) where st^t represents the step size. Note that the values of the parameters gamma$1_{ij'kj}^{t}$, gamma2_{ikj}^{t} , gamma$3_{ii'kj}^{t}$, and gamma$4_{ii'kj}^{t}$ are calculated as the violation of the relaxed constraints by Equations (28) - (44).

$$\text{gamma}1_{ij'kj}^{t} = \left[\left[-C_{ijk} + \sum_{k' \in k_{ij'}} C_{ij'k'} - M * (1 - Y_{ijk})\right] - (-ST_{ijk} - PT_{ijk})\right] \tag{28}$$

$$\text{gamma}2_{ikj}^{t} = \left[[-C_{ijk}] - \left(-Y_{ijk} * (ST_{ijk} + PT_{ijk})\right)\right] \tag{29}$$

$$\text{gamma}3_{ii'kj}^{t} = \left[[-C_{ijk} + C_{i'jk} - M * X_{i'ijk} - 2M + MY_{ijk} + MY_{i'jk}] - (-ST_{ijk} - PT_{ijk})\right] \tag{30}$$

$$\text{gamma}4_{ii'kj}^{t} = \left[[-C_{i'jk} + C_{ijk} - M * X_{ii'jk} - 2M + MY_{ijk} + MY_{i'jk}] - (-ST_{i1jk} - PT_{i1jk})\right] \tag{31}$$

$$\varepsilon_1^t = \begin{cases} \tau.\frac{\text{gamma1}_{ij'kj}^t.d1_{ij'kj}^{t-1}}{\left\| d1_{ij'kj}^{t-1} \right\|^2} & if\ \text{gamma1}_{ij'kj}^t.d1_{ij'kj}^{t-1} < 0 \\ 0\ other\ wise \end{cases} \tag{32}$$

$$d1_{ij'kj}^t = \text{gamma1}_{ij'kj}^t + \varepsilon_1^t.d1_{ij'kj}^{t-1} \tag{33}$$

$$UL_{kj}^{ij',t} = UL_{kj}^{ij',t-1} + st^t.d1_{ij'kj}^t \tag{34}$$

$$\varepsilon_2^t = \begin{cases} \tau.\frac{\text{gamma2}_{ikj}^t.d2_{ikj}^{t-1}}{\left\| d2_{ikj}^{t-1} \right\|^2} & if\ \text{gamma2}_{ikj}^t.d2_{ikj}^{t-1} < 0 \\ 0\ other\ wise \end{cases} \tag{35}$$

$$d2_{ikj}^t = \text{gamma2}_{ikj}^t. + \varepsilon_2^t.d2_{ikj}^{t-1} \tag{36}$$

$$VL_{kj}^{i,t} = VL_{kj}^{i,t-1} + st^t.d2_{ikj}^t \tag{37}$$

$$\varepsilon_3^t = \begin{cases} \tau.\frac{\text{gamma3}_{ii'kj}^t.d3_{ii'kj}^{t-1}}{\left\| d3_{ii'kj}^{t-1} \right\|^2} & if\ \text{gamma3}_{ii'kj}^t.d3_{ii'kj}^{t-1} < 0 \\ 0\ other\ wise \end{cases} \tag{38}$$

$$d3_{ii'kj}^t = \text{gamma3}_{ii'kj}^t + \varepsilon_3^t.d3_{ii'kj}^{t-1} \tag{39}$$

$$RL_{kj}^{ii',t} = RL_{kj}^{ii',t-1} + st^t.d3_{ii'kj}^t \tag{40}$$

$$\varepsilon_4^t = \begin{cases} \tau.\frac{\text{gamma4}_{ii'kj}^t.d4_{ii'kj}^{t-1}}{\left\| d4_{ii'kj}^{t-1} \right\|^2} & if\ \text{gamma4}_{ii'kj}^t.d4_{ii'kj}^{t-1} < 0 \\ 0\ other\ wise \end{cases} \tag{41}$$

$$d4_{ii'kj}^t = \text{gamma4}_{ii'kj}^t + \varepsilon_4^t.d4_{ii'kj}^{t-1} \tag{42}$$

$$SL_{kj}^{ii',t} = SL_{kj}^{ii',t-1} + st^t.d4_{ii'kj}^t \tag{43}$$

$$st^t = \theta^t.\left[\frac{UB-LB^*}{\sum_{ii'j'kj}\text{gamma1}_{ij'kj}^t+\text{gamma2}_{ikj}^t+\text{gamma3}_{ii'kj}^t+\text{gamma4}_{ii'kj}^t}\right] \tag{44}$$

