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# An EPQ Model for Deteriorating Products with Delayed Payments and Shortage

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***Abstract*** *– Credit incentives are crucial tools in supply chain and inventory management. Using this strategy, the buyer could pay the purchase cost with a delay. Therefore, it will increase the order quantity and the buyer's satisfaction. This paper investigates the economic production model considering the incentive conditions for supplier credit, variable demand, deteriorating items, and shortages. It is assumed that the supplier sends the ordered items to the manufacturer on time; however, he receives the purchase price of the products after a permitted delay. Furthermore, the deterioration rate is a fixed percentage of the inventory level. Therefore, a nonlinear programming model is proposed for figuring out replenishment policy by minimizing the total inventory cost. The best replenishment policy is examined by employing Wolfram Mathematica. Moreover, a genetic algorithm is suggested due to the model's nonlinearity. Numerical analyses show that while the results do not significantly differ, the proposed GA reaches near-optimum solutions in less CPU time.*

***Keywords*** *– Delay Payment, Variable demand, Deteriorating items, Shortage*

## I. INTRODUCTION

Inventory control, which is related to many aspects of the production organization, can affect the business and play an important role in operations management activities. Early and classical models of economic production systems had many limiting assumptions. However, many of the initial assumptions were modified over time. As a result, more complex and extensive assumptions such as delayed payment, deteriorating goods, dynamic and variable demand, or discounts have been considered.

One of the leading hypotheses in classical control systems is that the manufacturer pays the purchase cost when it receives the goods from the supplier. However, such an assumption will no longer work in today's highly competitive market. Delayed payment is now a valuable promotional tool for suppliers to increase their profits by further stimulating sales. It is an excellent opportunity for manufacturers or retailers to reduce demand uncertainty. In other words, when the supplier sends the ordered units to the retailer without payment, it transfers the responsibility of storage and its costs to the retailers while taking the risk of demand uncertainty. The supplier motivates his customers to trade and encourages them to place their orders in higher quantities. This method, known as trade credit, is used as an incentive policy to attract more customers and increase customer satisfaction. The vital point in this transaction is that the trade credit changes the costs of the existing system and requires a redesign of the inventory system. Therefore, the prices of the inventory system and its optimal policy must be recalculated.

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The delayed payment time can be during the production cycle or outside the production cycle. In cases where the purchase cost has not been paid, capital expenditures are not considered for goods in the warehouse because there is no capital involved in the inventory. At the same time, the cost of capital is higher than the interest rate of sold goods for which the money has not yet been paid.

Furthermore, one of the assumptions of classical inventory control models is that the demand rate is constant. However, demand may not be fixed in practice and may depend on time, price, inventory level, etc. Moreover, the classical inventory control models assume non-deterioration items. However, many goods are deteriorating and would deteriorate over time. Foods, medicine, and grains are some examples of deteriorating items. Hence considering the deterioration of things will give more accurate results. This paper addresses the above concerns and examines an inventory system considering inventory-dependent demand, delayed payment policy, deteriorating products, and back-ordering shortages.

## II. LITERATURE REVIEW

Credit trading was studied for the first time by Haley & Higgins (1973). They considered the impact of a two-part trade credit policy on the optimal balance and payment policy. Two-part commercial credit refers to items in which the manufacturer considers a cash discount paid over a while and a specified period in a more considerable credit period. Chapman et al. (1984) will develop optimal replenishment policies under the delayed payment for Economic Order Quantity (EOQ) model. They considered the EOQ model with constant demand in which the shortage is not allowed (Chapman et al.□ 1984).

Goyal (1985) examined the delayed payment in the EOQ system and assumed the manufacturer would allow the retailer to have a predetermined period for settling its order account. Then provide a mathematical model for determining the amount of economic order. Therefore, Goyal (1985) used credit purchasing in inventory control models as a mathematical model for the first time. Teng (2002) modified the Goyal (1985) model to assume the unit price and cost difference. They showed that the retailer should order smaller to take advantage of delayed payments and make more profit (Teng, 2002). Abad & Jaggi (2003) examined the seller-buyer inventory model in which the seller used trade credit, and the buyer used the EOQ model with no shortages. They formulated the seller-buyer relationship, considering the unit price, seller charges, and length of the credit period as decision variables (Abad & Jaggi, 2003). Chung & Huang (2003) developed the Goyal's (Goyal, 1985) model for Economical Production Quantity (EPQ) with delayed payments. Then, Chung & Huang (2006) extended a model to consider the defective items in the EPQ model with delayed payments and assumed the shortage is not allowed and the demand rate is constant. Chung (2009) studied an EOQ with deteriorating items and delays in payments. He assumed that the annual demand rate is constant, Shortages are not allowed, and the time horizon is infinite (Chung, 2009). Hu & Liu (2010) investigated the EOQ model with delays in payments and allowed shortages. They assumed that the unit selling price is not necessarily equal to the unit purchasing price and the demand is constant (Hu & Liu, 2010). Khanra et al. (2011) proposed an EOQ model with a constant rate of deterioration and time-dependent demand and delay payments. Then, Min et al. (2012) examined the EPQ model with deteriorating products and delayed payments, and demand dependent on the retailer's stock level. Li et al. (2014) studied the joint order of several retailers who buy similar goods from one supplier. Delays in payments were allowed, and the results showed that forming a large coalition of retailers was socially beneficial (Li et al.□ 2014). Sadeghi et al. (2016) considered an inventory control model with discrete demand, stochastic lead time, and periodic order quantity (POQ) policy. They assumed the shortage was permitted and that a fixed percentage of items would defect during production. Patoghi & Setak (2018) considered an EOQ model for noninstantaneous deteriorating items without shortage. They assumed that the demand depends on the frequency of advertisement and the selling price. Chaudhari et al. (2020) considered a single product with seasonal demand and time-dependent deteriorating items. They assumed that the retailer could pay the purchase cost before delivery (Chaudhari et al.□ 2020). Supakar & Mahato (2020) developed a deteriorating EPQ model for a single item with delayed payment. However, they assumed the shortage was not allowed. Sadeghi et al. (2021) proposed an optimal integrated production-inventory model with multi-delivery. They assumed that shortage is permitted and fully back-ordered (Sadeghi et al.□ 2021). Duary et al. (2021) assumed that the suppliers used an offer in the price discounts for payments made by their retailers. They assumed the backlogged shortage was allowed. Sundararajan et al. (2021) analyzed partially backlogged shortages in the EOQ inventory model.

