

Analyzing a Hybrid Approach of PCA-DEA in Two Different Modes

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Abstract – The present work compared input- and output-based integrated principal component analysis-data envelopment analysis (PCA-DEA) approaches. The approach minimizing the number of decision-making units (DMUs) identified as efficient would be the superior one (as it facilitates DMU ranking). This approach would somewhat handle a major drawback of DEA– i.e., the emergence of an excessively high number of DMUs. The input and output-based approaches were independently implemented in MATLAB and were compared to identify the superior one. A number of numerical examples were carried out to demonstrate the performance of the superior approach. The results show that the second approach (the output-based approach) is superior to the first approach (the input-based approach). Therefore, it is better to divide the outputs by the inputs to create the PCA-DEA indices. In order to achieve better results in this way, this point (of dividing the outputs by the inputs) is not specific to this research alone and can be used in other research (in case study research).

Keywords– DEA, Efficient DMUs, Performance Evaluation, PCA.

I. INTRODUCTION

According to Neely (2005), performance measurement is a frequently discussed concept with inadequate definitions. Therefore, Neely described performance measurement as a quantification process (Neely, 2005). Performance evaluation is defined as the process of measuring the performance of a business thoroughly in terms of efficiency, effectiveness, empowerment, and accountability within the framework of scientific principles and management concepts for organizational tasks and purposes in executive plans (Timothy & Gerald, 1993). Performance evaluation dates back to a very long time ago. In fact, humans have somehow considered evaluation since they started living together and practicing the division of labor in the very early form. However, the official use of evaluation systems dates from the 19th century, in which very primitive tools were employed to evaluate the output quality levels of organizations. However, performance evaluation has now become much more evolved than ever before. In other words, it has been developed in line with the management concept transformation model, its nature, and its functions (Gibbons & Murphy, 1990). The performance evaluation of a company has always been considered a challenge in management. Efficiency measurement has drawn a great deal of attention due to its importance, especially in the past two decades. Since 1957 when Farrell proposed a method for efficiency measurement, comprehensive reviews have been conducted

on this subject. Moreover, both parametric and non-parametric approaches are used extensively in efficiency evaluation (Battess & Coelli, 1995).

Today, in light of technological advances and the contributions of product and service organizations to human life, different and newer decision-making units (DMUs) are emerging in urban and rural areas. For an organization with numerous DMUs, it is essential for senior managers to identify the DMUs of higher efficiency— i.e., evaluating each DMU relative to the others (Ghalayini et al. 1997). The continuous evaluation of DMUs motivates employees and encourages DMUs to create added value in products and services. This reflects the necessity of deploying mechanisms to address the shortcomings of DMUs of poor efficiency and avoid resource waste (Vittorio et al. 2008).

Performance management refers to the deployment of a system to utilize information on organizational performance measurement using performance evaluation results to define objectives, allocate resources, and improve managers' awareness in order to maintain or change the current policy and achieve the objectives (Li, 2001). Data envelopment analysis (DEA) is an efficient tool for measuring and evaluating productivity. It is used as a nonparametric method for calculating the efficiency of decision-making units (DMUs) (Soofizadeh & Fallahnejad, 2022). The use of DEA is now rapidly growing. In fact, this technique is employed to evaluate different organizations and industries such as banking, post services, hospitals, education centers, power plants, and refineries (Omran et al., 2022). Abbreviated to DEA, data envelopment analysis is considered a novel method of operations research and economics intended to measure or estimate the performance efficiency of production units. However, a production unit may refer to a factory and also a service-providing company (Rakhshan & Alirezaee, 2019). Hence, DEA can be adopted in any economic activities to determine the efficiency of relevant DMUs (Moazeni et al., 2022).

In typical DEA methods, index dependency is less discussed (Dong et al. 2015) and the researchers often do not consider the assumption of data independency (Xiao et al., 2022), which leads to alteration in the results of implementation of this method (Razavi Hajiagha et al., 2022). Therefore, weaknesses and strengths of DEA method should be considered. In other words, the way to obtain better results by eliminating weaknesses of the DEA method should be sought. This will be elaborated in the following.

II. PROBLEM STATEMENT

Efficiency represents the extent to which an organization (or an organizational DMU) exploits its resources to maximize production quality (Golany & Roll, 1989). The efficiency levels of DMUs provide a clear representation of their statuses. DMU efficiency is dependent on several parameters (or criteria). This confuses managers and decision-makers in the organization (Choudhuri, 2014). In other words, a given DMU may be satisfactory in one criterion and poor in another one. This challenge can be somewhat handled using the importance (weights) of the criteria and multi-criteria decision-making (MCDM) techniques. However, the importance (weights) of decision criteria may be biased for some DMUs. To cope with the challenge, non-parametric methodologies, e.g., principal component analysis (PCA), are employed (Charnes et al. 1978). However, PCA has drawbacks, e.g., the emergence of an excessively high number of efficient DMUs (Andersen, & Petersen, 1993). The integration of PCA and data envelopment analysis (DEA) is a popular technique to minimize efficient DMUs (Kardiyen & Örkücü, 2010).

The average of the absolute values of the correlation of indices (components) in the first approach, before and after implementation of PCA, was 0.59 and 0.11, respectively, and in the second approach, 0.56 and 0.11, before and after implementation of PCA, respectively. Therefore, it can be seen that in both studied approaches, the correlation of indices (components) were reduced, which confirms efficiency of PCA in reducing dependency of indices.

A hybrid PCA-DEA approach converts all the criteria into either inputs only or outputs only. An output-based approach converts all the criteria into profit indices, while an input-based one converts all the criteria into cost indices. It remains a challenge to realize whether the input-based or output-based approach is the superior one. To this end, both approaches are implemented to identify the outperforming one. The superior approach might not certainly be identified

through one or two numerical examples. Hence, different and new numerical examples will be simulated in one of the final cases to draw a general conclusion and identify the superior approach by comparing the results.

III. NECESSITY

There may be several approaches to solve a given problem, while they yield different results. Therefore, it is necessary to identify the approach that works best for the problem. The present study compared input- and output-based PCA-DEA approaches.

