Manufacturing Process Optimization to Increase Product Reliability by Control Charts

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Abstract *–The need to produce highly reliable products is essential for the survival of many industries in today's world. Consequently, one of the main quality characteristics of products is their lifespan or reliability. The production process must be monitored to produce highly reliable products, and in this regard, control charts are one of the widely used process monitoring tools. In this study, the lifetime of the products, which is monitored as a quality characteristic, follows the Weibull distribution with a variable scale parameter and a fixed shape parameter. Monitoring the average lifetime of the products is conducted by examining the Weibull distribution scale parameter. To design a control chart, first, the control limits of the chart for different parameters are obtained and presented. Relationships and values regarding control limits show that the control limit of a one-sided chart is always larger than the control limit of a two-sided chart. For evaluating the performance of the proposed control chart, the average run length in the out-of-control mode for various parameters is presented, and sensitivity analysis is performed. The computational results show that the one-sided control chart has a better performance in detecting reduced lifetime compared to the two-sided control chart. Failure in the type II censoring life test is an important parameter that increases the performance of the control chart. As the number of failures increases, the test continues for a longer time, which causes the chances of detecting the changes to be improved. Finally, to show the performance of the proposed control chart, a practical example is provided.*

Keywords *– Statistical process control, Control chart, Lifetime, Type II censoring, Weibull distribution.*

I. INTRODUCTION

With the start of the production era, customers began to compare different goods, which intensified competition between manufacturers. Quality control, one of the issues humans always face, is a process or set of functions to check whether a product or service conforms to a specific set of standards and responds to customer needs (Kale, 2014). There are three statistical methods for quality control and improvement; one of the most important and influential is the statistical process control, which includes seven tools. The most effective tools for statistical process control are control charts, and the average run length (ARL) is an essential criterion for evaluating their performance (Montgomery, 2020).

Today, in the industry, products with high reliability are produced, and the lifetime or reliability of products is considered as the main quality characteristic. Time and cost are important factors in the inspection. It is impossible to

observe the lifetime of all products due to the duration and cost of inspection. Therefore, the concept of lifetime tests has been introduced to minimize inspection costs and save time. A lifetime test usually involves testing a set of products to see how long they last (Klyatis & Klyatis, 2010).

Lifetime data have two important features that make it difficult to design a control chart for lifetime monitoring. First, they are often followed by non-normal distributions. Second, given that the components are designed and produced for a long time, obtaining the lifetime of a component requires a high time and cost. Life tests are used to solve this problem. In this research, the one and two-sided design of the Shewhart control chart has been studied under the censored life test assumption (Barlow & Proschan, 1975).

In some processes and in reliability tests to measure the lifetime of products, even using lifetime tests, more waiting time is required. The operator is usually interested in implementing a series of factors and conditions in the test to make the product's lifetime faster.

There are several different life tests. One of the most important ones used in quality control and process sampling is censoring. The most important censoring schemes are type I censoring life tests or time censoring, type II censoring life tests or failure censoring, hybrid censoring life tests, and progressive censoring life tests (Meeker & Escobar, 2014).

In the censored life test, there are two modes. In the first case, when a component fails, it is replaced with another healthy component, which is called a censored test with replacement. In the second case, after observing the failure, no component is replaced, and the failed component is removed from the test. It is called a censoring test without replacement (Pham, 2006).

Often, the underlying assumption of designing Shewhart control charts is that the quality characteristics in question follow a normal distribution. However, if in a process, the desired quality characteristics do not follow the normal distribution, using a control chart designed to monitor the quality characteristics with the normal distribution is inappropriate and causes an incorrect warning in the process or indicates the change in the process with a delay. Therefore, a late warning about the altered process will lead to an increase in non-compliant or defective cases. To solve this problem, control charts must be designed with the distribution that models the qualitative characteristic. One of the characteristics that often follow non-normal distributions is life, so the normal distribution has limited application in life data. The most important distributions in reliability and lifetime include Weibull, exponential, log-normal, Burnham Sanders, Gamma, and Pareto distributions (Birolini, 1994).

Rasay & Alinezhad (2022) researched the design of a new life test under continuous distributions. Falahi et al. (2022) studied the lifetime performance index and sampling plans. Using lifetime performance index data, they presented a QSS sampling plan under type II censoring life test.

Today, control charts are used in various industries and sciences to monitor the process and increase the lifetime and reliability of products. Gharib et al.(2021) designed a multivariate exponentially weighted moving average control chart with measurement errors. Jafarian-Namin et al. (2019) examined the economic-statistical design of acceptance control charts using a robust optimization approach. Rasay et al.(2019) considered a two-stage dependent process and defined a qualitative characteristic for each stage. Then, at each stage, they monitored the process by control charts. Finally, they presented an integrated model for maintenance and statistical quality control.

The type of control charts and the life test used have been two critical issues for creating innovation in past research. For example, Huang et al. (2017) assumed that the qualitative characteristic follows an exponential distribution. They presented a control chart under the failure censoring test and used the meantime to alert (ATS) for evaluating the performance of their proposed control chart. (Rao, 2018) studied the design of an NP control chart under the truncated life test when the product lifetime follows an exponentiated half-logistic distribution.

