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An Integrated Production-Distribution-Routing Problem under an Unforeseen Circumstance within a Competitive Framework

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Abstract – Nowadays, enhancing the products' quality and gaining market share are the primary purposes of any company in a competitive market. So, applying a proper management approach could help companies to make optimal decisions. One of the efficient approaches is supply chain management that can manage the flow of final products and services continually. The present study develops a supply chain with integrating production and distribution activities and a multi-period routing problem. Also, in this problem, a Stackelberg competition occurs between the suppliers under normal and critical situations (in which procurement costs of materials are increased and the suppliers encounter the shortage). Therefore, some parameters are considered uncertain, and a two-stage stochastic optimization model is constructed. The model is also multi-objective to reduce cost, lost sales, and defective products. The GAMS software is used for solving a case study for the medicine industry. Due to the NP-hardness, we consider Non-Dominated Sorting Genetic Algorithms II (NSGA_II), Multiple Objective Particle Swarm Optimization (MOPSO), and a hybrid algorithm for the large-sized instances. Subsequently, the performances of the proposed algorithms are considered. The obtained results reveal that the hybrid algorithm has a better function for solving the model in medium and large-sized instances.

Keywords– Production-distribution, Routing, Competition, Two-Stage Stochastic Optimization model, Crisis

I. INTRODUCTION

There are many critical situations due to natural or manmade causes in the economy section which can significantly affect the profitability of the active components in these markets, even leading to the bankruptcy of some of these sectors. These events result in main decreases in the performance of the economic activists. For example, Economic sanctions, inflation of prices, devaluation of the national currency, change of government market regulation policies, change of monetary and banking policies of central banks, entry of new competitors and blowing up of products due to rapid change of technologies, delay in receiving orders due to natural disasters such as floods, hurricane or earthquake or due to manmade disasters such as military attacks and terrorism, are some of the main known crises which threaten market participants.

These crises from an intra-organizational perspective may contribute to decreased profitability, bankruptcy, closure of manufacturing companies, layoffs, a large-scale loss of satisfaction of customers, loss of key customers and market share compared to rivals, and so on. Also, from an Extra-organizational perspective, this crisis in large-scale cases may seriously affect the well-being and satisfaction of the citizens, the economic growth and increase in unemployment, and so on. To eliminate or reduce the negative effects of these unforeseen crises, economic participants must take these events into account in their planning and be prepared to manage these crises in advance.

Our concern in this research deals with the cases where the suppliers encounter a sudden rise in prices of the raw materials for unpredictable reasons. One of the well-known solutions that could be applicable for dealing with and reducing the consequences of these issues, is cooperating and integrating the chain of supply in markets. A supply chain consists of some geographical facilities (belonging to suppliers, manufacturers, distributors, retailers, and markets), and these facilities work together to fulfill customer requirements (Abraham et al., 2015). Some integrated activities in a network are production, inventory, and distribution, and their planning is according to the preceding activities (Adulyasak et al., 2015). In the production section, decisions are related to labor hiring or firing, production in regular time or overtime, scheduling, and capacity of machines. Moreover, distribution planning determines which facilities distribute the volumes to the customers (Fahimnia et al., 2013). In this study, distribution planning consists of a multi-period routing problem between geographical facilities. This vehicle routing problem (VRP) selects routes to transfer materials and products to achieve the optimal cost in each period. Furthermore, fleet sizing that can determine the number of required vehicles helps to increase the level of quality (by speeding up service delivery) and minimize the transportation cost. These decisions could affect each other and integration of them could lead to more efficiency and decline the network's costs (Miranda et al., 2018).

The distributors are competing to improve their performance and intend to consolidate their position in the competitive market by gaining more market share. Two different types of horizontal competition occurred between the same levels of a chain including in-chain and chain-to-chain horizontal competition. The former expresses the competition at each level of a chain while the latter studies the competition between two different chains (Mahmoodi & Eshghi, 2014). The in-chain horizontal competition considered in this paper is based on a strategic game known as the Stackelberg competition between a leader and a follower that competes based on price. The leader begins the game based on his/her predictions of the distributor's strategies. Then, observing the leader's strategy, the follower responds. The contributors in the Stackelberg competition must eventually determine their equilibrium strategies (Mahmoodi 2020; Rafiei et al., 2018). So, in this study, we integrate the production and distribution problems in a chain including suppliers, manufacturers, and retailers, where a Stackelberg competition is formed between two competitive suppliers.

Due to the nature of uncertainties in crisis (Fang & Shou, 2015), most studies have modeled uncertain parameters with a two-stage stochastic programming model. The paper of (Barbarosoğlu & Arda, 2004) was the first study to use this model for humanitarian logistics. Furthermore, (Alem et al., 2016) stated that the model is efficient for solving disaster problems with different scenarios.

Many objectives must be optimized through this problem such as the minimization of the lost sales and the level of defective products to maximize the quality of the supply chain, and the minimization of the network cost including production, transportation, supplying, holding, and manpower.

Concerning the above-mentioned problem, we develop a multi-objective mixed integer nonlinear programming (MOMINLP) model. In this model, we investigate the impacts of the crisis (in our case, rises in raw materials' prices for the suppliers), on the Stackelberg competition between suppliers in an integrated production-distribution problem. We use a two-stage stochastic optimization model to consider parameters with different scenarios whose cost and available quantity will change dramatically under crisis conditions. Just by taking a glance at the related literature, the existing studies typically have considered one of these challenges, separately and their effects on each other in a competitive situation have been neglected. It is worth mentioning that in competitive markets the rivals, in addition to trying to survive and make profits through competition, try to counterbalance the crisis issues by cooperating with other participants of the

supply chain.

To solve the proposed mathematical model, we obtain the exact solutions for a small instance of case study. For solving large-sized instances, a hybrid meta-heuristic algorithm is developed that consists of MOPSO and NSGA-II.

The remainder of the paper is organized as follows. The relevant literature is reviewed in Section 2. The assumptions, problem definition, and the mathematical model are presented in Section 3. In Section 4, the developed meta-heuristic algorithms are explained. Afterward, the numerical example of the proposed model is illustrated in section 5. The results, conclusions, limitations, and ideas for future works are presented in section 6.

II. LITERATURE REVIEW

Supply chain management is known to be one of the most important topics for researchers. Several studies in the literature investigate it. In the following sub-section, we outline the major common lines of some existing research in P-D planning problems, the developed solution methods, crisis, routing, and competition and then, apply them to explain the main distinctions between our work and them that was the key triggers of the current research.

A. Production- distribution

Recently, a new important approach to supply chain analysis is based on integrating the various optimized sequentially functions such as purchasing, producing, distributing, and so on into a simultaneous optimization model (Park, 2005). In reviewing the literature, specific studies have considered the problems of integrated production and distribution. Here are some of them.

(Cohen & Lee, 1988) presented a model that can predict the network's efficiency through the cost of products, the level of services, and the responsiveness and flexibility for manufacturers.

(Erengüç et al., 1999) reported a review regarding a production and distribution planning in different scopes. Furthermore, (Lee et al., 2002) considered minimizing the total cost as an objective function in an integrated production-distribution problem. Moreover, they used a hybrid analytic and simulation model concerning the dynamic operation time as a major constraint in the real system.

Furthermore, (Park, 2005) presented a solution for an integrated production-distribution in a multi-plant, multi-retailer, and multi-period logistics.

Moreover, (Amorim et al., 2012) suggested a multi-objective integrated production and distribution planning of perishable products. They exhibited the economic benefits of using an integrated approach. (Fahimnia et al., 2013) reviewed the current P-D planning models and classified them into seven categories. (Ma et al., 2016) proposed an integrated bi-level production-distribution planning for a supply chain. In this problem, a leader manages the total cost.

In addition, (Khalili-Damghani & Ghasemi, 2016) proposed a P-D planning problem. Their study aimed to analyze the performance of two classes of the supply chain. They utilized fuzzy mathematical optimization for solving their model. (Nemati & Alavidoost, 2019) developed a multi-objective mixed-integer linear programming to integrate sale, production, distribution, and procurement planning. (Badhotiya et al., 2019) addressed an integrated production-distribution planning for a two-echelon supply chain network. This study took multiple manufacturers, multiple selling, and selling locations into consideration. Ultimately, (Ghadimi & Aouam, 2021) developed a production–distribution system with a guaranteed service approach (GSA) to optimize the production capacity and safety stocks. This system has one producer, one warehouse with limited storage capacity, and one retailer. Also, this integrated problem is formulated as a non-convex program and is solved using a nested Lagrangian relaxation heuristic algorithm.

The literature review demonstrates the previous study integrated with these activities to optimize the supply chain by minimizing the total cost that is presented in (Rafiei et al., 2018). Our study is also considered an integrated production and distribution to optimize the network. The literature review indicates the efficiency of this integration in this regard.

B. Production- distribution problem under Competition

The competitive nature of nowadays markets requires special consideration of the competition between different parts of the supply chain. Hence, some studies in the P-D area have been conducted to investigate them. For instance, (Giri & Sarker, 2016) considered a supply chain consisting of a single manufacturer and two retailers competing on the price and service of the same products. Moreover, (Yue & You, 2017) proposed a bi-level programming model as a solution algorithm for optimal design and operations for non-cooperative supply chains under the Stackelberg game with a leader and a follower. (Li & Chen, 2018) developed a Stackelberg game, in which the retailer sold a product with two brands to the customers who were homogenous in product valuation. (Sadjadi et al., 2018) studied the retailer Stackelberg game between two manufacturers and one retailer, who competed on the price, service and discount, simultaneously. Also, (Roy et al., 2018) presented a supply chain with one manufacturer and two retailers competing on their sales price and dependent demand.

(Rafiei et al., 2018) presented an integrated P-D planning problem considering Cournot, Stackelberg, and quality competition to minimize the total cost and maximize the service level. (Jena et al., 2019) also studied the impact of branding on the total profit for a chain including two competing manufacturers and one retailer. Moreover, (Aazami & Saidi-Mehrabad, 2021) considered fixed lifetime for a new multi-period P-D planning for perishable products in a vertical competition framework between a seller and a buyer. (Rafiei et al., 2021) developed a competitive P-D model that was unimodular. The considered competition was Cournot and Stackelberg's competition.

Our study considers a Stackelberg competition in a P-D planning problem between two suppliers. To the best knowledge of the authors, this competitive P-D planning problem considering routing and under a critical situation has not been studied yet.

C. Routing

Distribution planning is essential in logistic systems, involving the flow of products from manufacturing plants through the transportation network to consumers. It can impact the cost of the supply chain. So, scientists try to consider it in their problems for optimization in the network.

In this scope, (Chiang & Russell, 2004) developed a model to integrate purchasing and routing into the gas supply chain. The problem included location, routing, and fleet sizing, simultaneously. The fleet sizing has a significant impact on increasing the level of servicing and optimizing network costs, which is also considered in our study. (Christiansen & Lysgaard, 2007) proposed the capacitated single depot vehicle routing problem with stochastic customer demand. In their work, the customers were serviced according to their needs. Furthermore, (Osvald & Stirn, 2008) modeled a vegetables distribution problem with time windows, time-dependent travel times, time dependence on the distance, and time of day and developed a Tabu search to solve the model. In addition, (Liu & Chen, 2011) modeled an inventory-routing problem and considered the effects of pricing on the problem. (Guerrero et al., 2013) developed an inventory-location-routing problem to minimize the cost. The presented mixed-integer linear programming model was multi-depot and multi-retailer. In their study, they considered vehicle services further and more helpful than a retailer in some periods.

(Miranda et al., 2018) developed an integrated production-distribution-routing into small Brazilian furniture companies. In their paper, they considered some assumptions, such as multi-items, multiple time windows, distribution routes developing over one or more periods, and customers' due dates. (Marandi & Fatemi Ghomi, 2019) introduced a model for an integrated P-D problem. Their problem involved production, scheduling, and VRP. (Zheng et al., 2019) introduced an integrated location, inventory, and routing in a supply chain and proposed a benders decomposition to solve it. Finally, (Mhamedi et al., 2021) developed a routing problem with different capacity vehicles in a multi-echelon network

is proposed and an exact branch-price-and-cut (BPC) algorithm for solving the problem is applied. Moreover, they used some computational experiments on benchmark instances to show applicability of the model and some managerial insights are shown to make the structure of this routing problem.

We consider the multi-period and multi-depot routing and fleet sizing problem to minimize the transportation cost. According to the literature review, fleet sizing and routing together were less common.

D. Crisis

As mentioned previously, crises include all types of unforeseen natural and manmade occurrences that could disrupt the performance of the supply chain. (Bozorgi-Amiri et al., 2013) suggested a multi-objective model including location, allocation, and routing for an earthquake. The objectives of their study included minimizing the maximum amount of shortages, calculating the sum of the expected value of the total cost, and identifying the variance of the total cost. They also considered uncertainty with a robust stochastic optimization approach. In addition, (Wang et al., 2014) presented a multi-objective model for location, distribution, and routing in post-earthquake. The objective functions minimized the cost and time and maximized the minimal reliability of the routes. They used two meta-heuristic algorithms which were NSGA-II and non-dominated sorting differential evolution algorithm (NSDE) to solve the problem. Furthermore, (Rezaei-Malek et al., 2016) developed a new model for location, allocation, and inventory of perishable commodities in an urban area for a potential earthquake. The researchers used a scenario-based robust stochastic approach to determining uncertainty. The objective functions minimized the time and cost. An interactive approach (reservation level Tchebycheff procedure) was applied for solving the problem.