The parameters $d1_{ij'kj}^0$, $d2_{ikj}^0$, $d3_{ii'kj}^0$, and $d4_{ii'kj}^0$are initialized with 1. The value of the parameter θ, which is used to calculate st, is set to 0.1, and the value of τ is set to [0,2]. In this algorithm, θ is halved if no improvement is achieved after t consecutive iterations at the best value of the lower bound. The stopping criterion is the maximum number of iterations equal to 50. As mentioned, the LR algorithm is used to solve larger problems. In Table IV, sample problems are designed in which the data are set according to Table II.

Table IV. Test problems specifications

problem	*Set of all AVs*	*Set of all routes*	*Set of all stages*
1	8	4	2
2	9	5	2
3	9	6	2
4	9	6	3
5	9	7	3
6	10	7	3
7	10	7	4

8	15	8	4
9	15	9	4
10	15	10	5
11	15	10	5
12	20	11	5
13	25	11	6
14	30	12	6
15	30	12	6

Table V. Comparison Lagrangian and CPLEX

	GAMS		***Lower bound (LR)***	***Upper bound (UB)***		
	Objective function	***Runtime***	***Objective function***	***Objective function***	***Runtime***	***optimality gap***
1	751.351	1:24	724.324	918.919	0:36	22.3
2	1586.486	3:56	1518.919	1727.027	1:04	8.85
3	2047.568	7:18	1996.216	2334.054	1:22	13.99
4	2572.162	9:47	2492.973	2812.703	1:56	9.35
5	2769.459	12:35	2762.703	3155.946	3:58	13.95
6	3379.189	29:21:00	3365.135	3638.649	5:01	7.67
7	3739.730	48:10:00	3732.703	3815.405	6:12	2.02
8	3944.865	60:00:00	3844.865	4158.378	7:43	5.41
9	4063.243	60:00:00	3991.892	4368.649	10:05	7.51
10	5369.459	60:00:00	5329.189	5658.649	12:30	5.38
11	5823.514	60:00:00	5737.568	6005.946	12:30	3.13
12	6231.081	60:00:00	6140.811	6390.270	14:27	2.25
13	6597.027	60:00:00	6551.351	6931.081	13:40	5.06
14	6992.703	60:00:00	6926.757	7273.243	14:40	4.01
15	7342.432	60:00:00	7253.514	7727.027	16:01	5.23

The time of one hour (60 minutes) was considered as a time constraint to solve the problems according to Table IV which contains the data obtained by solving through CPLEX solver and LR algorithm. As can be seen in Table V, GAMS reached the optimal solution up to the seventh problem before the time constraint, but after it, the solution was obtained with a gap. However, the Lagrange method did not reach a time of 60 minutes in any of the problems and achieved a solution in a much shorter time than GAMS. The optimality gap is calculated using the following formula.

$$Optimality\ gap = \frac{fitness - LB}{fitness} \times 100 \tag{45}$$

The two methods are compared in terms of the value of the objective function according to Table V.