An NP quality control chart under the truncated life test was designed by Aslam et al. (2016), considering the

product lifetime following a Pareto type II distribution. A quantitative control chart under the time truncated life test for Weibull distribution is also presented by Khan et al. (2018). Shafqat et al. (2018) proposed a statistical model for the control chart, and obtained how to design an attribute control chart for different distributions and introduced the distribution with best performance. (Rao, 2018) designed an NP control chart under the truncated life test for when the product lifetime follows an exponential half-logistic distribution. Dickinson et al. (2014) presented a cumulative sum (CUSUM) control chart to monitor the lifetime quality characteristic and compared the proposed control chart with an EWMA control chart. Balamurali & Jeyadurga (2018) researched the economic design of an NP control chart to monitor average lifetime based on multiple deferred state sampling.

(Mohammadipour et al., 2021) design EWMA control charts under type II censoring life test. They assumed the lifetime follows the Weibull distribution with a fixed shape parameter and a variable scale parameter. Balamurali & Jeyadurga (2019) assumed that the qualitative characteristic is longevity and follows a continuous distribution. They then presented how to design a quality control chart. Gadde et al. (2019) considered the quality characteristic of a lifetime following the Daugm distribution and then designed attribute control charts under truncated life tests. (Goodarzi & Amiri, 2017) used the accelerated failure time model, and presented two control charts to monitor the quality characteristic in the second stage under the censored and non-censored reliability data.

According to the above-mentioned literature, the design and application of a control chart to monitor the average lifetime of products under lifetime tests have become an important issue in the application of lifetime tests in industrial engineering. Product lifetime is one of the characteristics whose reduction is often considered by the manufacturer. Therefore, in this research, in addition to two-sided control charts, the method of designing one-sided control charts is also presented. In this study, the failure censoring test is used to obtain the average lifetime of products. One and two-sided Shewhart control charts under the failure censoring test are designed to monitor the average lifetime of products when the lifetime of the Weibull distribution follows the fixed shape parameter and the variable scale parameter.

In this research, to design control charts, first, the relationships of control limits are presented and the related values are obtained for different parameters. Then, the way to get ARL and its relationships are also provided. The performance of the proposed control chart will be assessed using ARL in the out-of-control state. The tables of ARLs are presented for various shift constants and specified parameters. Then, a sensitivity analysis is performed based on ARL values in the out-of-control state. Finally, applications of the control charts are shown by practical examples.

II. PROBLEM STATEMENT

The product lifetime, which is considered its main qualitative characteristic, is assumed to be X and follows a Weibull distribution with a fixed shape parameter and an unknown scale parameter. The cumulative distribution function X is as follows:

$$F_x(x) = 1 - exp[-(x\theta)^m] \tag{1}$$

In Equation (1), m and θ are the shape and scale parameters, respectively. The average lifetime of Weibull distribution is as follows:

$$\mu = \frac{\Gamma\left(\frac{1}{m}\right)}{\theta m} \tag{2}$$

According to Equation (2), monitoring the average lifetime in the case where the shape parameter is constant will be the same as monitoring the scale parameter. The scale parameter is equal to θ_0 when the process is in control, and becomes $\theta_1 = c\theta_0$ when the process is affected by an assignable cause and goes out of control, which is called a constant c value. The statistics obtained by the failure censoring test are as follows:

$$V_i = \sum_{i=1}^{r} \left(\frac{x_i}{\mu_0}\right)^m + (n-r)\left(\frac{x_r}{\mu_0}\right)^m \tag{3}$$

In the above relation, x_i is the failure time of component i, and the μ_0 is equal to the average life expectancy and has a fixed and definite value. According to Jun et al. (2010), the value of V_i follows the gamma distribution with parameters r and W_0. The value of W_0 is considered as follows:

$$W_0 = (\theta_0\mu_0)^m = \left(\frac{\Gamma(1/m)}{m}\right)^m \tag{4}$$

In addition, due to the relationship between gamma distribution and chi-square distribution, the value of $2VW_0$ follows the chi-square distribution with a 2r degree of freedom (Jun et al., 2010).

III. DESIGNING SHEWHART CONTROL CHART

For designing a control chart, the control limits of the chart must be determined first, and then created a chart. In the next step, sampling will be done to monitor the process and the desired statistics will be calculated. Here, one and two-sided charts are investigated for designing; First, the way of designing a one-sided chart and then the method of designing a two-way control chart is discussed.

A. One-sided Shewhart control chart

Suppose only the deterioration or improvement of a qualitative characteristic is considered in a process. In this case, one-sided charts are used. If the deterioration of quality characteristics or reduction of lifetime is considered in a process, a lower control limit is utilized, and if the improvement of quality characteristics or increase of lifetime is considered, an upper control limit is applied. Lifetime is one of the characteristics whose deterioration or reduction is considered. Therefore, for monitoring only the deterioration of the process or the reduction of life in the one-sided control chart, the lower control limit is used. In this case, since there is only one control limit, the α error is as follows:

$$P(V_i < LCL \mid \theta = \theta_0) = \alpha \tag{5}$$

In the above relation, V_i is the same statistic obtained by the lifetime test according to Equation (3). Now, according to what was said about V_i, Equation (5) can be written as follows to obtain a lower control limit:

$$P(2W_0V_i < 2W_0 \times LCL) = \alpha$$

$$P(\chi_{2r}^2 < 2W_0 \times LCL) = \alpha \tag{6}$$

$$LCL = \frac{\chi_{2r,1-\alpha}^2}{2W_0}$$

The error β is obtained similarly, except that, the process has changed and the scale parameter has become $\theta_1 = c\theta_0$. So in this case, W_1 is used instead of W_0, which is equal to:

$$W_1 = (\theta_1\mu_0)^m = c^m W_0 \tag{7}$$

Therefore, the β-error relation is obtained as follows:

$$\beta = P\left(V_i > LCL | \theta_1 = C\theta_0\right)$$

$$\beta = 1 - P(\chi^2_{2r} < 2c^m w_0 \times LCL)$$

$$(8)$$

According to β, ARL_1 can be calculated by the following equation:

$$ARL_1 = \frac{1}{1 - \left(P\left(c^m * \chi^2_{2r,1-(\alpha/2)} < \chi^2_{2r} < c^m * \chi^2_{2r,\alpha/2}\right)\right)}$$

$$(9)$$

B. TWO-SIDED SHEWHART CONTROL CHART

As the chart in Section 3-1 uses only a lower control limit due to its design, it is not able to report process improvements and deteriorations (increase and decrease in life) at the same time. Now, suppose that the deterioration and improvement of the qualitative characteristic are considered in a process. In this mode, two-sided charts are used. Like the one-sided chart, the concept of α error is used to obtain the control limits. The probability of error α is as follows:

$$\alpha = P(V_i > UCL | \theta = \theta_0) + P(V_i < LCL | \theta = \theta_0)$$

$$(10)$$

Assuming that half of the α error is above the UCL and the other half is below the lower control limit, the above relation can be rewritten as follows and the UCL relation can be obtained as a one-sided chart:

$$P(V_i > UCL | \theta = \theta_0) = \frac{\alpha}{2}$$

$$UCL = \frac{\chi^2_{2r,\alpha/2}}{2W_0}$$

$$(11)$$

Similarly, Equation (10) will be rewritten as follows and the LCL relation will be obtained:

$$LCL = \frac{\chi^2_{2r,1-(\alpha/2)}}{2W_0}$$

$$(12)$$

In this case, the error ratio β will be as follows:

$$\beta = P\left(LCL < V_i < UCL | \theta_1 = C\theta_0\right)$$

$$\beta = P(2c^m W_0 \times LCL < 2W_1 V_i < 2c^m W_0 \times UCL)$$

$$(13)$$

The value of ARL_1 will also be obtained by Equation (14):

$$ARL_1 = \cfrac{1}{1 - \left(P\left(c^m * \chi^2_{2r,1-(\alpha/_2)} < \chi^2_{2r} < c^m * \chi^2_{2r,\alpha/_2} \right) \right)} \tag{14}$$

IV. COMPUTATIONAL RESULTS AND ANALYSIS

In this section, the control limits of two graphs are first obtained for different parameters, and then the process of changing the control limits for these parameters is analyzed. Next, to evaluate the performance of the proposed control charts, ARL_1 is used. The changes trend of ARL_1 for different parameters is then discussed and analyzed. For instance, as can be seen in Table (1), the lower control limits are shown for $m = 0.5, 1$, the $ARL_0 = 200, 370$, and the different values of r.

Table 1. LCL of the one-sided charts for different parameters

m=0.5							m =1					
ARL0= 370							ARL0=370					
r	1	2	3	4	5	6	r	1	2	3	4	5	6
LCL	0.001	0.053	0.191	0.398	0.657	0.955	LCL	0.003	0.08	0.31	0.65	1.08	1.57
ARL0= 200							ARL0=200					
LCL	0.003	0.073	0.238	0.475	0.762	1.086	LCL	0.005	0.10	0.33	0.67	1.07	1.53

By examining the parameters of Table (1), the following results can be achieved:

- By increasing the r value, the LCL value also increases.
- By increasing the ARL_0 value, the LCL value decreases.
- By increasing the m value, the value of LCL also increases, and this trend is true for different values of r and ARL_0.

Table (2) also shows the values of the control limits of the two-sided chart as well as the one-sided chart for different parameters of $m = 0.5, 1$, $ARL_0 = 200, 370$, and different values of r.

The following results can be achieved by examining the control limits of the two-sided Shewhart control chart, which has been reported as an example in Table(1):

Table 2. LCL and UCL of the twe_sided charts for different parameters

m=0.5							m=1					
ARL0=370							ARL0=370					
r	1	2	3	4	5	6	r	1	2	3	4	5	6
UCL	4.672	6.2934	7.685	8.966	10.17	11.33	UCL	7.703	10.37	12.67	14.78	16.77	18.69
LCL	0.001	0.0374	0.149	0.329	0.559	0.830	LCL	0.001	0.061	0.246	0.542	0.923	1.369
ARL0=200							ARL0=200					
UCL	4.236	5.806	7.159	8.405	9.585	10.71	UCL	6.985	9.574	11.80	13.85	15.80	17.67
LCL	0.001	0.0512	0.186	0.390	0.646	0.940	LCL	0.002	0.084	0.307	0.643	1.065	1.551

