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Solving a multi-objective hierarchical location model for the healthcare problem considering congestion by LP-metric and augmented epsilon-constraint approaches

S.A. Davoodi[1], A. Ghodratnama[1*], M. Mohammadi[1]

[1] *Department of Industrial Engineering, Faculty of Engineering, Kharazmi University, Tehran, Iran*

**** Corresponding Author:*** *A. Ghodratnama (Email: Ghodratnama@khu.ac.ir)*

Abstract *– This paper addresses optimal locating healthcare facilities problem regarding the essential role of these systems on expense and equity at the strategic level to decision-makers. As a result, a multi-objective model with a hierarchical structure and congestion consideration is proposed for the location issue, which is the main contribution of this study. A mixed-integer non-linear programming (MINLP) model is developed to reduce overall system expenses, such as setup, operating, travel costs, and total waiting time at facility levels, while concurrently maximizing the number of covered patients. Furthermore, two M/M/1/K and M/M/C/K queue systems are utilized at facility levels. Then, two LP-metric and Augmented epsilon-constraint methods are implied. Several examples are conducted and evaluated using statistical tests and the TOPSIS approach to assess the performance of the solution strategies. After that, a sensitivity analysis is carried out. The findings indicate that the proposed model may be used as a tool to assist decision-makers in the design of multi-level healthcare facilities.*

Keywords*– Hierarchical Location, Queue Theory, Multi-Objective, Augmented epsilon-Constraint, LP-metric, Uncertainty*

I. INTRODUCTION

Health care systems have an essential role in reducing or increasing public health countries' overall economic costs and equity. Furthermore, the optimization of these systems can significantly impact the macro-level of countries. Timely, easy access and efficient service delivery are critical and vital for these systems. Many issues influence the efficiency of healthcare systems. Locating healthcare facilities (HCFs) is one of the practical factors in optimizing healthcare systems which causes reducing system costs and increased public access to this type of service. Therefore, in recent decades, many studies have been conducted to discuss the location of HCFs.

The Leinbach and Gould research is perhaps one of the first studies on the placement and distribution of health care facilities (Rahman & Smith, 2000). The difficulty of finding and assessing the capacity of hospitals was discussed in this work. The issue was also solved using the transportation method. Ahmadi-Javid et al. (2017) reviewed the literature on HCF sites and proposed a paradigm for categorizing non-emergency and emergency HCF locations.

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According to various diseases and therapies and care of patients, HCFs provide different treatments and services regarding the condition of patients. As a result, healthcare facilities may be classed according to their therapeutic levels, such as low or high. Regarding location difficulties, hierarchical location characteristics may give facilities several degrees of service. A common hierarchical facility placement challenge involves determining the position of facilities in a multi-level network to meet consumers' needs at all levels of the hierarchy (Farahani, Hekmatfar, Fahimnia, Kazemzadeh, & Engineering, 2014).

In hierarchical location problems, flow patterns can be classified into single, multi, referral, and non-referral. Service availability is either nested or non-nested. A higher-level facility delivers all services supplied by a lower-level facility or facilities at each level that provide various services. Spatial configuration may be coherent (i.e., a lower-level service area is a subset of a higher-level service area) or non-coherent (i.e., a lower-level service area is not a higher-level service area). (Farahani and colleagues, 2014) (Figure 1(Şahin, Süral, & Research, 2007)). This paper proposes a hierarchical location model containing three levels of the facility level-one, level-two, and level-three related to physicians, clinics, and general hospitals, Super-special and Special hospitals, respectively, with multi-flow patients, nested system, and non-coherent structure.

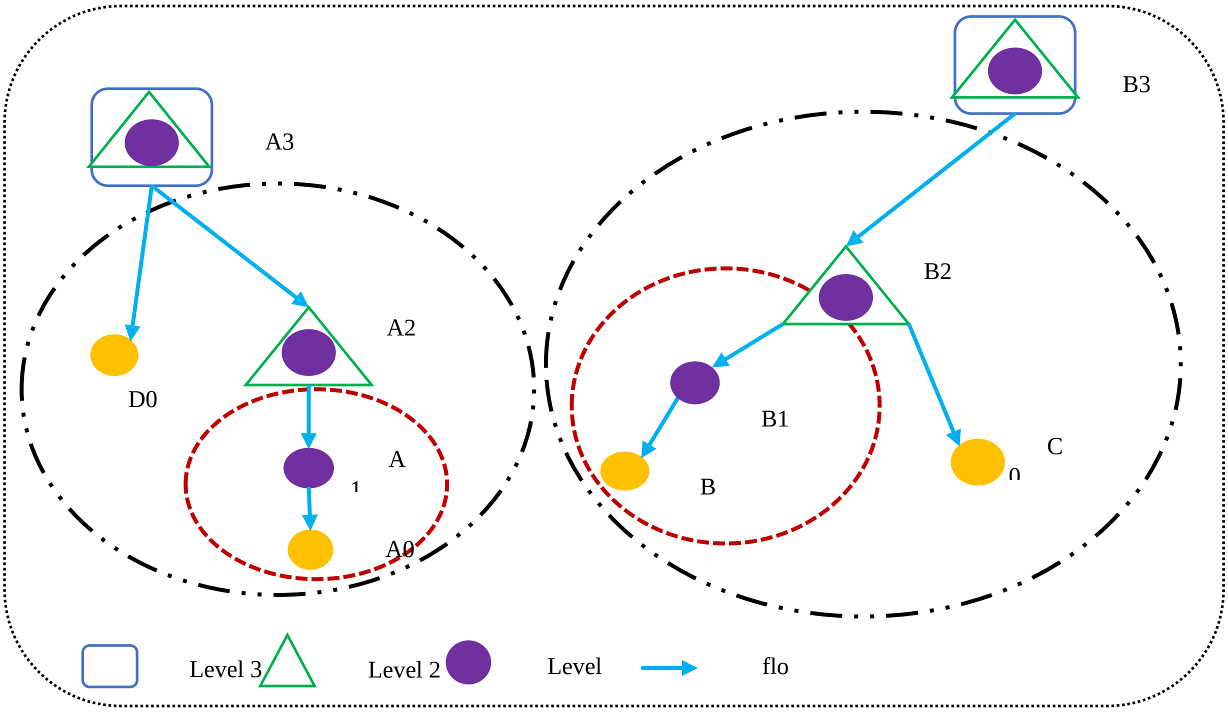

Fig 1. A multi-flow, nested, and coherent structure

Concerning service systems (e.g., post offices, banks, and hospitals), customers might suffer waiting time according to limitations of facilities such as insufficient capacity, interrupted, or unexpected demand. In healthcare facilities, patients must be treated as soon as possible and experience the shortest waiting time. The queuing theory has been used to address this aspect.

Larson (1974) developed a hypercube queuing model to determine the ideal sites for ambulance stations. As a result, the queuing theory has been applied in several investigations. To reduce overall waiting time in facilities, two queuing systems, M/M/1/K (Figure 2.a) and M/M/C/K (Figure 2.b), are studied in this research. With two LP-metric and Augmented epsilon-constraint solution approaches, this work describes and solves a mixed-integer non-linear programming (MINLP) model with three levels of facilities and congestion with demand uncertainty.

A. Motive

As mentioned, locating healthcare facilities is of great importance at high levels of decision-making by reducing costs and increasing equity in access to healthcare services. Motivated by this, the problem of locating healthcare facilities is addressed in this study. To this aim, a location model is presented to maximize covered patients and minimize total system costs and waiting time.

The hierarchical structure of healthcare facilities and the congestion of healthcare facilities were both taken into account. As a result, these characteristics are addressed by a mathematical model that combines a hierarchical location model with queuing theory.

Moreover, the number of patients is considered uncertain about getting close to the real world.

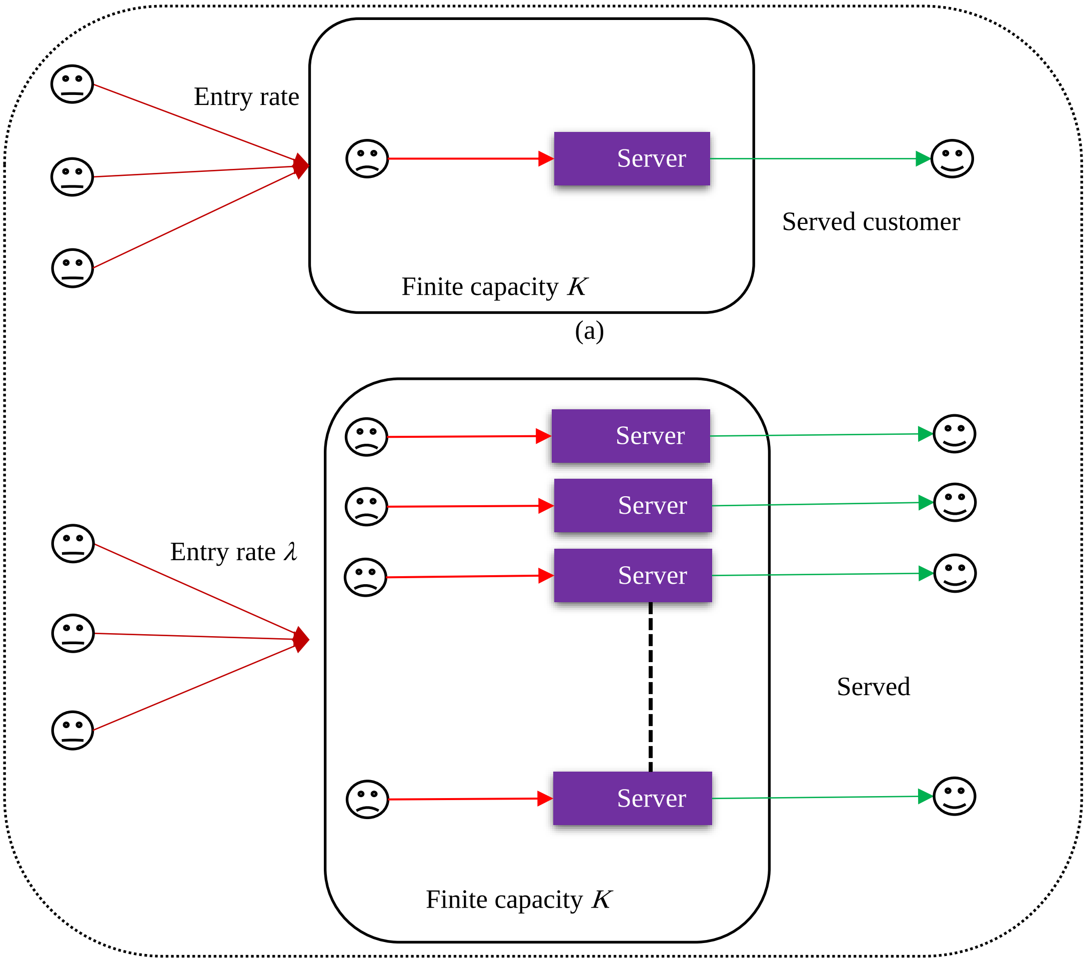

Fig 2. Structure of M/M/1/K (a) and M/M/C/K (b) queue system

The remainder of the paper is presented in the following sequence. The second section examines the relevant literature. Section III contains the problem description, mathematical model, and uncertainty. Section IV discusses the problem-solving approaches, while section V assesses the effectiveness of the solutions. Sensitivity analysis is covered in Section VI. Section VII concludes with findings and recommendations for further study.

II. LITERATURE REVIEW

Hierarchical and probabilistic location models in healthcare applications were investigated as part of the study subject.

A. Hierarchical location model

Narula and Ogbu are two early works on hierarchical location models (1979). They provided a two-level location model that reduced overall weighted journey distance and solved the issue using a heuristic strategy. Moore and ReVelle (1982) presented a hierarchical placement model that employed an exact solution approach based on the branch and bound methodology to maximize the number of covered demand nodes. Since then, a slew of relevant investigations has been carried out. Galvao et al. (2002) looked at a three-level location model with a nested service structure and referrals and a model for developing a maternal and perinatal care network in Rio de Janeiro. Ahin et al. (2007) offered two-level, multi-flow, layered, and coherent hierarchical location models to overcome challenges in the Turkish Red Crescent's regional blood services.

In recent decades, hierarchical location models have been developed to make more sophisticated judgments in system design. Mestre et al. (2012) suggested a hierarchical multi-service mathematical programming model to guide choices on the placement and supply of hospital services in the Portuguese south area, considering demand and hospital services uncertainty.

Smith et al. (2013) introduced hierarchical location models with bi-criteria efficiency/equity goals for the hospital environment. Baray and Cliquet (2013) investigated a three-level location strategy to optimise patients' geographical access to maternity care in France. Pehlivan et al. (2014) presented a multi-period hierarchical location model. They tackled the challenge of finding joint sites and ideal capacity levels at each institution to improve France's prenatal care network. In Shenzhen, Zhu et al. (2016) proposed a hierarchical location-allocation model that examined response, coverage, treatment, and cost capacity for low- and high-level trauma clinics.

The ideal solution was calculated using an ant colony optimization method. Paul et al. (2017) used the epsilon-constraint technique to produce a set of non-inferior solutions to a multi-objective hierarchical extension of the maximum covering location problem to maximize population coverage within a quick reaction window. Maleki et al. (2018) presented a two-level hierarchical model. The most important decisions were the optimal flow of patients between network levels, capacity planning, and the planning of required human resources. The model was solved using a credibility-based chance constraint programming method. By considering a referral system and using an augmented epsilon-constraint approach to solve the model, Maleki Rastaghi et al. (2018) suggested a multi-objective and multi-service location-allocation model with capacity planning to construct a healthcare facilities network.

Gitinavard et al. (2019) introduced a unique bi-objective multi-echelon supply chain model based on fuzzy demand to optimize the placement of perishable product distribution centers. Using the basic augmented epsilon-constraint technique, they used the probabilistic chance-constrained programming methodology to assess and rank the acquired Pareto optimum locations. Song et al. (2019) investigated the service availability, financial limitations, and quantity in a hierarchical facility-location issue for hybrid service availability. To tackle the case, they developed an upgraded genetic algorithm. Vakili et al. (2021) proposed two distinct scenario-based mathematical programming formulations for a Green Open Location- Routing Problem to reduce expenses connected with CO_2 emission and solved the problem using two probabilistic and resilient optimizing solution approaches.