Moreover, (Manopiniwes & Irohara, 2017) made a stochastic programming model with inventory prepositioning, evacuation, and relief vehicle planning for preparation and response phases. This study had three levels, and a case study for a flood disaster was conducted. (Torabi et al., 2018) proposed a model to determine the optimum level of relief commodities in strategic locations for preparation and procurements in the response phase of the earthquake in Iran. Therefore, a two-stage scenario-based combined with a fuzzy-stochastic programming model was determined. They considered a mixture of uncertain parameters. To solve this problem, a multi-step solution method was developed. (Beiki et al., 2020) designed an integrated network to optimize the pre and post-phases of an earthquake. Finally, Agarwal et al. (2021) proposed a mathematical model for decision-making in the pre- and post-disaster phases. This model consists of inventory, location and evacuation planning. Also, they used classical approach (CA), pattern search algorithm (PSA) and Genetic Algorithm (GA) to solve the proposed mathematical model and a real case study of India is applied to validate the proposed mathematical model.

The literature shows that most studies on supply chain and crisis are related to the inventory control and distribution of final products. Our article, however, is about the effects of the crisis on the production and distribution of final products in the network.

E. Solution methods for P-D problem

The literature review illustrates routing problems as NP-Hard problems. The meta-heuristic algorithms are utilized for solving NP-Hard problems with large instances (Wang et al., 2019). (Shimizu & Wada, 2003) studied the agile problem for a p-Hub location and routing with capacity constraints. The non-linear integrated model was solved with the Tabu search algorithm. (Yun et al., 2009) presented a supply chain with plants, distribution centers, and retailers. They adopted a Genetic algorithm with an adaptive local search to obtain an answer which is the closest to the optimized answer. Moreover, (Abraham et al., 2015) proposed a P-D problem considering multiple plants and products and adopted a Genetic algorithm to solve the model. Furthermore, (Senoussi et al., 2018) proposed a P-D problem. This supply chain consisted of a manufacturer, several retailers, and five GA-based heuristics algorithms, which were proposed to solve the problem. Li et al. (2019) developed a PSO and GA to solve their model. (Biuki et al., 2020) introduced a sustainable supply chain for perishable products. In this paper, location, routing, and inventory problem is developed. So, this NP-hard problem is solved by two hybrid metaheuristics Genetic Algorithm (GA) and Particle Swarm Optimization (PSO)).

(Emamian et al., 2021) developed an integrated model for production routing closed-loop supply chain with three-objective function to minimize supply chain costs, maximize social responsibility or social benefits, and finally, minimize environmental emissions. They used the Bee Colony Optimization (BCO) algorithm to solve the model.

Thus, in the current study, we develop a meta-heuristic algorithm to solve medium and large instances. In the following, we compare the common features of some previous papers with our study as could be seen in Table (I).

Table I. The comparison between studies of the literature review

<i>Solution</i>	<i>Other features</i>	<i>Inventory</i>	<i>Multi manufacturer</i>	<i>Heterogeneous vehicles</i>	<i>Objective function</i>	<i>Multi-Objective</i>	<i>Multi-period</i>	<i>Competition</i>	<i>Fleet sizing</i>	<i>Human resources</i>	<i>Production-Distribution</i>	<i>Routing</i>	<i>Uncertainty</i>	<i>Crisis</i>	<i>Authors</i>
Heuristic optimization algorithm	Location problem		<input type="checkbox"/>		Min (cost)						<input type="checkbox"/>		<input type="checkbox"/>		Cohen & Lee (1988)
Hybrid analytic-simulation method	Multi-shop production and distribution				Min(cost)		<input type="checkbox"/>				<input type="checkbox"/>				Lee et al. (2002)
Set partitioning approach	Purchasing planning, multi-product		<input type="checkbox"/>		Min(cost)				<input type="checkbox"/>			<input type="checkbox"/>			Chiang & Russell (2004)
Heuristic optimization algorithm			<input type="checkbox"/>	<input type="checkbox"/>	Max(net profit)		<input type="checkbox"/>		<input type="checkbox"/>		<input type="checkbox"/>				Park (2005)
branch-and-price algorithm					Min(cost)							<input type="checkbox"/>	<input type="checkbox"/>		Christiansen & Lysgaard (2007)
Tabu search algorithm	A perishable product, time window, time-dependent travel-times	<input type="checkbox"/>		<input type="checkbox"/>	Min(cost)				<input type="checkbox"/>			<input type="checkbox"/>			Osvold & Stirn (2008)
Hybrid Genetic algorithm with adaptive local search			<input type="checkbox"/>		Max(longest route time)						<input type="checkbox"/>				Yun et al. (2009)
Heuristic method	Pricing, vehicle dispatching	<input type="checkbox"/>			Max(profit)		<input type="checkbox"/>					<input type="checkbox"/>			Liu & Chen (2011)
CPLEX solver	Perishable product	<input type="checkbox"/>	<input type="checkbox"/>		Min(cost) Max(the delivered freshness of product)	<input type="checkbox"/>	<input type="checkbox"/>				<input type="checkbox"/>				Amorim et al. (2012)
Lingo	Multi-product, location, distribution planning				Min (cost, penalty of shortage)	<input type="checkbox"/>	<input type="checkbox"/>						<input type="checkbox"/>	<input type="checkbox"/>	Bozorgi-Amiri et al. (2013)
Heuristic method	Location problem	<input type="checkbox"/>		<input type="checkbox"/>	Min(cost)		<input type="checkbox"/>					<input type="checkbox"/>			Guerrero et al. (2013)
Game Theory					Max(expected profit)			<input type="checkbox"/>					<input type="checkbox"/>		Mahmoudi & Eshghi (2014)
Genetic Algorithm	Multi-product		<input type="checkbox"/>		Max (net benefit)		<input type="checkbox"/>		<input type="checkbox"/>		<input type="checkbox"/>				Abraham et al. (2015)

Game Theory (Cournot game)				Max(expected profit)			□						□	Fang & Shou (2015)	
Genetic algorithm	Core firm		□	Min(total cost)						□			□	Ma et al. (2016)	
A centralized and decentralized decision-making approach	Multi-product			Max(net profit)			□			□			□	Khalili-Damghani & Ghasemi (2016)	
Two-phase heuristic algorithm	Budget allocation, procurement, risk			□	Min(cost)		□		□				□	□	Alem et al. (2016)
Reformulation-and-decomposition algorithm	Multi-product, facility location, technology selection, and opening/shutting-down of production lines				Max(profit)			□		□				Yue & You (2017)	
Gurobi Optimizer and normalized weighted sum method	Location, distribution, lateral distribution, multi-product				Min (cost)								□	□	Manopinives & Irohara (2017)
Relax and Fix heuristic	Multiple products, customers 'due dates.				Min (cost)		□			□		□		Miranda et al. (2018)	
Genetic Algorithm	One-warehouse, multi-retailers, single product	□		□	Min(cost)		□			□		□		Senoussi et al. (2018)	
GAMS	Cournot, Stackelberg, and Quality competitions				Min(cost max(service level.)	□	□	□		□	□			Rafiei et al. (2018)	
Stackelberg game	Price discount, pricing, service decisions			□	Max(profit)			□						Sadjadi et al. (2018)	
CPLEX solver	Multi-product		□	□	Min(cost), Max(time)	□	□		□		□	□	□	Nemati & Alavidoost (2019)	
CPLEX solver	Multi-product		□	□	Min(time) Min(cost) Min(backorder level)	□	□		□		□			Badhotiya et al. (2019)	
Improved Imperialist competitive algorithm	Lateral transportation for manufacturers			□	Min(tardiness cost and transportation cost)					□		□		Marandi & Fatemi Ghomi (2019)	
Generalized Benders decomposition	Location, single-product	□		□	Min(cost)				□			□		Zheng et al. (2019)	
Genetic Algorithm (GA) and	sustainable supply chain, multi-product	□			Min(cost) , max(job creation,	□						□	□	Biuki et al. (2020)	

Particle Swarm Optimization (PSO)					environmental efficiency)											
Simultaneous optimization	Location of distribution centers	<input type="checkbox"/>		<input type="checkbox"/>	min (the number of casualties and financial damages)							<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>		Beiki et al. (2020)
branch-price-and-cut (BPC) algorithm	Routing with time window			<input type="checkbox"/>	Min (Cost)							<input type="checkbox"/>				Mhamedi et al. (2021)
Benders Decomposition Algorithm (BDA) and Genetic Algorithm (GA)	return, discount policies, and credit period	<input type="checkbox"/>			max (the seller's profit)			<input type="checkbox"/>			<input type="checkbox"/>					Aazami & Saidi-Mehrabad (2021)
classical approach (CA), pattern search algorithm (PSA) and Genetic Algorithm (GA)	A real case study of cyclone Fani, 2019 in Orissa, India	<input type="checkbox"/>			Min (cost)								<input type="checkbox"/>	<input type="checkbox"/>		Agarwal et al. (2021)
Lagrangian relaxation heuristic	Guaranteed service, Multi-echelon model, Safety stocks	<input type="checkbox"/>			Min (Lead time)							<input type="checkbox"/>				Ghadimi & Aouam (2021)
BCO algorithm	closed-loop supply chain, multi products			<input type="checkbox"/>	min (costs) max (social responsibility) min(enviromental emissions)							<input type="checkbox"/>				Emamian et al. (2021)
A hybrid algorithm based on NSGA-II and MOPSO		<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	Min(cost, lot sale, and deficient product)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	This paper

A review of the present literature shows that since considering all these features make the problem complicated to solve, all the present studies have neglected some of these assumptions and paid attention to an oversimplified version of this problem. To study the effects of these assumptions on each other and make the present research more applicable to solve the realistic issues, this is necessary that we study all these assumptions, simultaneously. To sum up, the main distinctions between the current work and the literature are outlined as follows:

- We study an integrated production and distribution problem along with considering a multi-period, multi-objective and multi-depot routing, fleet sizing, and inventory management.
- We consider a Stackelberg competition between suppliers where evaluates the impact of critical conditions that lead to an increase in cost or a lack of available raw materials on the competition. According to the present literature, this Stackelberg competition has not been studied in the previous studies.
- In the developed mathematical model in the current research, we propound some critical situations such as considering both certain and uncertain parameters and also studying human resources management herein, which plays an important role in network optimization and is less discussed in the production-distribution problem.

- Finally, in our research, the model is solved in the small-sized using GAMS software and a case study about pharmaceutical company is developed to show applicability of the model. Afterward, the hybrid algorithm is utilized for solving medium and large-sized instances and the efficiency and effectiveness of the proposed algorithm are studied.

In the next section, the problem description and assumptions are mentioned, and the model is presented.

III. PROBLEM DESCRIPTION

A multi-echelon supply chain including two suppliers, some manufacturers, some retailers, and a distributor are studied in this work. The suppliers, manufacturers, and retailers are considered as the nodes of a network that could connect through some arcs. The manufacturers procure their required raw materials from the suppliers and send them to the retailers after processing these materials and turning them into the final product.

In the first level, two suppliers compete on their prices to achieve more market shares of the manufacturers. The lower the price offered by a supplier for its raw materials, the more manufacturers will be willing to buy from this supplier. Due to the time difference of suppliers in entering the market, one of the suppliers in the market plays the role of leader and the other one is following him/her. Compared to simultaneous games, Stackelberg games can better model real-world competitions. As a matter of fact, in the real world markets, due to having more knowledge from the market circumstances, the leader is more powerful and could affect the decisions made by the follower to the extent. To model this situation, a Stackelberg game is developed which will be described in more detail later.

In the second level, the manufacturers, based on the prices offered by the suppliers, decide to buy their required raw materials from suppliers. They process this material and transform it into a product. Production planning can help the manufacturers to produce efficiently and at the lowest possible cost. Human resources management through hiring and firing the labor forces, managing the inventory levels, working in overtime hours, outsourcing, etc., are some of the common methods to plan optimally the production with the lowest costs. Accordingly, the holding cost, and warehouses' capacities of the manufacturers and the suppliers are considered to manage the inventory level of raw material and products. Since this model considers the critical situations, to increase the level of resiliency, safety stock is added to decrease the lost sales in a difficult situation. These safety stocks are determined by the coefficient of demand. In addition, if demand exceeds safety stocks, we penalize the lost sales to control it in the supply chain.

The manufacturers, after completing the production process of products, ship them to the retailers. Fig (1) represents how the parts of the supply chain are connected.

As known, transportation is one of the important parts of the supply chain which distributes items among the suppliers and manufacturers, and among the manufacturers and the retailers. Routing the carriers between these sections bring about the increased performance and decreased the cost of the total supply chain. In this work, the routing is done for multi-frequent periods, in which the distributor with multi-type of capacity-constrained vehicles tries to determine the fleet size and some optimum routes in each period to carry off the items from multi-depots to some predetermined nodes through the arcs. This routing problem for each period is the well-known capacity-constrained vehicle routing problem (CVRP).

This model has three objective functions. The first objective function minimizes the penalty cost of lost sales and the removal cost of defective final products. The costs associated with transportation, production, holding, hiring, firing, salary, and vehicles are considered in the second objective function. Moreover, due to forming a Stackelberg competition between the suppliers, the third objective function is dedicated to the minimization of the differences between equilibrium values on the material quantity in this competition.

