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A Hybrid Approach Based on a Cross-Efficiency Model to Measure the Efficiency of Bank Currency Units in a Competitive-Cooperative Environment

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Abstract – This study aims to evaluate and rank the performance of the currency units of the bank by using the integrated methods of the balanced scorecard, cross-efficiency data envelopment analysis, and game theory in a cooperative-competitive environment. In this regard, by studying the indices used to evaluate the efficacy of banks and with the help of experts in foreign exchange, seven indices are selected as inputs and outputs from four perspectives of the balanced scorecard approach. Then, by applying the proposed Nash bargaining game model in cross-efficiency in a competitive-cooperative environment, the efficiency of decision-making units is evaluated. In this way, the bank's branches compete in pairs. As a result, each branch tends to prioritize the other branch over the criteria in which they have a more significant advantage and allocate higher weight. This leads to the higher efficiency of the branch. Thus, the cross-performance matrix is complemented by the performance of the bargaining model, rather than being filled by the performance of the conventional data envelopment analysis model. The proposed approach presents a new aspect of measuring performance based on the cross-efficiency model. The real case study of Isfahan Bank Melli branches is used to show the process of implementation of the model as well as the ability of the proposed approach.

Keywords– Data envelopment analysis; Cross-efficiency; Bargaining game; Balanced scorecard; Bank branches.

I. INTRODUCTION

Nowadays, organizations are faced with conditions and challenges that affect their business environment. It can threaten their survival in the field of economics and commerce. Increasing numbers of competitors and alternative products, scarce resources and high costs, and rapid changes in consumption and demand patterns are examples of opportunities and threats that organizations and even countries face. In such an environment, the critical question is, what is the secret to the survival and sustainability of organizations in the business arena and what are some of the indicators that are conducive to competitive advantage for organizations?

Organizations need performance appraisal to understand the desirability of their activities, especially in complex and dynamic environments. The lack of a performance appraisal system in different dimensions of the organization is considered one of the symptoms of the disease of the organization (Eghbal, 2008).

Regarding the issue of Iran's accession to the World Trade Organization, in recent years, the country's banking system has faced new challenges, such as the entry of foreign banks, the launch of private banks, and the expansion of credit and financial institutions. Accordingly, these challenges have prompted the country's banks to improve their performance to survive and compete in this dynamic environment by identifying their strengths and weaknesses to improve the current situation (Manandhar and Tang, 2002).

The critical role of banks as a factor in the implementation of monetary policy in economic stability has prompted banks to compete in the market by establishing research centers and conducting research activities on their status compared to other banks to improve their performance in the domestic and foreign markets. In this regard, a rational and highly effective solution can determine the status of banks in terms of overall performance. As far as researchers are aware, performance appraisal and subsequent branch rankings are performed using traditional financial ratios and indices, which has challenged performance appraisal. The use of performance evaluation systems that rely only on financial indices can cause some challenges for the organization. However, one of the approaches which play a vital role in the analysis of strategic confrontation between banking activities is an analytical method based on game theory. In these types of games, banking units are trying to choose the best strategy against a competitor such that the best-balanced outcomes are achieved.

Performance is one of the most important indices of performance evaluation of organizations, especially financial institutions such as banks. It is the most essential and first step in improving its efficiency, measuring, and evaluating it. To achieve higher levels of performance, bank managers select suitable economic activities and increase their ability to compete with competitors, are willing, and have to make the maximum effort to achieve the highest levels of efficiency, respectively. Also, regulatory agencies such as the *Central Bank and the High Council of Banks*, as well as the general public of customers of these financial institutions, are interested in analyzing the performance of banks and their branches because of the increased efficiency that reduces the cost of service and enhances its quality (Wang and Chin, 2010). To explore the research gaps of the issue under investigation, the literature on the relevant subject is examined as follows.

The rest of this paper is organized as follows: In Section 2, the research gap is considered. In Section 3, the overall structure of the proposed approach is provided. In Section 4, the problem modeling is presented according to the proposed approach. In Section 5, the efficiency evaluation indices used in the proposed model are discussed. The analysis of results in different modes is provided in Section 6. Finally, the conclusions and development suggestions of this study are provided in Section 7.

II. LITERATURE REVIEW AND RESEARCH GAP

Game theory was first introduced in 1921 by a French mathematician named Emile Bourne. He first studied several familiar games, and papers focused on these games emphasized the predictability of results logically. In 1928, John Newman, together with Oscar Morgenstern, an Austrian economist, published a book named "Theory of Game and Economic Behavior."

Cross-efficiency evaluation is an effective method for ranking decision-making units (DMUs) in data envelopment analysis (DEA) and can be performed with different formulations (aggressive or benevolent), secondary goals, and models (Wang and Chin, 2010). In this paper, some alternative models for DEA cross-efficiency evaluation were proposed. Wu et al. (2009) developed a two-stage approach based on a bargaining game and a mixed-integer programming model for evaluating the cross-efficiency of DMUs. Du et al. (2011) used the bargaining game model to measure the performance of two-stage US network structures. Under the Nash bargaining theory, two stages are seen as players, and the data envelopment analysis (DEA) performance model is considered a cooperative game model. They showed that several breakdown points could be applied to calculate bargaining efficiency scores for two stages. They used Nash bargaining to evaluate 30 large US commercial banks and 24 Taiwanese insurance companies. Jahangoshai-Rezaee et al. (2012) presented a new approach to evaluating 24 thermal power plants in Iran. The study identified two categories of resources (operational and non-operational) to measure power plant performance. In their research, a new method based on game