The more relevant papers are categorized and shown in Table I. As could be seen, several articles examined the delayed payments in EOQ and EPQ systems. However, there are no papers considering delayed payments for the EPQ system of deteriorating items with shortages. This study tries to fill this gap by considering variable demands.

**Table I. Summary of The Relevant Publications**

| *Author(s)* | *Demand Rare* | | *Delay payments* | *Shortages* | *Inventory type* | | *Deteriorating item* |
|---|---|---|---|---|---|---|---|
| | *Constant* | *Variable* | | | *EPQ* | *EOQ* | |
| Liao (2007) | ✓ | | ✓ | | ✓ | | ✓ |
| Sana & Chaudhuri (2008) | | ✓ | ✓ | | | ✓ | |
| Ouyang et al. (2009) | ✓ | | ✓ | | | ✓ | ✓ |
| Chung (2009) | ✓ | | ✓ | | | ✓ | ✓ |
| Mahata (2011) | ✓ | | ✓ | | | ✓ | ✓ |
| Min et al. (2012) | | ✓ | ✓ | | ✓ | | ✓ |
| Sarkar (2012) | | ✓ | ✓ | | ✓ | | ✓ |
| RezaMaiham & Kamalabadi (2012) | | ✓ | | ✓ | ✓ | | ✓ |
| Soni (2013) | | ✓ | ✓ | | | ✓ | ✓ |
| Palanivel et al. (2015) | ✓ | | ✓ | | | ✓ | ✓ |
| Tavakoli & Taleizadeh (2017) | ✓ | | | ✓ | | ✓ | ✓ |
| Sadeghi (2019b) | | ✓ | | | ✓ | | |
| Dari & Sani (2019) | | ✓ | | | ✓ | | ✓ |
| Sadeghi (2019a) | | ✓ | | | ✓ | | |
| Taleizadeh et al. (2020) | ✓ | | ✓ | ✓ | ✓ | | |
| Singh et al. (2020) | | ✓ | ✓ | | | ✓ | ✓ |
| Sadeghi et al. (2021) | ✓ | | | ✓ | ✓ | | |
| Sundararajan et al. (2021) | | ✓ | ✓ | ✓ | | ✓ | ✓ |
| This paper | | ✓ | ✓ | ✓ | ✓ | | ✓ |

To conclude, in recent years, researchers have developed classical inventory models. Among these developments, the impact of delayed payments in repayments has indeed been one of the most important developments in economic development models, which has recently gained particular importance as an incentive technique for both retailers and producers among incentive techniques. Another issue is the variable demand combined with the permissibility of shortages and deteriorating goods in the economic production models addressed in this study. This paper considers economic production planning with delayed payments, variable demand, perishable products, and the permissibility of shortages. The paper's contributions are optimally considering the allowed shortage for the EPQ model with delayed payment, variable demand, and perishable products.

## III. NOTATIONS, PROBLEM DEFINITION, AND ASSUMPTIONS

We consider an economical production system with a fixed production rate in which the demand depends on the retailer's stock level. The product is deteriorating, and the rate of deterioration is a fixed percentage of the inventory level. The supplier sells its products to the manufacturer with a trade credit option. In other words, the supplier sends the manufacturer's order but receives its purchasing cost with a delay. Delay payment times can be at the time of production, consumption, shortage, or after the end of the cycle. Accordingly, there are five possible payment intervals for which modeling the problem is different. Fig.(1) show the trend of inventory level behavior over time. Furthermore, the shortage is allowed and fully backlogged.

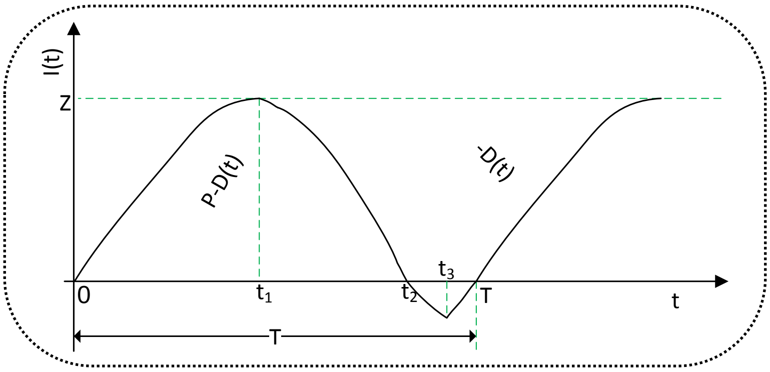


**Fig 1. Schematic of inventory level**

The used notations are provided in the following.

***Parameters:***

| | |
|---|---|
| $p$ | Production rate |
| $\lambda(t)$ | Demand rate |
| $K$ | Fixed setup cost |
| $C$ | purchasing cost per unit |
| $S$ | Selling price per unit |
| H | Holding cost per unit excluding interest received |
| V | Backordering cost per unit of product at a uint time |
| $I_e$ | Received interest rates |
| $I_c$ | Paid interest rates |
| M | Time of delayed payment |

***Dependent decision variable***

| | |
|---|---|
| Q | Order quantity per period |
| B | Backordering quantity per period |
| $I_t$ | Inventory level at time t |
| Z | The maximum inventory level |
| $t_2$ | The time of cycle in which the inventory level is equal to zero |
| $t_3$ | The time of cycle in which the back ordering level is maximum |

***Decision variable***

$t_1$ The length of production time per cycle

T The length of each cycle

## IV. MODEL DESCRIPTION

According to Fig.(1), the inventory level behavior of the problem is different at various intervals of time. Therefore, in modeling the problem, it is necessary to examine the inventory level behavior in other parts and determine the system costs. Consequently, we consider the inventory level in different parts to model the described problem.

### *A. Inventory level in time intervals* $[\mathbf{0}, \boldsymbol{t_1}]$

As shown in Fig.(1), the initial inventory is equal to zero, but the inventory level increases during the interval and reaches its maximum value $t_1$. Therefore, the maximum stock level is shown by $z$.