To promote accessibility and cost-efficiency, Karakaya and Meral (2022) suggested a bi-objective hierarchical location-allocation model for maternal-neonatal care regionalization. The model was solved using three innovative top-down heuristics and Lagrangian relaxation approaches. Korzebor et al. (2022) proposed a bi-objective model for placing and moving hospital facilities in a hierarchical method to optimize demand coverage while minimizing structural costs by considering the likelihood of disruptions and crises. The issue was also solved using the epsilon-constraint approach. Rouhani and Amin (2022) suggested a bi-objective hierarchical location-allocation approach for creating an organ transplant network to reduce overall time and costs while increasing regional parity. Then The model was solved using the augmented epsilon-constraint method. In addition, a convex robust optimization strategy was created to deal with uncertainty. Chouksey et al. (2022) suggested a hierarchical capacitated facility location-allocation model for developing maternal healthcare facilities in India to reduce overall expenditures and a sequential strategy to solving the issue. Additionally, recent researches tackle this issue correspondingly (Kamran Rad et al., 2021; Salimian and Mousavi, 2021; Khadem et al., 2021; Emami et al., 2021)

B. Probabilistic location model

More complex location models have recently supplemented this field. The challenge of identifying the location, service rate, and pricing of each service facility was studied by Abouee-Mehrizi et al. (2011). Their research looked at the maximum acceptable line length and gave patients the option of joining or not joining the queue. Vidyarthi and Jayaswal (2014) used an M/G/1 type queuing model to investigate a location issue. Mohammadi et al. (2014) presented a location model for designing a reliable healthcare network with uncertainty. A new hybrid solution approach based on queuing theory, interval programming, stochastic programming, fuzzy programming, and game theory was presented to solve the proposed model. Tavakkoli-Moghaddam et al. (2017) developed a novel Pareto-based multi-objective metaheuristic algorithm to solve a new multi-objective optimization model for the facility location problem with congestion (M/M/m/k system) and pricing policies.

Alumur et al. (2018) modeled service time limits and congestion in hub placement concerns. The models were evaluated on the Australia Post (AP) data set, and service time was computed by considering transit time on network connections and processing time at hubs. The placement of emergency service (ES) vehicles over fully linked networks was investigated by Akdoan et al. (2018). They used an approximation queueing model (AQM) to collect system performance metrics and a genetic algorithm to solve the model. Salmasnia et al. (2018) created a multi-objective competitive location issue with M/M/m/k queuing system for joining enterprises in a competitive setting.

The issue was solved using a non-dominated sorting genetic algorithm (NSGA-II) and a non-dominated ranked genetic algorithm (NRGA). The multiple allocation p-hub placement issues under congestion were suggested by zgün-Kibirolu et al.(2019), and the problem was solved using a heuristic approach called particle swarm optimization. Health post networks were created by Ahmadi-Javid and Ramshe (2020). They were inspired by the necessity of primary healthcare to decide the placement of facilities, the staff mix at each open facility, and their optimum number and capacity.

Bahrami et al. (2020) suggested a multi-objective maximum coverage facility placement model for emergency service centers inside an M(t)/M/m/m queuing system, considering various service levels and periodic demand rates. The model was solved using two enhanced epsilon-constraint and NSGA-II algorithm approaches. Using RFID technology, Hajipour et al. (2021) presented a bi-objective mathematical model to reduce supply chain costs and increase the number of undamaged items delivered to demand points. They used the multi-objective Vibration Damping Optimization (MOVDO) meta-heuristic method to overcome the issue. Fattahi et al. (2021) proposed a bi-objective mathematical model with M/MC/K queuing system for the location-pricing problem, which they solved using the NSGA-II and MOPSO methods.

C. Hierarchical and Probabilistic location model

Zhang and colleagues (2010) investigated a bi-level location model with an M/M/C queuing model. A case study that locates mammography clinics in Montreal to optimize the number of participating customers is discussed in their paper. Specifically, the number of customers seeking preventative treatment throughout the network increased. Zarrinpoor et al. (2017) presented a two-level multi-flow nested hierarchy with service referral, the risk of unanticipated disruptive occurrences, and future demand pattern changes. The Benders decomposition type technique was used to construct a queuing system that considers the uncertainty associated with demand and service. Pouraliakbarimamaghani et al. (2017) presented a hierarchical location-allocation model for a capacitated health care system that considered the M/M/C/K queuing system to determine the optimal number of facilities among candidates and optimal allocations of existing customers to operating health centers in a coverage distance.

Genetic and simulated annealing techniques, as well as their combination, were presented as meta-heuristic algorithms. Ghodratnama et al. (2018) created a novel bi-objective hub location-allocation model that considered M/M/C queueing systems and production schedules. To solve the bi-objective model, goal attainment and LP metric approaches were integrated to provide a more effective multi-objective methodology. Pouraliakbari et al. (2018) developed a probabilistic maximum covering location model to determine the best placement of facilities in congested (M/M/C queueing systems) healthcare systems with a referral hierarchy structure to reduce the overall amount of demand lost in the system. The model was solved using two meta-heuristic algorithms: population-based simulated annealing (PBSA) and ant colony optimization (ACO).

In Korea, Jang and Lee (2019) created a mathematical model that integrates a hierarchical newborn care services placement model with an M/G/S queuing system and allocates capabilities to neonatal intensive care facilities. Khodemani-Yazdi et al. (2019) proposed a bi-objective hierarchical hub placement issue with congestion to minimize the overall network cost and maximum trip time at the same time. Two M/M/C and M/M/1 queuing systems were examined in central and local hubs. A novel game theory variable neighborhood fuzzy invasive weed optimization was used to solve the model. Azimi and Asadollahi (2019) suggested a two-level location-allocation model that considered the M/M/1/K queue and was solved using NSGA II and MOPSO metaheuristics. Nasrabadi et al. (2020) proposed a bi-objective hierarchical location-allocation model for locating healthcare facilities, allocating service units, and calculating facility capacity. Furthermore, the M/M/C and M/M/C/C queue systems capture both short- and long-term uncertainty, and the issue is solved using an NSGA-II approach.

D. Contribution of our work

However, the areas mentioned above have been studied in detail and considering our model and applications, few studies are relevant to this research (Pehlivan et al. (Pehlivan et al., 2014)); Pouraliakbarimamaghani et al. (Pouraliakbarimamaghani et al., 2017); Salmasnia et al. (Salmasnia et al., 2018); Azimi and Asadollahi (Azimi & Asadollahi, 2019); H Jang et al. (Jang & Lee, 2019)).

Most of these studies were formulated as a minimization problem (i.e., costs, traveling time) or a maximization problem and applied different queue systems. Therefore, the contribution of this study is a hierarchical structure (multi-flow patients, non-coherent structure, and nested system) with three levels for locating the healthcare facilities and classifying patients into three classes. Moreover, due to the characteristics of healthcare facilities, two different queue systems are applied. In this regard, a multi-objective model is proposed to minimize total costs and waiting time and maximize covered patients simultaneously, and also, the number of patients is considered uncertain.

As a result, to the best of the authors' knowledge, this research does not precisely match the current literature.

III. PROBLEM MODELING

A. Problem definition

In this study, a multi-objective mixed-integer non-linear programming (MINLP) model for locating three levels of facilities with multi-flow patients (three classes of patients), nested system (higher-level facilities provide lower-level facilities services), and non-coherent structure is developed (lower-level service area is not a subset of a higher-level service area). Then, the location of the healthcare facilities problem is described, and the model is presented based on assumptions and two queue systems. Moreover, the number of patients is considered uncertain. In the following sections, two methods are applied to solve the proposed model, and these methods are compared statistically and ranked. In this detail, the structure of this study is illustrated in Figure 3 to describe the proposed approach.

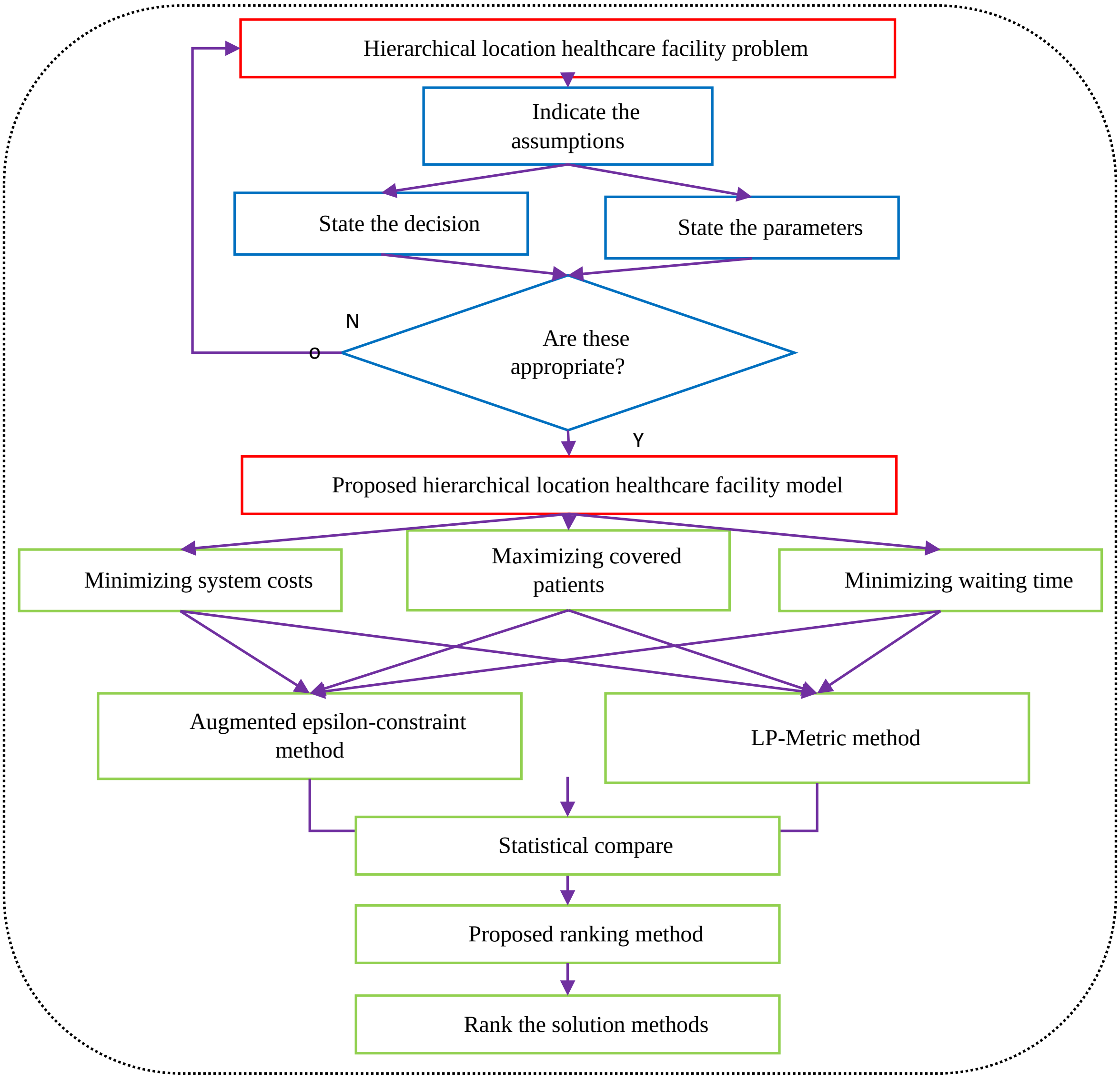

Fig 3. the structure of the proposed approach

B. Problem description in more detail

As mentioned before, the problem of locating healthcare facilities addressed in this study increases the overall number of patients covered while reducing total costs and waiting time. The facilities operate as three different levels of services which indexes J, K, and L denoted type and locations of facilities Physician and Clinic, General Hospitals and Specialty, Super specialty hospitals respectively (Figure 4). The types of patients are also defined in index B= [Ot, It, St] which Ot stands for outpatients, It refers to inpatients, and St is for patients with particular conditions;

Due to a seriously sick condition, this category ensures that a patient of this severity may be effectively managed by a level one, level two, or level three.

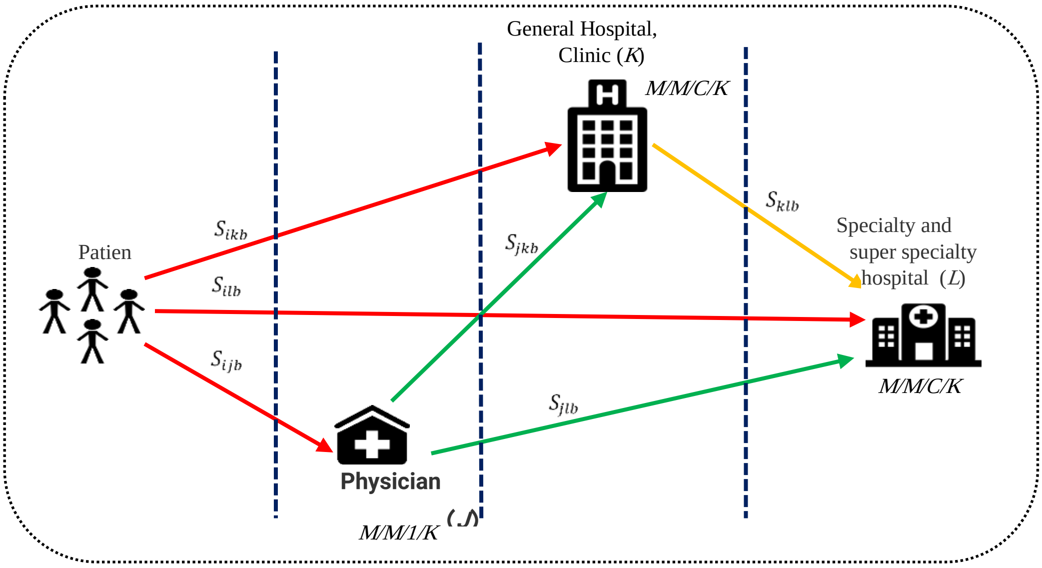

Fig 4. Conceptual model of patient flow, facilities levels, and queue systems

Index I defines demand at each location i, and the number of demand patients B is shown by dem_i^b.