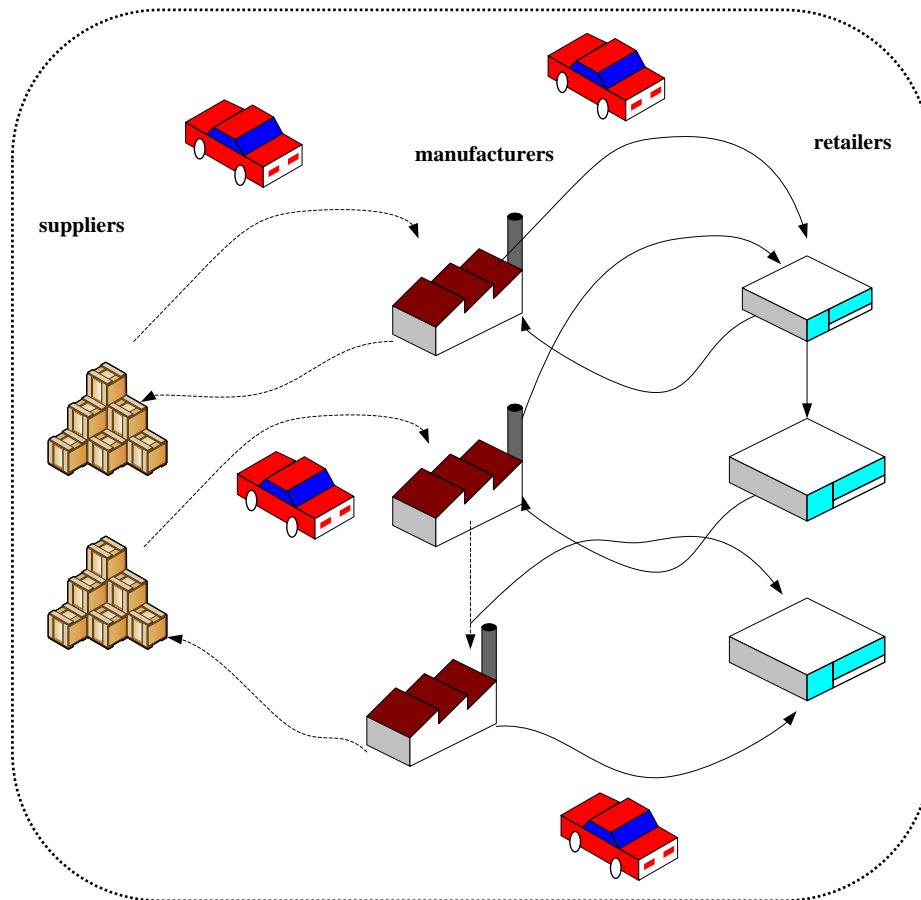


Fig 1.The network of the considered supply chain

Due to considering crisis conditions for the suppliers, some of the parameters are uncertain and include the material supplying cost, lost sale penalty cost, firing cost, hiring cost, salary cost, raw material cost for manufacturer, removal cost of defective final products, final products and raw materials holding cost for manufacturers and suppliers, the maximum available quantity of raw material, demand, allowed lost sale percentage of final products, the maximum and the minimum number of workers that are fired or hired, and the safety stock components.

To handle this uncertainty, we use two-stage linear stochastic programming. In the next sub-section, this model will be described.

A. Two-stage linear stochastic programming

The form of this model is below:

$$\min cx + E[Q(x, \zeta)] \tag{1}$$

$$Ax=b \quad x \geq 0 \tag{2}$$

In equations (1) and (2), E is the expected optimal value of the second stage of the problem, x is variable, a, b, c are the parameters, and ζ shows the data of the second stage. For solving the model, ζ has a finite number of possible realizations called scenarios. These scenarios $(\zeta_1, \zeta_2, \dots, \zeta_s)$ has respective probability masses $(\pi_1, \pi_2, \dots, \pi_s)$. Accordingly, we could write equation (3):

$$E[Q(x, \zeta)] = \sum_{s=1}^s \pi_s \times Q(x, \zeta_s) \tag{3}$$

Moreover, once the number of the scenarios is finite, two-stage stochastic linear programs could be modeled as the large-sized linear programming problems (Shapiro & Philpott, 2007). In this regard, we considered some scenarios, whose number is finite. Each of them has a probability made by the uniform distribution. Our objective function consists of definitive phrases and expected optimal value for uncertain variables.

Moreover, the constraints are written considering different scenarios. In these scenarios, uncertain parameters have different values, and we use the level of the available amount and cost of raw material for the supplier to make the critical scenarios. In our model, we normally assume that a critical situation such as natural, man-made disaster or economic crisis decreases the level of the available amount of raw material for the supplier. Therefore, the costs of raw materials increase, and a critical situation occur for the suppliers. The problem is developed under two situations.

1. No crisis exists. The first situation comprised a competition with different levels of costs and available quantity for raw material in the different scenarios.
2. The second situation involves the assumption of increasing the level of costs or decreasing the level of available quantity dramatically in different scenarios to create critical conditions for the suppliers.

In this study, the following assumptions are considered:

- a) Hiring and firing the labor force are allowed in each period.
- b) Routing is heterogeneous. (One distributor with some different types of vehicles)
- c) Storage is not allowed in the distribution centers.
- d) Some percentage of the final product is defective and must be removed.
- e) Hiring and firing have limitations.
- f) Over time working and outsourcing are not allowed.
- g) One type of raw material and the final product is transferred through the network.
- h) The maximum available quantity of material for each supplier is different.
- i) Consumption coefficient of per unit of the raw material in per unit of the final product
- j) Two routing problems are formed in each period. A routing problem among the suppliers as depots and the manufacturers and the other among the manufacturers as depots and the retailers.
- k) The retailers must be served by at least one of the manufacturers.
- l) The manufacturers must be served by only one supplier.
- m) The fleet size is unknown.
- n) The vehicles are based at multi central depot (suppliers and manufacturers),
- o) Only the capacity restrictions for the vehicles are imposed. We consider a fleet made of different vehicles with given capacity for vehicle v
- p) It is assumed that ships are received at the first of periods in the nodes dedicated to the manufacturers and the retailers.

q) In each period, every vehicle exits from a depot finally returns to that depot.

Notations of the mathematical model are defined as follows:

Sets:

- i : Set of suppliers $i \in \{1,2\}$
- m : Set of manufacturers $m \in \{1,2,3, \dots, M\}$
- v : Set of distributor's vehicles $v \in \{1,2, \dots, V\}$
- t : Set of periods $t \in \{1,2, \dots, T\}$
- s : Set of scenarios $s \in \{1,2, \dots, S\}$
- r : Set of retailers $r \in \{1,2,3, \dots, R\}$
- o, u : Set of suppliers and manufacturers $o, u \in \{IUM\}$
- o', u' : Set of manufacturers and retailers $o', u' \in \{MUR\}$
- o'', u'' : Set of all nodes in network $o'', u'' \in \{IUMUR\}$

Uncertain variables:

- L_{imvst} The quantity of material that manufacturer m purchases from supplier i and transports by distributor's vehicle type v in scenario s and period t
- w_{mst} The quantity of final product produced by manufacturer m in scenario s and period t
- Z_{mst} The initial inventory level of the final product for manufacturer m in scenario s and period t
- ZP_{mst} The remained inventory of the final product for manufacturer m in scenario s and period t
- L'_{mrvst} The quantity of final product that manufacturer m sends to retailer r in scenario s that transported by distributor's vehicle type v and period t
- SA_{rst} The quantity of the final product that retailer r sales in scenario s and period t
- D_{rst} The quantity of lost sale for retailer r in scenario s and period t
- F_{mst} The number of workers who fired by manufacturer m in scenario s and period t
- R_{mst} The number of workers who employed by manufacturer m in scenario s and period t
- N_{mst} The number of workers who work in manufacturer m in scenario s and period t
- φ_{mst} The quantity of defective final products for manufacturer m in scenario s and period t
- SS_{mst} The safety stock of final products for manufacturer m in scenario s and period t
- V_{vst} The required number of distributor's vehicles type v for scenario s and period t

IB_{mst}	The initial inventory of raw material for manufacturer m in scenarios s and period t
IP_{mst}	The remained inventory of raw material for manufacturer m in scenario s and period t
ω_{ist}	The amount of remained material in the warehouses of supplier i in scenario s and period t
Ω_{ist}	The quantity of material that supplier i supplies in scenario s and period t

Certain variables:

$RT_{o''nu''vt}$	If node o'' is passed after node u'' by vehicle v and period t is equal to 1 otherwise is equal 0
p'_{rv}	The variable for constraint of sub tour for retailer r and vehicle v
p''_{mv}	The variable for constraint of sub tour for manufacturer m and vehicle v

Uncertain parameters:

ε_{ist}	Maximum available quantity for supplier i in scenario s and period t
DM_{rs}	The demand of final product for retailer r in scenario s and period t
ψ_{st}	Allowed lost sale percentage of the final product in scenario s and period t
PR_{st}	Cost of supplying in scenario s and period t
SH_{rst}	The lost sale penalty cost of the final product for retailer r in scenario s and period t
θ_{mst}	The safety stock factor of material in manufacturer m in scenario s and period t
FC_{ms}	Cost of firing in manufacturer m in scenario s and period t
RC_{ms}	Cost of hiring in manufacturer m in scenario s and period t
NC_{ms}	Cost of salary for workers in manufacturer m in scenario s and period t
CM_s	Cost of raw material in scenario s
λ_s	Removing the cost of defective final products in scenario s
F_{mst}^{max}	The maximum number of workers that are fired by manufacturer m in scenario s and period t
F_{mst}^{min}	The minimum number of workers fired by manufacturer m in scenario s and period t
R_{mst}^{max}	The maximum number of workers hired by manufacturer m in scenario s and period t
R_{mst}^{min}	The minimum number of workers that are hired by manufacturer m in scenario s and period t
CH'_m	Cost of holding (final product) for manufacturer m in scenario s and period t
CH''_m	Cost of holding (raw material) for manufacturer m in scenario s and period t
CH_{ist}	Cost of holding (raw material) for supplier i in scenario s and period t

π_s Percentage probability of scenario s

Certain parameters:

cap_v The capacity of distributor’s vehicle v

CP_m The warehouse's capacity of manufacturer m for holding material

CPZ_n The capacity of manufacturer m for holding final product

CF_m fixed cost of production in manufacturer m

VF_v fixed cost of distributor’s vehicle v

VV_{vt} The variable cost of distributor’s vehicle v in period t

CV_{mt} The variable cost of production for manufacturer m in period t

ξ The production factor for workers

β The defective coefficient for manufacturers

$CT_{u''v}$ Transportation cost of distributor’s vehicle v for the interval (u'', o'') and period t

M_0 Big value

In the next sub-section, a multi-objective mixed-integer programming model is developed to model the pre-described problem.

B. MATHEMATICAL MODEL

$$\text{Min} \sum_{r=1}^R \sum_{t=1}^T \sum_{s=1}^S \pi_s \times (D_{rst} \times SH_{rst}) + \sum_{m=1}^M \sum_{s=1}^S \sum_{t=1}^T \pi_s \times (\varphi_{mst} \times \lambda_s) \tag{4}$$

$$\tag{5}$$

$$\begin{aligned} & \text{Min}(\sum_{m=1}^M \sum_{s=1}^S \sum_{t=1}^T \pi_s \times (w_{mst} \times (CF_m + CV_{mst}))) + (\sum_{i=1}^I \sum_{t=1}^T \sum_{s=1}^S \pi_s \times (\Omega_{ist} \times PR_{st})) + \\ & (\sum_{v=1}^V \sum_{u''=1}^{U''} \sum_{o''=1}^{O''} \sum_{t=1}^T CT_{o''u''vt} \times RT_{o''u''vt}) + (\sum_{m=1}^M \sum_{s=1}^S \sum_{t=1}^T \pi_s \times (F_{mst} \times FC_{mst})) + \\ & (\sum_{m=1}^M \sum_{s=1}^S \sum_{t=1}^T \pi_s \times (R_{mst} \times RC_{mst})) + (\sum_{m=1}^M \sum_{s=1}^S \sum_{t=1}^T \pi_s \times (N_{mst} \times NC_{mst})) + \\ & (\sum_{i=1}^I \sum_{m=1}^M \sum_{s=1}^S \sum_{v=1}^V \sum_{t=1}^T \pi_s \times (L_{imvst} \times CM_s)) + (\sum_{m=1}^M \sum_{s=1}^S \sum_{t=1}^T \pi_s \times (ZP_{mst} \times CH'_{mst})) + \\ & (\sum_{i=1}^I \sum_{s=1}^S \sum_{t=1}^T \pi_s \times (\omega_{ist} \times CH_{ist})) + (\sum_{m=1}^M \sum_{s=1}^S \sum_{t=1}^T \pi_s \times (IP_{mst} \times CH''_{mst})) + \\ & (\sum_{v=1}^V \sum_{t=1}^T \sum_{s=1}^S \pi_s \times (V_{vst} \times VV_{vt})) + (\sum_{v=1}^V \sum_{s=1}^S \sum_{t=1}^T \pi_s \times (V_{vst} \times VF_v) \end{aligned}$$

$$\sum_{u=1}^{U'} \sum_{v=1}^V RT_{uvrt} \geq 1 \quad \forall r \in R, t \in T \tag{6}$$

$$\sum_{u=1}^{U'} \sum_{v=1}^V RT_{ruvt} \geq 1 \quad \forall r \in R, t \in T \tag{7}$$

$$\sum_{r=1}^R RT_{mrvt} \leq 1 \quad \forall v \in V, t \in T, m \in M \tag{8}$$

$$\sum_{r=1}^R RT_{rmvt} \leq 1 \quad \forall v \in V, t \in T, m \in M \tag{9}$$

$$p'_{rv} - p'_{r'v} + (|R|) \times RT_{rr'vt} \leq |R| - 1 \quad \forall v \in V, t \in T, r \in R, r' \in R \tag{10}$$

$$\sum_{o'=1}^{O'} RT_{orvt} - \sum_{u'=1}^{U'} RT_{ruvvt} = 0 \quad \forall v \in V, t \in T, r \in R \quad (11)$$

$$\sum_{u=1}^U \sum_{v=1}^V RT_{umvt} = 1 \quad \forall m \in M, t \in T \quad (12)$$

$$\sum_{u=1}^U \sum_{v=1}^V RT_{muvt} = 1 \quad \forall m \in M, t \in T \quad (13)$$

$$\sum_{m=1}^M RT_{imvt} \leq 1 \quad \forall v \in V, t \in T, i \in I \quad (14)$$