In this interval time, the production rate is $P$, and at the same time, demand is applied to the system at a rate of $\lambda(t)$. Also, the examined product is deteriorating, and the number of deteriorated goods is expressed as a percentage of the stock level. It should be noted that when the inventory level is positive, the demand rate is described as a function of the stock level. On the other hand, if there is no inventory, the demand rate will be constant, so the demand function is defined as follows:

$$\lambda(t) = \begin{cases} a + b \times I(t) 0 \leq t \leq t_2 \\ a t_2 \leq t \leq T \end{cases}$$

Inventory level depends on demand, production, and the deterioration rate. Therefore, the differential equation representing the inventory level for this interval time is given by

$$\frac{\partial I_1(t)}{\partial t} = P - \lambda(t) - \theta \times I(t) 0 \leq t \leq t_1 \tag{1}$$

With boundary conditions $I_1(0) = 0$. By solving Eq. (1), it gives

$$I_1(t) = \frac{e^{-bt-t\theta}(-1+e^{t(b+\theta)})(P-a)}{b+\theta} \qquad 0 \leq t \leq t_1 \tag{2}$$

### *B. Inventory level in time intervals* $[\boldsymbol{t_1}, \boldsymbol{t_2}]$

At the time $t_1$, the inventory level is maximized, and production is stopped. As a result, from to the inventory is depleted by customer demand with a rate of $\lambda(t)$ and deterioration. Hence, the differential equation representing the inventory level for this interval time can be expressed as Eq. (3).

$$\frac{\partial I_2(t)}{\partial t} = -\theta \times I_2(t) - \lambda(t) \qquad t_1 \leq t \leq t_2 \tag{3}$$

With boundary conditions $I_2(t_1) = Z$. Solving Eq. (3) gives

$$I_2(t) = \frac{e^{-(t-t_1)(b+\theta)}(a - ae^{(t-t_1)(b+\theta)} + bZ + Z\theta)}{b+\theta} \qquad t_1 \leq t \leq t_2 \tag{4}$$

### *C. Inventory level in time intervals* $[t_2, t_3]$

During this time interval, there is no on-hand inventory. Still, the demand continues, and due to the backlogged shortages, customers wait until the inventory reaches the system. So differential equation representing the inventory level for this interval time can be expressed as Eq. (5).

$$\frac{\partial I_3(t)}{\partial t} = -\lambda(t) \qquad t_2 \le t \le t_3 \tag{5}$$

With boundary conditions $I_3(t_2) = 0$. By solving Eq. (5), it gives

$$I_3(t) = -a \times (t - t_2) \qquad t_2 \le t \le t_3 \tag{6}$$

### *D. Inventory level in time intervals* $[t_3, t_4]$

As shown in Fig.(1), the inventory level in the time interval $[t_3, t_4]$ is negative. Then, at a time $t = t_3$ when the shortage is maximum, the production operation begins, and the inventory level begins to increase. Over time $t_4 = T$, all existing shortages are compensated, and the inventory level reaches zero. In this case, the production system starts to produce at a rate of $P$, and at the same time, demand is applied to the system at a rate of $\lambda(t)$. So, the differential equation representing the inventory level for this interval time can be expressed as Eq. (7).

$$\frac{\partial I_4(t)}{\partial t} = P - a \qquad t_3 \le t \le t_4 \tag{7}$$

With boundary conditions $I_4(T) = 0$. By solving Eq. (7), it gives

$$I_4(t) = -(P - a) \times (T - t) \qquad t_3 \le t \le T \tag{8}$$

The maximum inventory level occurs at a time $t_1$ which $I_2(t_1) = I_1(t_1)$. Therefore, the maximum inventory level is as follows

$$I_1(t_1) = I_2(t_1) \Rightarrow Z = \frac{e^{-t_1 \times (b+\theta)}(-1 + e^{t_1(b+\theta)})(P - a)}{(b+\theta)}$$

Also, the inventory at the time $t_2$ equals zero. Thus, $t_2$ it is as follows

$$I_2(t_2) = 0 \Rightarrow e^{-(t_2 - t_1)(b+\theta)}(a - ae^{(t_2 - t_1)(b+\theta)} + bZ + Z\theta) = 0 \Rightarrow t_2 = -\frac{1}{(b+\theta)} Ln\left(\frac{a}{(a + bZ + Z\theta)}\right) + t_1$$

At the time $t_3$, $I_3(t_3) = I_4(t_3)$. Therefore, $t_3$ gives

$$I_3(t_3) = I_4(t_3) \Rightarrow -a \times (t_3 - t_2) = -(P - a) \times (T - t_3) \Rightarrow t_3 = \frac{(P-a) \times T + a \times t_2}{P}$$

Given the values of $t_1$, and $T$, we should calculate the per-cycle total cost of the proposed production-inventory system, which depends on the variables, as shown in Eq.(9).

$$TC(t_1, T) = HC(t_1, T) + BC(t_1, T) + PC(t_1, T) + SC(t_1, T) \tag{9}$$

### *E. Holding cost*

The holding cost rate is divided into two parts: the capital cost rate per unit of money per year, denoted by $i_1$, and the holding cost rate per unit of product per year, which does not include the capital cost indicated by $i_2$. In this section, holding costs are calculated without considering the cost of capital. In the proposed model, the inventory is held at time intervals $[0, t_2]$; therefore, the holding cost could be obtained as follows:

(10)

$$HC(t_1,T) = \square \times \left( \int_0^{t_1} I_1(t)dt + \int_{t_1}^{t_2} I_2(t)dt \right) =$$
$$i_2 \times c \times \left\{ \int_0^{t_1} \frac{e^{-bt-t\theta}(-1+e^{t(b+\theta)})(P-a)}{b+\theta} dt + \int_{t_1}^{t_2} \frac{e^{-(t-t_1)(b+\theta)}(a - ae^{(t-t_1)(b+\theta)} + bZ + Z\theta)}{b+\theta} dt \right\}$$

### F. Backordering cost

The shortage occurs at time intervals $[t_2, t_3]$ and $[t_3, t_4]$. Therefore, the back ordering cost is calculated as follows

(11)

$$\boldsymbol{BC(t_1,T) = \pi \times \left( \int_{t_2}^{t_3} I_3(t)dt + \int_{t_3}^{t_4} I_4(t)dt \right) = \pi \left\{ \int_{t_2}^{t_3} a \times (t-t_2)dt + \int_{t_3}^{T} (P-a) \times (T-t)dt \right\}}$$