All level-one and level-two medical services can be given by level-three, which implies that all level-three candidates may be considered level-two and level-one, and all level-two candidates can be considered level-one if required. A patient is covered if he or she can go to a healthcare institution that provides the required level of treatment within a reasonable distance of their home. i. If a patient is an outpatient, at least one level-one (Figure 5.a) (or level-two and level-tree) healthcare institution within driving distance may be covered. If a patient is an inpatient, at least one level-two (or level-three) (Figure 5.b) facility within the coverage distance, or at least one level-one facility within covered distance to level-one, with access to level-two within the available coverage distance, may be covered. In addition, if there is at least one level-three institution within the coverage distance, a patient with a particular condition may be covered. Or there exists at least one level-two (or level-one) facility within a covered distance to which the level-two (or level-one) has access to level-three within the available coverage distance (Figure 5.c).

When a patient is transported to healthcare facilities within coverage distance, he/she might experience delays in receiving treatment regarding congestion at facilities. So in this research, minimizing waiting time at facilities is considered to calculate the waiting time at the facility levels. Two queue systems The M/M/1/K (Figure 2. a) system with one server and finite capacity K for level-one, and The M/M/C/K (Figure 2.b) system with C servers and finite capacity K for level-two and level-three facilities are applied.

Summary of assumptions considered in the mode as follows

- Three levels of facilities are considered for the healthcare system.
- Patients are classified into three classes.
- Higher levels of facilities can provide lower-level services.
- The flow of patients might change from one level to another. (If they are within the facility's coverage range)
- The first level of facilities is considered M/M/1/K, and the second and third levels of facilities as M/M/C/K queue systems.
- Any institution-level may deliver services if the patient is within the coverage distance.

- The exact number of patients is unknown.

C. Mathematical model

C.A. Indices of model

I Patient node

J Potential location for facilities of level 1(Physician)

k Potential location for facilities of level 2(clinics, general hospitals)

l Potential location for facilities of level 2(special and super-special hospitals)

b Patient types

a Intersection of the levels $J \cap K \cap L$

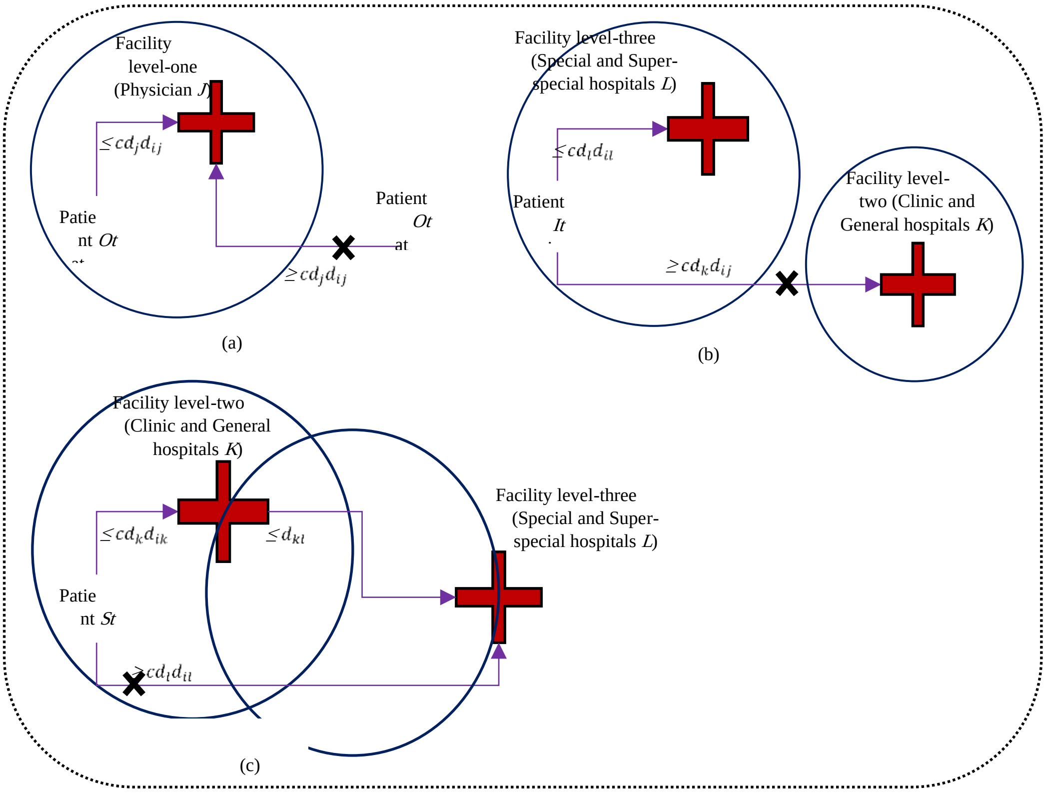

Fig 5. patient pathway to a healthcare facility with considering covering distance

C.B. parameters of model

dem_i^b Amount of patient types b at patients points i

fix_j Fixed cost of establishing facility j

fix_k Fixed cost of establishing facility k

fix_l Fixed cost of establishing facility l

oc_j Operating cost at facility j

oc_k Operating cost at facility k

oc_l Operating cost at facility l

d_{ij} Distance between patient points i to facility j

d_{ik} Distance between patient points i to facility k

d_{il} Distance between patient points i to facility l

d_{jk} Distance between facility j to facility k

d_{jl} Distance between facility j to facility l

d_{kl} Distance between facility k to facility l

cd_j Coverage distance of facility level-one j

cd_k Coverage distance of facility level-two k

cd_l Coverage distance of facility level-three l

tc_{ij} Traveling cost between patient points i to facility j

tc_{ik} Traveling cost between patient points i to facility k

tc_{il} Traveling cost between patient points i to facility l

tc_{jk} Traveling cost between facility j to facility k

tc_{jl} Traveling cost between facility j to facility l

tc_{kl} Traveling cost between facility k to facility l

num_j Maximum number of facilities j

num_k Maximum number of facilities k

num_l Maximum number of facilities l

MFP_j Minimum flow input to facility j

MFP_k Minimum flow input to facility k

MFP_l Minimum flow input to facility l

λ_j Entry rate of patient to facility j

λ_k Entry rate of patient to facility k

λ_l Entry rate of patient to facility l

μ_j Servicing rate at facility j

μ_k Servicing rate at facility k

μ_l Servicing rate at facility l

g_k Number of servers at facility k

g_l Number of servers at facility l

y_j Capacity at facility j

y_k Capacity at facility k

y_l Capacity at facility l

α big number

C.C. variables of model

S_{ijb} The expected number of patients type b who are transported to level one facility j from node i

S_{ikb} The expected number of patients type b who are transported to level two facility k from node i

S_{ilb} The expected number of patients type b who are transported to level three facility l from node i

S_{jkb} The expected number of patients type b who are transported from level one facility j to level two facility k

S_{jlb} The expected number of patients type b who are transported from level one facility j to level three facility l

S_{klb} The expected number of patients type b who are transported from level facility two k to level facility three l

W_{qj} Average waiting time at facility level one j

W_{qk} Average waiting time at facility level two k

W_{ql} Average waiting time at facility level three l

er_j Entry rate to facility j

er_k Entry rate to facility k

er_l Entry rate to facility l

C.D. Binary variables

xc_j 1 if a first-level facility is located at node j, 0 otherwise

xh_k 1 if a second-level facility is located at node k, 0 otherwise

xs_l 1 if a third-level facility is located at node l, 0 otherwise

C.E. Queue systems

As mentioned, two queue systems are used for three levels of facilities. The *M/M/1/K* queue system is assumed for first-level facilities, while the *M/M/C/K* queue system is inferred for second- and third-level facilities. (Shortle, Thompson, Gross, & Harris, 2018) These two systems are explained.

C.A.A. M/M/1/K

Customers arrive according to a Poisson process with the rate of λ, and the time to serve each customer by each server is exponentially distributed with the rate of μ. There is one server in the system with finite capacity k.

$$r_j = \frac{er_j}{\mu_j} \qquad \forall\ j \in J \qquad (1)$$

$$L_{qj} = \frac{r_j}{1-r_j} - \frac{r_j(y_j\, r_j{}^{y_j}+1)}{1-r_j{}^{y_j+1}} \qquad \forall\ j \in J \qquad (2)$$

Average queue length at facility level one j

$$W_{qj} = \frac{L_{qj}}{er_j} \qquad \forall\ j \in J \qquad (3)$$

Average waiting time at facility level one j

C.A.B. M/M/C/K

Customers come in a Poisson process at a rate of λ, and the time it takes each server to service each customer is exponentially dispersed at a rate of μ. The system has C servers, and its capacity is limited. Hence it is referred to as K.

$$r_k = \frac{er_k}{g_k \mu_k} \qquad \forall\ k \in K \qquad (4)$$

$$p_{0k} = \left[\frac{\left(\frac{er_k}{\mu_k}\right)^{g_k}}{g_k!} \left(\frac{1-r_k{}^{y_k-g_k+1}}{1-r_k}\right) + \sum_{n=0}^{g_k-1} \frac{r_k{}^n}{n!} \right]^{-1} \qquad \forall\ k \in K \qquad (5)$$

The probability p_{0k} , there is 0 patient at facility level two k

$$L_{qk} = \frac{P_{0k} r_k}{g_k!(1-r_k)^2} \left(\frac{er_k}{\mu_k}\right)^{g_k} \left[1 - r_k{}^{y_k-g_k+1} - (1-r_k)\left(y_k - g_k + 1\right) r_k{}^{y_k-g_k}\right] \qquad \forall\ k \in K \qquad (6)$$

Average queue length L_{qk} at facility level two k

$$W_{qk} = \frac{L_{qk}}{er_k(1\text{-}P_{nk})} \qquad \forall\ k \in K \qquad (7)$$

Average waiting time W_{qk} at facility level two k

$$r_l = \frac{\lambda_l}{g_l \mu_l} \qquad \forall\ l \in l \qquad (8)$$

$$P_{0l} = \left[\frac{\left(\frac{\lambda_l}{\mu_l}\right)^{g_l}}{g_l!} \left(\frac{1-r_l{}^{y_l-g_l+1}}{1-r_l}\right) + \sum_{n=0}^{g_l-1} \frac{r_l{}^n}{n} \right]^{-1} \qquad \forall\ l \in l \qquad (9)$$

The probabilityp_{0l} , there is 0 patient at facility level three l

$$L_{ql} = \frac{P_{0l}\, r_l}{g_l!(1-r_l)^2}\left(\frac{\lambda_l}{\mu_l}\right)^{g_l}\begin{bmatrix} 1 - r_l^{y_l - g_l + 1} - (1 - r_l) \\ (y_l - g_l + 1) r_k^{y_l - g_l} \end{bmatrix} \quad \forall\ l \in l \qquad (10)$$

Average queue length L_{ql} at facility level three l

$$W_{ql} = \frac{L_{ql}}{er_l(1\text{-}P_{nl})} \quad \forall\ l \in l \qquad (11)$$

Average waiting time W_{ql} at facility level three l

$$Min\ Z_1 = \left(\sum_j fix_j\, xc_j + \sum_k fix_k\, xh_k + \sum_l fix_l\, xs_l\right) + \left(\sum_i \sum_j \sum_b s_{ij}^{ot} oc_j\, xc_j + \sum_i \sum_j \sum_k \sum_b \left(s_{ik}^{ot} + s_{ik}^{it} + s_{jk}^{it}\right) oc_k\, xh_k + \sum_i \sum_j \sum_k \sum_l \sum_b \left(s_{il}^{ot} + s_{il}^{it} + s_{il}^{st} + s_{jl}^{it} + s_{jl}^{st} + s_{kl}^{st}\right) oc_l\, xs_l\right) + \left(\sum_i \sum_j \sum_b \left(s_{ij}^{ot} + s_{ij}^{it} + s_{ij}^{st}\right) tc_{ij}\, d_{ij}\, xc_j + \sum_i \sum_j \sum_k \sum_b \begin{matrix} \left(\left(s_{ik}^{ot} + s_{ik}^{it} + s_{ik}^{st}\right)\left(tc_{ik}\, d_{ik}\, xh_k\right) + \right. \\ \left. \left(s_{jk}^{it} + s_{jk}^{st}\right)\left(tc_{jk}\, d_{jk}\, xh_k\right)\right) \end{matrix} + \sum_i \sum_j \sum_k \sum_l \sum_b \left(\left(s_{il}^{ot} + s_{il}^{it} + s_{il}^{st}\right)\left(tc_{il}\, d_{il}\, xs_l\right) + \left(s_{jl}^{it} + s_{jl}^{st}\right)\left(tc_{jl}\, d_{jl}\, xs_l\right) + \left(s_{kl}^{st}\, tc_{kl}\, d_{kl}\, xs_l\right)\right)\right) \qquad (12)$$

$$Max\ Z_2 = \sum_i \sum_j \sum_b s_{ij}^{ot} + \sum_i \sum_k \sum_b s_{ik}^{ot} + \sum_i \sum_k \sum_b s_{ik}^{it} + \sum_i \sum_l \sum_b s_{il}^{ot} + \sum_i \sum_l \sum_b s_{il}^{it} + \sum_i \sum_l \sum_b s_{il}^{st} + \sum_j \sum_k \sum_b s_{jk}^{it} + \sum_j \sum_l \sum_b s_{jl}^{it} + \sum_j \sum_l \sum_b s_{jl}^{st} + \sum_k \sum_l \sum_b s_{kl}^{st} \qquad (13)$$

$$Min\ Z_3 = \sum_j w_{qj} xc_j + \sum_k w_{qk} xh_k + \sum_l w_{ql} xs_l \qquad (14)$$

$$\sum_j S_{ij}^{ot} + \sum_k S_{ik}^{ot} + \sum_l S_{il}^{ot} \le dem_i^{ot} \quad \forall\ i \in I, b \in B \qquad (15)$$

$$\sum_j S_{ij}^{it} + \sum_k S_{ik}^{it} + \sum_l S_{il}^{it} \le dem_i^{it} \quad \forall\ i \in I, b \in B \qquad (16)$$

$$\sum_j S_{ij}^{st} + \sum_k S_{ik}^{st} + \sum_l S_{il}^{st} \le dem_i^{st} \quad \forall\ i \in I, b \in B \qquad (17)$$

$$\sum_k S_{jk}^{it} + \sum_l S_{jl}^{it} \le \sum_i S_{ij}^{it} \quad \forall\ j \in J, b \in B \qquad (18)$$