$$\sum_{m=1}^M RT_{mivt} \leq 1 \quad \forall v \in V, t \in T, i \in I \quad (15)$$

$$p''_{mv} - p''_{m'v} + (|M|) \times RT_{mm'vt} \leq |M| - 1 \quad \forall v \in V, t \in T, m \in M, m' \in M' \quad (16)$$

$$\sum_{u=1}^U RT_{umvt} - \sum_{uu=1}^U RT_{muuvvt} = 0 \quad \forall v \in V, t \in T, m \in M \quad (17)$$

$$L_{o''uvvt} \leq RT_{o''uvvt} \times M_0 \quad \forall v \in V, t \in T, o'' \in O'', u'' \in U'' \quad (18)$$

$$IB_{mst} = \sum_{v=1}^V \sum_{i=1}^I L_{imvst} \quad \forall m \in M, t = 1, s \in S \quad (19)$$

$$IB_{mst} \geq w_{mst} \quad \forall s \in S, t \in T, m \in M \quad (20)$$

$$IB_{mst} - w_{mst} = IP_{mst} \quad \forall m \in M, t \in T, s \in S \quad (21)$$

$$IB_{mst} \geq \sum_{v=1}^V \sum_{i=1}^I L_{imvst} + IP_{mst-1} \quad \forall m \in M, t > 1, s \in S \quad (22)$$

$$\Omega_{ist} - \sum_{m=1}^M \sum_{v=1}^V L_{imvst} = \omega_{ist} \quad \forall s \in S, t \in T, i \in I \quad (23)$$

$$\Omega_{ist} \leq \varepsilon_{ist} \quad \forall s \in S, t \in T, i \in I \quad (24)$$

$$IB_{mst} \leq CP_m \quad \forall s \in S, t \in T, m \in M \quad (25)$$

$$Z_{mst} \leq CPZ_m \quad \forall s \in S, t \in T, m \in M \quad (26)$$

$$ZP_{mst} = Z_{mst} - \sum_{v=1}^V L_{mrvst} \quad \forall s \in S, t \in T, m \in M \quad (27)$$

$$\varphi_{mst} \leq \beta \times w_{mst} \quad \forall s \in S, t \in T, m \in M \quad (28)$$

$$Z_{mst} = w_{mst} - \varphi_{mst} \quad \forall s \in S, t = 1, m \in M \quad (29)$$

$$Z_{mst} = ZP_{mst} + w_{mst} - \varphi_{mst} \quad \forall s \in S, t > 1, m \in M \quad (30)$$

$$Z_{mst} \geq \sum_{v=1}^V \sum_{r=1}^R L_{mrvst} \quad \forall s \in S, t \in T, m \in M \quad (31)$$

$$SS_{mst} = \theta_{mst} \times \sum_{v=1}^V \sum_{r=1}^R DM_{rst} * RT_{mrvst} \quad \forall s \in S, t \in T, m \in M \quad (32)$$

$$IB_{mst} \geq SS_{mst} \quad \forall s \in S, t \in T, m \in M \quad (33)$$

$$SA_{rst} + D_{rst} = DM_{rst} \quad \forall r \in R, s \in S, t \in T \quad (34)$$

$$SA_{rst} \leq \sum_{v=1}^V \sum_{m=1}^M L_{mrvst} \quad \forall s \in S, t \in T, r \in R \quad (35)$$

$$D_{rst} \leq DM_{rst} \times \psi_{st} \quad \forall s \in S, t \in T \quad (36)$$

$$N_{mst} \geq 1/\xi \times w_{mst} \quad \forall s \in S, t \in T, m \in M \quad (37)$$

$$N_{mst} = R_{mst} - F_{mst} \quad \forall s \in S, t = 1, m \in M \quad (38)$$

$$N_{mst} = N_{mst-1} + R_{mst} - F_{mst} \quad \forall s \in S, t \in \{2, \dots, T\}, m \in M \quad (39)$$

$$F_{mst}^{min} \leq F_{mst} \leq F_{mst}^{max} \quad \forall s \in S, t \in T, m \in M \quad (40)$$

$$R_{mst}^{min} \leq R_{mst} \leq R_{mst}^{max} \quad \forall s \in S, t \in T, m \in M \quad (41)$$

$$V_{vst} = \sum_{m=1}^M \sum_{r=1}^R L'_{mrvst} + \sum_{m=1}^M \sum_{i=1}^I L_{imvst} / cap_v \quad \forall s \in S, t \in T, v \in V \quad (42)$$

$$L_{imvst}, w_{mst}, Z_{mst}, ZP_{mst}, L'_{mrvst}, SA_{rst}, \varphi_{mst}, SS_{mst}, IP_{mst}, IB_{mst}, \omega_{ist}, \Omega_{ist}, D_{rst} \geq 0$$

$$RT_{Ouvst} \in \{0, 1\}$$

$$F_{mst}, R_{mst}, N_{mst}, V_{vst} \geq 0 \text{ and integer}$$

Equation (4) expresses the minimization of the lost sales and the level of defective products, and it could warranty to maximize the quality of the supply chain. Equation (5) presents the minimization of the total cost containing the production cost, transportation costs in the different levels, supplying cost, holding cost at different levels, and hiring and firing cost.

Constraints (6) and (7) ensure that each retailer has to be served by at least one manufacturer and one vehicle in every period. This means that every retailer must be visited in each period. Constraints (8) and (9) guarantee that each vehicle can be used within at most one route in each period. It means that some of the existing vehicles may be left useless for some periods. We introduce constraint (10) to avoid sub-tours among the retailers and the manufacturers. If vehicle v enters an arc, it must leave that arc. Also, this vehicle could leave the manufacturer as a depot at the first of its route and return to the depot at the end, at most one time. Constraint set (11) covers this constraint.

Constraints (12)-(17) are the same as constraints (6)-(11), except that despite constraints (6)-(11), which relate to the routes between the manufacturers and the retailers, in these constraints, the constraints related to the routes between the suppliers and the manufacturers are addressed.

Constraint (18) explains that the materials and the final products could be transferred only in the determined routes. Constraint (19) states that the initial inventory of raw material for every manufacturer in the first period is equal to the purchased quantity of the raw material in this period by this manufacturer. Constraint (20) shows that the quantity produced by each manufacture in each period and scenario could not exceed the initial inventory of raw materials in that period.

Constraint (21) indicates the ending inventory of raw materials for every manufacturer at each period equals that portion of the initial inventories not used for production in that period.

Constraint (22) represents the level of inventory for each manufacturer in the next periods, which is equal to the remained inventory level of the former period with the assignment of this period.

Constraint (23) states that the quantity of the remained raw material in warehouses of each supplier in every period equals the quantity of the supplied materials not sold to the manufacturers in that period. Constraint (24) illustrates the maximum quantity of raw material that a supplier can supply couldn't exceed the available quantity in the market for him/her. Constraint (25) shows the initial inventory of raw material for every manufacturer in every scenario and each period is not allowed to exceed its warehouse's capacity.

Constraint (26) warrants that the initial inventory of the final product for every manufacturer in every scenario and each period is not allowed to exceed its warehouse's capacity. Constraint (27) indicates the ending inventory of the final

product for every manufacturer at each period equals that portion of the initial inventories not sold to the retailers that period.

Constraint (29) warrants that the number of defective final products in every period is not allowed to exceed a certain level. Constraints (29) and (30) depict the balance equation of the final products' inventory for the manufacturers in the first and the other periods, respectively. Constraint (31) expresses that the final products sold to the retailers equal to the level of inventory couldn't exceed the initial inventory of the final product in every period. Constraints (32) and (33) relate to the level of safety stock for the manufacturers in every period.

Constraint (34) expresses that the demand for the final product equals the sum of the sale and the lost sale amounts of the final product. Constraint (35) shows that the amount of final product sold by each retailer could not exceed the amount of final product received in every period.

Constraint (36) represents the maximum level of the lost sale for the retailers in each period. Constraint (37) warrants that the minimum level of manpower in every period depends on the production level.

Constraint (37) warrants that for producing a certain amount, a minimum level of manpower is required in every period. Constraints (38) and (39) are balanced equations of manpower that express the relation between the available manpower, the fired and hired manpower, for the first and the other periods, respectively. Constraints (40) and (41) represent the maximum and minimum levels of hiring and firing in every period, respectively. Constraint (42) reveals the required number of vehicles for the distributor.

C. Stackelberg equilibrium

In this study, the first and the second suppliers are considered as a leader and a follower, respectively that compare on their selling prices of the raw material for the manufacturers. The structure of the price could be defined as below:

P The price of the raw material

A The potential price when the sale is zero

v_1 The sale for the first supplier

v_2 The sale for the second supplier

A The intensity of the Stackelberg competition

r'_i The cost for supplier $i \in \{1,2\}$

μ_i The profit of supplier $i \in \{1,2\}$

$$P(v_1 + v_2) = A - \alpha(v_1 + v_2) \tag{43}$$

$$R_i(v_i) = r'_i \times v_i \quad i \in \{1,2\} \tag{44}$$

Equations (43) and (44) show the price and the cost of supplier i , respectively. Follower's profit is calculated from equation (45):

$$\mu_2 = P(v_1 + v_2) \times v_2 - R_2(v_2) = (A - (\alpha(v_1 + v_2))) \times v_2 - (r'_2 \times v_2) \tag{45}$$

Given the above-mentioned equations for the follower's profit, the differentiation is done and then set equal to zero to find the values of v_2 that maximizing the suppliers' profit.

$$\frac{\partial \mu_2}{\partial v_2} = 0 \rightarrow A - \alpha v_1 - 2 \alpha v_2 - r'_2 = 0 \quad (46)$$

$$v_2 = \frac{A - \alpha v_1 - r'_2}{2 \alpha} \quad (47)$$

By determining the best follower's strategy according to the leader's strategy, the leader also can determine its strategy in the most profitable way.

$$\mu_1 = (A - (\alpha(v_1 + v_2))) \times v_1 - (r'_1 \times v_1) = A v_1 - \alpha v_1 \left(\frac{A - \alpha v_1 - r'_2}{2 \alpha} \right) - \alpha v_1^2 - r'_1 v_1 \quad (48)$$

Equation (48) shows the profit function of the first supplier. We went through all steps to calculate the maximum profit of the follower with considering certain strategies for the follower to get the maximum profit of the leader by equation (49) and equation (50):

$$\frac{\partial \mu_1}{\partial v_1} = 0 \rightarrow \frac{A - 2\alpha v_1 + r'_2 - 2r'_1}{4} = 0 \quad (49)$$

$$v_1 = \frac{A + r'_2 - 2r'_1}{2\alpha} \quad (50)$$

$$v_1^* = \frac{A + r'_2 - 2r'_1}{2\alpha} \quad (51)$$

$$v_2^* = \frac{A - \alpha v_1 - r'_2}{2 \alpha} \quad (52)$$

Equations (51) and (52) show v_1^* (the equilibrium value of sale for the first supplier (leader)) and v_2^* (the equilibrium value of sale for the second supplier (follower)), respectively. (Rafiei et al., 2018).

In this model, we consider v_2^* , v_1^* as above, and OM_{ist} is defined as a competition variable, and the third objective function is added to the model via equation (54):

Parameters of competition:

$OM1^*$ Equilibrium value for the leader in Stackelberg competition

$OM2^*$ Equilibrium value for followers in Stackelberg competition

Variable of competition:

OM_{ist} Variable of competition that is equal to the quantity of sold materials
for supplier i in scenario s and period t

$$OM_{ist} = \sum_{m=1}^M \sum_{v=1}^V L_{imvst} \quad \forall i \in I, s \in S, t \in T \quad (53)$$

Equation (54) is the third objective function that tries to minimize the differences between equilibrium quantities of two suppliers in the Stackelberg competition.

$$\min \sum_{i=1}^I \sum_{s=1}^S \sum_{t=1}^T \pi_s * (|OM_{ist} - OM1^*| + \sum_{i=2}^I \sum_{s=1}^S \sum_{t=1}^T \pi_s * |OM_{ist} - OM2^*|) \quad (54)$$

D. Linearization

According to the absolute of the objective function, the proposed competitive objective is MINLP; therefore, the below procedure is applied to overcome the nonlinearity of the model (Bisschop, 2006).

$$\sum_i \sum_s \sum_t |x_{ist} - b| \tag{55}$$

An equation with an absolute value results in the nonlinearity of the model. Thus, binary variables are defined as follows:

Equation (55) could be rewritten as follow:

$$y_{ist}^+ = \begin{cases} x_{ist} - b & \text{if } x_{ist} - b > 0 \\ 0 & \text{ow} \end{cases}$$

$$y_{ist}^- = \begin{cases} b - x_{ist} & \text{if } x_{ist} - b < 0 \\ 0 & \text{ow} \end{cases}$$

And $y_{ist}^+, y_{ist}^- \geq 0$

So, equation (55) is as follows:

$$\sum_s \sum_i \sum_t y_{ist}^+ + y_{ist}^-$$

The constraint (56) was also added to the mathematical model:

$$x_{ist} - b = y_{ist}^+ - y_{ist}^- \tag{56}$$

y_{ist}^+, y_{ist}^- were defined in the model as below:

MP_{ist} : The variable of competition for linearization

MN_{ist} : The variable of competition for linearization

The third objective function could be rewritten as Equation (57):

$$\min \sum_{i=1}^I \sum_{s=1}^S \sum_{t=1}^T \pi_s * (MP_{ist} + MN_{ist}) \tag{57}$$

The constraints (58) and (59) are added to the model. Hence, the model is converted to a linear one by applying the below changes:

$$MP_{ist} - MN_{ist} = OM_{ist} - OM1 * \quad i = 1, \forall s \in S. t \in T \tag{58}$$

$$MP_{ist} - MN_{ist} = OM_{ist} - OM2 * \quad i = 2, \forall s \in S. t \in T \tag{59}$$

IV. SOLUTION METHODOLOGY

To solve the model, we consider some exact and meta-heuristic solution methods that will be described in this section.