### G. Production cost

he production cost in each cycle is equal to the production time multiplied by the production rate. The unit production cost is equal to $C$, then the production cost for each cycle is as follows

$$Pc = C \times P \times (t_1 + T - t_3) \tag{12}$$

### H. Setup cost

Fixed setup cost is constant during each cycle and assumed equal to K. Then, the total cost per cycle is given as follows

$$TC(t_1,T) = \frac{K + PC. + HC. + BC.}{T} = \frac{K}{T} + \frac{C \times P \times (t_1 + T - t_3)}{T}$$
$$+ \frac{i_2 \times c}{T} \times \left\{ \int_0^{t_1} \frac{e^{-bt-t\theta}(-1+e^{t(b+\theta)})(P-a)}{b+\theta} dt + \int_{t_1}^{t_2} \frac{e^{-(t-t_1)(b+\theta)}(a - ae^{(t-t_1)(b+\theta)} + bZ + Z\theta)}{b+\theta} dt \right\}$$
$$+ \frac{\pi}{T} \left\{ \int_{t_2}^{t_3} a \times (t-t_2)dt + \int_{t_3}^{T} (P-a) \times (T-t)dt \right\}$$

Where $Z = \frac{e^{-t_1 \times (b+\theta)}(-1+e^{t_1(b+\theta)})(P-a)}{(b+\theta)}$ , $t_2 = -\frac{1}{(b+\theta)} Ln\left(\frac{a}{(a+bZ+Z\theta)}\right) + t_1$ and $t_3 = \frac{(P-a)\times T + a \times t_2}{P}$

The purchased raw materials by the manufacturer follow the incentive system of delayed payment. In this case, the manufacturer orders the raw materials and pays the purchase cost at a time determined by the supplier, which could be during or outside the production cycle. Accordingly, five different cases may occur at the time of payment.

**Case 1: $M < t_1$**

In this case, the manufacturer receives the raw materials at the beginning of each period but pays the purchase cost at the time $M$where $M < t_1$. Fig.(2) shows the amount of interest received from the sale of products in $(0, t_1)$. Hence, the income from the delay can be calculated as follows.

$$IE_1 = S \times I_e \int_0^M \lambda(t)dt \tag{13}$$

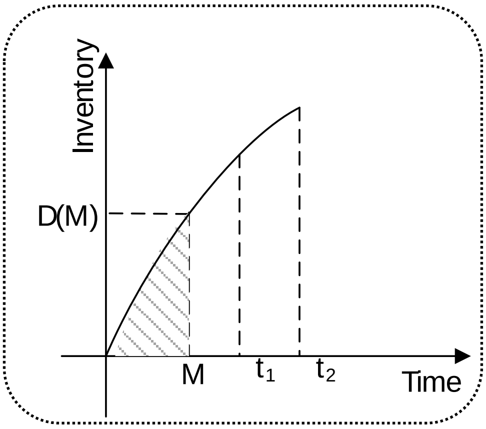


**Fig 2. The amount of inventory sold until M ($M < t_1$)**

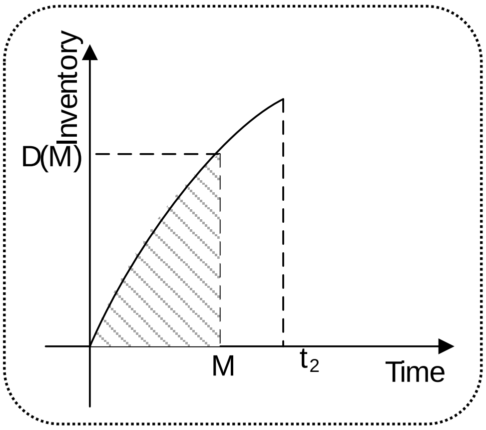


**Fig 3. The amount of inventory sold until M ($t_1 < M < t_2$)**

Furthermore, the amount of capital cost can be calculated as follows.

$$CC_1 = c \times i_1 \int_M^{t_1} I_1(t)dt + c \times i_1 \int_{t_1}^{t_2} I_2(t)dt = c \times i_1 \int_M^{t_1} \left(\frac{e^{-bt-t\theta}(-1+e^{t(b+\theta)})(P-a)}{b+\theta}\right) dt + c \times i_1 \int_{t_1}^{t_2} \left(\frac{e^{-(t-t_1)(b+\theta)}(a-ae^{(t-t_1)(b+\theta)}+bZ+Z\theta)}{b+\theta}\right) dt \quad (14)$$

**Case 2: $t_1 < M < t_2$**

In this case, the manufacturer receives the raw materials at the beginning of each period but pays the purchase cost at the time $M$where $t_1 < M < t_2$. Furthermore, the production of the product is stopped at $t_1$ and at a time interval, $(t_1, t_2)$ the product will be sold by rate $\lambda(t)$. Therefore, the inventory level is equal to zero at $t_2$.

$$IE_2 = S \times I_e \int_0^M \lambda(t)dt \quad (15)$$

In this period, the amount of capital cost can be calculated as follows.

$$CC_2 = c \times i_1 \int_M^{t_2} I_2(t)dt = c \times i_1 \int_M^{t_2} \left(\frac{e^{-(t-t_1)(b+\theta)}(a-ae^{(t-t_1)(b+\theta)}+bZ+Z\theta)}{b+\theta}\right) dt \quad (16)$$

**Case 3: $t_2 < M \leq t_3$**

During this time interval, no products are sold. Also, there is no on-hand inventory. Besides, a shortage occurs and is fully backlogged. Therefore, the income from delayed payment has been the same as the income from the sale of products until $t_2$. Fig.(4) shows the sales of products in this case. Thus, the income of delay payments can be calculated as follows.

$$IE_3 = S \times I_e \times \left( \int_0^{t_2} \lambda(t)dt + D(t_2) \times (M - t_2) \right) \tag{17}$$

In this case, the cost of capital is zero because all products produced in the production cycle have been sold, and there is no product left that includes capital involved.

**Case 4: $t_3 < M \leq T$**

At the time $t_2$, the system is in its maximum shortage, at which the system begins to produce. Then, the products made at a rate $P$are sent to customers to compensate for the shortage. Therefore, in this period, products are sold at a rate of $P$. Fig.(5) shows the sales of products in this case, based on which the amount of income from the delayed payment can be calculated as follows.