$$\sum_k S_{jk}^{st} + \sum_l S_{jl}^{st} \le \sum_i S_{ij}^{st} \quad \forall\ j \in J, b \in B \qquad (19)$$

$$\sum_l S_{kl}^{st} \le \sum_i S_{ik}^{st} + \sum_k S_{jk}^{st} \quad \forall\ k \in K, b \in B \qquad (20)$$

$$\sum_k \sum_b (S_{jk}^{it} + S_{jk}^{st}) \le \alpha\ er_j\ xc_j \quad \forall\ j \in J \qquad (21)$$

$$\sum_l \sum_b (S_{jl}^{it} + S_{jl}^{st}) \le \alpha\ er_j\ xc_j \quad \forall\ j \in J \qquad (22)$$

$$\sum_l \sum_b S_{kl}^{st} \le \alpha\ er_k xh_k \quad \forall\ k \in K \qquad (23)$$

$$\sum_i \sum_b S_{ij}^{ot} \le y_j\ xc_j \quad \forall\ j \in J \qquad (24)$$

$$\sum_i \sum_b S_{ik}^{ot} + \sum_i \sum_b S_{ik}^{it} + \sum_j \sum_b S_{jk}^{it} \le\ y_k\, xh_k \quad \forall\ k \in K \qquad (25)$$

$$\sum_i \sum_b S_{il}^{b} + \sum_j \sum_b S_{jl}^{it} + \sum_j \sum_b S_{jl}^{st} + \sum_k \sum_b S_{kl}^{st} \le\ y_l\ xs_l \quad \forall\ l \in L \qquad (26)$$

$$\sum_i \sum_b S_{ij}^{b}\, xc_j \ge MFP_j \quad \forall\ j \in J \qquad (27)$$

$$\sum_i \sum_b S_{ik}^{b}\ xh_k \ge MFP_k \quad \forall\ k \in K \qquad (28)$$

$\left(\sum_j \sum_b S_{jk}^{it} + \sum_j \sum_b S_{jk}^{st}\right) xh_k \geq MFP_k$	∀ k∈K	(29)
$\sum_i \sum_b S_{il}^{b}\, xs_l \geq MFP_l$	∀ l∈L	(30)
$\left(\sum_j \sum_b S_{jl}^{it} + \sum_j \sum_b S_{jl}^{st}\right) xs_l \geq MFP_l$	∀ l∈L	(31)
$\sum_k \sum_b S_{kl}^{st}\, xs_l \geq MFP_l$	∀ l∈L	(32)
$xc_a + xh_a + xs_a \leq 1$	∀ a∈J∩K∩L	(33)
$\sum_{j \in J} xc_j \leq num_j$		(34)
$\sum_{k \in K} xh_k \leq num_k$		(35)
$\sum_{l \in L} xs_l \leq num_l$		(36)
$d_{ij} xc_j \leq cd_j$	∀ i∈I,j∈J	(37)
$d_{ik} xh_k \leq cd_k$	∀ i∈I,k∈K	(38)
$d_{jk} xh_k \leq cd_k$	∀ j∈J,k∈K	(39)
$d_{il}\, xs_l \leq cd_l$	∀ i∈I,l∈L	(40)
$d_{jl}\, xs_l \leq cd_l$	∀ j∈J ,l∈L	(41)
$d_{kl}\, xs_l \leq cd_l$	∀k∈K,l∈L	(42)
$er_j \geq \lambda_j\, xc_j$	∀ j∈J	(43)
$er_k \geq \lambda_k\, xh_k$	∀ k∈K	(44)
$er_l \geq \lambda_l\, xs_l$	∀ l∈L	(45)
$xc_j,\ xh_k,\ xs_l \in \{0,1\}$	∀ i∈I,j∈J,k∈K,l∈L	(46)
$S_{ijb},\ S_{ikb},\ S_{ilb},\ S_{jkb},\ S_{jlb},\ S_{klb} \geq 0$	∀ i∈I,j∈J,k∈K,l∈L,b∈B	(47)

The first objective function (12) tries to reduce overall system costs, including facility setup expenses, operational costs at the facility level, and travel costs. Whereas the second objective function (13) increases the overall number of patients served by healthcare facilities. The third objective function (14) reduces the average waiting time at the facilities

According to constraints, the flow of patient b from i to facilities must be less than or equal to the number of requests (15-17). All predicted patients referred from level-one j to level-two k and level-three l should be fewer than or equal to the entrance flow to level-one j facility, according to constraints (18) and (19). All predicted patients referred from level-two k to level-three l should be fewer than or equal to the entire entrance flow to the level-two k facility, according to Constraint (20).

Constraint (21) states that patients referred to facility level-two k can only be accepted if facility level-one j was selected. Constraints (22) and (23) state that patients referred to facility l can only be accepted if facilities level-one j and level-two k were selected.

The number of patients I who attend facility-level one j must be fewer than or equal to the facility's capacity, according to Constraint (24). The number of patients i and level-one j who attend facility level-two k must be fewer than or equal to its capacity, according to Constraint (25). The number of patients I level-one j, and level-two k who enter the facility level-three l must be fewer than or equal to its capacity, according to Constraint (26). Constraints (27-32) ensure that the level-one, level-two, and level-three healthcare facilities can meet patient demand. According to Constraint (33), healthcare institutions qualified to be classified as level one, level two, or level three may only be allocated to one of the three levels. Constraints (34-36) guarantee the maximum number of level one, level two, and level three facilities are available. Constraint (37) assures that the distance between the patient point I and facility level-one j must be less than or equal. The distance between patient point i and facility-level one must be less than or equal to the coverage distance of facility-level two k, according to constraints (38) and (39). Constraints (40-42) guarantee that the distance between the patient point k, facility-level one i, and facility-level two k must be less than or equal to the facility level-three coverage distance l.Constraints (43-45) state the inflow rate of patient types b to the healthcare facility level-one, level-two, and level-three, respectively. Constraint (46) denotes binary variables, and Constraint (47) is related to the positive variables.

C.A.C Uncertainty

In this model, uncertainty related to the demand of patients (dem_i^b) is considered using the Scenario-based stochastic programming technique and three basic scenarios, pessimistic and optimistic, the number of dem_i^b.is implemented with probabilities $p_1 = 0.5$, $p_2 = 0.25$, and $p_3 = 0.25$, respectively.

IV. SOLUTION METHODS

For the hierarchical location healthcare issue, the mathematical model in section 3 is a multi-objective mixed-integer non-linear programming. As is evident in the multi-objective models, the objectives conflict with each other so that one objective cannot reach optimal value without declining the other objective functions. Multi-criteria decision-making (MCDM) strategies are used to address issues by taking into account conflicting goals. Multi-objective issues may be solved using a variety of approaches. The suggested model is solved using two multi-objective decision-making techniques: augmented epsilon-constraint and LP-metric approaches. The chosen solution approaches are discussed in the following sections.

A. Augmented epsilon-constraint method

The epsilon-constraint algorithm aims to optimize one objective function while treating the others as constraints. The epsilon-constraint approach is one of the most efficient ways for generating a collection of Pareto solutions for multi-objective optimization problems, giving managers and decision-makers a range of options. However, the result can be inefficient Pareto solutions.

As a result, the new form of the issue has been adjusted by including the slack variables in the objective function constraints as a second term in the objective function and the weight for the objective functions. (Esmaili, Amjady, & Shayanfar, 2011).

$$\begin{aligned} &Min\ \ f_1(x) - \sum_{i=2}^{n} \left(\frac{w_i}{w_1}\right)\phi_i s_i \\ &f_i(x) + s_i = e_i \ \ i = 2,3,..,n \\ &\qquad x \in X \\ &\qquad s_i \geq 0 \end{aligned} \tag{48}$$

Where s_i is a negative variable for deficiency, the range of e_i are determined based on the Lexicography method, and then the e_i are quantified. w_i ($\sum_{i=1}^{n} w_i = 1$) is the weight factors of the decision-maker. ϕ_i is a parameter that normalizes the first objective function's value concerning objective i. ($\phi_i = \frac{R(f_1)}{R(f_i)}$)

B. LP-metric method

The LP-metric approach solves multi-objective issues with conflict in the objective functions. This strategy aims to find a solution that minimizes the deviation from the ideal criteria. In other words, if the answer F^* is considered as the ideal answer, then the minor digression in the answer F^A from the answer F^* , the better the performance of the method. (In this study, p is considered 1)

$$Norm_p(F^*, F^A) = |F^* - F^A|_p = \left(\sum_{i=1}^{n} \left(f_i^* - f_i(x^A)\right)^p\right)^{\frac{1}{p}} \tag{49}$$

V. PERFORMANCE EVALUATION AND COMPARISON

To indicate validation and evaluate the performance of the solutions as mentioned above in terms of the objectives function value and CPU times, twenty hypothetical problems with different sizes (Table III) are solved with the BARON solver of the GAMS software on a laptop Intel(R), Core (TM) i7 CPU, 2.8 GHz, 8.00 GB of RAM. Parameters of the mathematical model are randomly generated using uniform distributions, and the parameters with the same values in all problems and the parameters with different values in the problems are shown in Tables I and II (a,b,c), respectively. Moreover, the weights of the objectives function are W_1=0.3, W_2= 0.5, W_3=0.2, and the Minimum flow input rate (MFP) to each level of facilities is considered 0.5 based on the capacity of the facilities.

Table III shows the values of the objective functions (Z_1, Z_2, and Z_3) and CPU times in the solution methods for twenty distinct issues, and the outcomes of the solution methods, as shown in Figures 6-9, demonstrate that the two suggested techniques follow the same pattern.

In multi-objective models, one objective function can't attain its optimum value without causing other objective functions to deteriorate. According to the augmented epsilon-constraint method's Pareto optimum positions (Table IV), the first and third objective function values increased by increasing the second objective function value. The conflict between objective functions is proved. This conflict is depicted in Figure 10. These results can help decision-makers reach their optimal objective functions regarding the degree of importance of objective functions.

Table I. Parameter with the same values in all problems

Parameter	*Value*	*Parameter*	*Value*
fix_j	~ uniform (100000 ,150000)	cd_l	~ uniform (40,50)
fix_k	~ uniform (500000,550000)	tc_{ij}	~ uniform (10,12)
fix_l	~ uniform (1000000,1500000)	tc_{ik}	~ uniform (10,12)
oc_j	~ uniform (100,150)	tc_{il}	~ uniform (10,12)
oc_k	~ uniform (500,550)	tc_{jk}	~ uniform (12,14)
oc_l	~ uniform (100,1500)	tc_{jl}	~ uniform (12,14)
cd_j	~ uniform (30,35)	tc_{kl}	~ uniform (14,16)
cd_k	~ uniform (35,40)		

Table IIa. Parameter with the different values in the problems

Parameter	*Problem number*						
	1	*2*	*3*	*4*	*5*	*6*	*7*
$\mu_j \sim uniform$	(0.7,0.8)	(0.8,0.9)	(0.7,0.8)	(0.8,0.9)	(0.7,0.8)	(0.8,0.9)	(0.7,0.8)
$\lambda_j \sim uniform$	(0.6,0.7)	(0.7,0.8)	(0.6,0.7)	(0.7,0.8)	(0.6,0.7)	(0.7,0.8)	(0.6,0.7)
$\mu_k \sim uniform$	(0.5,0.6)	(0.6,0.7)	(0.6,0.7)	(0.5,0.6)	(0.6,0.7)	(0.6,0.7)	(0.6,0.7)
$\lambda_k \sim uniform$	(0.4,0.5)	(0.5,0.6)	(0.5,0.6)	(0.4,0.5)	(0.5,0.6)	(0.5,0.6)	(0.5,0.6)
$\mu_l \sim uniform$	(0.3,0.4)	(0.4,0.5)	(0.3,0.4)	(0.4,0.5)	(0.4,0.5)	(0.4,0.5)	(0.4,0.5)

$\lambda_l \sim uniform$	(0.2,0.3)	(0.3,0.4)	(0.2,0.3)	(0.3,0.4)	(0.3,0.4)	(0.3,0.4)	(0.3,0.4)
$g_k \sim uniform$	(10,15)	(16,20)	(10,15)	(10,15)	(16,20)	(16,20)	(16,20)
$g_l \sim uniform$	(10,15)	(16,20)	(16,20)	(16,20)	(10,15)	(16,20)	(16,20)
$y_j \sim uniform$	(30,40)	(41,50)	(51,60)	(51,60)	(30,40)	(41,50)	(51,60)
$y_k \sim uniform$	(50,60)	(61,70)	(71,80)	(50,60)	(71,80)	(61,70)	(71,80)
$y_l \sim uniform$	(50,60)	(61,70)	(71,80)	(61,70)	(50,60)	(61,70)	(71,80)

Table IIb. Parameter with the different values in the problems

Parameter	***Problem number***						
	8	***9***	***10***	***11***	***12***	***13***	***14***
$\mu_j \sim uniform$	(0.8,0.9)	(0.7,0.8)	(0.7,0.8)	(0.7,0.8)	(0.8,0.9)	(0.7,0.8)	(0.7,0.8)
$\lambda_j \sim uniform$	(0.7,0.8)	(0.6,0.7)	(0.6,0.7)	(0.6,0.7)	(0.7,0.8)	(0.6,0.7)	(0.6,0.7)
$\mu_k \sim uniform$	(0.5,0.6)	(0.5,0.6)	(0.5,0.6)	(0.6,0.7)	(0.6,0.7)	(0.5,0.6)	(0.6,0.7)
$\lambda_k \sim uniform$	(0.4,0.5)	(0.4,0.5)	(0.4,0.5)	(0.5,0.6)	(0.5,0.6)	(0.4,0.5)	(0.5,0.6)
$\mu_l \sim uniform$	(0.4,0.5)	(0.3,0.4)	(0.4,0.5)	(0.3,0.4)	(0.3,0.4)	(0.3,0.4)	(0.3,0.4)
$\lambda_l \sim uniform$	(0.3,0.4)	(0.2,0.3)	(0.3,0.4)	(0.2,0.3)	(0.2,0.3)	(0.2,0.3)	(0.2,0.3)
$g_k \sim uniform$	(10,15)	(10,15)	(10,15)	(16,20)	(10,15)	(10,15)	(10,15)
$g_l \sim uniform$	(16,20)	(10,15)	(16,20)	(10,15)	(10,15)	(10,15)	(16,20)
$y_j \sim uniform$	(51,60)	(30,40)	(30,40)	(41,50)	(51,60)	(30,40)	(51,60)
$y_k \sim uniform$	(50,60)	(50,60)	(71,80)	(61,70)	(61,70)	(50,60)	(71,80)
$y_l \sim uniform$	(61,70)	(50,60)	(61,70)	(71,80)	(50,60)	(50,60)	(71,80)