A. Input-based PCA-DEA

The inputs were divided by the outputs, obtaining a large number of new input indices ($Z_{m(r-1)+i,j} = \frac{x_{i,j}}{y_{r,j}}$; $i = 1, 2, \dots, m$ and $r = 1, 2, \dots, s$ and $j = 1, 2, \dots, n$). To exclude the insignificant inputs, PCA can be employed. The efficiency of DMUs can be calculated using DEA (During this research we call this approach the first approach).

B. Output-based PCA-DEA

The outputs were divided by the inputs, leading to numerous new output indices ($Z_{m(r-1)+i,j} = \frac{y_{r,j}}{x_{i,j}}$; $i = 1, 2, \dots, m$ and $r = 1, 2, \dots, s$ and $j = 1, 2, \dots, n$). Likewise, PCA could be utilized to exclude the insignificant outputs. DMU efficiency may be measured using DEA (During this research this approach is called the second approach).

IV. LITERATURE REVIEW

Wu (2009) employed an integrated model of a decision tree (DT), DEA, and an artificial neural network (ANN) to evaluate suppliers. They classified suppliers into efficient and inefficient and trained the DT and ANN using a dataset. The trained DT was applied to new suppliers.

Ahmadvand *et al.* (2011) evaluated the road safety level performance in different provinces of Iran. They employed the data envelopment analysis (DEA) to calculate the efficiency score of each province. They also adopted the principal component analysis (PCA) to improve the DEA distinguishability, create independent variables, and prevent the information overlap in decision-making units (DMUs). An innovation of their paper was to develop the PCA when some inputs or outputs were undesirable.

Hosseinzadeh Lotfi *et al.* (2012) evaluated the performance of banks. They aimed to propose a framework for evaluating the general performance of bank branches in terms of profitability effectiveness and efficiency by using the bi-level DEA model. In this model, all outputs of the first level are used as the inputs of the second level to help evaluate DMUs in the best way possible.

Mohaghar *et al.* (2013) studied the supplier selection problem. They employed DEA and the VIKOR method to calculate the efficiency of suppliers.

Sadraei-Javaheri and Ostadzad (2014) estimated the efficiency of Iranian fossil-fueled and renewable power plants using DEA. The DEA network was developed with several inputs (generation inputs) and an output (power generation). They used the fuel cost (zero for renewable power plants, labor cost, and operation costs as the inputs, whereas annual power generation was treated to be the output. The Iranian power plants were classified based on their efficiency, proposing solutions to improve power plant efficiency under different scenarios.

Shokrollahpour *et al.* (2016) investigated the relative efficiency calculation and pattern determination of Tejarat Bank branches in Iran through a hybrid DEA-ANN model. They sought to tackle some drawbacks of DEA. The hybrid

model was demonstrated to improve DEA.

Rostamy-Malkhalifeh *et al.* (2018) analyzed uncertainty in the calculation of efficiency weight at DMUs. They used fuzzy numbers for modeling to handle the uncertainty. They also employed rank functions to solve the mathematical models. In other words, a fuzzy problem can be converted into a certain (crisp) problem through the rank functions of fuzzy numbers. It is then solved through classical methods. In this paper, some numerical examples were presented to better perceive the proposed method.

Heidary *et al.* (2018) developed a hybrid model for efficiency evaluation. The model consisted of two stages: (1) evaluating DMUs using DEA and (2) identifying DMUs with an efficiency weight of 1 using an ANN. It was found that the hybrid model could introduce only one efficient DMU.

Jafari and Ehsanifar (2020) studied a widely used technique in multi-attribute decision-making (MADM) problems. They developed the VIKOR method under non-crisp (grey) conditions. Their proposed method can evaluate decision alternatives under crisp (interval) conditions. The potential application of the proposed method was illustrated by a numerical example.

Firoozishahmirzadi (2020) introduced a new DEA approach that could rank efficient DMUs. It was found to outperform some earlier methodologies.

Tsolas *et al.* (2020) proposed an integrated artificial intelligence (AI)-DEA approach to calculate the efficiency of bank branches. DEA was used for preprocessing. Then, the branches were classified into efficient and inefficient groups using an ANN. The integrated approach was found to be significantly helpful in future decision-making.

Rahimpour *et al.* (2020) evaluated the efficiency of organizational DMUs. They aimed to develop an evaluation model for organizational DMUs based on intellectual capital (IC) and employee loyalty using PCA and DEA. The operation, design and manufacturing, production planning, internal, quality control, and security units were identified to be efficient. The operation, internal, and quality control units had the first, second, and third ranks, respectively, while the human resource unit had the last rank.

Jafarigorzin and Asadi Talooki (2021) analyzed the ranking problem of efficient units through the DEA. Their proposed approach was to use the Malmquist index (MI) based on common weights in addition to calculating the final efficiency of DMUs. According to numerical examples, their proposed approach performed a complete ranking process on DMUs.

Salehi *et al.* (2022) assessed DEA in specific conditions. By specific conditions, they refer the conditions that assessment of decision-making units is done without any input index so that all the indices are of output type. Through proving a mathematical theorem, the authors showed that a simple mathematical model can solve the problem.

Moazeni *et al.* (2022) used the DEA technique to evaluate industrial units. In fact, they aimed to propose a network mode for calculating the efficiency of partial and total DMUs. They also employed the principal component analysis (PCA) technique to improve the distinguish ability of network DEA results. The proposed model was executed in 26 stone factories. The results indicated acceptable differences between industrial units.

A. Research Innovation

According to the literature review, each of the previous studies has somehow tried to fully rank DMUs (*i.e.*, to reduce the number of efficient units) (Amirteimoori *et al.*, (2014), Soltani *et al.*, (2022) and Zanboori *et al.*, (2014)). In some studies, the PCA–DEA method was employed to reduce the quantity of efficient units (Mohammadnazari *et al.*, 2022). However, none of those studies compared different execution modes of the PCA–DEA method, a problem which is partially addressed in this study. In fact, this study aims to compare two different approaches to the execution of PCA–DEA.