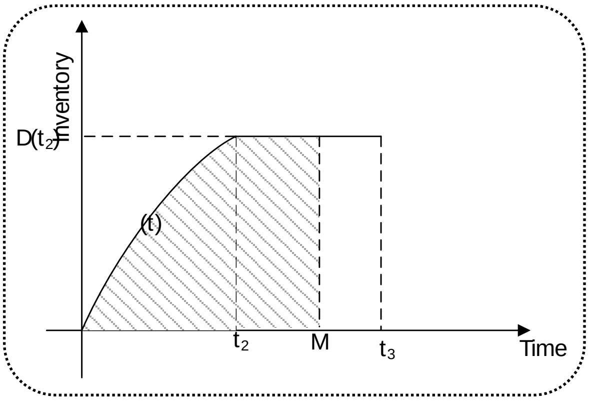


**Fig 4. The amount of inventory sold until M ($t_1 < M < t_2$)**

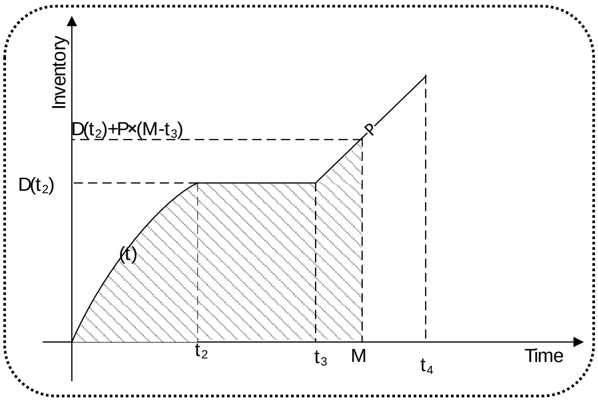


**Fig 5. The amount of inventory sold until M ($t_3 < M \leq T$)**

$$IE_4 = S \times I_e \times \left( \int_0^{t_2} \lambda(t)dt + D(t_2) \times (M - t_2) + P \times (M - t_3) \right) \tag{18}$$

Since no product includes capital in this period, the cost of capital is zero.

**Case 5: $M > T$**

In this case, all the products produced in one production cycle have been sold, yet it is not time to pay for raw materials. Therefore, interest will be delivered to all products sold until payment.

Fig.( 6) shows the sales of products in this case. Therefore, the amount of income from delays in payments can be calculated as follows.

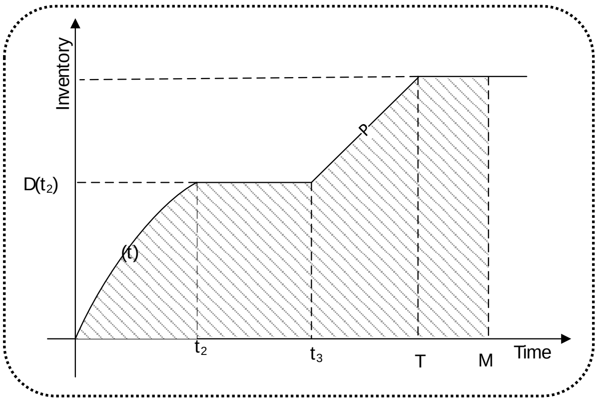


**Fig 6. The amount of inventory sold until M ($M > T$)**

$$IE_5 = S \times I_e \times \left(\int_0^{t_2} \lambda(t)dt + D(t_2) \times (T - t_2) + P \times (T - t_3)\right) \times (M - T) \tag{19}$$

In this case, the production cycle is over. Thus, the amount of capital involved is zero.

Based on the stated cost function and revenue from delayed payment, the total cost function for the five different payment time modes is expressed as follows.

$$AC(t_1, T) = \begin{cases} AC_1(t_1, T) = TC(t_1, T) + \frac{1}{T}(CC_1 - IE_1) M \leq t_1 \\ AC_2(t_1, T) = TC(t_1, T) + \frac{1}{T}(CC_2 - IE_2) t_1 < M \leq t_2 \\ AC_3(t_1, T) = TC(t_1, T) + \frac{1}{T}(CC_3 - IE_3) t_2 < M \leq t_3 \\ AC_4(t_1, T) = TC(t_1, T) + \frac{1}{T}(CC_4 - IE_4) t_3 < M \leq T \\ AC_5(t_1, T) = TC(t_1, T) + \frac{1}{T}(CC_5 - IE_5) M > T \end{cases}$$

## V. SOLUTION APPROACH

The proposed model is a nonlinear programming problem. Due to the existence of exponential and logarithmic functions and integrals in the mathematical model, we could not apply exact or classical methods to find the optimal solution. Therefore, a genetic meta-heuristic algorithm is used to overcome the problem's complexity and find a suitable and satisfactory answer. The Genetic Algorithm(GA) is very efficient in solving nonlinear problems. The algorithm process begins by generating multiple random solutions to the problem. This set of answers is called the initial population, and each generated response is called a chromosome. Then, the chromosomes are combined using a crossover operator. Next, after selecting better chromosomes, a mutation operator is applied to them. Finally, the current and new populations resulting from the Crossover and mutation operators are combined.

One of the essential steps in designing and implementing a meta-heuristic algorithm is parameter setting. The parameters in the GA include population size, number of iterations, mutation rate, and intersection/crossover rate. These parameters could be determined by designing an experiment using the Taguchi methods.

The main decision variables in this research are $t_1$ and $T$, and besides these variables, other dependent variables significantly affect the difficulty of the problem. For example, the dependent variables of the issue include $t_2$, $t_3$, and the maximum inventory level (Z).

The random numbers are used to initialize the chromosomes. A high limit is set to prevent the search for useless numbers in determining the value. In this case, a random number is specified for the specified interval. Then, the values of the dependent variables are determined. Therefore, we first randomly initialize $t_1$ and $T$. Then, the maximum inventory level and $t_3$ could be determined.

After determining the value of the decision variables, the value of the objective function is calculated for each chromosome produced in the initialization step. Then, after determining the target function, the probability value of each chromosome is determined. The more valuable the chromosome is, the greater the chance and probability of being selected to produce the next generation. For the selection process, the roulette wheel method has been used, for which the accumulation probability values of chromosomes must be calculated.