Table IIc. Parameter with the different values in the problems

Parameter	***Problem number***					
	15	***16***	***17***	***18***	***19***	***20***
$\mu_j \sim uniform$	(0.7,0.8)	(0.8,0.9)	(0.8,0.9)	(0.7,0.8)	(0.8,0.9)	(0.7,0.8)
$\lambda_j \sim uniform$	(0.6,0.7)	(0.7,0.8)	(0.7,0.8)	(0.6,0.7)	(0.7,0.8)	(0.6,0.7)
$\mu_k \sim uniform$	(0.6,0.7)	(0.5,0.6)	(0.6,0.7)	(0.5,0.6)	(0.5,0.6)	(0.6,0.7)
$\lambda_k \sim uniform$	(0.5,0.6)	(0.4,0.5)	(0.5,0.6)	(0.4,0.5)	(0.4,0.5)	(0.5,0.6)
$\mu_l \sim uniform$	(0.3,0.4)	(0.4,0.5)	(0.4,0.5)	(0.4,0.5)	(0.3,0.4)	(0.4,0.5)
$\lambda_l \sim uniform$	(0.2,0.3)	(0.3,0.4)	(0.3,0.4)	(0.3,0.4)	(0.2,0.3)	(0.3,0.4)
$g_k \sim uniform$	(16,20)	(10,15)	(16,20)	(10,15)	(16,20)	(16,20)
$g_l \sim uniform$	(10,15)	(16,20)	(16,20)	(16,20)	(10,15)	(16,20)
$y_j \sim uniform$	(41,50)	(51,60)	(41,50)	(30,40)	(41,50)	(51,60)
$y_k \sim uniform$	(61,70)	(50,60)	(61,70)	(71,80)	(50,60)	(71,80)
$y_l \sim uniform$	(71,80)	(61,70)	(61,70)	(61,70)	(50,60)	(71,80)

Table III. The results obtained by the solution methods

	Number of potential location	***LP-Metric***				***Augmented Epsilon-Constraint***			
Problem No	***i-j-k-l***	Z_1	Z_2	Z_3	***time***	Z_1	Z_2	Z_3	***time***
1	6-2-1-1	2287100	207	2.09	1.6113	2287473	210	2.2	1.23
2	6-2-2-2	4974991	438	3.78	7.6956	4975737	444	4.01	5.83
3	6-3-2-1	3778429	336	8.53	7.8792	3780294	351	9.55	5.88

4	6-3-2-2	5240563	459	8.69	17.94	5242428	474	9.72	13.8
5	6-3-3-3	8215866	546	8.41	46.9431	8407376	612	10.6	36.39
6	8-2-2-2	5253735	435	3.78	12.9536	5254854	444	4.13	10.12
7	8-3-2-2	6005747	519	9.34	27.8901	6008358	540	10.76	20.97
8	8-3-3-2	6798597	492	9.32	49.5232	6801581	516	10.98	38.69
9	8-3-3-3	8401605	513	7.83	94.2975	8403843	531	9.03	74.25
10	8-4-3-2	7694281	501	13.64	80.7	7696519	519	15.45	64.56
11	8-4-3-3	10703987	741	18.34	153.02	10707717	771	21.49	109.3
12	8-4-4-4	13467511	762	22.66	341.8314	13469749	780	24.61	260.94
13	10-3-2-2	5869531	447	8.25	41.1734	5994725	456	8.84	32.42
14	10-3-3-3	10835682	825	11.63	126.7712	10842396	879	15.29	99.04
15	10-4-3-2	8614366	552	17.23	128.2905	8616977	573	19.46	99.45
16	10-4-3-3	10340546	672	19.82	222.391	10343903	699	22.74	171.07
17	10-4-4-4	16032118	957	20.25	521.56	16035102	981	22.77	401.2
18	10-5-3-3	11836430	672	21.01	339.0542	11841652	714	26.34	258.82
19	10-5-4-3	12252863	609	24.42	527.275	12255474	630	27.22	402.5
20	10-5-4-4	19341026	1137	34.42	805.3356	19344383	1164	38.06	614.76

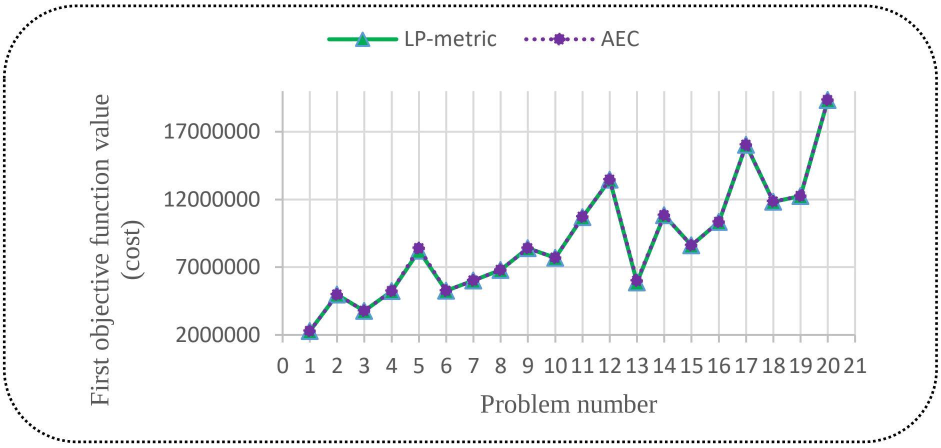

Fig 6. The first objective function values in both proposed methods for the generated problems

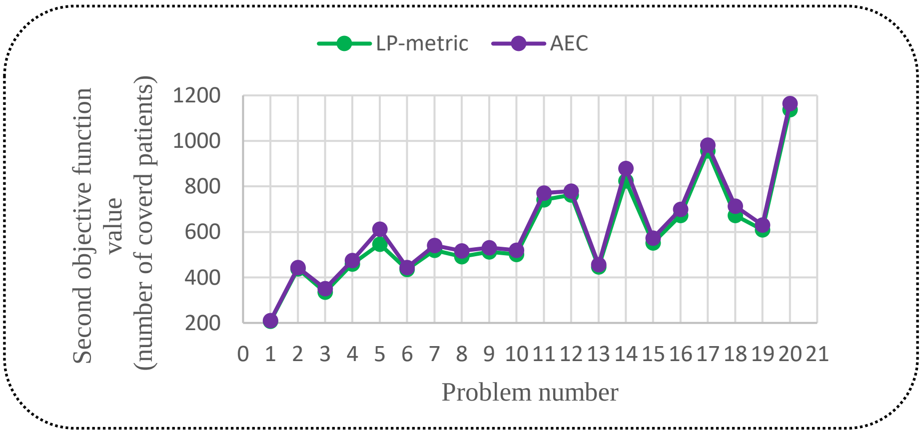

Fig 7. The second objective function values in both proposed methods for the generated problems

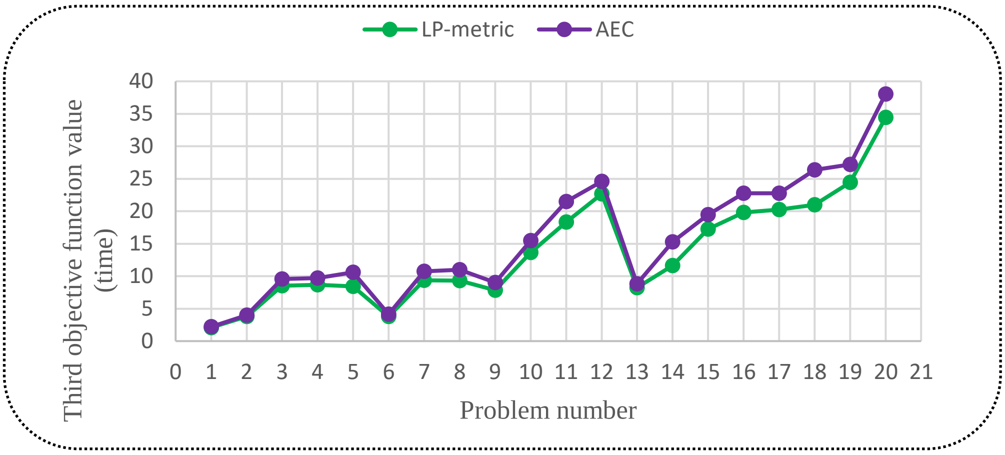

Fig 8. The third objective function values in both proposed methods for the generated problems

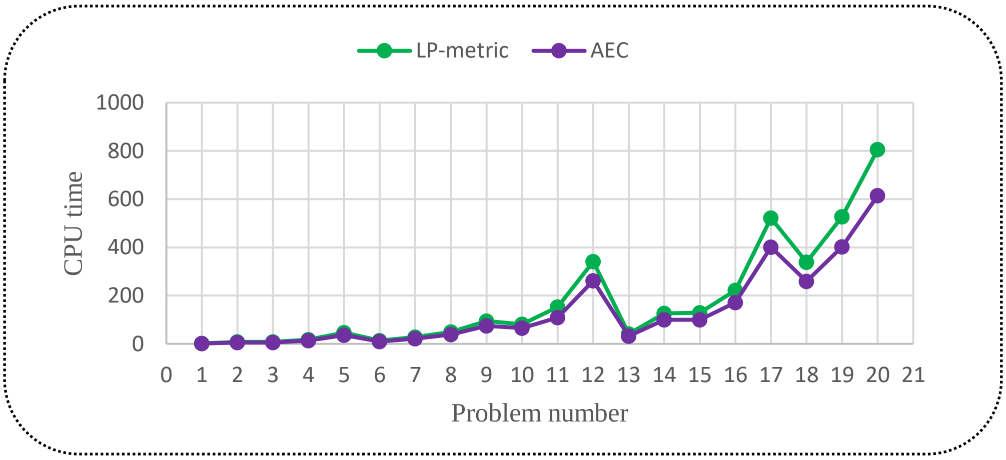

Fig 9. CPU time in both proposed methods for the generated problems

Table IV. The obtained Pareto optimal points by the Augmented epsilon-constraint method

Pareto optimal points	***First objective value***	***Second objective value***	***Third objective value***
1	8403843	531	9.03
2	8404589	537	9.42
3	8405335	543	9.82
4	8405708	546	10.02
5	8423864	558	10.62
6	8594980	594	11.02
7	8680718	612	11.12
8	8784020	630	11.22
9	8907642	633	12.02

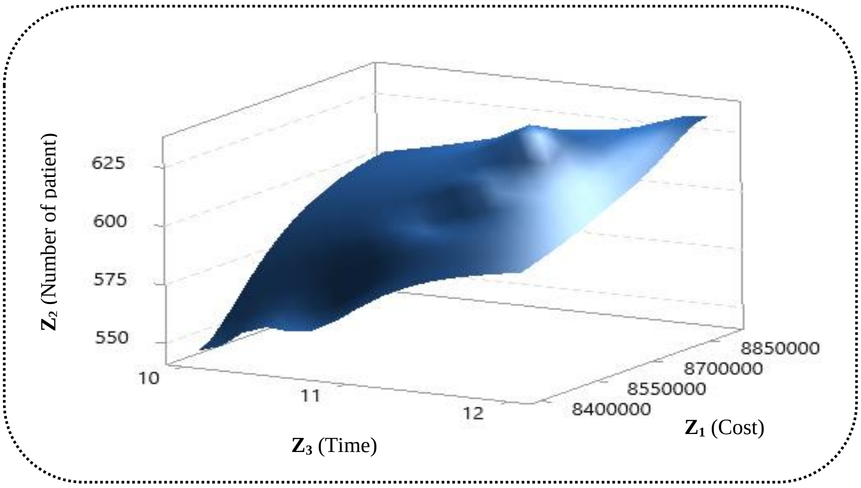

Fig 10. The conflict of the objective functions

A. Statistical comparison

To analyze and compare the results of the solution methods with each other, paired t-test at the 95% significant level is performed.

The paired *t*-test is a statistical method for comparing two population means when the samples are matched pairs. 1) The test can only be performed with matched pairings. 2) It is assumed that normal distributions exist. 3) The two samples have the same variance. 4) Cases must be distinct from one another.

For the mean objectives function values and CPU times comparison, the hypotheses are as follows:

H_0: $\mu_{LP-m} = \mu_{AEC}$

H_1: $Otherwise$ 0

The test results using MINITAB 19.2 software for the equality of means for the objectives function values (OFV) and CPU times are presented in Tables V and VII.

There is no significant difference between the results of the provided approaches regarding the values of the First objective function since the significance for the equality of the first objective function is more than 0.05. The importance of the second, third, and CPU time means being equal is less than 0.5; hence, the equality of means assumption is rejected.