V. METHODOLOGY

This paper adopted a descriptive-analytical methodology. In fact, mathematical-statistical analyses and descriptions were used to compare the input- and output-based approaches. In simpler terms, two numerical examples are first presented to compare the two approaches. Further numerical examples will then be presented to improve the reliability of comparison results (through a simple numerical simulation). These numerical examples will be created randomly. Finally, their results will be discussed. For this purpose, the two approaches are first introduced and then compared.

A. Data Envelopment Analysis

DEA is employed to measure the relative technical efficiency of organizational DMUs. It was developed in 1976 and introduced by Charnes et al. (1978) as the CCR model in the paper “Measuring the Efficiency of DMUs.” The DEA model allocates importance weights to the criteria to maximize the performance of DMUs (Xiuli et al., 2010). Several mathematical models can be used for efficiency calculation, including the DEA-CCR method. It is a strict method in the calculation of efficiency. Therefore, the present work adopted DEA-CCR.

B. DEA-CCR Method

CCR seeks to allocate optimal weights to the inputs and outputs to maximize the efficiency fraction of a DMU such that the efficiency of the other DMUs would not exceed 1. Constant returns to scale mean a portion of inputs producing the same portion of the output. The CCR model assumes constant returns to scale. Hence, large and small DMUs are compared. For a total of n DMUs with m inputs and s outputs, the CCR model is written as (Abbasi, 2022):

$$Max E_o = \sum_{r=1}^s u_r y_{ro} \tag{1}$$

Subject to:

$$\sum_{i=1}^m v_i x_{io} = 1 \tag{1-2}$$

$$\sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0 ; j = 1, \dots, n \tag{1-3}$$

$$v_i, u_r \geq \varepsilon ; i = 1, 2, \dots, m \text{ and } r = 1, 2, \dots, s \tag{1-4}$$

As can be seen, it is an input-based model that keeps constant inputs and seeks to maximize the output (Premachandra, 2001).

C. Principal Component Analysis

PCA is a transformation in the vector space. It is often employed to downsize datasets. PCA was introduced by Pearson (1901) and analyzes the eigenvalues of the covariance matrix (Sarkar, 2015). It is an orthogonal linear transformation that maps data into a new coordinate system in which, for example, the largest variance lies on the first coordinate axis, and the second-largest covariance rests on the second coordinate axis. PCA can be employed to downsize a dataset. Thus, the dataset components with the highest contribution to the variance are maintained (Han et al., 2020).

D. Principal Component Analysis algorithm

The implementation of PCA on a data matrix $Z = [Z_1, Z_2, \dots, Z_n]_{m \times n}$ includes a number of steps (Omrani et al., (2015), MendonçaPeixoto et al., (2020) and Saqafi et al., (2018)):

1) Calculate the normalized data matrix $D = [D_1, D_2, \dots, D_n]_{m \times n}$;

Matrix Z is normalized using $D_j = \frac{z_j - \bar{z}_j}{\sqrt{\text{var}(Z_j)}}$; $j = 1, 2, \dots, n$ as:

$$\begin{cases} \bar{z}_j = \frac{1}{m} \sum_{i=1}^m z_{i,j} \\ \text{Var}(Z_j) = \frac{1}{m-1} \sum_{i=1}^m (z_{i,j} - \bar{z}_j)^2 \end{cases} \quad (2)$$

2) Calculate the variance-covariance matrix as:

$$\begin{cases} c_{i,j} = \text{Cov}(D_i, D_j) \\ \text{Cov}(D_i, D_j) = \frac{1}{m-1} \sum_{k=1}^m (d_{k,i} - \bar{D}_i)(d_{k,j} - \bar{D}_j); i = 1, 2, \dots, n \text{ and } j = 1, 2, \dots, n \end{cases} \quad (3)$$

3) Calculation of the matrix of correlation coefficients ($R = (r_{i,j})_{n \times n}$) as follows:

$$r_{i,j} = \frac{c_{i,j}}{\sqrt{c_{i,i} \times c_{j,j}}}; i = 1, 2, \dots, n \text{ and } j = 1, 2, \dots, n \quad (4)$$

4) Calculate the eigenvalues of matrix R ;

To calculate the eigenvalues of C , it is required to find the roots of the next determinants:

$$\det(R - \lambda \times I_{n \times n}) = 0 \quad (5)$$

Let $\lambda_1, \lambda_2, \dots, \lambda_n$ be the roots of Eq. (4).

5) Calculate the eigenvector matrix V ;

To calculate the eigenvectors, a system of linear equations is solved as:

$$R \times V_j = \lambda_j \times V_j; j = 1, 2, \dots, n \quad (6)$$

6) Exclude the insignificant eigenvectors;

The eigenvectors whose eigenvalues are smaller than 1 are excluded from matrix V (a total of p eigenvectors remain in matrix V , $p \leq n$).

7) Calculate the principal components $PC = [PC_1, PC_2, \dots, PC_p]_{m \times p}$;

The principal components are calculated as:

$$PC = D \times V \tag{7}$$

8) End.

DEA assumes all the data to be higher than zero. Therefore, when matrix PC contains a non-positive element, $-\min\{pc_{i,j}\} + 1$ is added to matrix PC so that the negative and zero elements could become positive (Rahimpour et al., 2020).

VI. MATHEMATICAL MODEL FOR EFFICIENCY CALCULATION IN THE FIRST APPROACH (PCA-DEA(1))

It is recommended that the following mathematical model with a virtual output of 1 be employed in the first approach (Because there aren't any indicators of the output type in the first approach.):

$$Max E_o = u_1 \tag{8-1}$$

Subject to:

$$\sum_{i=1}^m v_i pc_{i,o} = 1 \tag{8-2}$$

$$u_1 - \sum_{i=1}^m v_i pc_{i,j} \leq 0 \ ; \ j = 1, \dots, n \tag{8-3}$$

$$u_1, v_i \geq \varepsilon \ ; \ i = 1, 2, \dots, m \tag{8-4}$$

VII. MATHEMATICAL MODEL FOR EFFICIENCY CALCULATION IN THE SECOND APPROACH (PCA-DEA(2))

It is recommended that the following mathematical model with a virtual input of 1 be employed in the second approach (Because there are not any input type index in the second approach):

$$Max E_o = \sum_{r=1}^s u_r pc_{r,o} \tag{9-1}$$

Subject to:

$$v_1 = 1 \tag{9-2}$$

$$\sum_{r=1}^s u_r pc_{r,j} - v_1 \leq 0 \ ; \ j = 1, \dots, n \tag{9-3}$$

$$u_r, v_1 \geq \varepsilon \ ; \ r = 1, 2, \dots, s \tag{9-4}$$

A. Numerical Example (1)

Table 1 provides the data of twenty DMUs.