We use a pointcut method for crossover operator, in which a place in the parent chromosome is randomly selected, and then the sub-chromosomes obtained from this point are replaced in two units. Parent chromosomes paired together are chosen randomly, and the number of paired chromosomes is determined by a parameter called the cut-off rate. The mutation rate parameter specifies the number of chromosomes that mutate. The mutation process is performed by randomly changing the value of a gene in a random position to a new value. At the end of the mutation process, an iteration or a generation of genetic algorithms is created, which must be re-evaluated by the objective function.

### *A. Parameters setting*

In metaheuristic algorithms, assigning an appropriate value to the algorithm parameters can significantly impact the efficiency and performance of the algorithm. Like other evolutionary algorithms, the genetic algorithm first randomly selects a population of answers and then tries to improve these answers by using selection operators, combination and mutation operators, and the elitism operator to arrive at a suitable solution. To quickly find the proper response and the proper search for the algorithm, choosing the appropriate parameter for the algorithm is crucial. One of the most appropriate methods of determining the parameters of a meta-heuristic algorithm is the Taguchi method, which has been used in this research.

For this purpose, first, several different scenarios are designed for the value of the parameters. Then, the genetic algorithm is performed by considering these scenarios. Next, the necessary effects are examined according to the rules of the Taguchi method. Then, the appropriate values of the parameters are selected. Genetic algorithm parameters that have a significant impact on algorithm performance include initial population size (P), number of iterations (N), crossover rate (p (c)), and mutation rate (p (m)).

In this study, the five scenarios shown in Table II have been employed to determine the appropriate values of the GA parameters.

**Table II. Taguchi Test Scenarios**

| *Parameters* | *Code* | *Scenario 1* | *Scenario 2* | *Scenario 3* | *Scenario 4* | *Scenario 5* |
|---|---|---|---|---|---|---|
| N | A | 100 | 80 | 60 | 40 | 25 |
| Papulation size | B | 80 | 60 | 52 | 40 | 20 |
| $p(c)$ | C | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
| $p(m)$ | D | 0.1 | 0.2 | 0.25 | 0.3 | 0.35 |

The Taguchi test is designed using Minitab 19 software. After running the program, the signal-to-noise diagram is provided for different modes. Then, the appropriate estimate for the parameters is obtained. Finally, the parameter estimation results are given in Table III.

**Table III. The Value Of Ga Parameters**

| *Parameters* | *values* |
|---|---|
| N | 80 |
| Population size | 52 |
| $p(c)$ | 0.9 |
| $p(m)$ | 0.3 |

It should be noted that due to the high complexity of the model, we cannot prove the convexity of the total cost function. However, in numerous numerical examples, convexity is held. Considering the numerical example and examining their convexity, the (near-)optimal solution is determined using the Matematica software and GA Algorithm. Then, the obtained answers are compared based on the RPD (relative percent difference) index, and the results are expressed.

## VI. NUMERICAL EXAMPLE

The supplier of raw materials sends its product to the manufacturer at the price of 100$ per product unit. However, due to the manufacturer's credit and the supplier's incentive system, it receives the cost of purchasing raw materials from the manufacturer with a delay. Therefore, we consider the following demand function for the manufacturer's product.

$$\lambda(t) = \begin{cases} 1000 + 0.2 \times I(t) 0 \leq t \leq t_2 \\ 1000 t_2 \leq t \leq T \end{cases}$$

Furthermore, we assume the production rate is limited and equal to 5000, the rate of capital cost per currency unit per unit time is equal to 15%, and the holding cost rate per unit of product per year (does not include the cost of capital) is equal to 10%. Also, the cost of shortage of each unit per unit time equals 6$. Moreover, it is assumed that the selling price per unit is 140$, and the rate of interest received by the manufacturer is equal to 8%. Further, assume that 1% of the manufacturer's inventory is corrupt.

The above example is solved for different amounts of delayed payment for the purchase using the Mathematica software (optimal solution) and the genetic algorithm. The results are given in Table IV.

According to Table IV, the average value of the RPD index is 0.38%. Therefore, the genetic algorithm is efficient in solving the problem.

**Table IV. The Solution Results With Wolfram Mathematica And Ga Algorithm**

| *No.* | *M* | *Wolfram Mathematica* | | | *GA algorithm* | | | *RPD* |
|---|---|---|---|---|---|---|---|---|
| | | *t1* | *T* | *Cost* | *t1* | *T* | *Cost* | |
| 1 | 0 | 0.02 | 0.20 | 101966 | 0.04 | 0.355 | 101973.956 | 0.01% |
| 2 | 0.01 | 0.02 | 0.20 | 101885 | 0.004 | 0.047 | 101893.069 | 0.01% |
| 3 | 0.1 | 0.01 | 0.06 | 100681 | 0.056 | 0.61 | 100836.729 | 0.15% |
| 4 | 0.15 | 0.02 | 0.21 | 100220 | 0.005 | 0.211 | 100418.946 | 0.12% |
| 5 | 0.2 | 0.01 | 0.06 | 95050.9 | 0.03 | 0.428 | 100314 | 0.15% |
| 6 | 0.25 | 0.02 | 0.20 | 101966 | 0.003 | 0.054 | 99040.2 | 1.86% |

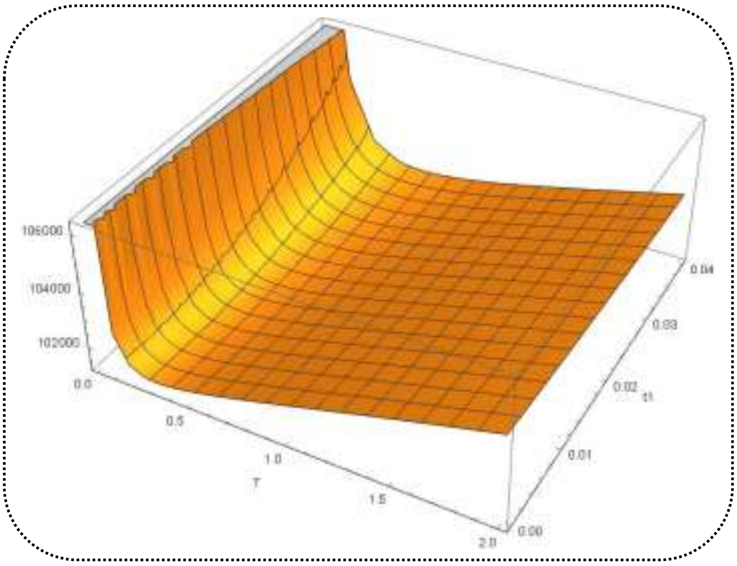


**Fig 7. Relationship between T, $t_1$, and total cost**

## VII. SENSITIVITY ANALYSIS

This section analyzes the sensitivity of the model to the main parameters. For this purpose, the main parameters of the problem are changed from minus 80% to plus 80% (if possible). It should be noted that the delayed payment is equal to 30 days after purchasing raw materials.