Table 3. The ARL$_1$ of the one-sided control charts for different parameters

m=1							m=1.5					
ARL$_0$=370							ARL$_0$=200					
r	1	2	3	4	5	6	r	1	2	3	4	5	6
c	ARL$_1$						c	ARL$_1$					
1	370.37	370.37	370.37	370.37	370.37	370.37	1	200.00	200.00	200.00	200.00	200.00	200.00
1.1	336.75	307.62	283.89	264.40	248.02	233.97	1.1	173.42	151.85	135.32	122.36	111.87	103.15
1.2	308.73	259.78	223.08	195.10	173.10	155.30	1.2	152.27	118.25	95.20	78.95	66.96	57.78
1.3	285.02	222.46	178.99	148.02	125.06	107.44	1.3	135.10	94.08	69.22	53.29	42.45	34.71
1.4	264.69	192.77	146.19	114.99	93.06	77.00	1.4	120.93	76.22	51.76	37.38	28.27	22.14
1.5	247.08	168.76	121.25	91.17	71.02	56.88	1.5	109.09	62.74	39.67	27.12	19.65	14.89
1.6	231.67	149.07	101.91	73.57	55.41	43.14	1.6	99.07	52.35	31.06	20.26	14.18	10.48
1.7	218.07	132.70	86.66	60.30	44.08	33.48	1.7	90.51	44.22	24.78	15.53	10.58	7.67
1.8	205.98	118.96	74.47	50.11	35.67	26.52	1.8	83.11	37.76	20.10	12.19	8.13	5.83
1.9	195.17	107.30	64.58	42.16	29.31	21.39	1.9	76.68	32.55	16.56	9.76	6.41	4.57
2	185.44	97.32	56.47	35.86	24.41	17.54	2	71.03	28.30	13.83	7.97	5.18	3.69
2.5	148.45	63.84	31.90	18.21	11.52	7.88	2.5	50.97	15.62	6.62	3.62	2.37	1.77
3	123.79	45.44	20.35	10.86	6.62	4.47	3	38.90	9.82	3.88	2.17	1.52	1.24

Table4. The ARL$_1$ of the two-sided control charts for different parameters

m=2.5							m=2					
ARL$_0$=370							ARL$_0$=200					
r	1	2	3	4	5	6	r	1	2	3	4	5	6
c	ARL$_1$						c	ARL$_1$					
0.2	1.13	1.01	1.00	1.00	1.00	1.00	0.2	1.27	1.05	1.01	1.00	1.00	1.00
0.4	1.95	1.29	1.11	1.04	1.02	1.01	0.4	2.61	1.61	1.29	1.14	1.07	1.04
0.6	6.30	3.43	2.40	1.89	1.60	1.41	0.6	8.58	4.85	3.39	2.63	2.17	1.86
0.8	42.48	26.45	18.86	14.41	11.51	9.49	0.8	43.08	29.69	22.53	17.99	14.86	12.58
1	370.37	370.37	370.37	370.37	370.37	370.37	1	200.00	200.00	200.00	200.00	200.00	200.00
1.2	463.30	302.67	206.32	147.58	109.88	84.48	1.2	264.75	193.44	144.71	112.00	89.23	72.79
1.4	319.67	144.24	72.95	41.28	25.58	17.02	1.4	204.00	109.00	63.97	40.91	27.97	20.15
1.6	229.10	76.39	30.87	15.04	8.51	5.40	1.6	156.55	65.76	32.20	18.10	11.31	7.68
1.8	170.79	44.05	15.10	6.80	3.79	2.48	1.8	123.80	42.40	18.07	9.35	5.61	3.77
2	131.36	27.21	8.34	3.69	2.16	1.53	2	100.38	28.83	11.07	5.48	3.29	2.27
2.5	75.41	10.30	2.87	1.48	1.12	1.03	2.5	64.42	13.12	4.38	2.21	1.48	1.20
3	47.99	5.00	1.56	1.07	1.01	1.00	3	44.89	7.18	2.37	1.37	1.10	1.02

- By increasing the *r* value, the values of the control limits also increase.
- By increasing the ARL0 value, UCL values also increase, and LCL values decrease.
- Similar to a one-sided control chart, as the value of *m* increases, the control limits also increase.

After obtaining the control limits, ARL_1 is used to evaluate the performance of the proposed control charts. To obtain ARL_1, first, the β error values are calculated for different parameters. The parameters affecting the ARL_1 in the Shewhart control chart are the shape parameter (*m*), in-control average run length (ARL_0), the number of failures in the failure censored test (*r*), and the change constant (*c*). The trend of ARL_1 changes in one-sided and two-sided control charts for different parameters is the same. These trends are shown as an example of a two-sided control chart, and the analyses are presented based on charts derived from ARL_1 values.

For instance, ARL_1 values for different parameters in one and two-sided charts are shown in Tables (3) and (4).

- According to Figure (1), as *r* increases, the value of ARL_1 decreases. The reason for this trend is that the higher the *r* value in the failure censoring test, the longer the test time and the more information is collected.
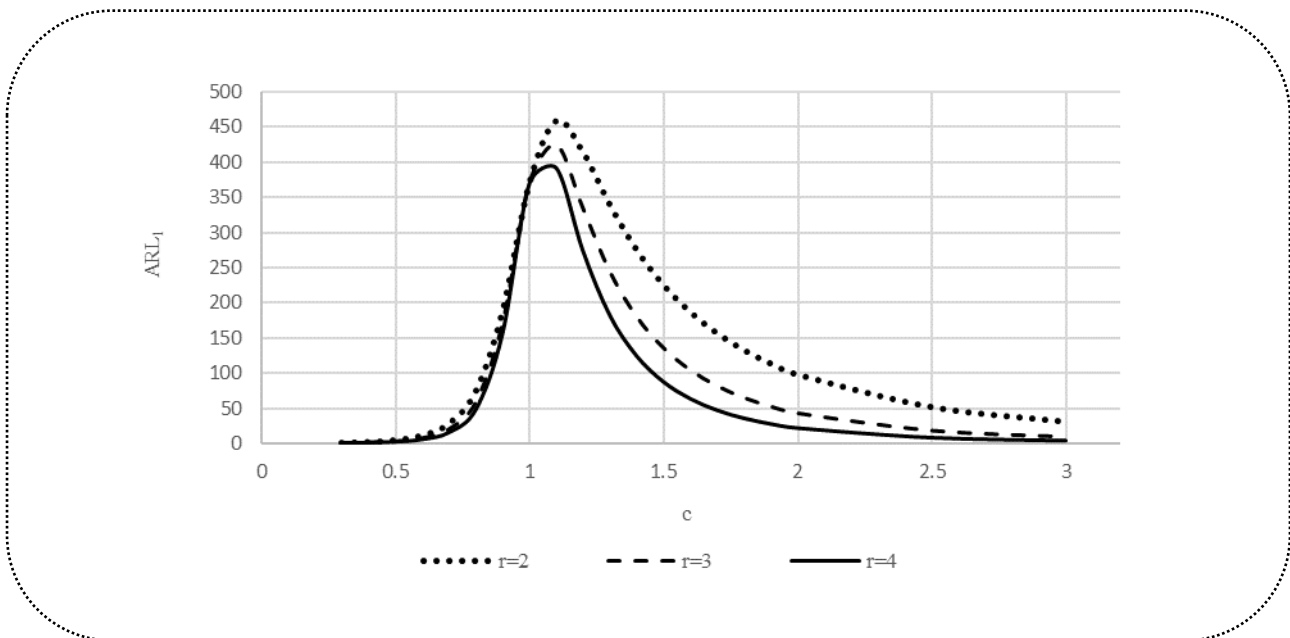