theory and DEA is presented to evaluate decision-making units (DMUs) with a large scale of values. The proposed way was more effective regardless of the number of DMUs to discriminate between DMUs. DMUs were compared with several categories in the competitive environment. Yang and Moriata's (2013) focused on improving the performance of banking systems from various perspectives, with different input/output definitions of various features of the banking system. This study uses the DEA and Nash bargaining games to improve inefficient banks. They aimed to evaluate the performance of 65 Japanese banks that have proposed a new model of DEA that included Nash bargaining (Yang and Moriata, 2013). Rezaee et al. (2016) combined DEA and Nash bargaining game as a cooperative game theory approach to evaluate the performance of the city transportation system by applying a large scale of inputs/outputs. The proposed approach, regardless of the number of units, discriminates among the units more effectively. In this paper, two categories of inputs, including operational and spatial measures to evaluate the performance of transportation systems. The results depict in which category each bus line has a better performance and in which lines it is inefficient. In another article by Jahangoshai Rezaee and Shokry (2017), they evaluated the performance of 17 Iranian cement companies based on two different approaches of multi-objective DEA model and DEA-game model. In their study, the DEA-game model uses the concept of bargaining to link views to the BSC, while each aspect is considered separately as a goal in the multi-objective DEA model. Their findings show that the DEA-game model can separate cement companies more effectively. Contreras et al. (2020) proposed the secondary goals that enable the optimal weighting vectors to be chosen. In this paper, a two-stage method has been proposed to discriminate between optimal weighting profiles based on bargaining problems and the Kalai-Smorodinsky solution.

Another category of using the bargaining game in DEA is the use of it in the two-stage structure of DMUs evaluation. Two-stage network structures or processes, where all the outputs from the first stage are intermediate measures that make up the inputs to the second stage. Under Nash bargaining theory, the two stages are viewed as players, and the DEA efficiency model is a cooperative game model. In other words, in this structure, each stage is a player that competes with another stage (Zhou et al., 2013; Jahangoshai Rezaee and Moini, 2015).

Other forms of using game theory other than the bargaining game include the satisficing game (Tchangani, 2006), Egoist's dilemma (Nakabayashi and Tone, 2006), horizontal cooperation for information sharing (Lozano, 2012), Shapley value (Rezaee, 2015), and Leader-follower structure (Naini et al., 2013; Chu et al., 2020).

As can be seen, the volume of studies in the cross-efficiency areas of the DEA, the BSC, and the game theory is enormous. This demonstrates the superiority of these methods and their advantages over other performance evaluation methods in organizations. Only a handful of these studies, however, have integrated these two approaches for performance evaluation. However, all these studies have pointed out the advantages and strengths of these approaches. All previous studies show that most of the studies done to evaluate the performance of banks' branches and rank them based on cross-efficiency techniques of DEA, BSC, and game theory has been done separately. If a combined approach can be used, it can take into account the strengths of each of these techniques and also cover their weaknesses. Their study seeks to provide an efficient hybrid model by considering the strengths and weaknesses of each of these techniques. Accordingly, and considering the financial and non-financial indicators, the performance of the bank's foreign exchange units is evaluated and ranked. None of these studies have addressed competition between decision-making units using the Nash bargaining game model in cross-efficiency. In the present study, this defect has been eliminated. For the first time, a model has been used under the Nash bargaining game in cross performance and balanced scorecard (BSC) in a competitive and cooperative environment. In today's fast-paced economic world, there is much competition between bank branches not considered in previous studies. Another significant discussion of this study is that after examining the efficiency and ranking by considering the Nash bargaining game model in cross performance, it examines the relationships between the bank's currency units and improves the entire banking system by influencing other factors. Given the research gaps, comprehensive modeling to consider the competition among decision-makers in calculating performance ratings and static ranking can bring the modeling world closer to the real world. Therefore, this study has been prepared in this direction so that the results are practical and realistic.

III. OVERALL STRUCTURE

In this section, the approaches used in this research are provided sufficiently based on the type of use in the cross-efficiency model based on the bargaining game.

A. The Balanced Scorecard (BSC)

The BSC combines performance appraisal criteria that include current, past, and future performance indices, and puts non-performance measures alongside financial measures (Arab Mazar et al., 2009) .

Kaplan and Norton proposed the concept of BSC in 1992. BSC is widely used in evaluating the performance of organizations from the four aspects of finance, customer, internal processes, and learning and development (Wu, 2012) .

To develop DEA models in 1986, a cross-performance model (CEM) was introduced to identify the best performance of decision-making units and rank them (Sexton et al., 1986). The primary origin of this method is based on Sexton et al. (1986) and reviewed by Doyle and Green (1994).

Cross performance evaluation has two main advantages:

1. Providing a unique ranking for decision-making units.
2. Eliminating unrealistic weight plans without the need for weight restrictions resulting from specialized scopes.

In cross-efficiency, unlike the basic DEA model, where the same unit weights determine unit scores, the other units are used to assess efficiency scores. Thus, by using the CCR model, the masses of the outputs and inputs for different groups are calculated, and then, using these weights, the efficiency score of the group is specified. The final score of the group will be the average of the earned points.

Doyle and Green have pointed out that the uniqueness of the possible optimal weights of DEA may reduce the usefulness of cross-performance. The cross-efficiency measures obtained from basic models of DEA are generally not unique. Depending on which of the optimal linear programming solutions are used in DEA, the performance (cross-efficiency) of a decision unit may improve as the performance of the other groups deteriorates (Karimi and Khorram, 2016).

B. Data Envelopment analysis

The basic model of DEA is presented as follows:

$$\text{Max } \Theta_k = \frac{\sum_{r=1}^s u_r y_{rk}}{\sum_{i=1}^m v_i x_{ik}} \quad (1)$$

$$\frac{\sum_{r=1}^s u_r y_{rk}}{\sum_{i=1}^m v_i x_{ik}} \leq 1 \quad (2)$$

$$u_r, v_i \geq 0 \quad (j = 1, 2, \dots, n), (r = 1, 2, \dots, s), (i = 1, 2, \dots, m) \quad (3)$$

The model is known as the CCR pattern in honor of Charles, Cooper, and Rhodes. In this model, j , r , and i represent the production unit, product (y), and input (x). In the model, there are n production units or decision units that produce s number of outputs by using m inputs. The template's purpose is to specify the efficiency score of the unit k , denoted by θ_k . It is equal to the ratio of the total weight of inputs to the total weight of data, provided that the maximum performance score of other generating units by using the weights of u_r and v_i , which are the output weights and inputs of k units, respectively, is not higher than one. Therefore, the purpose of the planning model is to determine the masses of data to achieve a maximum performance score (Nash, 1950a) .