Initially, GAMS software is used to solve small-sized instances utilizing a computer with a Core i5- 6200U CPU and a 2.3 GHz 4GB RAM. The results are shown to describe the behavior of the model. A sensitivity analysis is carried out with GAMS software. This part indicates the impact of various input parameters on the objective functions and variables and aims at investigating the impacts of different scenarios in critical or non-critical cases.

The previous studies have implied that routing is one of the complicated and well-known NP-hard problems (Farrokhi-Asl et al. □ 2017). Their results showed the incapability of this software for large instances to solve the model in a reasonable and acceptable amount of time. As a result, meta-heuristic algorithms could be useful once the problem has several nodes. Therefore, three meta-heuristic algorithms (NSGA-II, MOPSO, and a combination algorithm) are developed for solving the model. The results obtained from these algorithms are compared with ones obtained from

GAMS software to validate the efficiency and effectiveness of the developed meta-heuristics.

To this end, eight stochastically produced instances in various sizes are solved by the algorithms and the obtained results are analyzed and discussed in the next sections.

A. The developed NSGA-II

Genetic and PSO algorithms are successful algorithms regarding the optimization problem. NSGA-II and MOPSO algorithms are derivatives of the mentioned algorithms for solving the multi-objective models. NSGA-II algorithm generates high quantity solutions and MOPSO algorithm can reach logical solutions with less computational time (Farrokhi-Asl et al. □ 2017). NSGA-II was introduced by Deb et al. (2002). It is based on the Pareto solution and comprises the steps below (Deb et al. □ 2002 and Farrokhi-Asl et al. □ 2017):

Step 1. The initial solutions are generated randomly. The chromosomes have two types. The first type shows the value of inventory, which is a continuous type. The second type is a discontinuous chromosome that is related to the routing problem of the model. Fig (2) depicts a discontinuous sample.

Step 2. The initial solutions are evaluated by a non-dominated rating and crowding distance. To sort them, a non-dominated rating is assigned due to the objective function quantities. The first-ranked solution is the best one. Rate two is more dominated. The same trend continues for the next levels.

Step 3. Once the ranks between two members are similar, we calculate crowding distance with the average distance of two points on either side of the point along with each of the objectives. Crowding distance is used to estimate the density of members surrounding a particular member of the population. The distribution of Pareto solutions is made uniformly by applying this operator. Between two members with the same rates, the priority goes to the member located in a less crowded region.

Step 4. Some of the members are chosen. Subsequently, crossover and mutation are applied to them. Children are created and they merge with parents. There are different types of crossover and mutation operators that could be used to generate children. For example, we use swap mutation for routing chromosomes as Fig (3).

In this type of mutation, two points are selected randomly and their places are changed to create a new child (Yang □ 2020). To produce children from the inventory's chromosomes (continuous type) arithmetic crossover is employed. This crossover operator combines two parents linearly to produce a child according to the equations (60) (Jin et al. □ 2017).

$$\text{Child1} = a' * \text{Parent1} + (1 - a') * \text{Parent2} \quad (60)$$

a' is generated randomly between 0 and 1.

Children and parents are combined to make a new population. Afterward, non-dominated rating and crowding distance are determined for them.

Step 5. Sorting the merged population is done based on these two factors.

Step 6. The non-dominated members positioned at the top of the ranking are saved in the archive as Pareto solutions, and additional populations are eliminated.

Step 7. These steps continue until meeting the stop condition.

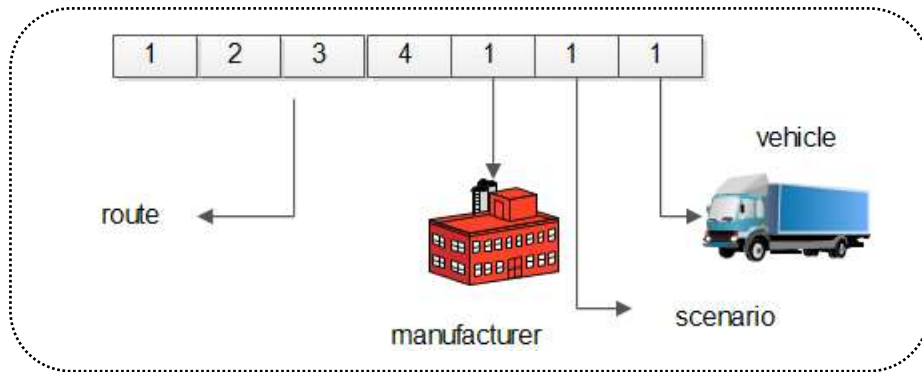


Fig 2. The chromosome of routing

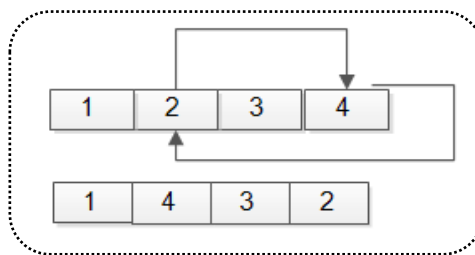


Fig 3. The swap mutation

B. The developed MOPSO

MOPSO is an efficient algorithm developed by (Coello and Lechuga, 2002). The literature shows that this algorithm has been successful in solving several complex routing problems and makes the Pareto front closer to the desired Pareto fronts. In this algorithm, the structure of the chromosome of NSGA-II is used, yet this algorithm is for continuous problems. Therefore, the uniform random numbers between 0 and 1 are used (Farrokhi-Asl et al. 2017). The steps of this algorithm are as follows:

Step 1. A population is generated randomly. Each member (particle) of this population has one velocity vector. For each particle, the velocity vector consists of a speed that indicates the direction in which the current position of the particle could be improved. Moreover, each particle is evaluated, and its position is stored by personal best (pbest), which shows the best position of particles.

Step 2. Non-dominated solutions are determined and saved in a group called a repository. The members of a repository are the leaders which lead the other particles toward a better place.

Step 3. The velocity and position of every solution are up to date as follows:

$$VE_i = W \times VE_i + c1r1 \times (pbest_i - pop_i) + c2r2 \times (rep \cdot pop_i - pop_i) \tag{61}$$

$$pop_i = pop_i + VE_i \tag{62}$$

In equation (61), W is the inertia weight and can control the impact of velocity history. $r1$ and $r2$ are random numbers between (0,1). $c1$ and $c2$ represent social learning factors that are associated with the particle’s successes and its neighborhood success. $pbest_i$ shows the best place for i th particle. $rep \cdot pop_i$ is the value gained from the repository and pop_i is the position of the particle. Afterward, the new position is determined with equation (62).

Step 4. In each iteration, new non-dominated solutions are saved in the repository, and the dominated ones are deleted. This process continues according to the determined criteria to end the algorithm.

C.A. hybrid algorithm of MOPSO and NSGA-II

Additionally, in this study, a hybrid algorithm is developed. Fig (4) depicts the process of the presented algorithm. This process is a hybrid of MOPSO and NSGA-II algorithms. In this hybrid algorithm, the initial population with continuous and discontinuous chromosomes is generated. Then, the factors of NSGA-II are calculated, and members of the population are sorted by these factors. Parents are selected and the operators of NSGA-II are calculated to make a new population. In the next step, MOPSO is applied to the new population, and this process continues until the finishing factors of the algorithm.

D. Discussion and experimental results

In this section, we validate the efficiency and effectiveness of the proposed algorithm to solve the model. Moreover, there are no similar models in the literature; therefore, we cannot compare our developed algorithms with the results obtained from the other works. Thus, we generate several random numerical examples in which their parameters are generated in some predetermined intervals of uniform distributions. These intervals and the value of constant parameters are inspired by a real-life case medicine industry to show the applicability of the proposed model. This industry produces Vitamin tablets, and this product has two exclusive suppliers that compete. Without loss of generality, the model could be useful in other supply chains applications.

This company has two suppliers, two manufacturers and two type of vehicles for transportation. Moreover, we study this case for three time periods and two retailers. When Covid-19 started, the demand for vitamins, to boost human immunity increased. So, this industry has chaotic years and face with some challenges. For example, the biggest challenge in this industry is market forecasting to grow customer service levels and be ahead in the competition. Furthermore, in this supply chain quality and cost of transportation is essential and choosing an appropriate strategic approach could optimize the network cost.

Also, human resource management to make accurate estimates and prevent production shortages, which is one of the main goals of the company. As a result, the company needs integrated planning to increase coordination and meet uncertain demand to minimize shortages and reduce network costs.

Table (II) represents the determined range of input parameters for the numerical example. This table is related to the first problem in this section solved with GAMS. Furthermore, we use a parameter M as a large enough parameter to solve the LP model. This parameter is useful when it is large enough. Precise evaluation is complex. Meanwhile, a value bigger than needed may generate a loss of precision and numerical instabilities. The value of this parameter depends on the other parameters of the model (Song, 2015). So, we determine M_0 according to the parameters and size of each problem to avoid infeasibility.

Since this model is multi-objective, to solve the model in GAMS software, each objective function is normalized by solving the model with each objective function, individually and obtaining their optimal values. Then the Weighted Sum method is used to turn the model into a single objective one. In this method, a weight coefficient is assigned for each objective function and the weighted sum of the objective functions are calculated as displayed in Equation (63):

$$Z4 = w1 \times Z1 + w2 \times Z2 + W3 \times Z3 \quad (63)$$

We employ MACBETH method to determine $W1$, $W2$, and $W3$. This method needs qualitative judgments for each objective function (Bana e Costa & Chagas 2002). We suppose that the lost sale penalty cost of defective final products is important and two other objective functions have the same importance.

The obtained results in Fig (5) show that the demand points are served at least by one supplier in route 1 and one manufacturer in route 2. Moreover, no sub-tour is made, and each vehicle comes back to its source. Also, this Figure shows the supplied materials transferred through the specified routes due to the supply restrictions in situations both suppliers are active in each period and demonstrates the quantities of the products sent to the retailers based on their

demands through the specific routes.

Table (III) shows the value of competitive variables. In this case, the suppliers prefer to remain in the competition and when these variables get a certain quantity, it means the suppliers could leave the competition. On several occasions, although the equilibrium value is more than the demand for the supply chain, the supplier prefers to remain. But sometimes, the cost or shortage in the available quantity makes the suppliers decide to leave the market.

Table (IV) represents the production amount and level of the defective final products. As seen, the level of intact products is equal to the total amount of the final products produced, subtracting the number of defective final products in every period. On top of that, in this instance, the amount of the produced final product is equal to the procurement amount of the raw materials from the supplier.

Table (V) shows some raw materials in the manufacturer’s warehouses as a safety stock in every period, which is a function of demand. This table also shows the primary levels of the raw materials for every manufacturer, which are equal to the received materials from the suppliers.

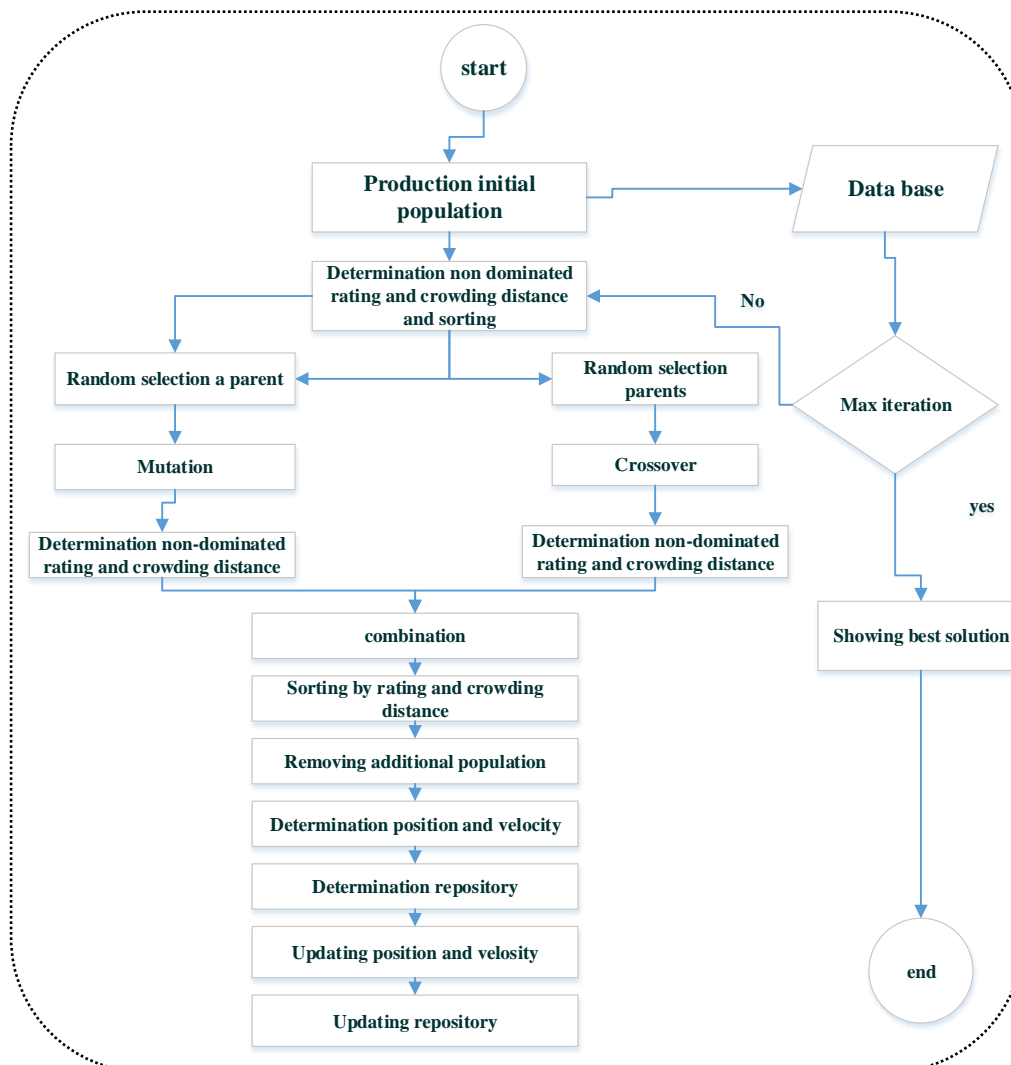


Fig 4.The process of the developed hybrid algorithm

Table II. The range of the parameters for the first sample in GAMS software

<i>parameter</i>	<i>value</i>	<i>Parameter</i>	<i>Value</i>	<i>parameter</i>	<i>value</i>	<i>parameter</i>	<i>Value</i>
ξ	10	ε_{ist}	(120,130)	CF_m	(10,12)	$CT_{vwio't}$	(5,15)
DM_{rst}	(80,100)	ψ_{st}	(0.1,0.2)	CV_{mt}	(5,10)	SH_{rst}	(50,70)
PR_{ist}	(25,27)	CM_{ms}	(30,38)	FC_{mst}	(10,15)	CH_{mst}	(20,25)
θ_{mst}	(0.05,0.1)	RC_{mst}	(15,17)	λ_s	(10,20)	cap_v	(50,120)
F_{mst}^{max}	(3,10)	R_{mst}^{max}	(3,10)	CP_m	(110,130)	CH'_{mst}	(10,15)
F_{mst}^{min}	(1,2)	R_{mst}^{min}	(1,2)	CPZ_m	(110,120)	CH''_{mst}	(12,14)
$OM1 *$	120	VF_v	(5,10)	VV_{vt}	(2,4)	$OM2 *$	110
M_o	1.2×102	α	(0.1,1)	$W1$	0.58	$W2$	0.21
$W2$	0.21	r'_i	(25,27)	β	0.05		