**Table V. The Sensitivity Analysis Of Raw Material Price**

| *Change%* | *Parameters* | | | | | | *Solution* | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | *Purchased price* | *Selling Price* | *UBC* | *I1* | *I2* | *ie* | *t1* | *T* | *Total cost* | *Total cost Change%* |
| -80% | 20 | 140 | 20 | 15% | 10% | 8% | 0.04 | 0.08 | 23,926 | -73% |
| -60% | 40 | 140 | 20 | 15% | 10% | 8% | 0.03 | 0.249 | 40,412 | -55% |
| -40% | 60 | 140 | 20 | 15% | 10% | 8% | 0.023 | 0.227 | 60,590 | -32% |
| -20% | 80 | 140 | 20 | 15% | 10% | 8% | 0.012 | 0.148 | 80,722 | -10% |
| -10% | 90 | 140 | 20 | 15% | 10% | 8% | 0.006 | 0.063 | 88,779 | -1% |
| 0% | 100 | 140 | 20 | 15% | 10% | 8% | 0.021 | 0.129 | 89,498 | 0% |
| 10% | 110 | 140 | 20 | 15% | 10% | 8% | 0.009 | 0.065 | 110,905 | 24% |
| 20% | 120 | 140 | 20 | 15% | 10% | 8% | 0.016 | 0.227 | 120,954 | 35% |

In Table V, the changes in the initial purchase cost are examined. As can be seen, the purchase cost of raw materials is changed from 80% to 20%, and its effects on the model are examined. The maximum price change is considered to be 20% of the initial amount because, by an increase of more than 20%, the purchase price of raw materials exceeds the selling price. Therefore, it can be stated that with the reduction of the purchase price, both the purchase cost and the maintenance cost of the product unit are reduced. As a result, the cost of the entire system is reduced.

The effect of the purchase cost on the cost of the entire system is almost exponential. The chart shown in Fig.(8) indicates the total cost changes trend relative to cost changes.

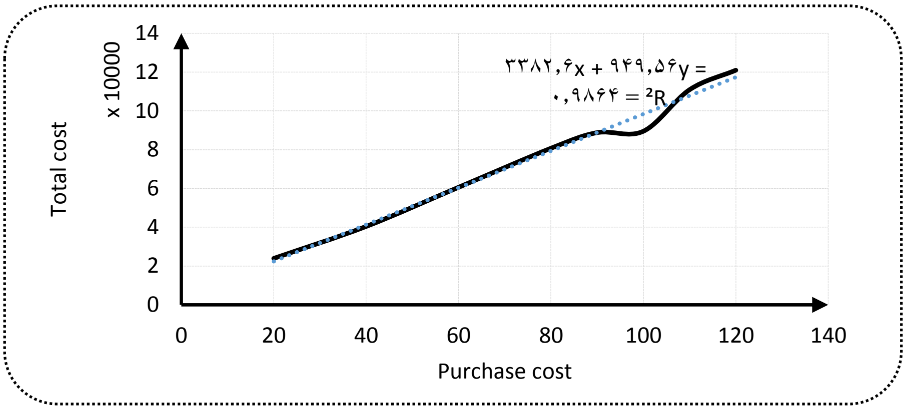


**Fig 8. Relationship between unit purchased cost and total cost**

**Table VI. Sensitivity Analysis Of The Selling Price**

| *Change%* | *Parameters* | | | | | | *Solution* | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | *Selling price* | *Purchase price* | *UBC* | *I1* | *I2* | *ie* | *t1* | *T* | *Total cost* | *Total cost Change%* |
| -20% | 112 | 100 | 20 | 15% | 10% | 8% | 0.005 | 0.059 | 93,357.10 | 4% |
| -10% | 126 | 100 | 20 | 15% | 10% | 8% | 4% | 43% | 92,282.62 | 3% |
| 0% | 140 | 100 | 20 | 15% | 10% | 8% | 2% | 13% | 89,498.49 | 0% |
| 10% | 154 | 100 | 20 | 15% | 10% | 8% | 3% | 28% | 87,831.23 | -2% |
| 30% | 182 | 100 | 20 | 15% | 10% | 8% | 6% | 30% | 87,284.61 | -2% |
| 50% | 210 | 100 | 20 | 15% | 10% | 8% | 0.014 | 0.266 | 87,057.73 | -3% |
| 80% | 252 | 100 | 20 | 15% | 10% | 8% | 0.035 | 0.554 | 61,249.19 | -32% |

Table VI examines the changes in the selling price of the produced product. The selling price of each product unit is changed from 20% to 80%, and its effects on the model are examined. The minimum change in the selling price is -20% since more than 20% causes the selling price to be lower than the buying price, making the model uneconomical.

According to the obtained results, reducing the selling price increases the system costs due to lowering the profit received for the sold units. Fig) show the effects of the selling price on system costs. The chart shows that system costs have a non-increasing trend with the selling price. It should be noted that the purpose of this study is to minimize costs, and the effect of the selling price is given to the manufacturer only to determine the amount of interest received for late delay payment.