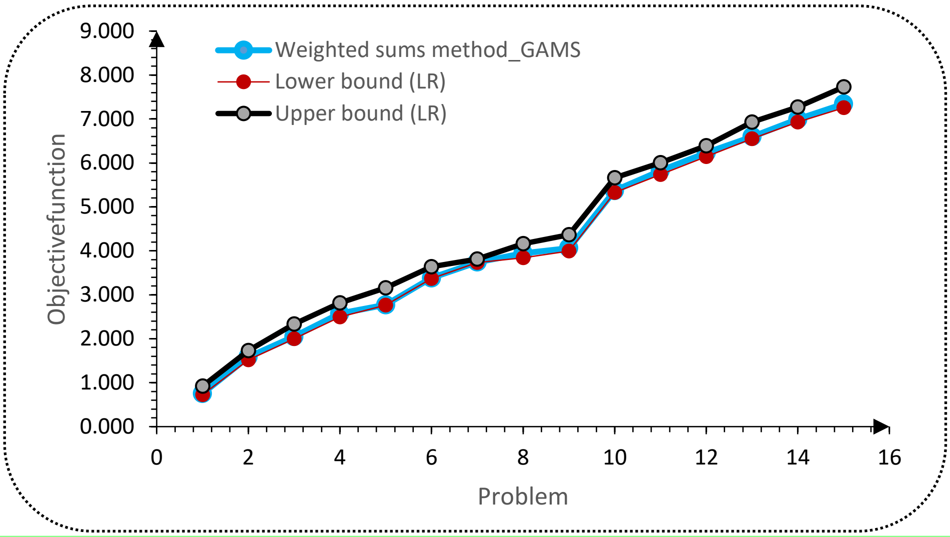

Fig 5. Comparing the solutions obtained from GAMS and the algorithm

The comparison of the objective function can be seen in Fig. (5). As can be seen, the solutions obtained from GAMS and the algorithm are close to each other, and the solutions are reliable. Besides, the Lagrange method performs better in terms of solution time based on the Fig. (6).

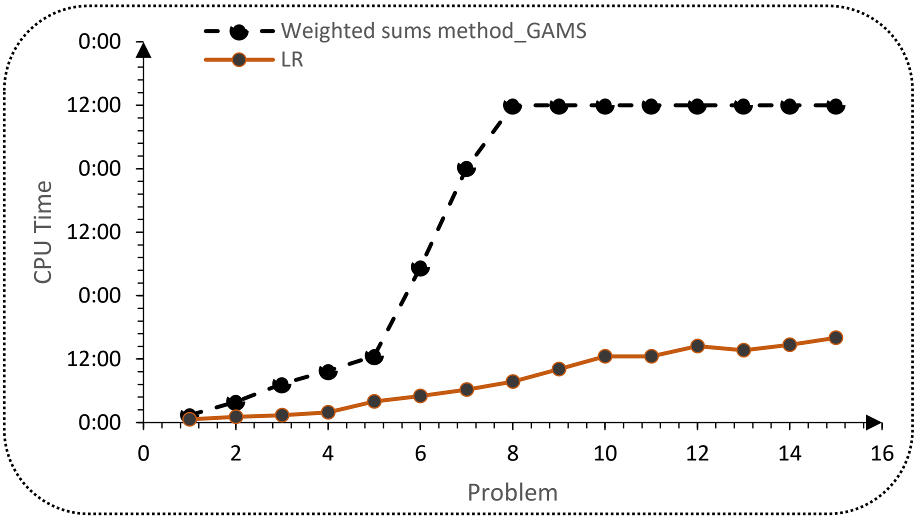

Fig 6. Comparing the run time

The comparison of the run time can be seen in Fig. (6). As can be seen, the Lagrange method performs better.

VIII. CONCLUSIONS AND FUTURE RESEARCH

In this research, a mathematical model was proposed for the autonomous vehicle routing problem. This research differs from previous studies in that it takes into account routes while considering traffic conditions for the routing problems of autonomous vehicles. The mathematical model's objectives are to minimize travel time and costs. The weighted sum method was used to solve the bi-objective mathematical model. The results show that including traffic in the model makes it more accurate, and the sensitivity analysis of the model shows that if decision-makers prioritize reducing travel time, the system will incur more costs; on the other hand, if they prioritize reducing cost, the travel time will be longer. The value of the objective function tends to increase when travel and traffic times increase. Since the mathematical model for this research is in the NP-hard category, the Lagrange algorithm was used to solve 15 designed cases. The results of solving the model in large dimensions show the efficiency of the Lagrange algorithm. This mathematical model can be useful for programming autonomous vehicles.

Future study could provide an uncertainty approach, a multi-objective mathematical model based on energy consumption and the number of stops along the route, or other approaches to solving the real issue, including metaheuristic algorithms.

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