Table (VI) indicates that the level of raw materials and the product's inventory are equal to zero at the end of the period for each manufacturer. Table (VII) illustrates the condition of the human resources of each supplier in every period expressing that some of the man powers are fired or hired and the number of human resources is at the optimal level due to the production. Table (VIII) expresses the level of sales for each retailer. These findings indicate that since the level of lost sales is zero, the sales are equal to the demands. Table (IX) shows the number of the supplied materials by each supplier. This table also demonstrates some of the remaining materials in the supplier's warehouses and the supplier prefers to retain this number of materials to remain in the competition. Table (X) depicts the number of required vehicles in every period.

Table III. The competitive variables

<i>Variable 1</i>	<i>indexes</i>	<i>value</i>	<i>Variable 2</i>	<i>indexes</i>	<i>value</i>	
MN_{ist}	$i = 1 \quad s = 1$	$t = 1$	0	MP_{ist}	$t = 1$	0
		$t = 2$	0		$t = 2$	0
		$t = 3$	0		$t = 3$	0
	$i = 2 \quad s = 1$	$t = 1$	0		$t = 1$	0
		$t = 2$	0		$t = 2$	0
		$t = 3$	0		$t = 3$	0

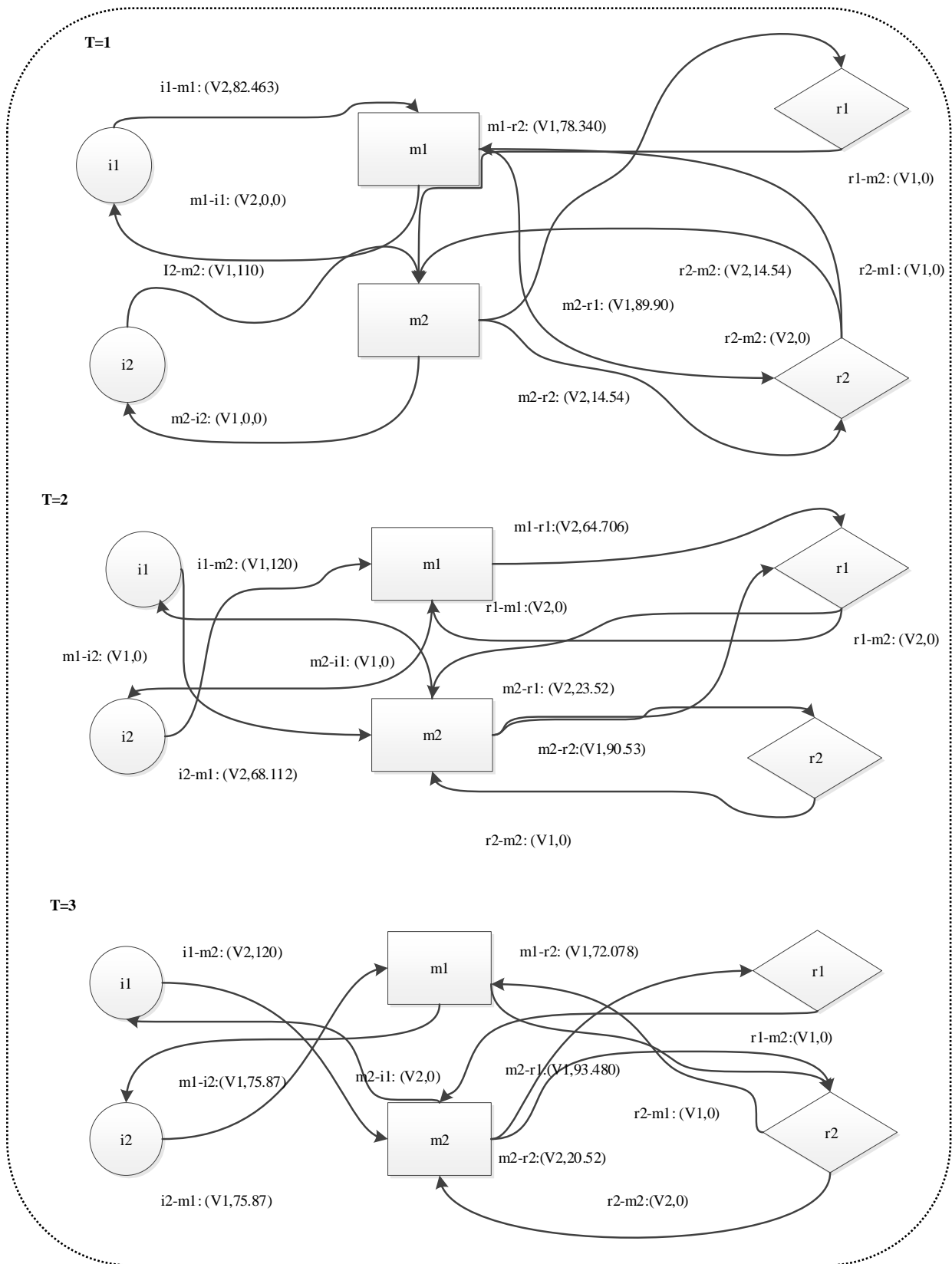


Fig 5.The result of GAMS software

Table IV. Level of inventory, defective products, and production

Variable1	Indexes			value	Variable2	indexes			value	Variable3	indexes			value	
w_{mst}	$m = 1$	$s = 1$	$t = 1$	82.463	ϕ_{mst}	$m = 1$	$s = 1$	$t = 1$	4.12	z_{mst}	$m = 1$	$s = 1$	$t = 1$	78.340	
			$t = 2$	68.112				$t = 2$	3.4				$t = 2$	64.706	
			$t = 3$	75.872				$t = 3$	3.7				$t = 3$	72.078	
	$m = 2$			$t = 1$	110	$m = 2$			$t = 1$	5.5	$m = 2$			$t = 1$	104
				$t = 2$	120				$t = 2$	6				$t = 2$	114
				$t = 3$	120				$t = 3$	6				$t = 3$	114

Table V. Level of safety stock and raw materials

Variable 1	indexes			value	Variable 2	indexes			value
SS_{mst}	$m = 1$	$s = 1$	$t = 1$	4.73	IB_{mst}	$m = 1$	$s = 1$	$t = 1$	82.46
			$t = 2$	7.47				$t = 2$	68.11
			$t = 3$	7.93				$t = 3$	75.87
	$m = 2$	$s = 1$	$t = 1$	16.52		$m = 2$	$s = 1$	$t = 1$	110
			$t = 2$	14.40				$t = 2$	120
			$t = 3$	18.08				$t = 3$	120

Table VI. Level of raw materials and inventory

Variable 1	indexes			value	Variable 2	indexes			value
IP_{mst}	$m = 1$	$s = 1$	$t = 1$	0	ZP_{mst}	$m = 1$	$s = 1$	$t = 1$	0
			$t = 2$	0				$t = 2$	0
			$t = 3$	0				$t = 3$	0
	$m = 2$	$s = 1$	$t = 1$	0		$m = 2$	$s = 1$	$t = 1$	0
			$t = 2$	0				$t = 2$	0
			$t = 3$	0				$t = 3$	0

Table VII. Human resources conditions

Variable 1	indexes			value	Variable 2	indexes			value	Variable 3	indexes			value
F_{mst}	$m = 1$	$s = 1$	$t = 1$	2	R_{mst}	$m = 1$	$s = 1$	$t = 1$	10	N_{mst}	$m = 1$	$s = 1$	$t = 1$	8
			$t = 2$	4				$t = 2$	1				$t = 2$	8
			$t = 3$	2				$t = 3$	2				$t = 3$	8
	$m = 2$	$s = 1$	$t = 1$	2		$m = 2$	$s = 1$	$t = 1$	13		$m = 2$	$s = 1$	$t = 1$	11
			$t = 2$	2				$t = 2$	3				$t = 2$	12
			$t = 3$	2				$t = 3$	2				$t = 3$	12

Table VIII. Level of sale and lost sale

Variable1	indexes			value	Variable2	indexes			value
SA_{rst}	$r = 1$	$s = 1$	$t = 1$	89.90	D_{rst}	$r = 1$	$s = 1$	$t = 1$	0
			$t = 2$	88.22				$t = 2$	0
			$t = 3$	93.48				$t = 3$	0
	$r = 2$	$s = 1$	$t = 1$	92.881		$r = 2$	$s = 1$	$t = 1$	0
			$t = 2$	90.537				$t = 2$	0
			$t = 3$	92.59				$t = 3$	0

Table IX. Remaining products in supplier's warehouses and the amount of supplied products

<i>Variable1</i>	<i>indexes</i>			<i>value</i>	<i>Variable2</i>	<i>indexes</i>			<i>value</i>
Ω_{ist}	i = 1	s = 1	t = 1	120	ω_{ist}	i = 1	s = 1	t = 1	37.53
			t = 2	120				t = 2	0
			t = 3	120				t = 3	0
	i = 2	s = 1	t = 1	110		i = 2	s = 1	t = 1	0
			t = 2	110				t = 2	41.88
			t = 3	110				t = 3	34.128

Table X. Number of required vehicles

<i>variable</i>	<i>indexes</i>			<i>value</i>
V_{vst}	v = 1	s = 1	t = 1	2
			t = 2	3
			t = 3	4
	v = 2	s = 1	t = 1	4
			t = 2	4
			t = 3	3

E. Sensitivity Analysis

For sensitivity analysis, the impact of critical situations (increasing cost or decreasing available material) on the model behavior is investigated in different scenarios with different costs. Once the cost of raw material rises due to a crisis, the cost of purchase from the supplier increases consequently. We investigate the impacts of the increase in these two parameters on the model behavior at first in Table (XI).

Table XI. Variation of objective functions

	<i>Interval1</i>	<i>Interval2</i>	<i>Interval3</i>	<i>Interval4</i>	<i>Interval5</i>	<i>Interval6</i>	<i>Interval7</i>	<i>Interval8</i>	<i>Interval9</i>
PR_{st}	(25,27)	(25.5,27.5)	(26,28)	(26.5,28.5)	(27,29)	(27.5,29.5)	(28,30)	(28.5,30.5)	(29,31)
CM_{ms}	(30,38)	(30.5,38.5)	(31,39)	(31.5,39.5)	(32,40)	(32.5,40.5)	(33,41)	(33.5,41.5)	(34,42)
Z2	21940	22314	22706	22427	21874	21735	21653	21532	21491
Z3	0	0	0	18	24	37	114	125	134

According to Table (XI) and Figures (6) and (7), with an increase in the purchasing cost of raw material, the purchasing cost of material for the manufacturer increases. However, the model decided to stay in the competition and this trend continued up to one interval. Therefore, when this cost increases more than this interval, the suppliers prefer to supply less, and the cost function starts to decrease.