Scoring using this template is accompanied by criticism. The criticism is that the objective function is a fraction and, in addition to the need for non-linear methods, can provide a set of many weights as a result. To solve the problem, the objective function of the CCR pattern is linearized with an equal face or denominator. If the denominator is equal to one, the constant return pattern relative to the scale with the input-oriented approach, if equal to one, is called the fixed return pattern relative to the scale with the output-oriented approach. In this study, since the input-oriented approach is considered, all the models introduced based on this approach are presented. By placing the denominator of the fraction in the objective function of the base model, the CCR implicit model is obtained. This model is shown as follows (Cooper et al., 2011):

$$\text{Max } \Theta_k = \sum_{r=1}^s u_r y_{rk} \tag{4}$$

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$$\sum_{i=1}^m v_i x_{ik} = 1 \tag{5}$$

$$\sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0, (j = 1, 2, \dots, n) \tag{6}$$

$$u_r, v_i \geq 0 \quad (r = 1, 2, \dots, s), (i = 1, 2, \dots, m) \tag{7}$$

Where u and v represent the weights of the outputs and inputs, respectively. In addition, this model creates a fixed set of importance, it is possible to estimate the pattern using a linear programming method.

x_{ij} : Input i for unit j ($i = 1, 2, \dots, m$)

y_{rj} : Output r for unit j ($r = 1, 2, \dots, s$)

u_r : weight given to output of r (output price of r)

v_i : Weight given to the input of i (input cost of i)

C. Cross-Efficiency

Researchers have welcomed DEA models because of the need not to specify production functions and specific weights for outputs and inputs. However, it became apparent very soon that some of the disadvantages of the issues that happened when using DEA patterns, their inability production units were completed in rank. In other words, the basic designs were only capable of separating the production units into efficient and inefficient ones. The ranking of inefficient units was based on performance ratings. However, the efficient ranking units assigned a number, which means full performance is impossible (Mecit and Alp, 2012).

Therefore, to calculate the efficiency score using this method, first, the set of input and output weights for k units ($u_{1k}, u_{2k}, \dots, u_{sk}$) and ($v_{1k}, v_{2k}, \dots, v_{mk}$) are specified and then calculated using this set of weights and unit information j through the relation (8).

E_{kj} is the efficiency of unit j using the unit weights of k and obtained from the following equation:

$$E_{kj} = \frac{\sum_{r=1}^s u_{rk} y_{rj}}{\sum_{i=1}^m v_{ik} x_{ij}} \quad (k, j = 1, 2, \dots, n) \tag{8}$$

Using Equation (8), for unit j , calculate the number of units to be evaluated, and then by averaging these numbers, the unit efficiency score is obtained by using Equation (9):

$$\bar{E}_j = \frac{\sum_{k=1}^n E_{kj}}{n} \tag{9}$$

Where \bar{E}_j represents the efficiency of unit j , and n is the number of units (Wu et al., 2016).

Another disadvantage of the standard DEA model is achieving performance values of 1, which is not analytically justified. Also, conventional data coverage analysis may not discriminate between decision-making units, and the resulting performance scores may not be significant, especially when the number of DMUs is insufficient; these defects are remedied by cross-efficiency. Cross-efficiency consistently ranks DMUs differently; it effectively detects excellent and poor performance among DMUs (Wu et al., 2016).

The choice of weights in the DEA linear programming model is such that the unit under investigation can maximize its efficiency over other groups. The Performance measurement of each group with the best set of weights calculated by the model is called pure ability. The calculated pure ability for unit k , E_{kk} , is maximized based on estimated weights and the desired request of the unit model k . If another unit's efficiency, such as j , is computed with the selected weights of the k , it will be represented as E_{kj} and called cross efficiency. In the crossover efficiency model, the efficiency score for each unit is calculated n times for the weights obtained for the other groups. Finally, the average of these efficiencies for the importance of the other groups is cross efficiency. In contrast to the usual ability, this method utilizes weights obtained from other groups. A pair of comparisons are made between two sets of groups. To rank groups, the cross-performance matrix for groups is formed as follows.

Table I. Cross-efficiency matrix (CEM) [23]

<i>Unit</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
A	E_{AA}^*	E_{AB}	E_{AC}	E_{AD}
B	E_{BA}	E_{BB}^*	E_{BC}	E_{BD}
C	E_{CA}	E_{CB}	E_{CC}^*	E_{CD}
D	E_{DA}	E_{DB}	E_{DC}	E_{DD}^*

$$* E_{AA} = E_{BB} = E_{CC} = E_{DD} = 1$$

D. Game theory

What is called game in the game theory is "the conditions in which each person's decision affects the decision of the other person and all the people in that situation are aware of that," thus "competition" and "cooperation" can be seen as a game. Game theory is the science of studying games and wants to show players the principles and rules of decision-making in interactive terms. It can be used in many ways, the most important of which is real-world game analysis, event prediction, and strategy, to make the right choice (Osborne, 2009).

Game theory is the formal study of the rational behavior of participants' engagement in strategic situations by describing conflicts of interest and interdependence. The formal review of the rational behavior of participants' game theory is a mathematical derivation that analyzes the cognitive abilities of players' strategies. The aim of game theory arguments is not to predict the outcome of a game but to study how a game is played and how rational players pursue their interests to make strategic decisions in response to other players' strategies (Sharma and Bhattacharya, 2013).

In game theory, the value of a result is expressed as "utility." In a game, utility represents a player's motivation. The utility function is the value assigned to each player for the possible outcomes of each game. As the utility function grows, the corresponding result is considered more desirable. For example, a player prefers the L_1 result to L_2 if and only if the expected utility of L_1 is higher than L_2 (Bailey et al., 2010).

Game theory is a science that studies people's decisions in terms of interacting with others. In other words, it is the science of studying conflicts and cooperation between rational players. The primary purpose of game theory is to give an attitude and perspective on which players should act rationally. It is divided into two main indices: 1 – non-cooperative games, and 2 – cooperative games. In non-cooperative games, it is assumed that the players act rationally and think only of their interests, and there is no cooperation between them. In cooperative games, players have the possibility of

cooperation, and the main goal of the game provided a way for a fair division of the benefits of collaboration (Osborne, 2009).