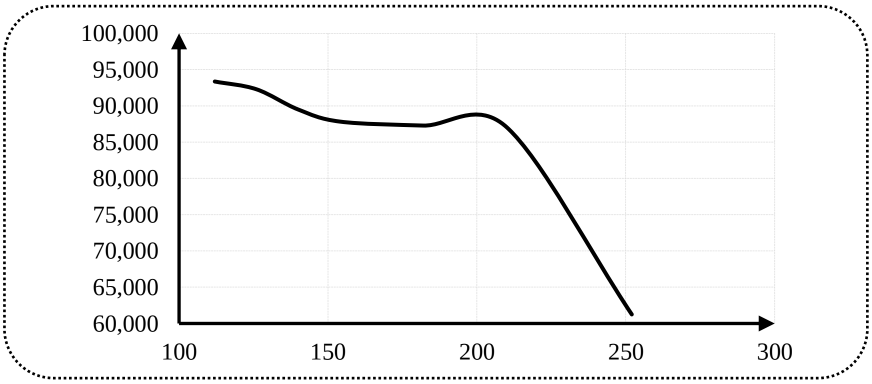


**Fig 9. Relationship between the unit selling price and total cost**

Another parameter affecting system costs and the replenishment policy is the received interest rate. The behavior of received interest from the sale of products is similar to that of the product selling price.

Table VII shows the sensitivity analysis of the model to the shortage cost. With the increase in the unit shortage cost, the system total cost has increased. Fig) shows the relation between the back-ordering cost and the system total cost.

**Table VII. Sensitivity Analysis Of The Unit Back-Ordering Cost**

| *Change%* | *Parameters* | | | | | | *Optimal Solution* | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | *UBC* | *Purchase price* | *Selling price* | *I1* | *I2* | *ie* | *t1* | *T* | *Total cost* | *Total cost Change%* |
| -20% | 16 | 100 | 140 | 15% | 10% | 8% | 0.045 | 0.197 | 85,241 | -5% |
| -10% | 18 | 100 | 140 | 15% | 10% | 8% | 0.075 | 0.503 | 86,252 | -4% |
| 0% | 20 | 100 | 140 | 15% | 10% | 8% | 0.021 | 0.129 | 89,498 | 0% |
| 10% | 22 | 100 | 140 | 15% | 10% | 8% | 0.025 | 0.527 | 89,855 | 0.40% |
| 30% | 26 | 100 | 140 | 15% | 10% | 8% | 0.044 | 0.413 | 91,246 | 1.95% |
| 50% | 30 | 100 | 140 | 15% | 10% | 8% | 0.006 | 0.1 | 91,415 | 2% |
| 80% | 36 | 100 | 140 | 15% | 10% | 8% | 0.065 | 0.372 | 91,206 | 2% |

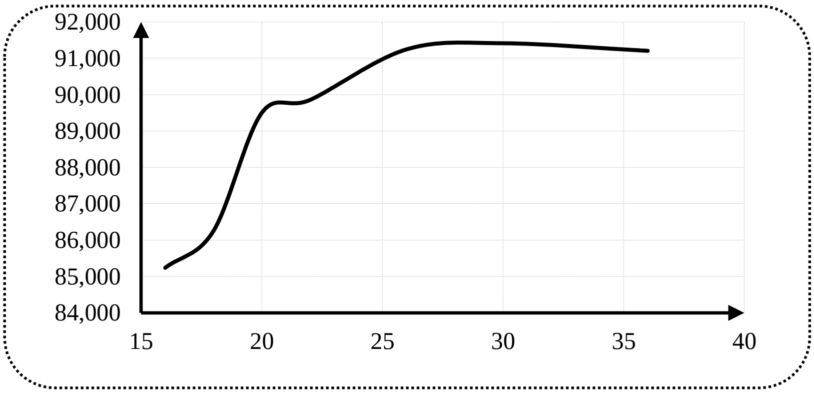


**Fig 10. Relationship between unit back-ordering cost and total system cost**

Finally, we are interested in comparing the proposed model and the classic model without delayed payment. For this purpose, several different issues are considered, and the results are compared. In the classic model, the time of delay payment is zero. The best solution for the classic model (*M*=0) is shown in Table IV.

$$t_1^* = 0.016, Q^* = t_1^* \times P = 0.016 \times 5000 = 80, T^* = 0.204$$

$$TotalOrderperyear = \frac{Q^*}{T^*} = 392.16, Total\ Cos\ t = 101966$$

**Table VIII. Result Of Comparing The Proposed Model With The Classic Model**

| *No.* | *M* | $t_1^*$ | $T^*$ | $Q^*$ | *Order (Per Year)* | *Total Cost* |
|---|---|---|---|---|---|---|
| 1 | 0 | 0.02 | 0.20 | 80 | 392.16 | 101966 |
| 2 | 0.01 | 0.02 | 0.20 | 75 | 380.71 | 101885 |
| 3 | 0.1 | 0.01 | 0.06 | 25 | 390.63 | 100681 |
| 4 | 0.15 | 0.02 | 0.21 | 88 | 414.12 | 100220 |
| 5 | 0.2 | 0.01 | 0.06 | 27.67 | 432.87 | 95050.9 |
| 6 | 0.25 | 0.01 | 0.06 | 28.59 | 446.26 | 92235.1 |

As shown in Table VIII, as the time of delay payment increases, the order quantity increases compared to the classic model. However, the total system cost decreases despite the increase in order quantity. This behavior shows the usefulness of delayed payment in inventory control systems.

## VIII. CONCLUSION

In the classical model of economic production, it is assumed that demand is fixed and constant. Still, in reality, there are some products whose demand depends on inventory level, and demand increases with on-the-shelf inventories. This paper proposed an economical production system with demand dependent on inventory level and supplier incentive conditions. It was assumed that the supplier sent his product to the manufacturer but received the purchasing cost from the manufacturer with a delay. Furthermore, we considered the deteriorating items whose deterioration rate is a fixed percentage of the inventory level. Many deteriorating products such as alcohol, fruits, and vegetables have the same trend. Therefore, this paper has developed a single-product economic production system model considering inventory-dependent demand, delayed payment policy, deteriorating products, and back-ordering shortages. First, a mathematical model has been designed to deal with the problem. Then the optimal solution was determined using Mathematica software. However, choosing the optimal solution using the Mathematica software was time-consuming. Therefore, a metaheuristic algorithm has been employed, which could quickly provide the solution to the proposed model.

The model's primary purpose was to determine the optimal production volume, optimal shortage volume, and cycle length so that the cost of the entire system is minimized. Our numerical results showed that the proposed algorithm works rather well. One of the most critical issues that have not been considered in our study is the delayed payment in different parts of the supply chain. If the chain is integrated, the delayed payment can significantly impact the analysis of chain results. Therefore, an attractive direction for future research is investigating the trade credit among different supply chain parties. Another suggestion for future research is to consider the multi-delivery strategy for the proposed model.

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