Table V. Paired sample t-test for the equality of means for the first objective function values of Augmented epsilon-constraint and LP-metric

		Paired difference					*T*	*Df*	*Sig. (2-tailed)*
		Mean	*Std. deviation*	*Std. error mean*	*95% confidence interval of the difference*				
					Lower	*Upper*			
Pair z1	LP-m ,AEC	-18278	49118	10983	-41266	4709	-1.66	19	0.112

Table VI. Paired sample t-test for the equality of means for the second objective function values of Augmented epsilon-constraint and LP-metric

		Paired difference					*T*	*Df*	*Sig. (2-tailed)*
		Mean	*Std. deviation*	*Std. error mean*	*95% confidence interval of the difference*				
					Lower	*Upper*			
Pair z2	LP-m ,AEC	-23.40	15.47	3.46	-30.64	-16.16	-6.76	19	0.000

Table VII. Paired sample t-test for the equality of means for the third objective function values of Augmented epsilon-constraint and LP-metric

		Paired difference					*T*	*Df*	*Sig. (2-tailed)*
		Mean	*Std. deviation*	*Std. error mean*	*95% confidence interval of the difference*				
					Lower	*Upper*			
Pair z3	LP-m ,AEC	-1.990	1.341	0.300	-2.618	-1.363	-6.64	19	0.000

Table VIII. Paired sample t-test for the equality of means for CPU times of Augmented epsilon-constraint and LP-metric

		Paired difference					*T*	*Df*	*Sig. (2-tailed)*
		Mean	*Std. deviation*	*Std. error mean*	*95% confidence interval of the difference*				
					Lower	*Upper*			
Pair time	LP-m ,AEC	-41.6	52.2	11.7	17.2	66.1	3.56	19	0.002

B. The Best solution method

The technique for order of preference by similarity to the ideal solution (TOPSIS) is a multi-criteria decision analysis method developed by Hwang, Lai, and Liu (Hwang, Lai, & Liu, 1993) to determine the best alternative based on the concept that the chosen alternative should have the shortest geometric distance from the positive ideal solution and the longest geometric distance from the negative ideal solution. This method's stages are as follows:

Step1: Create evaluation matrix X consisting of m alternatives and n criteria

$$X = [r_{ij}] \qquad i=1,\dots,m \quad j=1,\dots,n \tag{50}$$

Step2: normalized matrix X to the matrix R by the normalization method r_{ij}

$$R = [r_{ij}] \qquad i=1,\dots,m \quad j=1,\dots,n \tag{51}$$

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \qquad i=1,\dots,m \quad j=1,\dots,n \tag{52}$$

Step3: Calculate the weighted normalized decision matrix V_{ij}

$$V_{ij} = W_j * r_{ij} \qquad i=1,\dots,m \quad j=1,\dots,n \tag{53}$$

Where W_j is the weight of indicator j and also $\sum_1^n W_j = 1$

Step 4: Determine the worst alternative $A^-$and the best alternative A^+

$$A^+ = \{(max\, V_{ij} | j \in J+), (min\, V_{ij} | j \in J-) | i = 1,2 \dots, m\} = \{V_1^+, V_2^+, \dots, V_n^+\} \tag{54}$$

$$A^{-} = \{(min\, V_{ij} | j \in J+), (max\, V_{ij} | j \in J-) | i = 1,2 \dots, m\} = \{V_1^{-}, V_2^{-}, \dots, V_n^{-}\} \quad (55)$$

Step5: Calculate the distance between alternative *i* and the best A^{+} and the worst A^{-} alternative

$$d_i^{+} = \sqrt{\sum_{j=1}^{n} (V_{ij} - V_i^{+})^2} \qquad i=1,2,\dots,m \quad (56)$$

$$d_i^{-} = \sqrt{\sum_{j=1}^{n} (V_{ij} - V_i^{-})^2} \qquad i=1,2,\dots,m \quad (57)$$

Step6: Calculate the similarity to the worst condition

$$CL_i^{*} = \frac{d_i^{-}}{d_i^{-} + d_i^{+}} \qquad i=1,2,\dots,m \quad (58)$$

Step7: Ranked the alternatives

According to four criteria (first, second and third objective functions and CPU time) and two alternatives (solution methods), TOPSIS method results show that the Augmented epsilon-constraint method (rank one) is a better solution than the LP-metric method (rank two).

VI. SENSITIVITY ANALYSIS

The impact of changes in the weights of objective functions and the entry rate on the values of the objective functions are specified using sensitivity analysis in this section. The augmented epsilon-constraint approach is utilized to do the sensitivity analysis based on the results of the preceding section.

A. Changes in the weights of objective functions

The weights of the two objective functions are adjusted, while the third objective function's weight remains constant.

According to Tables IX-XI, the weights of the first objective versus the second objective, the first objective versus the third objective, and the second objective versus the third objective are tested, respectively. All three tests are performed with the Augmented epsilon-constraint method, selected in section 5 as a better solution, and problem no 8.

Table IX. Computational results for test 1. (problem no.8)

Scenario	*Weights*			*Objective function*			*CPU-time*
	W1	*W2*	*W3*	*Z1*	*Z2*	*Z3*	
1	0.8	0.1	0.1	5940906	243	5.1558	37.49
2	0.7	0.2	0.1	5940906	243	5.1558	37.03
3	0.6	0.3	0.1	5950231	318	6.5061	37.32
4	0.5	0.4	0.1	5950231	318	6.5061	37.29
5	0.4	0.5	0.1	6756921	558	11.2192	37.18
6	0.3	0.6	0.1	6851204	586	11.9892	37.39
7	0.2	0.7	0.1	7237377	642	13.1350	37.85
8	0.1	0.8	0.1	7237377	642	13.1350	37.58

Table X. Computational results for test 2. (problem no.8)

Scenario	*Weights*			*Objective function*			*CPU-time*
	W1	*W2*	*W3*	*Z1*	*Z2*	*Z3*	
1	0.8	0.1	0.1	5940906	243	5.1558	37.49
2	0.7	0.1	0.2	6000315	245	5.1558	38.54
3	0.6	0.1	0.3	6059724	250	5.1558	38.39
4	0.5	0.1	0.4	6089429	252	5.1558	38.51
5	0.4	0.1	0.5	6101310	267	5.3627	38.35
6	0.3	0.1	0.6	6119133	280	5.5286	38.50
7	0.2	0.1	0.7	6125074	292	5.5842	38.43
8	0.1	0.1	0.8	6335106	362	7.4063	38.23

Table XI. Computational results for test 3. (problem no.8)

Scenario	*Weights*			*Objective function*			*CPU-time*
	W1	*W2*	*W3*	*Z1*	*Z2*	*Z3*	
1	0.1	0.8	0.1	7237377	642	13.1350	38.46
2	0.1	0.7	0.2	7237377	642	13.1350	38.47
3	0.1	0.6	0.3	7227942	567	11.2092	37.99
4	0.1	0.5	0.4	7227942	567	11.2092	38.19
5	0.1	0.4	0.5	7127087	539	11.0854	37.87
6	0.1	0.3	0.6	7086745	527	10.9800	38.54
7	0.1	0.2	0.7	7066574	521	10.9800	38.50
8	0.1	0.1	0.8	6335106	363	7.4063	38.47

The value of the first objective function rises with the steady drop of w_1 and growth of w_2 in test 1 (Figure 11). The value of the objective function rose in the second test (Figure 12) by reducing w_1 and increasing w_3. The weight of the objective function (w_1) is deemed fixed in the third test (Figure 13); yet, its value, which has an almost constant slope at first, declines towards the conclusion. The numerical findings for the first objective function include mean values and standard deviations of 6483144.125 and 598115.3107 in test 1, 6096374.625, and 115355.4012 in test 2, and 7068268.75 and 304611.6362 in test 3. Therefore, the highest dispersion of the first objective function is related to test 1, and the lowest is in test 2.

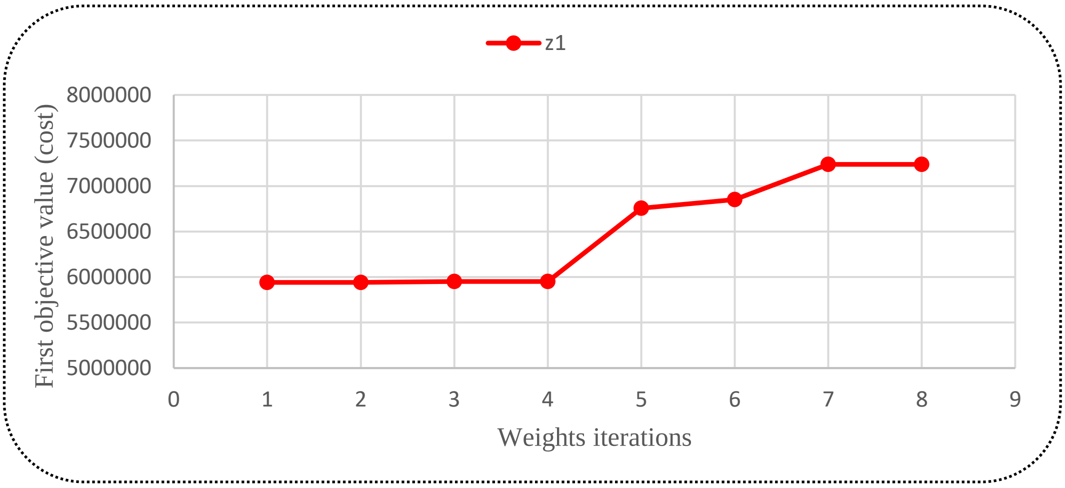

Fig 11. First objective function values in test 1

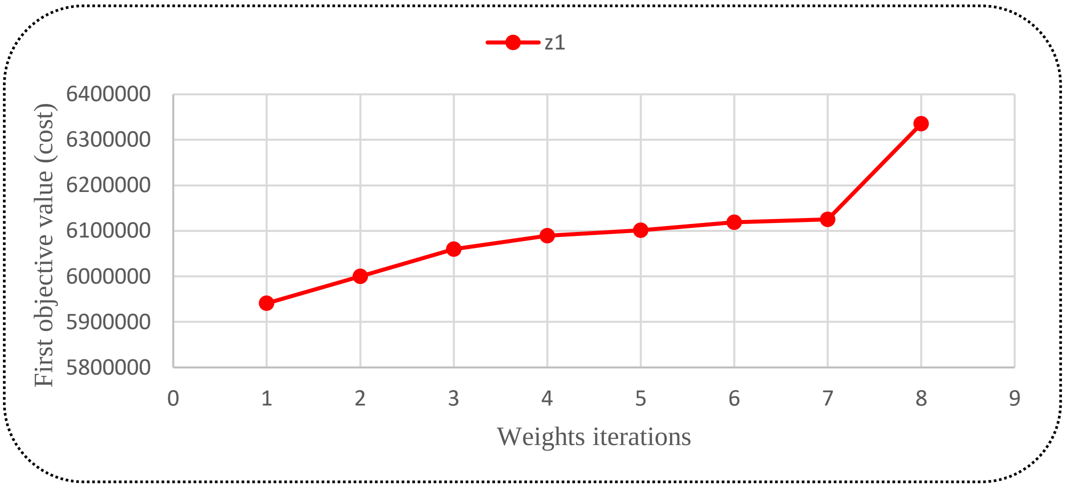

Fig 12. First objective function values in test 2

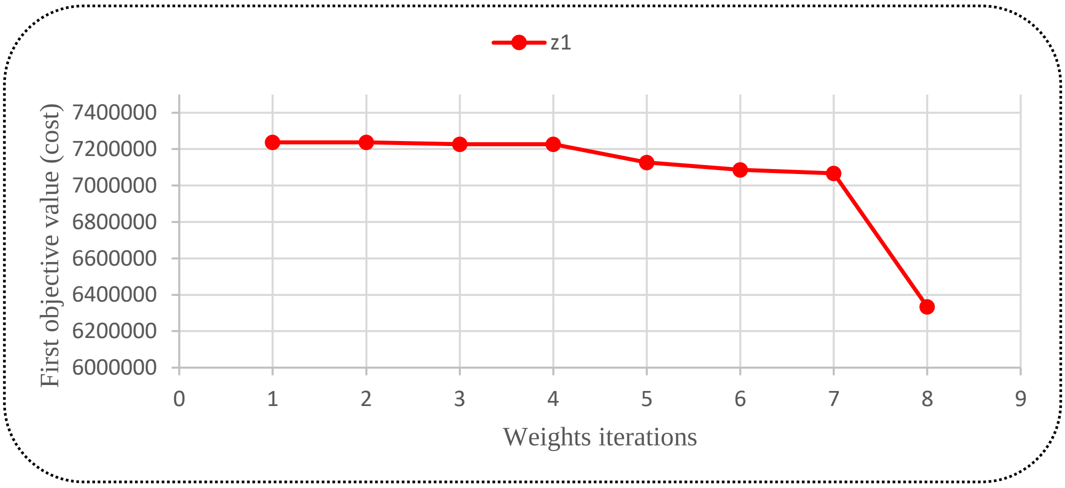

Fig 13. First objective function values in test 3

According to test 1 (Figure 14), the second objective function value increased by increasing w_2= 0.1 to 0.8 and declining w_1. In test 2 (Figure 15), the amount of the second weight is considered fixed, but the value of the second objective function grows when $w1$ decreases to 0.1 and $w3$ increases to 0.8. In test 3 (Figure 16), the second objective weight is reduced, and the third objective weight increases.

Although the second objective function's value remains constant at the start, it decreases significantly at the end. The numerical findings for the second objective function had mean values and standard deviations of 443.75 and 178.9339 in test 1, 273.875 and 39.66804 in test 2, and 546 and 87.69916 in test 3. Thus, the highest dispersion of the second objective function is in test 1, and the lowest one is in test 2.