E. Nash Bargaining Game (NBG)

over a problem, each party seeks to achieve its desired outcome and realize that outcome. It must employ strategy and discipline. In the bargaining process, the parties continue to negotiate their bids, so bargaining is one of the most critical issues in game theory (Osborne, 2009).

In 1950, John Nash invented and established the most crucial concept of equations in bargaining issues and about cooperative game theory. In a study presented by Nash in this regard, a theoretical discussion of bargaining issues is raised (Sharma and Bhattacharya, 2013) .

In conventional models of DEA, there is no possibility of ranking. Also, in the crossover efficiency model, unit efficiency for each unit is not achieved, and the obtained results are not Pareto solutions. In their proposed method, each group has the mandate of a player, and the solution obtained from the classic bargaining game is a Pareto solution. After configuring the model with a BSC and cross-efficiency given the competitive environment to gain more advantage over competitors, the game theory, and, more specifically, the Nash bargaining model are added.

Bargaining is known as a cooperative game. In the bargaining game, two players compete to earn more. There will be no problem if both players agree on a payoff. The competition will mean when each of these players wants to win more. In the present study, players are banking units that compete in a cooperative environment.

To obtain a Nash bargaining solution, the solution space must be compact, convex, and include payment vectors. Payment vectors should be such that the payouts for each player must be higher than their respective breakdown points. Breakdown points are the minimum earnings that a player expects to earn. In other words, breakdown points occur when a player is playing against the best competitive strategy. Let $U(x)$ and $V(x)$ be the utility functions for players 1 and 2, respectively. Assuming U and V are also traded in this case, the logical decision would be to multiply the utility by multiplying the breakdown point values for each player. In other words, the following equation can be defined:

$$\text{Maximize } |u(x) - u_d||v(y) - v_d| \tag{10}$$

The solution to this relationship is called the Nash solution, and hence the multiplication is called the Nash multiplication. Nash's relationship can be extended to more than two players. Now, if it is assumed that $U_i(x)$ to be a utility function for player i and U_i is the breakdown point value for player i when there is not a game. In this case, the following equation is derived [30]:

$$\text{Maximize } \prod_{i=1}^n |u_i(x) - u_i(d)| \tag{11}$$

In the Nash bargaining game, there are n players (bank units), assuming X is the decision space and $f_i: X \rightarrow R$. As the objective function of the decision-maker i , the criterion space is defined as follows:

$$H = \{u \mid u \in R^n, u = (u_i), u_i = f_i(x) \text{ with some } x \in X\} \tag{12}$$

When players cannot reach an agreement, they achieve lower objective function values. If d_i is this value for the decision-maker i and $d = (d_1, d_2, \dots, d_n)$, the conflict is defined by the pair (H, d) .

If H is convex and compact and at least one $f \in H$ such that $f \geq d$, the Nash bargaining solution $f^* = \varphi(H, d)$ can be obtained as the optimal single-problem solution.

$$\text{Maximize } (f_1 - d_1) \times (f_2 - d_2), \dots, (f_n - d_n) \tag{13}$$

$$f_i \geq d_i, \quad (i = 1, 2, \dots, n) \quad (14)$$

$$(f_1, f_2, \dots, f_n) \in H \quad (15)$$

This solution fulfills all the principles of the Nash solution conditions. It should be noted that in this case, the followers (players) make the decision simultaneously (Safari et al., 2014).

The advantage of using the Nash bargaining game model is: (I) It provides the best overall performance ratings (Du et al., 2011). (II) Nash Bargaining, as a cooperative game, divides the profit between the two players based on their competition (Rezaee et al., 2012).

V. PROBLEM MODELING

A. Assumptions

The assumptions of the study are as follows. (A) Foreign and domestic government policies on the policies and performance of the branches do not have a significant impact on the proposed model. (B) Analyzing bank branch data is done in static mode. (C) Investigating and evaluating the branches of the Melli Bank of Isfahan in a competitive/cooperative environment. (D) The period used in the model is one year. (E) It is assumed that all indices of decision-making units have equal value. (F) The low-performance limit is considered zero (i.e., the tendency of a player to have a minimum efficiency).

B. Indices, parameters, and decision variables

Mathematical modeling indices for Bank Currency Units are as follows:

i: Input index for DMU $i = 1, 2, 3$

r: Output index for DMU $r = 1, \dots, 4$

j: Index for DMU $j = 1, \dots, 30$

o: DMU index (Base)

p: Competitor DMU index

In the proposed model for each DMU_j, the purpose is to evaluate the efficiency of n decision units, and each unit has m inputs and s outputs.

x_{ij} : Input rate i for unit j ($i = 1, 2, \dots, m$)

y_{rj} : Output rate r for unit j ($r = 1, 2, \dots, s$)

U_r : The weight given to output r

V_i : Weight is given to input i

Θ_o : The point of failure or low-performance of a base player

Θ_p : Breakdown point or low-performance of an opposing player

x_{i0} : input rate of i for a base player

x_{ip} : input rate of i for an opposing player

y_{ro} : output r for a base player

y_{rp} : Output r for an opposing player

α : Coefficient assigned to the inputs and outputs of the second player (the coefficient of correlation between the data of two base players and competitors).

This model evaluates relative efficiency. By solving this model, the banking units compete for one by one, and the input and output weights for each index are obtained. Using the obtained weights, the efficiency score of each group is received over the other groups. If the efficiency score is equal to one, it means that the two players are equal, and subsequently, the game is balanced. If the performance score is less than one, the rival player is defeated or lost to the base player. Also, if the performance score is higher than one, it means that the opposing player wins over the base player. Input variables related to variables x are as follows; the number of staff, total training hours, and total costs. The output variables of the problem related to the variables y are as follows; branch operating profit, number of currency accounts, number of Rial accounts, and average number of essential currency services per day.