Figure1. ARL $_1$ two-sided charts (various *r*)

- As can be seen in Figure (2), the values of ARL_1 decrease by reducing the ARL_0. The reason for this trend, as explained in the one-sided chart, is that in a two-sided chart, the lower the ARL_0 value, the closer the control limits will be, this allows the control chart to detect the change sooner if there is a change in the process.
- As the trend of Figure (1) shows, the relationship between ARL_1 and the change constant (*c*) is such that with increasing constant change value up to $c = 1$, the ARL_1 trend is upward, and from $c = 1$ onwards, the trend will be downward. The reason for this trend is that the higher the amount of change in a process, the greater the probability of detecting that change and the lower the likelihood of type II error, and consequently, the lower the value of ARL_1.
- By increasing the parameter value in Figure (2), the changes in ARL_1 are as follows. The reason for this trend is that as the shape parameter increases, the amount of type II error decreases, and ARL_1 will have a decreasing trend. This trend is shown in Figure (3).
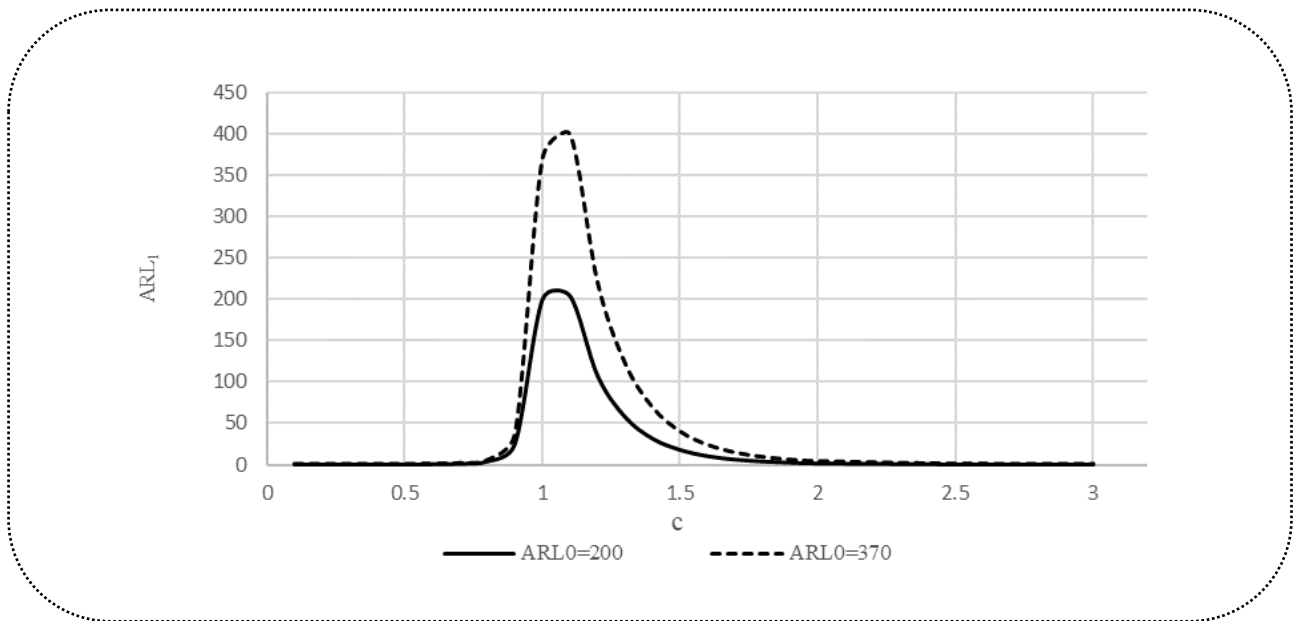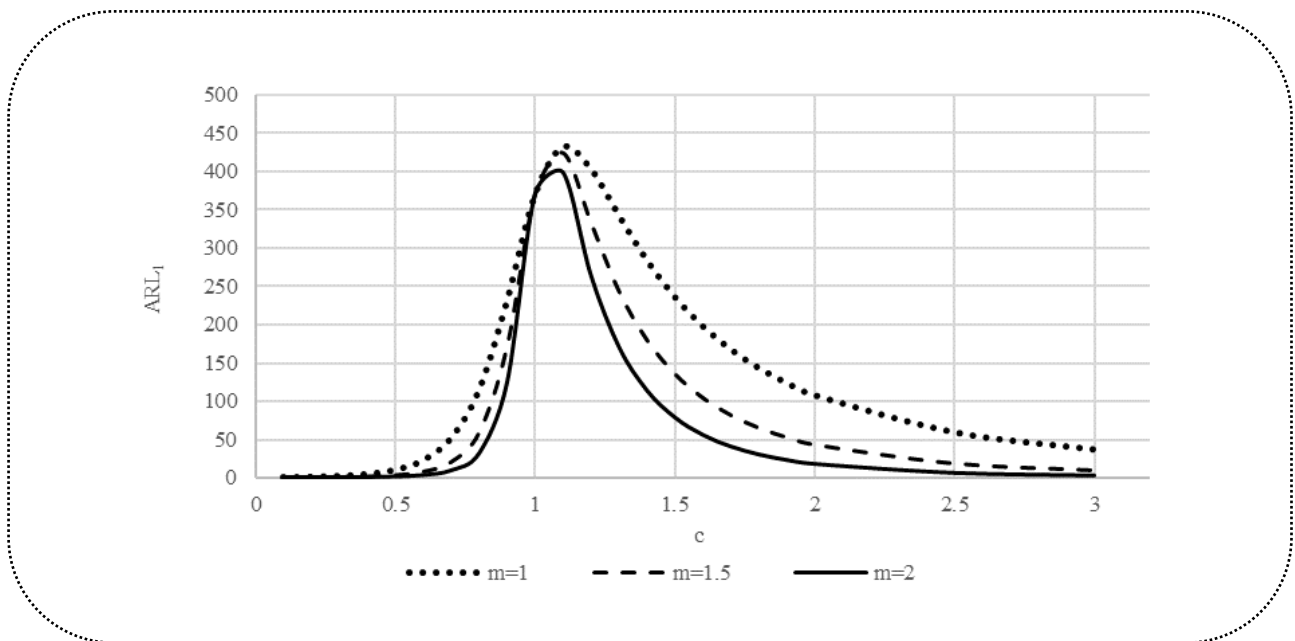
Figure 2. ARL $_1$ two sided charts (various ARL $_0$)