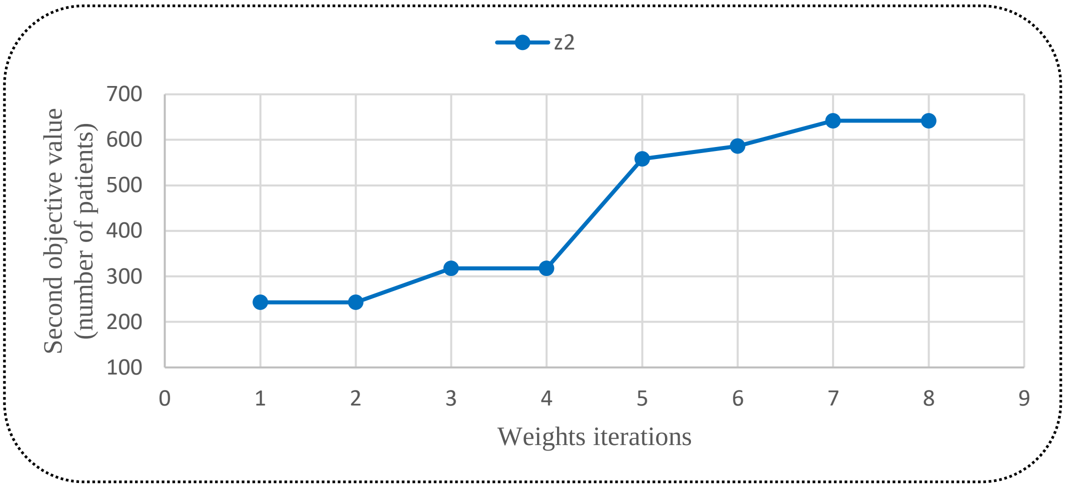

Fig 14. Second objective function values in test 1

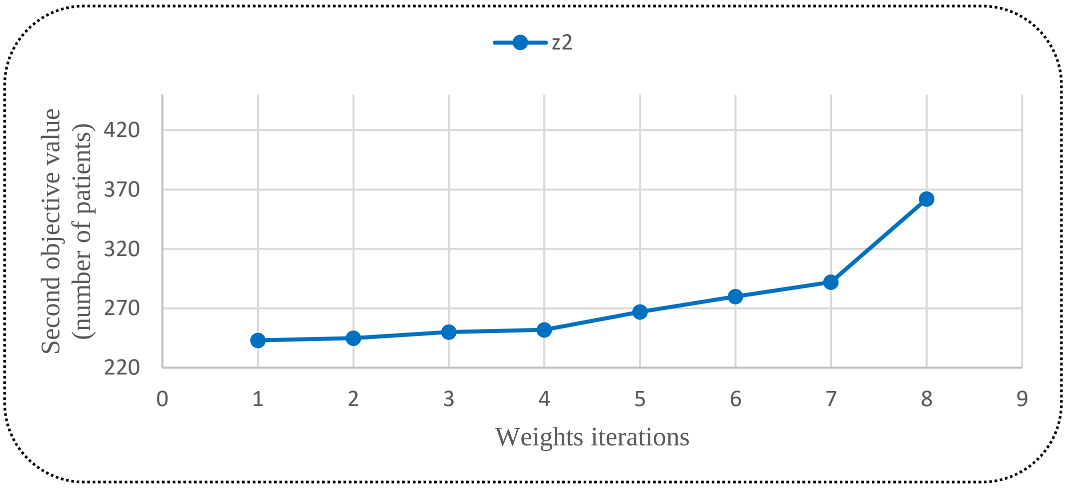

Fig 15. Second objective function values in test 2

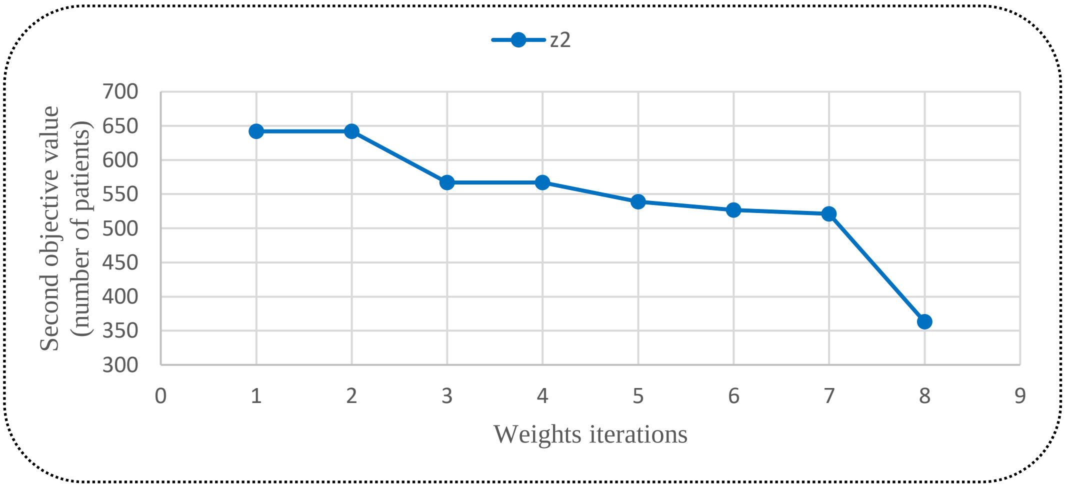

Fig 16. Second objective function values in test 3

In contrast to its constant weight, the value of the third objective function in test 1 (Figure 17) displays an increasing tendency.

In test 2 (Figure 18), by increasing *w3* and declining w1, the value of the third function is approximately constant, and just, in the end, the value increased. In test 3 (Figure 19), by increasing the third objective weight from 0.1 to 0.8 and (*w2* from 0.8 to 0.1), The value of the third objective function decreased. However, it has a constant slope in the middle. The numerical findings for the third objective function include mean values and standard deviations of 9.10 and 3.585102 in test 1, 5.563 and 0.765589 in test 2, 11.1425 and 1.771351 in test 3, respectively. The third objective function has the highest dispersion in test 1, and the lowest is in test 2.

The second, first, and third objective functions significantly influenced the dispersion of objective function values when the objective function weights were connected.

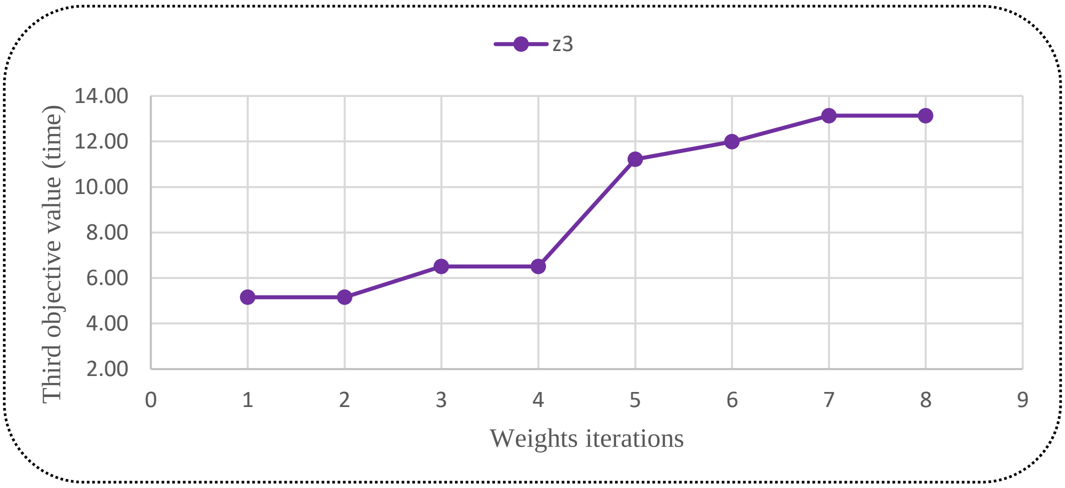

Fig 17. Third objective function values in test 1

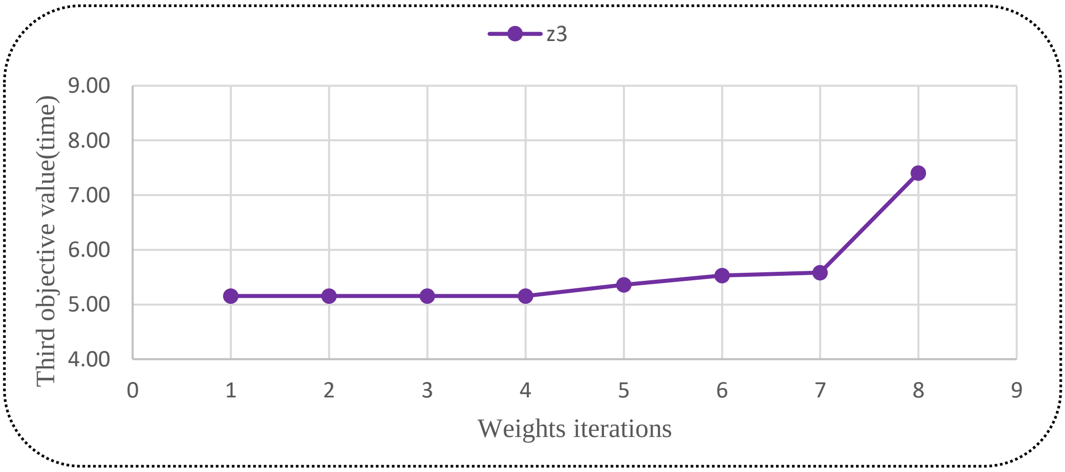

Fig 18. Third objective function values in test 2

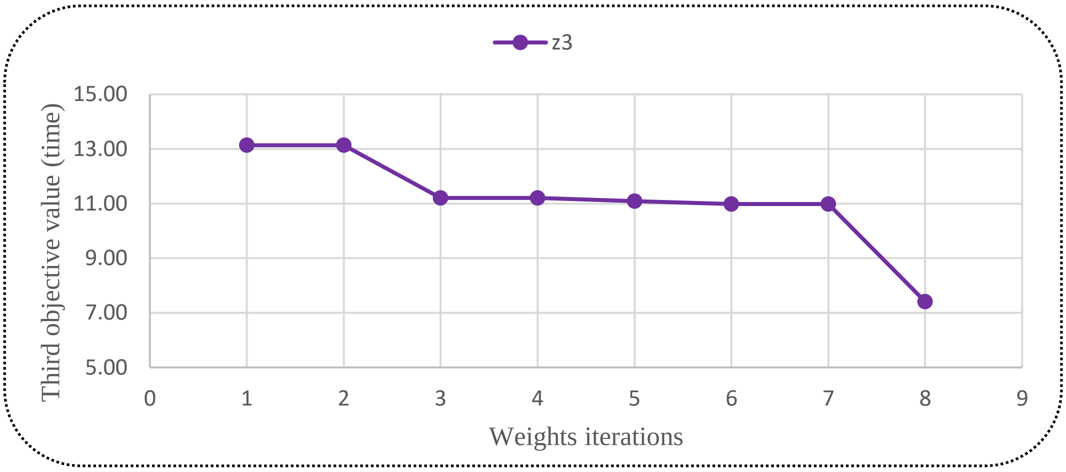

Fig 19. Third objective function values in test 3

B. Changes in the parameter λ

The impact of modifications in the entry rate parameters $(\lambda_j, \lambda_k, \lambda_l)$ on the objective functions is studied in this section (Figures 20-28). All tests are performed with the Augmented epsilon-constraint method, selected in Section V as a better solution, and problem no 9.

Increasing the value of the parameter λ_j at all of the rates increased all three objective function values.

The mean values and standard deviations for the first, second, and third objective function values are 8434292.02 and 20194.28, 549 and 14.468, 8.808 and 0.233, respectively (Figures 20-22).

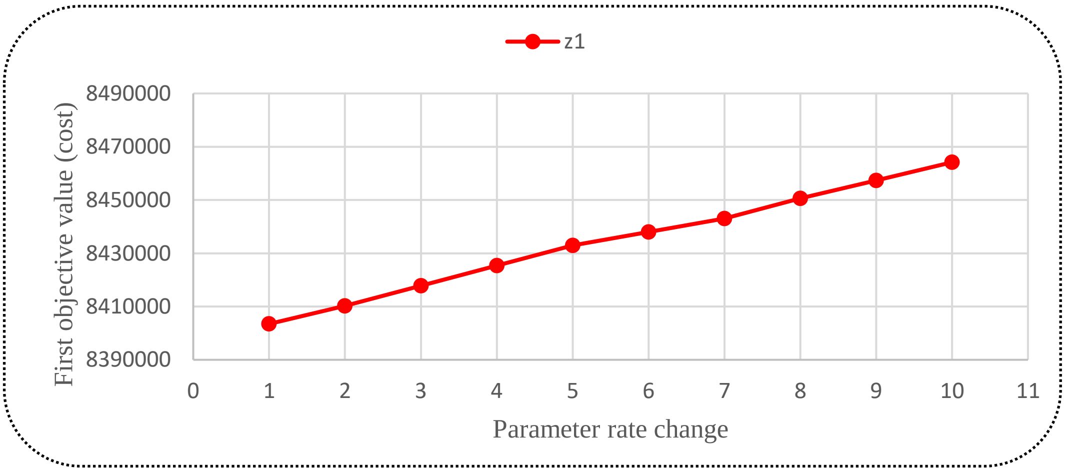

Fig 20. First objective function values by the changes in parameter λ_j

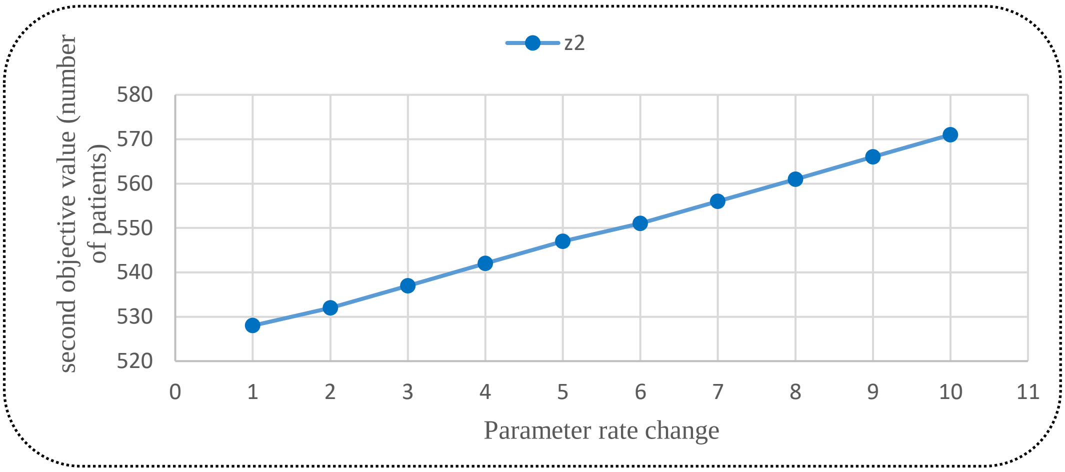

Fig 21. Second objective function values by the changes in parameter λ_j

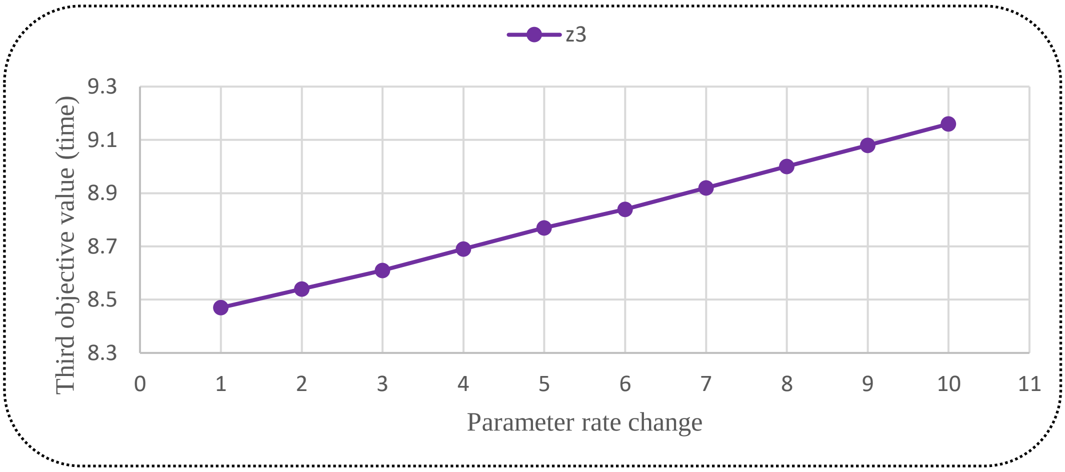

Fig 22. Third objective function values by the changes in parameter λ_j

By increasing the value of the parameter λ_k the first objective function value increased, moreover, the second and third objective function values increased slightly. The mean values and standard deviations for the first, second, and third objective function values are 8066351.97 and 16844.25, 506.3 and 1.059, 8.844 and 0.011, respectively (Figures 23-25).