C. Mathematical modeling

To create a model for the problem at hand, the model, assumptions, parameters, indices, and decision variables are first introduced in absolute terms. Finally, the goals and limitations of the definitive model will be discussed. First, a consolidated model is proposed using Nash bargaining in cross-efficiency. Its primary purposed function is as follows:

$$\text{Max} \left(\frac{\sum_{i=1}^s u_r y_{ro}}{\sum_{i=1}^m v_i x_{io}} - \Theta_o \right) \times \left(\frac{\sum_{i=1}^s u_r y_{rp}}{\sum_{i=1}^m v_i x_{ip}} - \Theta_p \right) \tag{16}$$

The simplification of the objective function requires several variables' changes as follows.

$$t_1 \times \sum_{i=1}^m v_i x_{io} = 1 \tag{17}$$

$$\frac{1}{\sum_{i=1}^m v_i x_{io}} = t_1 \tag{18}$$

Now the following variable is changed again:

$$\frac{1}{\sum_{i=1}^m v_i x_{ip}} = t_2 \tag{19}$$

$$2 \times \sum_{i=1}^m v_i x_{ip} = 1 \tag{20}$$

Also, the problem inputs are:

$$t_1 \times v_i = V_i \tag{21}$$

$$t_2 \times v_i = V'_i \tag{22}$$

$$\frac{V_i}{V'_i} = \frac{t_1}{t_2} \tag{23}$$

$$V'_i = \frac{t_2}{t_1} \times V_i \tag{24}$$

The above relationship can be set by changing the variable to the following value:

$$V'_i = \alpha \times V_i \quad (25)$$

Finally, the problem outputs will be:

$$t_1 * u_r = U_r \quad (26)$$

$$t_2 * u_r = U'_r \quad (27)$$

$$\frac{U_r}{U'_r} = \frac{t_1}{t_2} \quad (28)$$

$$U'_r = \frac{t_2}{t_1} * U_r \quad (29)$$

The above relationship can be set by changing the variable to the following value:

$$U'_r = \alpha \times U_r \quad (30)$$

The basic model of the problem is as follows:

The objective function of the proposed model is as follows:

$$\text{Max } (\sum_{i=1}^s U_r y_{ro} - \theta_o) \times (\alpha \times \sum_{i=1}^s U_r y_{rp} - \theta_p) \quad (31)$$

St.

$$\sum_{i=1}^s U_r y_{rj} - \sum_{i=1}^m V_i x_{ij} \leq 0 \quad (j = 1.2 \dots n) \quad (32)$$

$$\sum_{i=1}^s U_r y_{ro} \geq \theta_o \quad (33)$$

$$\sum_{i=1}^s U_r y_{rp} \geq \theta_p \quad (34)$$

$$\sum_{i=1}^m V_i x_{io} = 1 \quad (35)$$

$$\sum_{i=1}^m V_i x_{ip} = \frac{1}{\alpha} \quad (36)$$

$$U_r, V_i \geq 0 \quad (37)$$

Constraint (33) represents that the efficiency of all smaller units is equal to one. In other words, the ratio of the weighted sum of the outputs to the weighted sum of the inputs is more minor than zero. Constraint (34) indicates that the weighted sum of the outputs of the base player is higher than its low efficiency (breakdown point). Constraint (35) suggests that the weighted sum of the outputs of the competing (rated) player is higher than its low-efficiency level. Constraint (36) shows that the fair amount of data of the base player is equal to one. Constraint (37) implies that the fair amount of data of the base player is similar to a coefficient of α . α is considered positive and the connection between the base and competitor player inputs. Constraint (38) expresses the positivity of the given weights to the output of r and input i .

In the present study, the breakdown points related to DMUs or low-efficiency limits are considered. Also, the correlation coefficient between the inputs and outputs of the two base and competing DMUs is deemed to be 0.5.

V. PROBLEM DEFINITION

Given the significant role of banks and financial and credit institutions in each country, as well as the privatization process of state-owned banks, the evaluation of banks' performance has become crucial. With increasing competition in the banking system, banks must continuously monitor their performance each year using appropriate models to assess the game. Accordingly, banks can significantly guarantee their success in competing with other rivals, due to their competitive advantages and strengths. Existing methods of evaluating bank branch performance are often empirical and lack reliable scientific support, and because of their lack of standardization, their results in different banks are not comparable. The overwhelming methods have merely made the bank's output a benchmark for performance evaluation.

There are many reasons for the decline of the banking system, including state-owned banks, the inefficiency of government management, commercial banks, and so on. Since several stakeholders in the network have sought to improve the efficiency of the banking system, it is essential to conduct research that compares the performance of the banking system over a given time. Western countries' restrictions and sanctions on some banks have introduced new arrangements in the foreign exchange market of the banking system and increased the market share of other banks in foreign exchange services. Knowing the current conditions of a bank's currency performance will help senior bank managers, currency managers, and currency unit staff to be able to determine and identify the causes of efficiency and inefficiency of these units, to develop appropriate plans to achieve the vision, strategies, and goals of banks in their field of currency. The development of integrated, desirable, and accepted indices of all banks, which can be cited in the evaluation and ratings and assessed the banks from different dimensions, is one of the requirements of the banking system. To achieve the goals of the project, the banks' performance should not only be considered from the financial aspect, but it should also be thoroughly evaluated by the banks' performance in all its dimensions (financial and non-financial). Melli Bank of Iran, as one of the specialized banks of the country, like other banks, considering the opportunities in the field of foreign trade and Rial services, seeks to increase foreign exchange and Rial services and improve their quality, and ultimately increase their share in the foreign exchange market.

To achieve the mentioned goals, the game theory approach has been used to rank bank branches based on the cross-efficiency model. In the conventional cross-efficiency model, the weight obtained for inputs and outputs from the evaluation of each DMU is used for calculating the efficiency score of other DMUs. Other DMUs except under evaluation DMU do not interfere with the efficiency score. In other words, only the DMU under evaluation determines the efficiency score of the other DMUs. This is repeated for the other DMUs. As a result, the cross-efficiency table contains values that none of the DMUs are involved in determining their efficiency relative to the target DMU. In this paper, to create a cross-efficiency table, the DMUs compete in pairs based on the bargaining game to maximize their efficiency by determining their desired weights. In this method, the comparison of the two DMUs is made more realistically.