Table 1. Data and outputs of twenty DMUs

	Inputs			Outputs		
	x_1	x_2	x_3	y_1	y_2	y_3
DMU ₁	43	36	27	141	133	122
DMU ₂	47	11	25	146	101	119
DMU ₃	15	44	41	106	143	139
DMU ₄	47	48	42	146	147	140
DMU ₅	35	37	17	132	134	109
DMU ₆	13	41	30	104	138	124
DMU ₇	21	40	28	114	137	122
DMU ₈	32	26	36	127	120	132
DMU ₉	49	36	39	148	133	136
DMU ₁₀	49	17	40	149	108	138
DMU ₁₁	16	38	21	108	136	114
DMU ₁₂	49	11	37	149	101	134
DMU ₁₃	49	21	36	148	114	133
DMU ₁₄	29	11	16	124	102	108
DMU ₁₅	42	13	14	140	104	106
DMU ₁₆	15	43	30	107	141	125
DMU ₁₇	27	38	49	121	135	148
DMU ₁₈	47	23	23	146	116	117
DMU ₁₉	42	48	33	140	148	129
DMU ₂₀	49	11	19	148	101	111

The PCA algorithm was implemented, calculating the principal components through the two approaches, as shown in Table 2.

Table 2. Principal components in the input- and output-based approaches

	Input-based		Output-based		
	PC_1	PC_2	PC_1	PC_2	PC_3
DMU ₁	3.99	4.35	6.07	4.48	4.56
DMU ₂	4.70	1.53	3.33	3.84	7.50
DMU ₃	5.44	8.31	3.21	4.73	1.00
DMU ₄	5.90	5.62	5.98	3.10	3.72
DMU ₅	2.11	4.64	6.58	7.11	5.21
DMU ₆	3.75	7.83	3.05	5.93	1.01
DMU ₇	3.61	6.78	4.79	5.13	2.83
DMU ₈	5.62	4.87	4.76	3.39	4.03
DMU ₉	5.94	4.40	5.77	3.08	4.20
DMU ₁₀	6.94	2.75	4.20	2.53	5.57
DMU ₁₁	2.30	6.79	4.35	6.92	2.61
DMU ₁₂	6.77	2.00	2.75	2.45	7.07
DMU ₁₃	6.10	2.95	4.88	2.97	5.20
DMU ₁₄	2.53	2.61	3.54	6.44	7.46
DMU ₁₅	2.44	1.53	4.91	7.35	8.12
DMU ₁₆	3.70	7.75	3.65	5.65	1.52
DMU ₁₇	7.03	6.92	4.61	3.06	2.75
DMU ₁₈	3.86	2.64	5.65	4.78	5.68
DMU ₁₉	4.52	5.66	6.12	3.96	3.88
DMU ₂₀	3.68	1.00	3.84	5.15	8.11

Table 3 shows the efficiency weights under three scenarios: (1) DEA approach, (2) input-based PCA-DEA approach, and (3) output-based PCA-DEA approach.

Table 3. Efficiency weights under the three scenarios

	Efficiency		
	DEA	PCA-DEA(1)	PCA-DEA(2)
DMU ₁	0.737	0.585	0.922
DMU ₂	1.000	0.727	0.924
DMU ₃	1.000	0.411	0.645
DMU ₄	0.608	0.400	0.909
DMU ₅	1.000	1.000	1.000
DMU ₆	1.000	0.567	0.807
DMU ₇	0.885	0.601	0.727
DMU ₈	0.884	0.424	0.739
DMU ₉	0.635	0.406	0.876
DMU ₁₀	0.801	0.449	0.773
DMU ₁₁	1.000	0.916	0.941
DMU ₁₂	1.000	0.525	0.871
DMU ₁₃	0.708	0.462	0.822
DMU ₁₄	1.000	0.926	0.919
DMU ₁₅	1.000	1.000	1.000
DMU ₁₆	0.954	0.574	0.769
DMU ₁₇	0.920	0.335	0.700
DMU ₁₈	0.775	0.628	0.931
DMU ₁₉	0.680	0.508	0.929
DMU ₂₀	1.000	1.000	0.999

According to Table 3, a total of nine efficient DMUs were identified using DEA. The integrated input-based PCA-DEA approach identified three efficient DMUs. The output-based PCA-DEA approach identified two efficient DMUs. Although both integrated PCA-DEA approaches reduced the number of efficient DMUs, the output-based approach introduced fewer efficient DMUs and, therefore, outperformed the input-based one.

The average of the absolute values of the correlation of indices(components) in the first approach, before and after implementation of PCA, was 0.59 and 0.11, respectively, and in the second approach, 0.56 and 0.11, before and after implementation of PCA, respectively. Therefore, it can be seen that in both studied approaches, the correlation of indices (components) were reduced, which confirms efficiency of PCA in reducing the dependency of indices. More details are included in Table 10 to Table 13.

B. Numerical Example (2)

Table 4 reports the data of thirty DMUs.