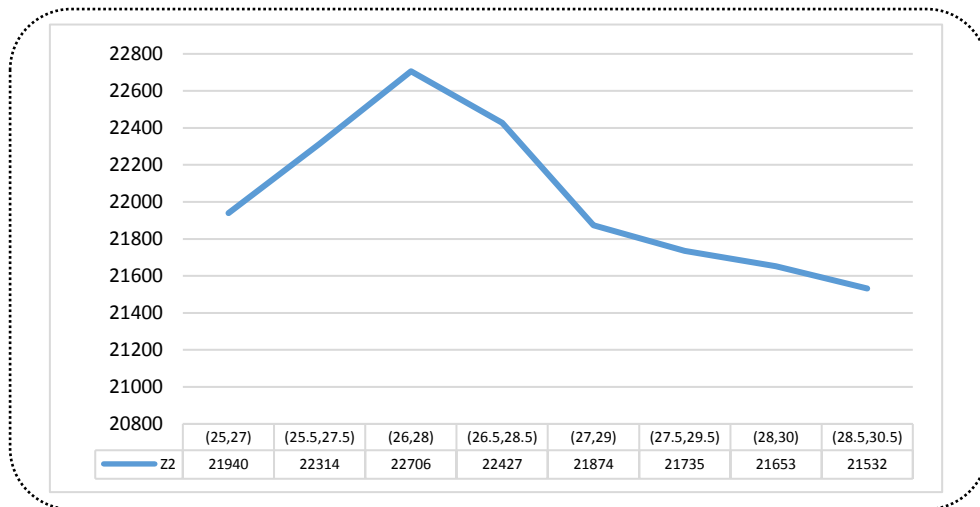


Fig 6. Effect of increasing raw materials costs on the second objective function

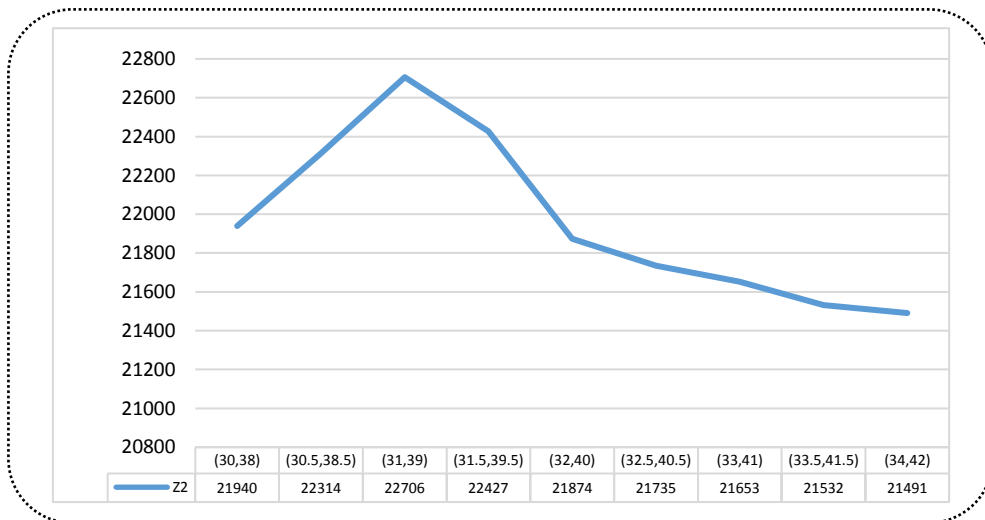


Fig 7. Effect of increasing procurement raw materials costs on the second function

Figures (8) and (9) represent that the internal quantity of the supplied products increased from competitive values due to the increase in the prices. Accordingly, the third objective function increases.

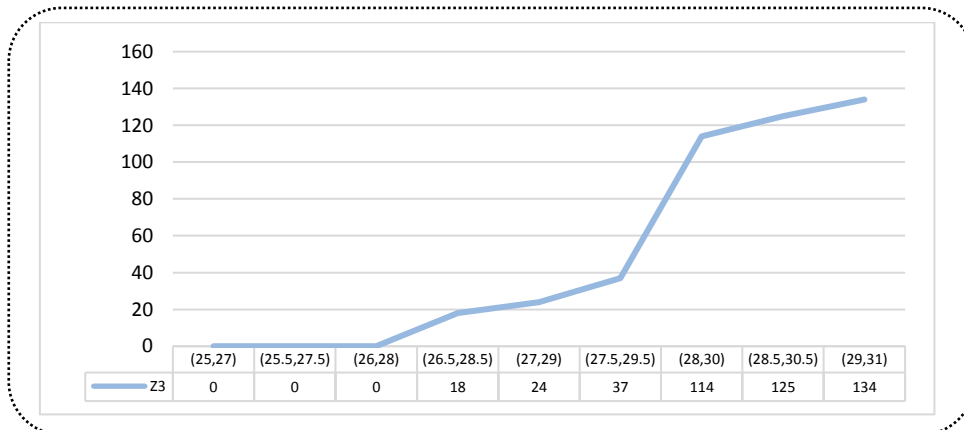


Fig 8. Effect of increasing raw materials costs on the third function

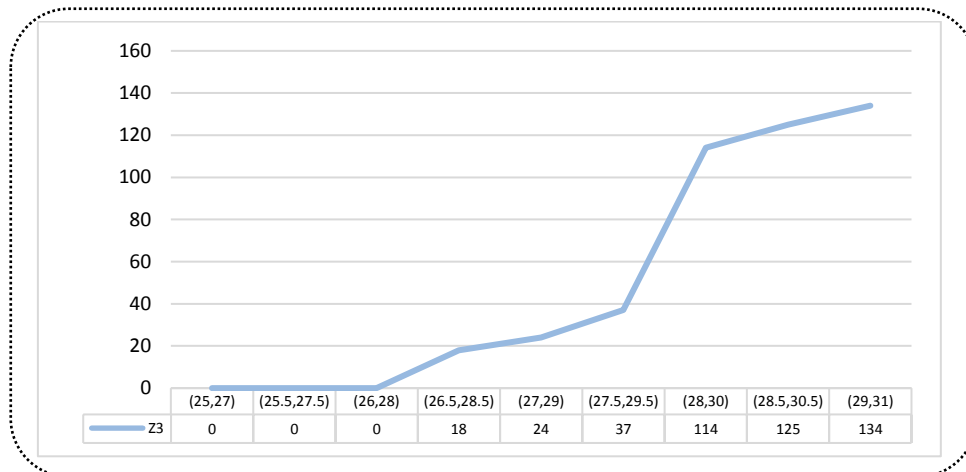


Fig 9. Effect of increasing procurement raw materials costs on competitive function

According to Table (XII) and Figures (10-12), the accessibility to the raw materials may become low under a crisis; the lower this initial level is, the higher the amount of lost sale would be (first objective function); the cost reduces (the second objective function) and the provided raw materials get far from the equilibrium values. Therefore, there is no competition in the model.

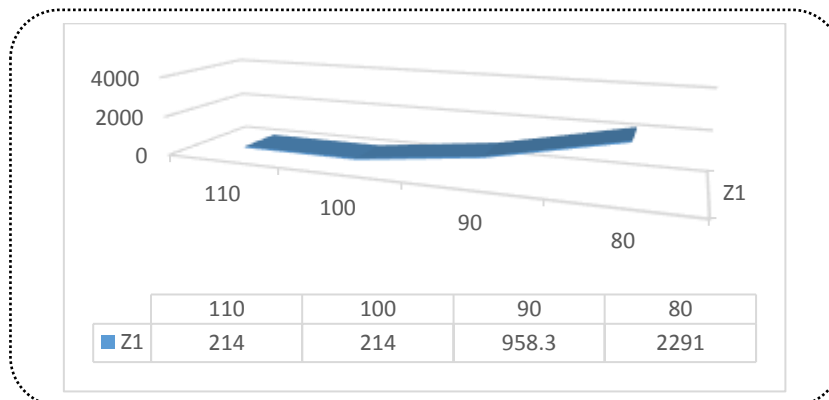


Fig 10. Effect of decreasing available inventory of raw materials on the first objective

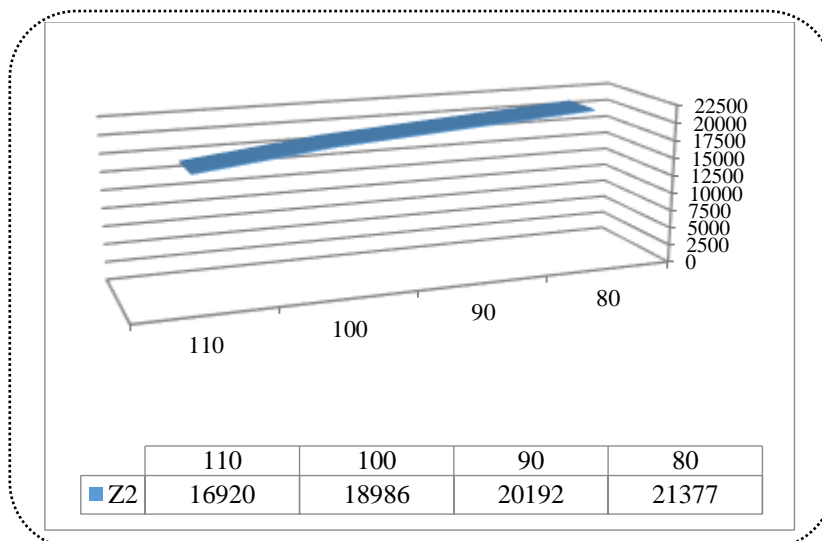


Fig 11. Effect of decreasing available inventory of raw materials on the second objective

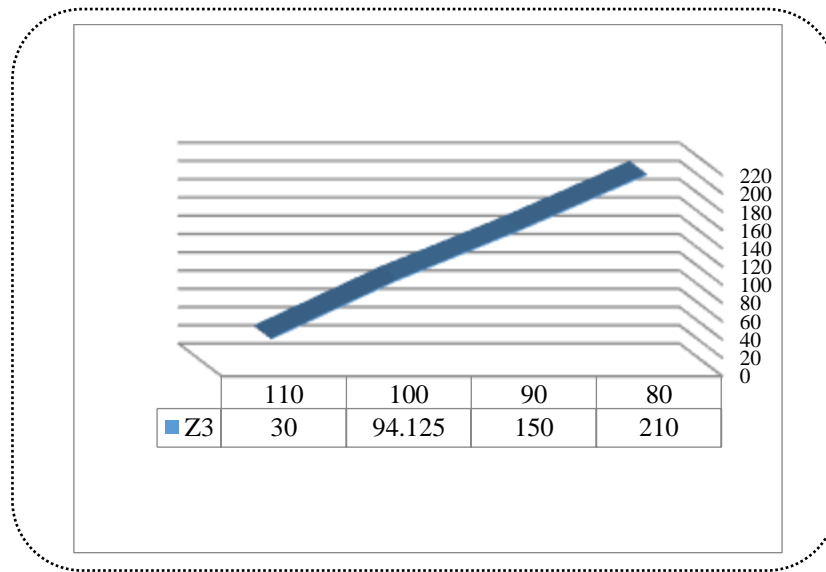


Fig 12. Effect of decreasing available inventory of raw materials on the third objective

Table XII. Effect of decreasing available inventory of raw materials

ϵ_{ist}	110	100	90	80
Z1	214	214	958.3	2291
Z2	21377	20192	18986	16920
Z3	30	94.125	150	210

In the following, we examine the parameters affecting human resources in the network. In Tables (XIII) to (XIX), these parameters and human resource variables are specified. It should be noted that the numbers in the tables are equal to the sum of the matrix numbers of the variable

Table (XIII) shows by increasing the quantity of demand, the quantity of production increase simultaneously. So, the number of human resources and recruitments increases too, but the number of dismissals does not change much, because the network needs these human resources.

Table XIII. Effect of demand on manpower

DM_{rst}	(30,40)	(40,50)	(50,60)	(60,70)	(70,80)
N_{mst}	22	30	37	43	49
F_{mst}	13	16	16	16	16
R_{mst}	20	25	28	30	32

Table XIV. Effect of penalty of lost sale on manpower

SH_{rst}	(40,50)	(50,60)	(70,80)	(80,90)	(90,100)
N_{mst}	32	36	37	37	37
F_{mst}	14	15	16	16	16
R_{mst}	25	27	28	28	28

Table (XIV) demonstrate increasing the quantity of lost sale penalty cost (for example in critical situation and the importance of commodities) grows the number of requirements, human resources and dismissals.

Table (XV) demonstrates by growth in the salary, the number of human resource and requirements decrease gradually. But the number of dismissals increases until its maximum level.

Table XV. Effect of salary cost on manpower

<i>NC_{mst}</i>						
	<i>(1,10)</i>	<i>(10,20)</i>	<i>(20,30)</i>	<i>(30,40)</i>	<i>(40,50)</i>	<i>(50,60)</i>
<i>N_{mst}</i>	44	40	37	37	36	34
<i>F_{mst}</i>	12	15	16	16	16	16
<i>R_{mst}</i>	30	29	28	28	27	24

Also, Table (XVI) shows, when the cost of dismissal enhances, the number of human resources increase, too. Because keeping human resources in this situation is more economical. Moreover, Table (VVII) reveals that the growing cost of requirements could decrease the number of requirements and fired persons.

Table XVI. Effect of salary cost on manpower

<i>FC_{mst}</i>						
	<i>(5,10)</i>	<i>(10,15)</i>	<i>(15,20)</i>	<i>(20,25)</i>	<i>(25,30)</i>	<i>(30,35)</i>
<i>N_{mst}</i>	37	37	37	38	38	44
<i>F_{mst}</i>	16	16	16	15	15	12
<i>R_{mst}</i>	28	28	28	28	28	28

Table XVII. Effect of requirement cost on manpower

<i>RC_{mst}</i>							
	<i>(10,15)</i>	<i>(15,20)</i>	<i>(20,25)</i>	<i>(25,30)</i>	<i>(30,35)</i>	<i>(35,40)</i>	<i>(50,60)</i>
<i>N_{mst}</i>	38	38	38	38	38	38	38
<i>F_{mst}</i>	15	15	15	15	15	15	12
<i>R_{mst}</i>	28	28	28	28	28	28	25

Moreover, Table (XVIII) shows that the efficiency of human resource is important and when production rate increase the number of required workers reduce and it can impact on cost function.

Table XVIII. Effect of production rate on manpower

<i>ξ</i>	5	10	15	20	25
<i>N_{mst}</i>	NA	38	26	20	16
<i>F_{mst}</i>		16	12	12	12
<i>R_{mst}</i>		28	21	19	14

Moreover, when the production rate is 5, the number of requirements is higher than the Maximum level and the problem is infeasible.

Also, in this section, the impact of material cost in Table (XIX) is demonstrated. Results shows, the high level of commodities cost decreases the number of production and the number of required workers for production decrease too. So, we must control this cost to prevent lost sales and rising unemployment.