VI. DETERMINATION OF EFFICIENCY EVALUATION INDICES

A. Initial indices

At this stage by reviewing studies on student theses and articles, foreign and domestic journals, consulting with experts in the field of currency, and based on the opinions of academic professors and also, given the performance nature of the bank branches, the initial performance evaluation indices are set out in four BSC perspectives as described in Table 2. To evaluate its effectiveness, its indices must first be examined, and finally, the final indices are selected using the banking experts.

Table II. Early efficiency evaluation indices

<i>BSC Perspectives</i>	<i>Index</i>	<i>Input/Output</i>	<i>Reference</i>
Financial	Number of Staff	Input	Mohaghar et al. (2015)
Financial	Profit margin	Output	Kianfar et al. (2016)
Financial	Direct costs	Input	Bazrkar and Iranzadeh (2017)
Financial	Unpaid loans	Input	Khaki et al. (2012)
Financial	Operating costs	Input	Yang and Liu (2012)
Financial	Operating Profit	Output	Mohaghar et al. (2015)
Financial	Total costs	Input	Najafi et al. (2011)
Customers	Rial Accounts	Output	Kashanipour and Ghazizadeh (2009)
Customers	Number of Staff	Input	Bazrkar and Iranzadeh (2017)
Customers	Foreign currency accounts	Output	Mohaghar et al. (2015)
Customers	closed Deposits	Input	Najafi et al. (2011)
Customers	rate of new customers	Output	Khaki et al. (2012)
Internal processes	The average number of important currency services provided per day	Output	Mohaghar et al. (2015)
Internal processes	Customer response time	Output	Bazrkar and Iranzadeh (2017)
Internal processes	The rate of use of labor	Input	Bazrkar and Iranzadeh (2017)
Growth and learning	The average of staff experience	Input	Shahroodi and Bahraloom (2014)
Growth and learning	Total hours of employees training	Input	Mohaghar et al. (2015)
Growth and learning	Number of employees with higher qualifications	Output	Bazrkar and Iranzadeh (2017)

B. Final indices

A questionnaire containing the indicators extracted in the previous step was designed in four BSC perspectives. Each currency expert was then asked to give points 1 to 5 on each index according to the Likert scale. Indices with a mean score of less than three were omitted. In the next step, experts were asked to score indicators recognized significantly previously, from one to five according to the availability of index information. Again, signs with an average score of less than three were removed. According to a survey of academic staff and interviews with professional experts, the number of indicators was reduced to seven ones. Finally, the final nominated signs were located in the aspects of BSC, as shown in Table 3.

Table III. Final BSC Indices for efficiency Evaluation

<i>BSC Perspectives</i>	<i>Index</i>
Financial	Number of Staff
	Total costs
	Operating profit
Customer	Number of currency accounts (opened in 2018)
	Number of Rial accounts (opened in 2018)
Internal processes	The average number of essential currency services provided per day
Growth and learning	Total hours of employees training

Since a questionnaire with several questions (a 5-point Likert scale) is like a test, reliability can be obtained using Cronbach's alpha method. It is said that if the alpha coefficient is more significant than 0.7, the proof is acceptable. In this study, the questionnaire Cronbach's alpha was calculated using SPSS software, and the value is 0.721, which indicates the optimal reliability of the questionnaire. The validity of the survey content was evaluated by the managers and experts of the branches of foreign exchange. After discussing and consulting with them, the questionnaire questions were finalized. Morgan table was used to determine the number of required questionnaires. According to the statistical population of the research, the number of DMUs is 30. The number of questionnaires needed is 28.

Finally, seven of the most essential indices were selected to evaluate the efficiency of MFIs in four BSC perspectives. After a discussion with various experts and previous studies in the field of a hybrid model, three variables were identified as inputs and four variables as outputs of the model.

The input variables of the problem are the number of staff, total hours of employee training, and total costs.

The following variables are considered output variables: operating Profit, number of currency accounts, number of Rial accounts, and the average number of essential currency services provided per day.

The table of normalized data and information on the performance indices of the 30 Melli Bank currency units is presented below. It should be noted that to maintain the confidentiality of Melli Bank's information, the names of these units are stated as DMUX.

Table IV. Data and information of currency units of Melli Bank of Isfahan

<i>Row</i>	<i>Currency unit name</i>	<i>Number of staff</i>	<i>Total training hours</i>	<i>Total costs</i>	<i>Operating profit</i>	<i>Number of currency accounts</i>	<i>Number of Rial accounts</i>	<i>The average number of essential currency services provided per day</i>
1	DMU1	0.145	0.142	0.213	0.172	0.086	0.147	0.088
2	DMU2	0.278	0.208	0.257	0.443	0.299	0.314	0.353
3	DMU3	0.133	0.168	0.324	0.195	0.046	0.101	0.112
4	DMU4	0.181	0.218	0.146	0.083	0.059	0.178	0.064
5	DMU5	0.097	0.123	0.138	0.082	0.042	0.109	0.037
6	DMU6	0.121	0.157	0.098	0.051	0.051	0.105	0.049
7	DMU7	0.193	0.194	0.143	0.108	0.046	0.166	0.078
8	DMU8	0.266	0.209	0.154	0.210	0.372	0.305	0.399
9	DMU9	0.217	0.145	0.145	0.178	0.422	0.219	0.282
10	DMU10	0.157	0.121	0.128	0.055	0.053	0.056	0.059
11	DMU11	0.109	0.124	0.148	0.201	0.147	0.248	0.306
12	DMU12	0.157	0.203	0.110	0.070	0.048	0.120	0.071
13	DMU13	0.254	0.149	0.378	0.485	0.332	0.324	0.308
14	DMU14	0.157	0.175	0.180	0.040	0.042	0.113	0.062
15	DMU15	0.193	0.214	0.165	0.077	0.150	0.043	0.092
16	DMU16	0.242	0.229	0.313	0.252	0.378	0.299	0.189
17	DMU17	0.133	0.164	0.136	0.065	0.046	0.070	0.067
18	DMU18	0.193	0.200	0.158	0.074	0.086	0.048	0.032
19	DMU19	0.169	0.196	0.177	0.099	0.156	0.047	0.070
20	DMU20	0.193	0.210	0.149	0.108	0.150	0.085	0.075
21	DMU21	0.169	0.179	0.207	0.254	0.319	0.252	0.360
22	DMU22	0.205	0.205	0.054	0.044	0.044	0.091	0.052
23	DMU23	0.229	0.219	0.090	0.039	0.037	0.108	0.036
24	DMU24	0.145	0.098	0.133	0.057	0.037	0.061	0.068