Table 4. Inputs and outputs of thirty DMUs

	Inputs			Outputs			
	x_1	x_2	x_3	y_1	y_2	y_3	y_4
DMU ₁	17	27	11	259	210	264	131
DMU ₂	39	53	59	146	275	138	165
DMU ₃	13	22	59	220	108	124	163
DMU ₄	51	39	16	122	281	265	279
DMU ₅	46	57	33	203	126	228	149
DMU ₆	57	52	43	268	267	203	162
DMU ₇	35	12	24	285	260	280	182

Continue Table 4. Inputs and outputs of thirty DMUs

	Inputs			Outputs			
	x_1	x_2	x_3	y_1	y_2	y_3	y_4
DMU ₈	43	11	48	200	284	203	242
DMU ₉	55	44	38	155	127	209	128
DMU ₁₀	37	40	31	231	201	221	275
DMU ₁₁	24	15	23	284	181	252	116
DMU ₁₂	59	50	58	102	134	171	192
DMU ₁₃	11	41	55	295	215	176	106
DMU ₁₄	26	13	47	139	221	117	251
DMU ₁₅	59	13	30	122	143	247	240
DMU ₁₆	28	16	57	159	204	166	143
DMU ₁₇	25	50	23	179	298	268	236
DMU ₁₈	16	14	37	184	198	174	212
DMU ₁₉	56	22	58	162	239	266	270
DMU ₂₀	16	22	23	239	182	135	212
DMU ₂₁	26	15	22	118	106	126	281
DMU ₂₂	55	53	57	180	158	276	184
DMU ₂₃	35	45	13	159	261	108	171
DMU ₂₄	41	47	25	161	169	238	198
DMU ₂₅	39	43	40	121	116	247	151
DMU ₂₆	45	36	20	219	202	187	286
DMU ₂₇	11	26	42	156	173	176	193
DMU ₂₈	36	43	50	131	248	296	151
DMU ₂₉	11	15	35	100	205	180	186
DMU ₃₀	52	17	43	157	261	188	241

The principal components were found through the input- and output-based approaches, as shown in Table 5.

Table 5. Principal components under the two approaches

	Input-based			Output-based		
	PC_1	PC_2	PC_3	PC_1	PC_2	PC_3
DMU ₁	5.54	2.74	1.20	5.84	10.43	9.83
DMU ₂	5.19	5.68	7.19	4.82	5.36	3.13
DMU ₃	4.56	8.37	4.07	3.23	4.39	5.82
DMU ₄	3.63	1.61	3.76	7.52	9.03	5.83
DMU ₅	5.80	2.86	7.12	5.40	6.30	3.32
DMU ₆	4.98	3.31	5.75	5.56	5.99	3.60
DMU ₇	3.64	3.18	1.21	8.71	4.47	9.30
DMU ₈	2.48	4.57	2.49	8.40	2.29	7.49
DMU ₉	4.11	3.12	8.14	5.60	5.76	2.87
DMU ₁₀	4.60	3.24	3.18	5.58	6.42	4.94
DMU ₁₁	4.39	3.63	1.64	6.56	5.47	7.96
DMU ₁₂	2.51	4.79	10.40	5.36	5.29	2.33
DMU ₁₃	7.15	6.96	3.98	1.00	5.32	7.31
DMU ₁₄	2.65	5.92	2.94	6.33	3.30	6.11
DMU ₁₅	1.00	2.69	4.40	8.18	4.22	5.62
DMU ₁₆	3.21	6.69	3.83	5.77	3.86	5.08
DMU ₁₇	5.72	2.94	2.63	4.79	7.74	6.41
DMU ₁₈	3.97	5.07	1.40	4.91	3.87	7.86
DMU ₁₉	2.05	4.60	4.16	6.63	4.20	4.72
DMU ₂₀	4.87	3.89	1.51	4.48	6.03	7.36
DMU ₂₁	3.01	3.58	2.91	6.63	5.21	5.66

Continue Table 5. Principal components under the two approaches

	Input-based			Output-based		
	PC ₁	PC ₂	PC ₃	PC ₁	PC ₂	PC ₃
DMU ₂₂	4.24	4.42	7.20	5.30	5.49	3.04
DMU ₂₃	5.78	1.62	4.83	6.58	8.96	5.29
DMU ₂₄	4.93	2.51	4.79	5.77	6.90	4.18
DMU ₂₅	4.44	4.10	7.02	5.23	5.68	3.24
DMU ₂₆	4.11	2.23	3.29	6.74	7.66	5.33
DMU ₂₇	4.94	5.72	2.44	2.11	4.92	7.28
DMU ₂₈	4.64	4.78	5.26	4.92	5.51	4.02
DMU ₂₉	3.96	5.44	1.76	3.28	4.20	8.22
DMU ₃₀	2.05	3.92	3.71	7.15	4.12	5.22

Table 6 represents the efficiency weights under the three scenarios.

Table 6. Efficiency weights under three scenarios

	Efficiency		
	DEA	PCA-DEA(1)	PCA-DEA(2)
DMU ₁	1.000	1.000	1.000
DMU ₂	0.508	0.458	0.630
DMU ₃	1.000	0.575	0.592
DMU ₄	1.000	1.000	1.000
DMU ₅	0.397	0.591	0.713
DMU ₆	0.472	0.618	0.721
DMU ₇	1.000	1.000	1.000
DMU ₈	1.000	1.000	0.963
DMU ₉	0.361	0.645	0.719
DMU ₁₀	0.739	0.767	0.745
DMU ₁₁	1.000	0.843	0.845
DMU ₁₂	0.331	0.532	0.682
DMU ₁₃	1.000	0.445	0.744
DMU ₁₄	1.000	0.896	0.727
DMU ₁₅	1.000	1.000	0.939
DMU ₁₆	0.731	0.714	0.686
DMU ₁₇	0.987	0.809	0.767
DMU ₁₈	1.000	0.905	0.804
DMU ₁₉	0.705	0.836	0.782
DMU ₂₀	1.000	0.801	0.750
DMU ₂₁	1.000	0.857	0.809
DMU ₂₂	0.420	0.530	0.682
DMU ₂₃	1.000	0.991	0.945
DMU ₂₄	0.607	0.725	0.766
DMU ₂₅	0.489	0.546	0.680
DMU ₂₆	1.000	0.915	0.885
DMU ₂₇	1.000	0.662	0.741
DMU ₂₈	0.588	0.561	0.645
DMU ₂₉	1.000	0.853	0.836
DMU ₃₀	0.792	0.900	0.832

According to Table 4, DEA identified sixteen efficient DMUs. The input- and output-based integrated approaches yielded five and three efficient DMUs. Hence, the output-based PCA-DEA approach showed higher performance.