In the end, the impact of penalty for lost sales on objective functions is investigated in Table (XX).

This investigation shows in this network penalty cost could reduce the lost sale quantity and increase production quantity. So, the second objective function enhances. Moreover, this penalty cost can bring the amount of supply closer to the equilibrium value. But this trend stops because the lost sales become zero after the seventh interval.

F. Comparison among the results obtained from the exact and meta-heuristic algorithms

In this section, to select the best method among the mentioned approaches to solve the large-sized instances, we compare the results of these algorithms with the obtained results from GAMS software. So, some samples are created and the answer to the second objective function and CPU time for solving these problems are compared. For the investigation of the results, we use Equation (64) to calculate the proximity of the answer to the optimal answer. The results are shown in Table (XXI)

$$GAP\% = \frac{meta\ z_2 - gams\ z_2}{gams\ z_2} * 100 \tag{64}$$

Each sample is repeated four times. The obtained results reveal that the function of the hybrid algorithm has better performance in this sub-section. Moreover, CPU time for MOPSO is less than that of the other meta-heuristic algorithms, and CPU time for the hybrid algorithm is higher than others. Due to the quality of the solutions of the hybrid algorithm and the small-time difference of the solution compared to the other two algorithms, this difference can be ignored.

Table XIX. Effect of cost of production rate on manpower

CM_{mst}					
	(10,20)	(20,30)	(30,40)	(40,50)	(50,60)
N_{mst}	38	38	38	38	34
F_{mst}	15	15	15	15	14
R_{mst}	28	28	28	28	26

Table XX. Effect of penalty on objective function

SH_{rst}									
	(1,10)	(10,20)	(20,30)	(30,40)	(40,50)	(50,60)	(60,70)	(70,80)	(80,90)
Z1	573.59	1103	1641	2193	1251	390	202.96	202.96	202.96
Z2	21152	21152	21152	21152	23360	24881	25179	25179	25179
Z3	96.34	96.34	96.34	96.42	42	42	38	38	38

In the next step, a method (Moattar Husseini et al. □ 2015) is applied to evaluate the results of three different meta-heuristics used in medium and large sizes. This method consists of some steps as follows:

1. Keeping the non-dominated answers for three algorithms in an archive,
2. Calculating rank, crowded distance, and the ratio of non-dominated solution

The results are shown in Table (XXII). Also, the number of non-dominated solutions of each algorithm is shown in this table.

3. Calculating the ratio of non-dominated solution by Equation (65).

$$Y_{MOPSO} = \frac{|\{x \in B_{MOPSO}\} \cap \{x \in UB_{ND}\}|}{|B_{MOPSO}|} \quad UB_{ND} = \{B_{(MOPSO)}, B_{(NSGA_II)}, B_{(Hybrid)}\} \quad (65)$$

$B_{(MOPSO)}$: (The set of non-dominated answers of MOPSO)

$B_{(NSGA_II)}$: (The set of non-dominated answers of NSGA_II)

$B_{(Hybrid)}$: (The set of non-dominated answers of Hybrid)

The results demonstrate that the solutions of the hybrid algorithm can dominate the solution of NSGA_II more. This shows the quality of answers and applicability of hybrid algorithm.

Table XXII. Comparison between metaheuristic algorithms

Problem	No. experiment	Average ratio of non-dominated solution		
		MOPSO	NSGA-II	Hybrid algorithm
8	1	0.62	0.74	1
	2	0.49	0.66	0.99
	3	0.77	0.71	0.89
9	1	0.47	0.59	0.86
	2	0.23	0.48	1
	3	0.49	0.52	1
10	1	0.49	0.46	0.98
	2	0.48	0.72	0.79
	3	0.54	0.75	0.88
11	1	0.77	0.92	0.81
	2	0.28	1	0.77
	3	0.69	0.77	0.95
12	1	0.74	0.53	1
	2	0.55	0.47	0.94
	3	0.49	0.74	1

G. The results of the developed hybrid algorithm

Based on the findings in the previous sub-section, the hybrid algorithm has an appropriate and acceptable performance. Thus, we could apply it to solve the instances in different sizes. Table (XXIII) represents different sizes of the problem solved with MATLAB, whose results are shown in Table (XXIV). The number of iteration, population and probability of Mutation and Crossover are 100,50,0.5 and 0.5 respectively.

Table XXIII. Size of solved problems in MATLAB

number	Size
13	(i = 2; m = 4; r = 8; v = 2; t = 3; s = 3)
14	(i = 2; m = 6; r = 12; v = 2; t = 3; s = 3)
15	(i = 2; m = 8; r = 16; v = 2; t = 3; s = 3)
16	(i = 2; m = 10; r = 20; v = 2; t = 3; s = 3)
17	(i = 2; m = 12; r = 24; v = 2; t = 3; s = 3)
18	(i = 2; m = 14; r = 28; v = 2; t = 3; s = 3)
19	(i = 2; m = 16; r = 32; v = 2; t = 3; s = 3)
20	(i = 2; m = 18; r = 36; v = 2; t = 3; s = 3)

Given the results of the competition variables, the model uses the competitive value in some scenarios and does not use it in some other scenarios. In addition, MP_{ist} equals zero indicate that the amount of the supplied materials is less than or equal to that of the competitive value.

Furthermore, L_{imvst} shows the amount of flow from the suppliers to the manufacturer, and w_{mst} states that each supplier produces based on the capacity and input materials. Z_{mst} is also equal to the amount of the products minus the defective final products. These results reveal that this interval is close to the produced goods. One of the objectives of this study decreases defective products. Hence, being close to this interval expresses that the waste is low in this network.

Moreover, the results of the variables L'_{mrvhst} , SS_{mst} , and IB_{mst} indicates that the manufacturers produce their entire raw materials. A part of the model decides on human resources levels and F_{mst} , R_{mst} , and N_{mst} implies that they are considered based on the production of the manufacturers.

Additionally, our findings reveal that the variables φ_{mst} , SA_{rst} , and D_{rst} have the permitted value in each instance. Furthermore, the results display that on several occasions, Ω_{ist} is equivalent to the competitive amount, and sometimes, it is lower than it. Also, ω_{ist} shows that in some instances, the supplier preferred to buy more than the required amount to remain in the competition.

Ultimately, V_{vst} demonstrates the number of vehicles and it takes the value according to the transported materials and products. Furthermore, in this problem, the second objective function (cost) has a conflict with the first objective function (minimizing lost sale) shown in the charts Fig (13). In this regard, we assume that the percentage of failure is equal to zero. On the other hand, according to Fig (14), the second objective function has a conflict with the third objective function. Based on our research, if the amount of the supplied materials is close to the competitive values, the amount of the third objective function decreases. On the contrary, the amount of the second objective function increases due to the increase in procurement and cost.

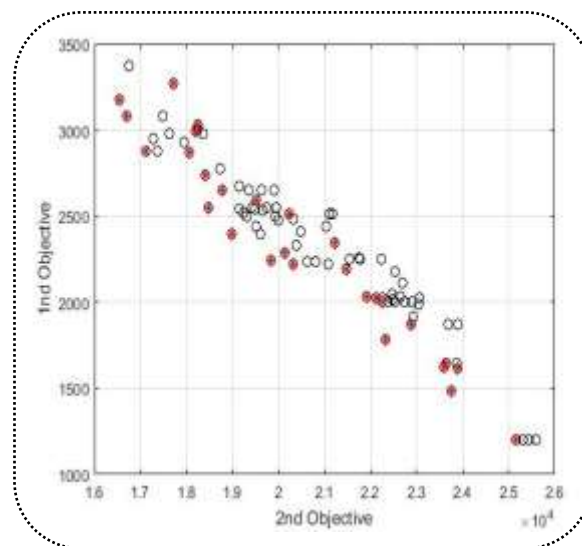


Fig 13. Relation between 1st and 2nd objectives

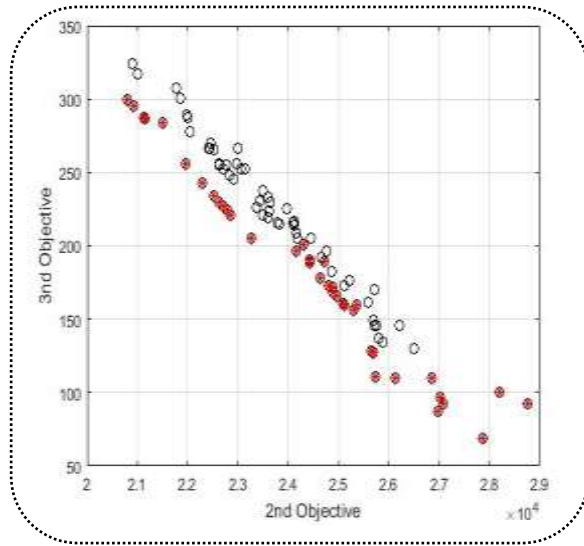


Fig 14.Relation between 3nd and 2nd objectives

Table XXIV. Results of hybrid Algorithm for large instances

	13	14	15	16	17	18	19	20
MN_{ist}	(0,7.14)	(0,18.9)	(0,27.2)	(0,36.5)	(0,41.58)	(0,47.06)	(0,56)	(0,61)
MP_{ist}	0	0	0	0	0	0	0	0
L_{imvst}	(0,173.92)	(0,281)	(0,337)	(0,428)	(0,442)	(0,458)	(0,493)	(0,536)
w_{mst}	(184.5,234.23)	(58,175)	(54,240)	(58,238)	(110,342)	(128,358)	(145,373)	(116,386)
Z_{mst}	(175.28,222.25)	(55.2,166.25)	(49.14,228)	(55.1,226.1)	(104.5,324.9)	(121.6,340.1)	(137.75,354.35)	(110.2,366.7)
L'_{mrvst}	(0,118.2)	(0,59.08)	(0,112.4)	(0,59.57)	(50,110.78)	(52.3,59.9)	(50,59.75)	(52.3,114.67)
SS_{mst}	(16.34,17.01)	(4.5,18.74)	(4.3,21.23)	(5.1,27.2)	(8.4,33.2)	(9.1,42.4)	(10.08,49.5)	(12.2,50.1)
IB_{mst}	(184.5,234.23)	(58,175)	(54,240)	(58,238)	(110,342)	(128,358)	(135,373)	(116,386)
F_{mst}	(2,3)	(2,6)	(2,7)	(2,6)	(2,7)	(2,8)	(2,8)	(2,7)
R_{mst}	(2,5)	(6,13)	(4,18)	(3,17)	(3,5)	(4,7)	(2,9)	(5,8)
N_{mst}	(19,24)	(6,18)	(5,25)	(6,24)	(11,35)	(13,36)	(14,36)	(11,39)
φ_{mst}	(9.2,11.98)	(2.68,16.22)	(2.79,24.91)	(3.14,23.3)	(8.7,32.1)	(9.12,36.58)	(10.24,35.11)	(11.3,38.6)
SA_{rst}	(45.24,59.48)	(51.1,59.08)	(48.30,59.68)	(47.25,59.7)	(50,59.78)	(46.5,59.9)	(48.68,59.75)	(52.3,59.67)
D_{rst}	(1.19,5.84)	(0.87,5.24)	(0.42,6.3)	(0.12,7.8)	(0,7.2)	(0,0.53)	(0,0.67)	(0,0.81)
Ω_{ist}	(167.23,173.92)	(185,281)	(208,337)	(225,428)	(152,342)	(178.2,358)	(147.2,373)	(120,386)
ω_{ist}	0,12.5)	(0,15.8)	(0,11.30)	(0,17.5)	(0,25.5)	(0,22.4)	(0,20.13)	(0,27.2)
V_{vst}	(1,7)	(1,10)	(0,13)	(0,15)	(1,19)	(1,21)	(0,23)	(0,25)
Z1	5.99*103	8.01*103	9.5*103	1.01*104	1.45*104	1.7*104	1.95*104	2.26*104
Z2	6.8*104	7.05*104	8.9*104	1.2*105	1.34*105	1.48*105	1.56*105	1.78*105
Z3	4.3*10	6.97*10	1.3*102	1.9*102	2.3*102	2.5*102	3.10*102	3.3*102

V. Conclusions And Future Research Directions

Given the importance of supply chain management to companies, this study considered a multi-echelon supply chain with a competition between the suppliers and crisis, simultaneously. The results shed light on the fact that integrating production and distribution could help to reduce defective and additional products, increase the quantity level of servicing, and decrease the holding cost. Moreover, this integration could optimize the number of required vehicles and the amount of needed material and have several economic benefits for companies. Furthermore, tackling the vehicle routing problem decreases the cost of transportation for a distribution company. Moreover, critical cases in this problem were shown with an increasing cost or decreasing available amount of raw material. Our results revealed that the model remained in Stackelberg competition at a specific level. After this level of cost or the available amount of raw material, each supplier decided to supply the amount of raw material that was different from the competitive value. In addition, the model aimed at minimizing the costs, defective products, lost sales, and competitive differences. The findings herein exhibited that the lost sale and cost's function has a conflict. This opposing relationship was also observed between the cost function and competitive function.

To solve the model, it was primarily solved with GAMS software and the results showed that we could use this software for models with small instances in a reasonable time. However, due to the complexity of the NP-Hard model, we utilized meta-heuristic algorithms. The results of GAMS software were compared to those of MOPSO, NSGA-II, and the combination of them. This comparison revealed that the hybrid algorithm demonstrated a better solution. Thus, we employed a hybrid algorithm for solving eight large instances and the associated results showed the reasonable performance of this algorithm. Further research is recommended to focus on the following areas to overcome the limitations of this study. For example, considering different objective functions, a different method of competition, and different types of final products could be taken into account for ameliorating the model. Moreover, attending to the risk concept or other uncertainty approaches, such as robust or fuzzy optimization approaches, could help to develop the study.

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