25	DMU25	0.097	0.197	0.277	0.362	0.248	0.332	0.367
26	DMU26	0.157	0.203	0.095	0.085	0.068	0.175	0.058
27	DMU27	0.097	0.124	0.079	0.040	0.040	0.048	0.047
28	DMU28	0.109	0.222	0.194	0.112	0.158	0.205	0.078
29	DMU29	0.121	0.164	0.081	0.056	0.084	0.065	0.086
30	DMU30	0.302	0.210	0.112	0.139	0.090	0.254	0.221

VII. ANALYSIS OF RESULTS

The main contribution of this research is in creating the game structure for evaluating bank branches. In this way, the bank's branches compete in pairs. As a result, each branch tends to prioritize the other branch over the criteria in which they have a more significant advantage and allocate higher weight. This leads to the higher efficiency of the branch. Thus, the cross-performance matrix is complemented by the performance of the bargaining model, rather than being filled by the performance of the conventional data envelopment analysis model. For example, the *i*th unit in the row is considered as the base unit, and the *j*th unit determines based on the model that the efficiency of this unit is higher than the *i*th unit or equal to or lower.

Unlike cross-efficiency where other DMUs do not have the same performance as the unit, in performance evaluation in a competitive-cooperative environment, both DMUs enter the game concurrently and compete. This type of play is called a Nash bargaining game, and the game's solution is called Nash equilibrium. The proposed model helps to discriminate more effectively among DMUs. Also, decision-making units compete in pairs (two by two), and one can figure out which one wins and which one fails. In the Nash bargaining game, if the performance score is higher than one, the competing player wins the base player; if it is smaller than one, it means defeat, and if it is equal to 1, it means a draw. Finally, the units can be ranked and determined in the competitive-collaborative space using the scores obtained and averaging for each decision unit.

After formulating assumptions and variables, problem models with the relevant software results will be solved. The performance points obtained by solving the model fall into the $n \times n$ square matrix, which is 30 in the present study; because the number of DMUs is 30. If it is considered that each column is a competing DMU performance score over the base DMU and the average of each column is the performance value of each unit based on the proposed model solution to rank the banking units, the form of the model solution will be 30×30. This is done by using Gams, Matlab, and Excel software, and analyzes are given.

The results of the first ten DMUs' performance using a cross bargaining game using the relevant software are as follows:

Table V. Rival DMUs efficiency score over base DMU (first 10 DMUs)

Base	Rival									
	DMU1	DMU2	DMU3	DMU4	DMU5	DMU6	DMU7	DMU8	DMU9	DMU10
DMU1	1	2.094	0.837	0.474	0.485	0.499	0.649	2.472	2.141	0.619
DMU2	0.301	1	0.252	0.173	0.177	0.182	0.237	1.125	1.024	0.287
DMU3	0.721	3.25	1	0.42	0.451	0.464	0.48	4.043	1.543	0.446
DMU4	0.943	3.134	1.175	1	0.612	1.053	1.143	4.242	3.675	1.052
DMU5	1.541	4.77	1.514	0.927	1	1.027	1.06	5.392	5.024	0.985
DMU6	1.686	4.344	1.912	0.873	0.536	1	0.998	4.072	3.209	0.922
DMU7	0.757	2.518	0.903	0.73	0.492	0.769	1	3.712	3.216	0.845
DMU8	0.159	0.53	0.617	0.154	0.103	0.162	0.21	1	0.75	0.178
DMU9	0.212	0.706	0.548	0.151	0.138	0.159	0.207	0.982	1	0.237
DMU10	0.896	4.027	1.773	0.602	0.582	0.634	0.825	3.915	3.458	1
DMU11	0.2	1.102	0.485	0.119	0.122	0.125	0.144	1.265	0.463	0.134

DMU12	0.64	3.164	1.393	0.679	0.415	0.766	0.845	3.317	2.874	0.714
DMU13	0.3	0.821	0.323	0.142	0.146	0.15	0.195	0.924	0.941	0.236
DMU14	1.199	3.216	1.004	0.829	0.778	0.872	1.023	3.799	3.291	0.952
DMU15	0.741	2.463	0.62	0.683	0.481	0.719	0.848	3.147	2.726	0.788
DMU16	0.684	1.626	0.572	0.356	0.364	0.374	0.487	1.921	1.664	0.481
DMU17	0.839	3.407	1.5	0.702	0.544	0.756	0.802	4.043	2.58	0.746
DMU18	2.04	6.782	1.75	1.835	1.324	1.932	2.438	9.047	7.838	2.267
DMU19	1.045	3.066	0.874	0.822	0.678	0.865	0.976	3.622	3.138	0.907
DMU20	0.821	2.729	0.903	0.822	0.533	0.865	1.04	3.86	3.344	0.916
DMU21	0.238	0.596	0.199	0.146	0.15	0.154	0.19	0.704	0.61	0.176
DMU22	1.954	5.034	2.216	0.978	0.932	0.513	1.227	4.228	4.795	0.625
DMU23	2.205	5.679	2.5	1.096	0.67	1.235	1.364	6.477	4.862	1.152
DMU24	0.808	4.04	1.711	1.459	0.434	0.445	0.579	5.027	2.771	0.703
DMU25	0.16	0.612	0.223	0.093	0.101	0.106	0.107	0.75	0.851	0.099
DMU26	1.012	3.043	1.147	0.718	0.439	0.809	0.893	4.061	3.186	0.755
DMU27	2.15	5.537	2.437	0.73	0.451	0.815	0.834	4.65	5.275	0.775
DMU28	0.848	2.263	0.86	0.494	0.533	0.561	0.565	2.558	1.816	0.525
DMU29	1.536	3.955	1.741	0.413	0.253	0.465	0.514	2.346	1.829	0.434
DMU30	0.209	1.661	0.701	0.222	0.136	0.250	0.276	1.313	0.986	0.234

For instance, as can be seen in Table (5), DMU₂₅ won over DMU₁ but lost the game over DMU₂ in the game. It has a performance score of 3.006 against DMU₁, which is higher than one, but it has a performance score of 0.965 against DMU₂, which is less than one.