The average of the absolute values of the correlation of indices(components) in the first approach, before and after implementation of PCA, was 0.43 and 0.08, respectively, and in the second approach, 0.35 and 0.08, before and after implementation of PCA, respectively. Therefore, it can be seen that in both studied approaches, the correlation of indices(components) was reduced, which confirms the efficiency of PCA in reducing dependency of indices.

D. Numerical Example(3)

Table 7 provides twenty random examples solved by the two integrated approaches. As can be seen, no approach outperformed the other one in all the cases; however, the output-based PCA-DEA approach showed higher performance in most cases.

Table 7. Twenty random examples

Example	Number of Inputs	Number of Outputs	Number of DMUs	Number of Efficient DMUs		
				DEA	PCA-DEA(1)	PCA-DEA(2)
1	3	3	21	9	8	6
2	2	4	19	4	4	2
3	2	5	30	9	9	3
4	2	5	27	7	6	1
5	2	4	19	9	5	3
6	3	4	26	11	6	5
7	2	4	19	5	3	2
8	5	2	26	8	3	3
9	2	3	16	6	6	2
10	3	4	29	7	7	4
11	6	3	35	25	10	9
12	4	4	29	18	10	5
13	3	3	25	8	8	5
14	6	5	35	22	21	5
15	3	2	24	6	2	2
16	6	3	30	20	18	17
17	5	5	39	25	13	12
18	5	5	40	30	13	10
19	6	5	35	22	21	5
20	2	3	22	6	4	2
Sum	72	76	546	257	177	103

According to Table 7, it can be said that the output-based PCA-DEA approach identified a fewer total efficient DMUs than the input-based one ($103 < 177$). As a result, the output-based PCA-DEA approach outperformed the input-based PCA-DEA approach in general. For further clarity, Figure (1) indicates the number of efficient units. Evidently, the second approach is superior to the first approach. In fact, it has fewer efficient units than the first approach and the classical DEA.

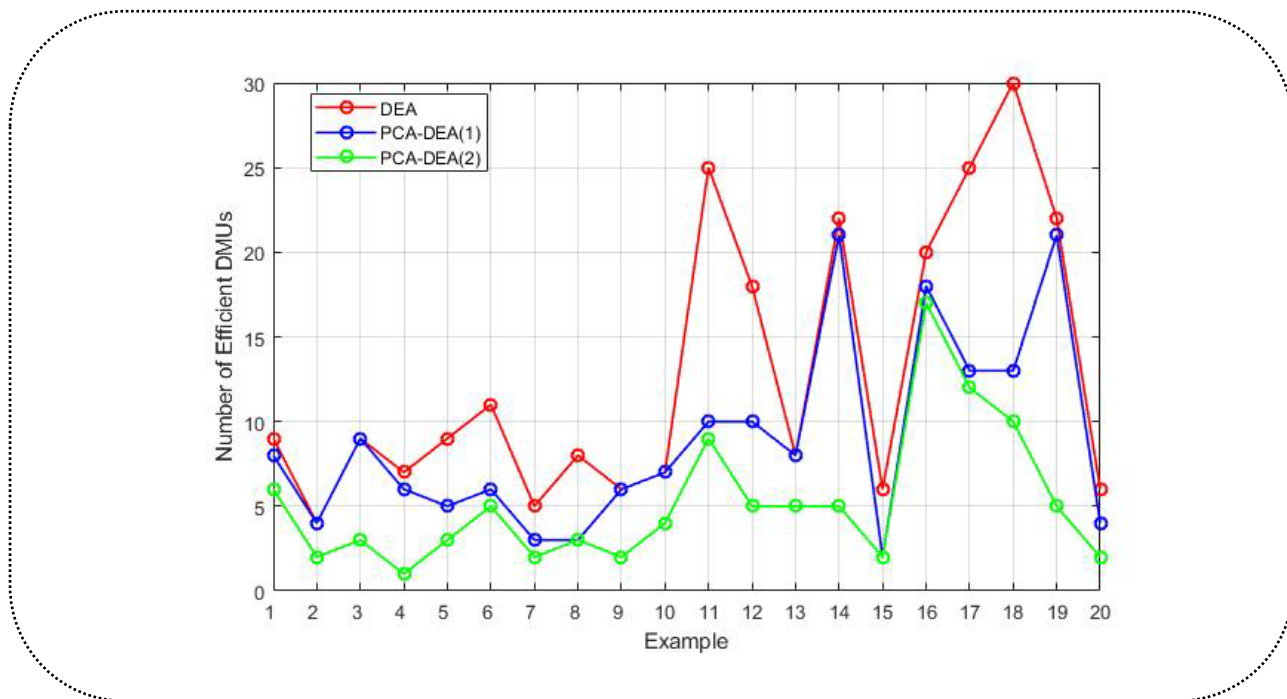


Figure 1. The number of efficient units in the simulated examples

Table 8. The mean absolute values of correlations between indices(components)

Example	PCA-DEA(1)		PCA-DEA(2)	
	Before	After	Before	After
1	0.42	0.11	0.35	0.11
2	0.37	0.13	0.49	0.13
3	0.35	0.10	0.46	0.10
4	0.32	0.10	0.50	0.10
5	0.42	0.13	0.39	0.13
6	0.28	0.08	0.32	0.08
7	0.27	0.13	0.42	0.13
8	0.44	0.10	0.40	0.10
9	0.35	0.17	0.45	0.17
10	0.28	0.08	0.31	0.08
11	0.30	0.06	0.27	0.06
12	0.27	0.06	0.33	0.06
13	0.38	0.11	0.34	0.11
14	0.21	0.03	0.22	0.03
15	0.53	0.17	0.38	0.17
16	0.31	0.06	0.22	0.06
17	0.25	0.04	0.23	0.04
18	0.24	0.04	0.27	0.04
19	0.21	0.03	0.22	0.03
20	0.44	0.17	0.45	0.17

In plain language, the performance distinguish ability of DMUs is higher in the second approach than the other two approaches. As a result, the second approach is better than the first. It is better to divide the outputs by the inputs to create the PCA indices.