Figure 3. ARL$_1$ two-sided chart (various values of *m*)

Here, the ARL_1 in one and two-sided charts are compared. A one-sided chart monitors the process from one direction because it has only one control limit. However, due to having two control limits, the two-sided control chart can monitor the process from both sides. Thus, the ARL_1 curve of the two-sided chart has skewness, and one-sided charts have no skewness towards ARL_1 and have a uniform decreasing or increasing trend, which is also shown in Figure (4).
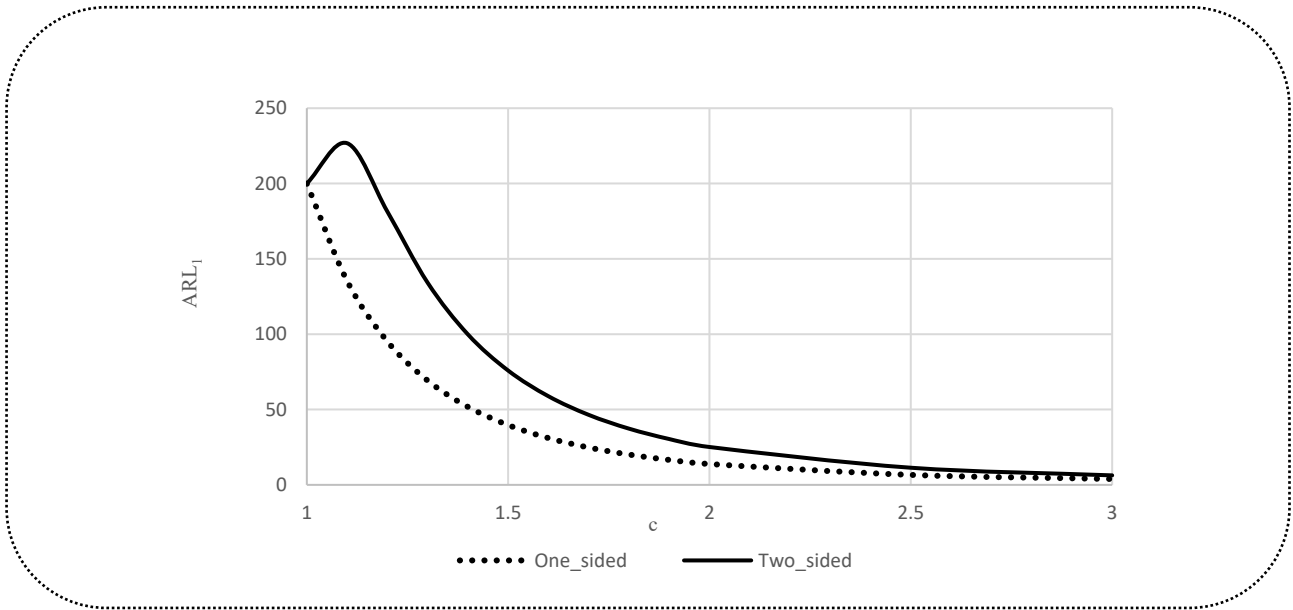
Figure 4. ARL $_1$ one and two sided chart for various values of c

On the other hand, the ARL_1 in the one-sided chart is always lower than the ARL_1 in the two-sided chart. Because there is only one control limit in a one-sided control chart, it is assumed that the α error exists on only one side, and therefore, the LCL one-sided chart is larger than the LCL two-sided chart. This causes the one-sided control chart to detect the deviation sooner if there is a deviation in the process.