The first objective function value grew when the value of the parameter λ_k was raised, and the second and third objective function values also increased marginally. The mean values and standard deviations for the first, second, and third objective function values are 8066351.97 and 16844.25, 506.3 and 1.059, and 8.844 and 0.011, respectively (Figures 23-25).

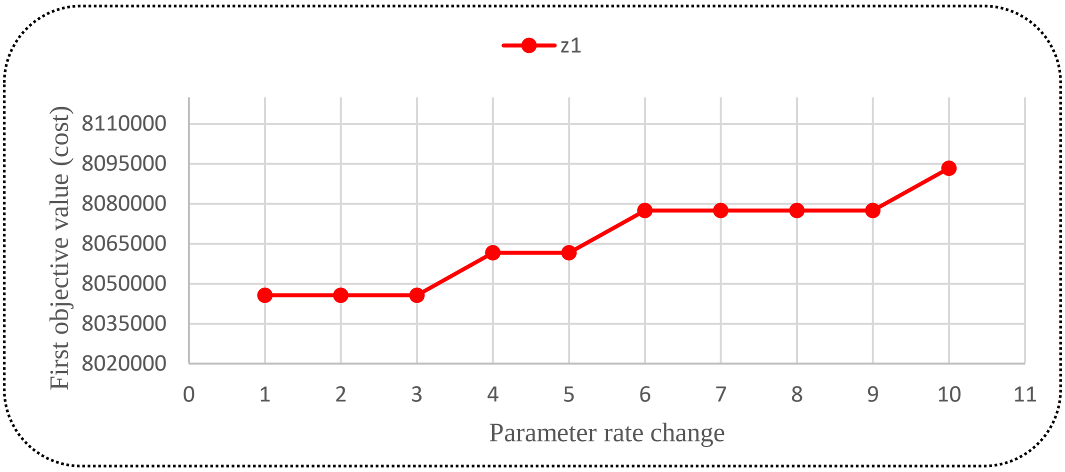

Fig 23. First objective function values by the changes in parameter λ_k

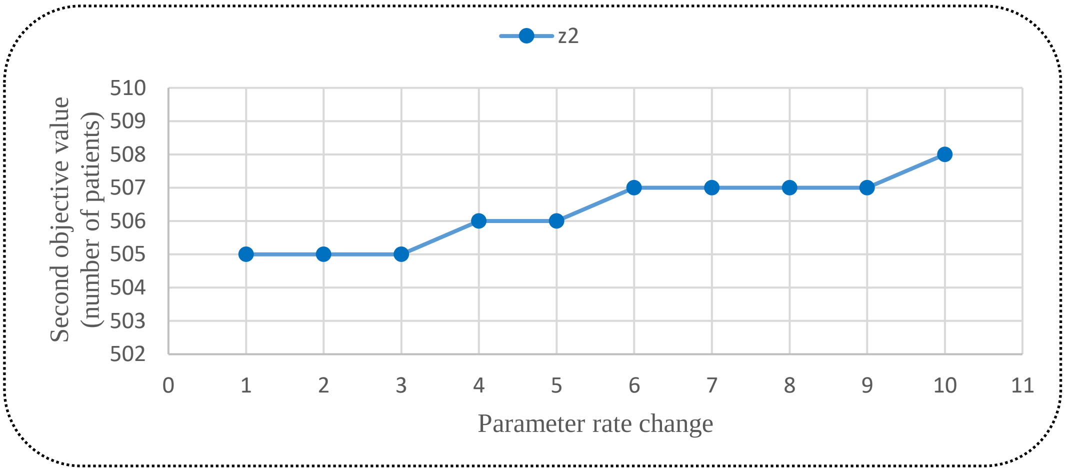

Fig 24. Second objective function values by the changes in parameter λ_k

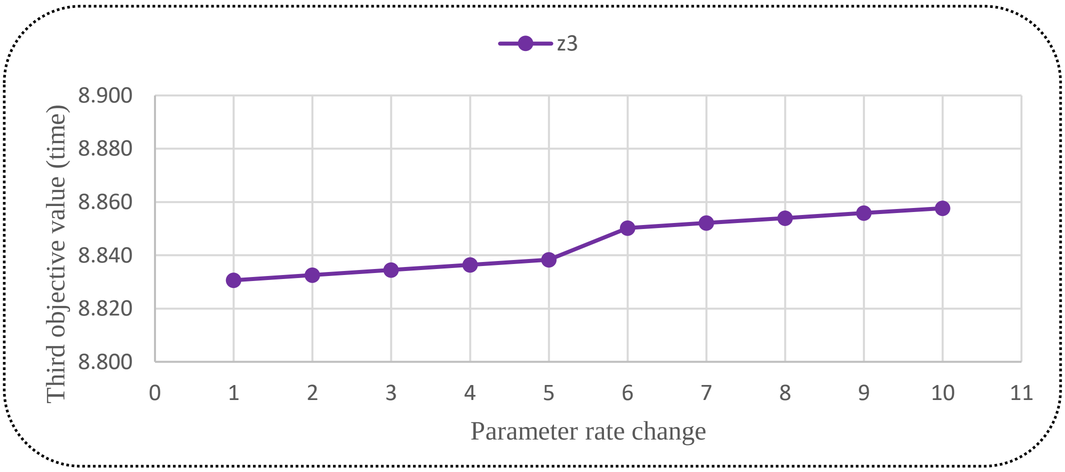

Fig 25. Third objective function values by the changes in parameter λ_k

Increasing the value of the parameter λ_l result in almost the same as the parameter λ_k. Slightly increased in the second and third objective function values and increased in the first objective function value. The mean values and standard deviations for the first, second, and third objective function values are 8060170.70 and 16892.89, 505.7 and 0.675, 8.850

and 0.012, respectively (Figures 26-28).

Increasing the value of the parameter λ_l has roughly the same outcome as increasing the value of the parameter λ_k. The values of the second and third objective functions rose somewhat, but the value of the first objective function increased. For the first, second, and third objective function values, the mean values and standard deviations are 8060170.70 and 16892.89, 505.7 and 0.675, and 8.850 and 0.012, respectively (Figures 26-28).

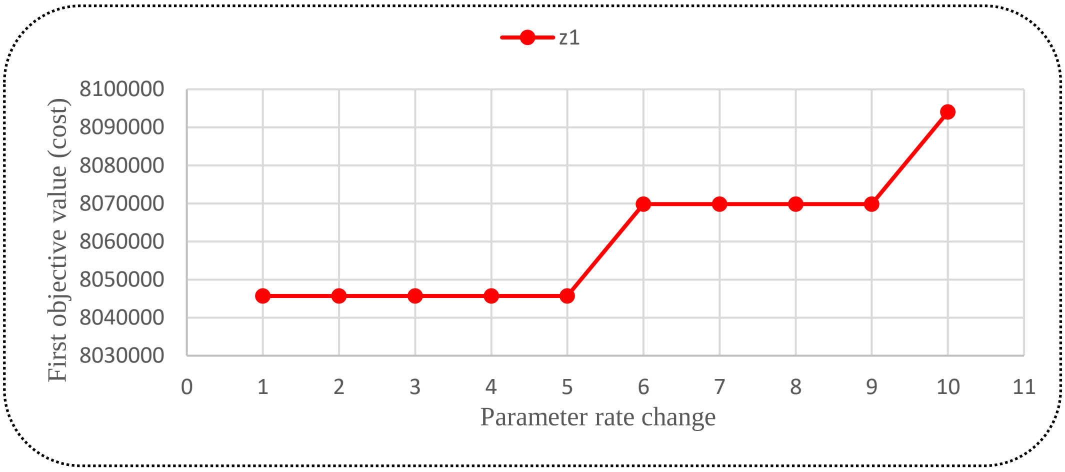

Fig 26. First objective function values by the changes in parameter λ_l

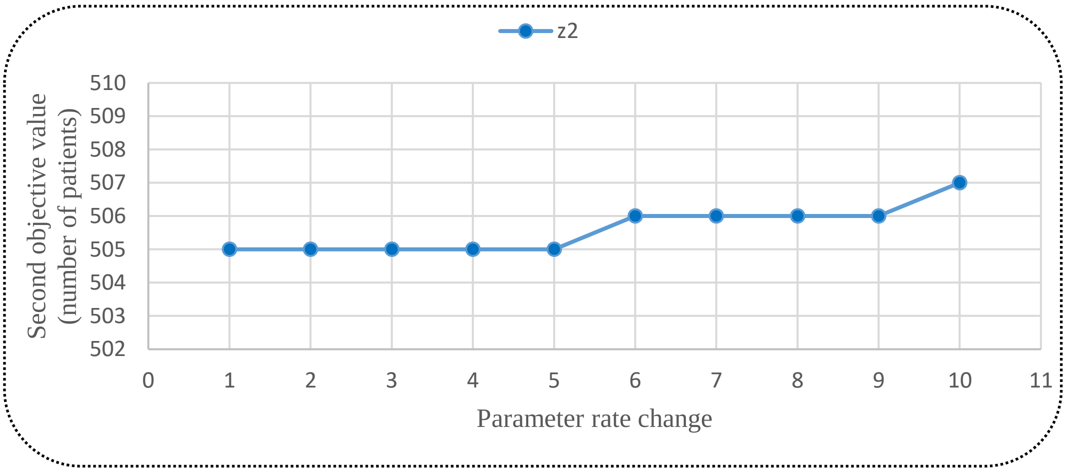

Fig 27. Second objective function values by the changes in parameter λ_l

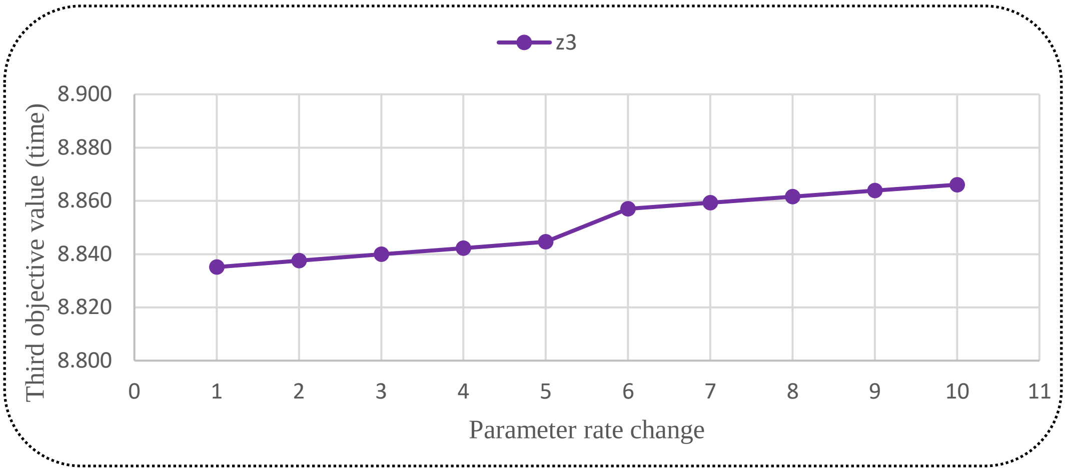

Fig 28. Third objective function values by the changes in parameter λ_l

VII. CONCLUSIONS AND FUTURE STUDY

The optimum placing healthcare facilities problem was explored in this research, as well as the relevance of this issue in decreasing expenditures and enhancing equality at the strategic level for decision-makers. As a result, a hierarchical structure placement model that considers congestion is presented. The issue is described as a mixed-integer non-linear model with three goal functions: decrease overall expenditures (establishing, operating, and transportation costs) and waiting time while concurrently increasing the number of insured patients. Two queuing systems are used for facility levels: M/M/1/K and M/M/C/K. The hierarchy structure consists of three levels of the facility, level one for the physician, level two for Clinics and General hospitals, and level three for Special and Super-special hospitals, and considering three types of patients (outpatient, inpatient, patient with special treatment). Furthermore, patients within the coverage distances of institutions may be referred to all levels and moved from a lower to a higher level.

The number of patients is considered uncertain about making the model more realistic.

The following sections solve the model using LP-metric and Augmented epsilon-constraint techniques. Twenty different hypothetical problems are employed to evaluate the efficacy of the solution methods regarding the objective function values and CPU time. The Pareto optimal points are obtained to show the conflict between the objective functions. Then, paired t-tests and TOPSIS methods are used to compare the solution methods. The first objective function indicated no significant difference, but the second, third, and CPU time showed substantial differences. The TOPSIS results also illustrate that the augmented epsilon constraint is better than the LP-metric method. In the end, the influence of modifying the entry rate parameters and the objective function weights on the model results is tested via sensitivity analysis. According to the findings, the admission rate to the facility level one (λ_j) had a greater impact on the values of the objective functions, whereas the entry rates to the facility level two and three (λ_k and λ_l) had almost the same impact. The changes in the weights of the first and second objective functions (test 1), the second and third objective functions (test 3), and the first and third objective functions (test 2), respectively, caused the most dispersion in the objective function values.

It is worth considering some recommendations to investigate for future research of the present work. In this study, the number of patients as the main parameter considers uncertain. Other elements to consider in this respect include patient travel costs, fixed installation costs, and queue characteristics (service time, entrance rate, capacity) would be uncertain. According to patient conditions, different transport mods could take into account to transfer patients from demand points to healthcare facilities and between facilities levels. Results indicate that the solution approach can only solve the suggested model with small-size instances in a fair amount of time, which is the study's primary restriction. As a result, using metaheuristic algorithms to deal with large-scale cases is advantageous.

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