In previous studies, an important goal of integrating PCA with DEA was to decrease the number of efficient units. According to Figure (1), the number of efficient units in the second approach (*i.e.*, the greed diagram) is smaller than or equal to the number of efficient units in the first approach (*i.e.*, the blue approach) in all simulation examples. In other words, the second approach (*i.e.*, PCA-DEA (2)) could further reduce the number of efficient DMUs. For instance, the first approach (*i.e.*, PCA-DEA (1)) had six efficient DMUs in this simulation example; however, the second approach (*i.e.*, PCA-DEA (2)) had two. Other examples are interpreted similarly.

Moreover, Table (8) presents the mean absolute values of correlations(MAC) between indices (components) before and after the execution of PCA.

For further clarity, Figure (2) demonstrates these data.

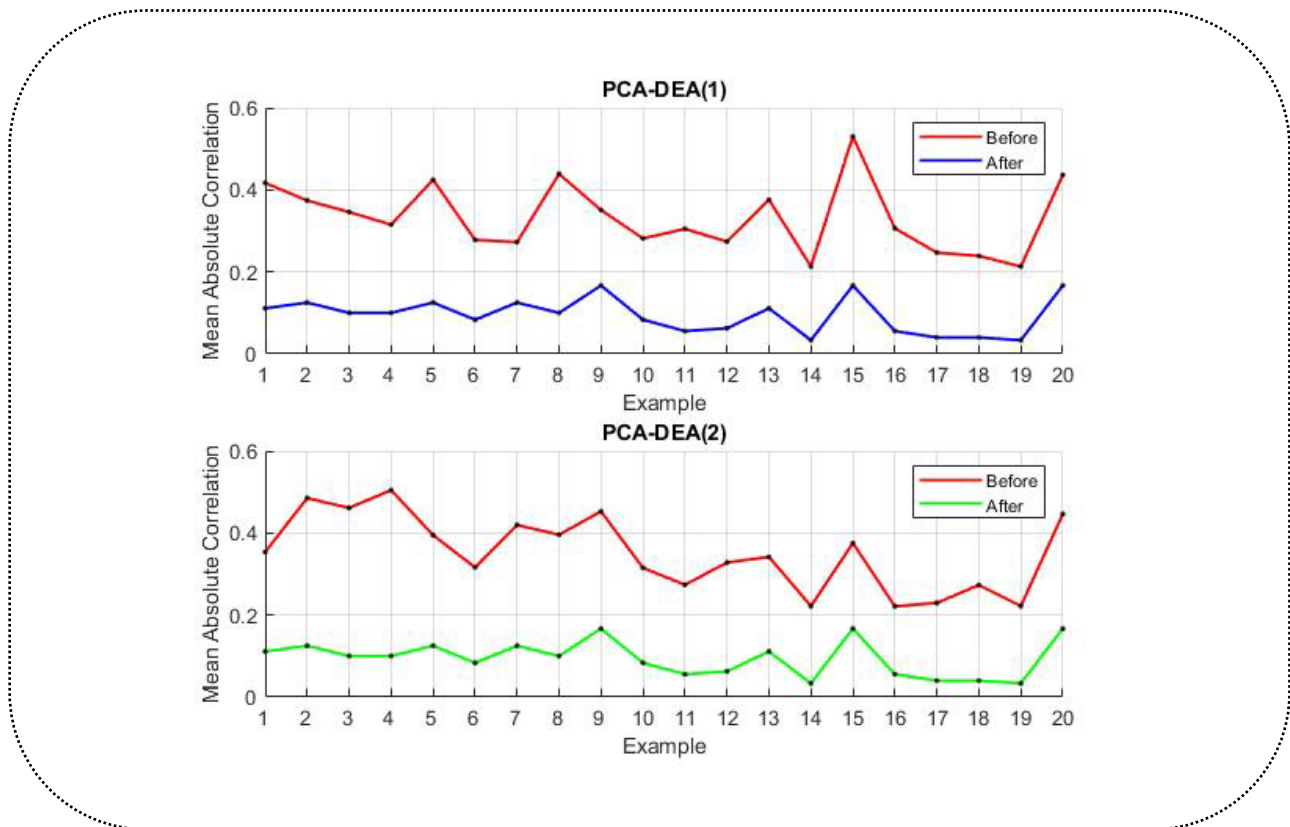


Figure 2. The mean absolute values of correlations between indices(components)

According to Figure (2), both approaches analyzed in this study reduced the correlations between criteria (components). Therefore, PCA is confirmed to be efficient in decreasing the correlations between indices (components).

Table (9) presents the mean squared errors (MSEs) of each of the twenty random examples($MSE = \frac{1}{n} \sum_{j=1}^n (Efficiency_j^{DEA} - Efficiency_j^{PCA-DEA})^2$).

Table 9. The MSEs of each of the twenty random examples

Example	MSE	
	PCA-DEA(1)	PCA-DEA(2)
1	0.16	0.20
2	0.25	0.01
3	0.26	0.16
4	0.19	0.10
5	0.07	0.04
6	0.18	0.14
7	0.15	0.18
8	0.09	0.04
9	0.08	0.04
10	0.13	0.06
11	0.02	0.02
12	0.03	0.05
13	0.26	0.20
14	0.04	0.03
15	0.14	0.13
16	0.04	0.05
17	0.04	0.04
18	0.03	0.04
19	0.04	0.03
20	0.27	0.06
Sum	2.48	1.64

According to Table (9), the second approach had smaller MSEs than the first approach in most of the random examples. In the cases where the second approach had greater MSEs, the two approaches were slightly different in terms of MSE. In total, the first and second approaches had 2.48 and 1.64 MSEs, respectively, in the entire simulation. Therefore, this comparison indicates that the efficiency scores of the second approach were closer to the DEA efficiency scores. It is now possible to conclude that the second approach outperformed the first approach by further separating DMUs and yielding higher accuracy in efficiency calculation.