V. DESIGNING A CONTROL CHART IN A PRACTICAL EXAMPLE

In this section, the actual data of a Korean car manufacturer is used to design control charts. The data belongs to the operational time of a part of the machine until the breakdown in one month, which follows the Weibull distribution with $m = 2.5$ and $\theta_0 = 1$. It is assumed that $ARL_0 = 370$ and failure number $=3$. The values of V_i are reported in Table (5) (Aslam & Jun, 2015).

Table 5. The V$_{(i)}$ statistical values for real data

sample	1	2	3	4	5	6	7	8	9	10	11
V$_{(i)}$	8.2600	9.6870	3.2270	3.7560	5.1410	1.3090	2.9770	5.9220	3.4210	4.7180	4.5950
sample	12	13	14	15	16	17	18	19	20	21	22
V$_{(i)}$	7.9940	9.7640	5.3940	2.4050	3.8080	3.8600	1.6550	1.1080	3.1160	4.2380	3.1210
sample	23	24	25	26	27	28	29	30	31	32	33
V$_{(i)}$	7.1010	5.5620	6.4440	4.1890	3.4480	7.2690	4.6650	0.6960	1.9340	2.9680	5.0930
sample	34	35	36	37	38	39	40	41	42	43	44
V$_{(i)}$	5.3030	10.2320	4.6860	3.2180	2.6830	4.6460	10.5180	2.5220	1.1100	4.0510	8.8780
sample	45	46	47	48	49	50					
V$_{(i)}$	3.2530	2.0900	5.2360	1.3630	4.5920	3.1770					

The process is monitored with a two-sided Shewhart control chart. Figure (5) demonstrates the process to be in control and no factors have caused the process to deviate.
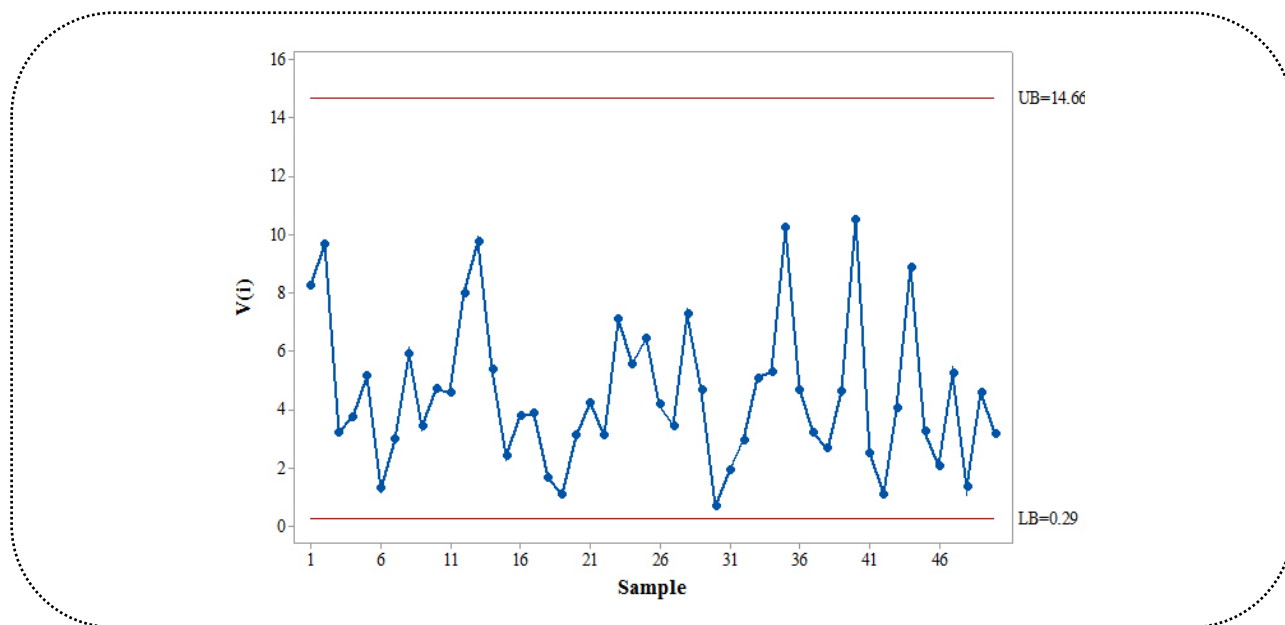

Figure 5. The proposed Shewhart control chart for real data

VI. CONCLUSION

One of the most important and widely used control charts is the Shewhart control chart. The main quality characteristic of reliability is the product's lifetime. Reducing lifetime is an important issue in reliability. Therefore, in this study, in addition to two-sided control charts, one-sided control charts that monitor lifetime reduction are presented. In this research, the failure censored test without replacement has been used to obtain the statistics. Next, the relationship control limits, α and β errors, and ARL_1 are presented. Then, the trend of changes in ARL and control limits for different values of effective parameters are presented and analyzed. The effective parameters in the Shewhart control charts are the number of failures, the shape parameter, the change constant, and the average run length of the in-control state. According to the results, the one-sided Shewhart control chart performs better than the two-sided control chart in detecting reduced lifetime. It is recommended to use one-sided control charts in industries that only reduce lifetime. In one-sided and two-sided control charts, by increasing each of the values r, m, and decreasing ARL_0, the performance of the control chart enhances. One of the most important parameters for enhancing the performance of the control chart is the number of lifetime test failures, and the more it is considered, the faster the chart reports the change in the process. The control charts presented in this research can be used to monitor the production process of products in industries whose product lifetime is long, such as mechanical components and electrical and electronics components. Designing CUSUM-Shewhart control charts for hybrid censoring life tests or other censoring life tests is a promising direction for future studies.