VIII. CONCLUSION

DEA is a mathematical programming-based technique to evaluate the efficiency of a congruent set of DMUs. A DMU receives inputs and produces a set of outputs. However, problems mostly involve an excessively high number of DMUs and cannot be solved using basic methods to identify the efficient DMUs. In such cases, PCA can be exploited to substantially handle the challenge. This paper implemented input- and output-based integrated PCA-DEA approaches. The approaches were compared in the number of efficient DMUs to identify the superior approach through a number of examples. It was found that both integrated approaches improved DEA; however, the output-based PCA-DEA approach had higher performance. Therefore, it is suggested that future works focus on output-based approaches. Researchers are recommended to analyze the problem addressed by this study in the conditions with data uncertainty (e.g., Fuzzy data or Grey data). And also the investigation of this challenge for DEA-BCC models.

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Appendix

Table 10. Correlation value of indicators before implementing the PCA method in the first approach

	Z ₁	Z ₂	Z ₃	Z ₄	Z ₅	Z ₆	Z ₇	Z ₈	Z ₉
Z ₁	1	0.94	0.96	-0.73	-0.51	-0.55	-0.39	0.24	0.00
Z ₂	0.94	1	0.95	-0.89	-0.75	-0.77	-0.46	0.24	-0.11
Z ₃	0.96	0.95	1	-0.79	-0.61	-0.60	-0.59	0.02	-0.24
Z ₄	-0.73	-0.89	-0.79	1	0.96	0.95	0.58	-0.08	0.34
Z ₅	-0.51	-0.75	-0.61	0.96	1	0.98	0.55	-0.02	0.41
Z ₆	-0.55	-0.77	-0.60	0.95	0.98	1	0.39	-0.21	0.23
Z ₇	-0.39	-0.46	-0.59	0.58	0.55	0.39	1	0.73	0.91
Z ₈	0.24	0.24	0.02	-0.08	-0.02	-0.21	0.73	1	0.90
Z ₉	0.00	-0.11	-0.24	0.34	0.41	0.23	0.91	0.90	1

Table 11. Correlation value of the components after the implementation of the PCA method in the first approach

	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈	C ₉
C ₁	1	1.18E-12	5.78E-13	2.03E-13	-6.2E-15	3.69E-14	3.32E-14	3.85E-15	-5.1E-15
C ₂	1.18E-12	1	1.75E-13	-4.2E-13	1.81E-14	-5.9E-14	-5.1E-15	-1.4E-14	9.26E-15
C ₃	5.78E-13	1.75E-13	1	3.45E-14	1.08E-14	-1.1E-15	4.09E-15	1.93E-15	-2.4E-15
C ₄	2.03E-13	-4.2E-13	3.45E-14	1	1.5E-14	2.64E-14	1.28E-15	-1.2E-15	-8.6E-16
C ₅	-6.2E-15	1.81E-14	1.08E-14	1.5E-14	1	-5.5E-15	1.93E-16	-7E-16	2.23E-15
C ₆	3.69E-14	-5.9E-14	-1.1E-15	2.64E-14	-5.5E-15	1	-1.1E-16	9.48E-16	-8E-16
C ₇	3.32E-14	-5.1E-15	4.09E-15	1.28E-15	1.93E-16	-1.1E-16	1	-5.3E-17	-1.7E-16
C ₈	3.85E-15	-1.4E-14	1.93E-15	-1.2E-15	-7E-16	9.48E-16	-5.3E-17	1	-1.8E-16
C ₉	-5.1E-15	9.26E-15	-2.4E-15	-8.6E-16	2.23E-15	-8E-16	-1.7E-16	-1.8E-16	1

Table 12. Correlation value of indicators before implementing the PCA method in the second approach

	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7	Z_8	Z_9
Z_1	1	-0.53	-0.34	0.99	-0.45	0.13	0.98	-0.48	-0.08
Z_2	-0.53	1	0.54	-0.62	0.99	0.03	-0.57	0.99	0.40
Z_3	-0.34	0.54	1	-0.41	0.57	0.81	-0.47	0.43	0.96
Z_4	0.99	-0.62	-0.41	1	-0.55	0.09	0.99	-0.57	-0.16
Z_5	-0.45	0.99	0.57	-0.55	1	0.10	-0.50	0.98	0.46
Z_6	0.13	0.03	0.81	0.09	0.10	1	-0.02	-0.07	0.92
Z_7	0.98	-0.57	-0.47	0.99	-0.50	-0.02	1	-0.50	-0.23
Z_8	-0.48	0.99	0.43	-0.57	0.98	-0.07	-0.50	1	0.29
Z_9	-0.08	0.40	0.96	-0.16	0.46	0.92	-0.23	0.29	1

Table 13. Correlation value of the components after the implementation of the PCA method in the second approach

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
C_1	1	-4.6E-13	-1E-13	-2.1E-13	-9E-14	6.28E-14	7.59E-15	-1.8E-15	9.72E-15
C_2	-4.6E-13	1	3.84E-14	-1.2E-13	-1.6E-13	-2.1E-14	5.18E-15	2.67E-15	9.99E-15
C_3	-1E-13	3.84E-14	1	-1E-14	-1E-14	-2.1E-16	2.74E-15	9.4E-16	4.85E-15
C_4	-2.1E-13	-1.2E-13	-1E-14	1	-3E-14	1.55E-14	-2.6E-15	-2.5E-15	7.13E-15
C_5	-9E-14	-1.6E-13	-1E-14	-3E-14	1	-1.1E-15	1.4E-15	-1E-16	2.73E-15
C_6	6.28E-14	-2.1E-14	-2.1E-16	1.55E-14	-1.1E-15	1	-2.2E-15	5.75E-16	5.15E-16
C_7	7.59E-15	5.18E-15	2.74E-15	-2.6E-15	1.4E-15	-2.2E-15	1	1.8E-16	2.24E-17
C_8	-1.8E-15	2.67E-15	9.4E-16	-2.5E-15	-1E-16	5.75E-16	1.8E-16	1	3.34E-16
C_9	9.72E-15	9.99E-15	4.85E-15	7.13E-15	2.73E-15	5.15E-16	2.24E-17	3.34E-16	1