References

Aslam, M., & Jun, C.-H. (2015). Attribute Control Charts for the Weibull Distribution under Truncated Life Tests. Quality Engineering, 27(3), 283-288. doi:10.1080/08982112.2015.1017649

Aslam, M., Khan, N., & Jun, C.-H. (2016). A control chart for time truncated life tests using Pareto distribution of second kind. Journal of Statistical Computation and Simulation, 86(11), 2113-2122. doi:10.1080/00949655.2015.1103737

Balamurali, S., & Jeyadurga, P. (2018). Economic design of an attribute control chart for monitoring mean life based on multiple deferred state sampling. Applied Stochastic Models in Business and Industry, 35(3), 893-907. doi:10.1002/asmb.2419

Balamurali, S., & Jeyadurga, P. (2019). An attribute np control chart for monitoring mean life using multiple deferred state sampling based on truncated life tests. *International Journal of Reliability, Quality and Safety Engineering*, *26*(01), 1950004.

Barlow, R. E., & Proschan, F. (1975). Statistical Theory of Reliability and Life Testing, Holt, Reinhart and Winston. *Inc., New York*.

Birolini, A. (1994). Reliability and Quality of Technical Systems. In: Springer-Verlag, Berlin.

Dickinson, R. M., Roberts, D. A. O., Driscoll, A. R., Woodall, W. H., & Vining, G. G. (2014). CUSUM charts for monitoring the characteristic life of censored Weibull lifetimes. *Journal of Quality Technology*, *46*(4), 340-358.

Falahi, S., Farughi, H., & Rasay, H. (2022). Applications of lifetime performance index in acceptance sampling plans under censoring. Journal of Quality Engineering and Production Optimization.

Gadde, S. R., Fulment, A., Josephat, P. J. L. C. R., & Engineering, S. (2019). Attribute control charts for the Dagum distribution under truncated life tests. 8(4), 329-335.

Gharib, A., Amiri, A., & Jalilibal, z. (2021). Designing a multivariate exponentially weighted moving average control chart with measurement errors %J Journal of Quality Engineering and Production Optimization. -. doi:10.22070/jqepo.2021.5667.1163

Goodarzi, A., & Amiri, A. (2017). Monitoring Lognormal Reliability Data in a Two-Stage Process Using Accelerated Failure Time Model. Journal of Quality Engineering and Production Optimization, 2(1), 17-26. doi:10.22070/jqepo.2016.349

Huang, S., Yang, J., Xie, M. J. Q., & International, R. E. (2017). A study of control chart for monitoring exponentially distributed characteristics based on type-II censored samples. 33(7), 1513-1526.

Jafarian-Namin, S., Fallahnezhad, M. S., Tavakkoli-Moghaddam, R., & Mirzabaghi, M. (2019). Robust Economic-Statistical Design of Acceptance Control Chart %J Journal of Quality Engineering and Production Optimization. 4(1), 55-72. doi:10.22070/jqepo.2018.3646.1078

Jun, C.-H., Lee, H., Lee, S.-H., & Balamurali, S. J. J. o. A. S. (2010). A variables repetitive group sampling plan under failure-censored reliability tests for Weibull distribution. 37(3), 453-460.

Kale, S. S. (2014). Quality assurance and quality control of pressure relief systems. *Process Safety Progress*, *33*(2), 136-142.

Khan, N., Aslam, M., Khan, M. Z., & Jun, C.-H. J. T. (2018). A Variable Control Chart under the Truncated Life Test for a Weibull Distribution. 6(2), 55.

Klyatis, L. M., & Klyatis, E. (2010). Accelerated quality and reliability solutions: Elsevier.

Meeker, W. Q., & Escobar, L. A. (2014). Statistical methods for reliability data: John Wiley & Sons.

Mohammadipour, P., Farughi, H., Rasay, H., & Arkat, J. (2021). Designing Exponentially Weighted Moving Average Control Charts under Failure Censoring Reliability Tests. International Journal of Engineering, 34(11), 2398-2407. doi:10.5829/ije.2021.34.11b.03

Montgomery, D. C. (2020). Introduction to statistical quality control: John Wiley & Sons.

Pham, H. (2006). Handbook of reliability engineering: Springer Science & Business Media.

Rao, G. S. (2018). A control chart for time truncated life tests using exponentiated half logistic distribution. *Appl. Math. Inf. Sci*, *12*(1), 125-131.

Rasay, H., & Alinezhad, E. (2022). Developing an adaptable sequential probability ratio test applicable for lifetime analysis of different continuous distributions. Quality Technology & Quantitative Management, 1-20.

Rasay, H., Fallahnezhad, M. S., & Zaremehrjerdi, Y. (2019). An integrated model of statistical process control and maintenance planning for a two-stage dependent process under general deterioration. European Journal of Industrial Engineering, 13(2), 149-177.

Shafqat, A., Hussain, J., Al-Nasser, A. D., & Aslam, M. (2018). Attribute control chart for some popular distributions. Communications in Statistics - Theory and Methods, 47(8), 1978-1988. doi:10.1080/03610926.2017.1335414.