Table VI. Ranking of the DMUs the cooperative-competitive environment based on efficiency scores (game gains)

<i>Unit</i>	<i>Efficiency (gain)</i>	<i>Rank</i>	<i>Unit</i>	<i>Efficiency (gain)</i>	<i>Rank</i>	<i>Unit</i>	<i>Efficiency (gain)</i>	<i>Rank</i>
DMU1	0.9282	12	DMU11	3.4779	1	DMU21	3.0006	4
DMU2	2.9056	5	DMU12	0.7495	19	DMU22	0.5334	27
DMU3	1.1230	10	DMU13	3.0691	3	DMU23	0.3705	29
DMU4	0.6281	24	DMU14	0.5896	26	DMU24	0.7865	15
DMU5	0.4687	28	DMU15	0.9149	13	DMU25	2.7726	6
DMU6	0.6242	25	DMU16	2.3872	8	DMU26	0.6452	23
DMU7	0.7385	21	DMU17	0.7411	20	DMU27	0.7609	17
DMU8	3.2671	2	DMU18	0.3651	30	DMU28	0.8716	14
DMU9	2.6960	7	DMU19	0.7632	16	DMU29	1.1149	11
DMU10	0.6730	22	DMU20	0.7583	18	DMU30	1.6872	9

Table (6) represents the efficiency value (game gains) of each unit and its rank among the other DMUs. According to the above table, the first rank is DMU11, and the last level is DMU18. Other DMUs are also ranked according to Table (6). Each efficiency score (game gain) is calculated based on Table (5) and is equal to the average efficiency obtained in each column of this table for the DMU under evaluation.

Table VII. Ranking of the DMUs the cooperative-competitive environment based on the number of games won

<i>Unit</i>	<i>No. games won</i>	<i>Rank</i>	<i>Unit</i>	<i>No. games won</i>	<i>Rank</i>	<i>Unit</i>	<i>No. games won</i>	<i>Rank</i>
DMU1	20	11	DMU11	25	6	DMU21	28	4
DMU2	25	7	DMU12	18	14	DMU22	12	24
DMU3	21	10	DMU13	28	3	DMU23	5	29
DMU4	9	27	DMU14	11	26	DMU24	12	23
DMU5	6	28	DMU15	19	13	DMU25	29	1
DMU6	13	22	DMU16	22	9	DMU26	15	20

DMU7	18	16	DMU17	18	15	DMU27	11	25
DMU8	28	2	DMU18	0	30	DMU28	20	12
DMU9	27	5	DMU19	14	21	DMU29	17	17
DMU10	17	18	DMU20	15	19	DMU30	23	8

Table (7) represents the ranking of DMUs according to the number of games won. According to the above table, the first rank is DMU25, and the last level is DMU18. Other DMUs are also ranked according to Table (7). Some of the differences between the rankings of DMUs based on the efficiency score (game gains) and the number of winning games is because some DMUs, despite the smaller number of slave games, the total gain earned in the games is high and vice versa, some DMUs despite a large number of winning games, the total gain earned in the games is low.

A. Investigating results in the competitive-cooperative environment

According to the competition results between players in pairs, for example, in Table 5, DMU2 wins over DMU1 but loses to DMU8. Because it has an efficiency score of 2.094 against DMU1, which is higher than one. Nevertheless, in the game against DMU8 efficiency score is 0.53, which is less than one. According to Table (6), the average efficiency score of DMU2 compared to other DMUs is 2.9056, indicating its position among other DMUs. According to the results, DMU2 ranks fifth among DMUs. It should be noted that the numbers on the original diagonal are one because it means that each player is competing with himself.

B. Suggestions for improvement in a competitive - cooperative environment

In the Nash bargaining game, when rival player indices are better than base player indices or, in other words, take better indicator weight. The opposing player overcomes the base player and wins. If you want to make suggestions about improvement using the proposed model, in competition, the number of losses, wins, and weights associated with the indicators should be considered. For example, the DMUX has been a loser or winner in several competitions. Losing or winning is due to certain indices. Improvement is computationally challenging to achieve with an efficient unit. In this method, we deal with the weights of the indices. Consider, for example, the DMU24 game status that has been ranked 15th (middle) in the competitive arena. The indices are specified having the highest weight.s

Table VIII. Results of the game of DMU24 with other DMUs

<i>Currency unit name</i>	<i>Result of game</i>	<i>Number of staff</i>	<i>Total training hours</i>	<i>Total costs</i>	<i>Operating profit</i>	<i>Number of currency accounts</i>	<i>Number of Rial accounts</i>	<i>The average number of essential currency services provided per day</i>
DMU1	Lost	✓						✓
DMU2	Won	✓						✓
DMU3	Won	✓						✓
DMU4	Won			✓				✓
DMU5	Lost	✓						✓
DMU6	Lost			✓				✓
DMU7	Lost			✓				✓
DMU8	Won			✓				✓
DMU9	Won			✓				✓
DMU10	Lost			✓				✓
DMU11	Won	✓						✓
DMU12	Lost			✓				✓
DMU13	Won		✓					✓
DMU14	Lost	✓						✓
DMU15	Won			✓				✓

DMU16	Won	✓						✓
DMU17	Lost	✓						✓
DMU18	Lost			✓				✓
DMU19	Won	✓						✓
DMU20	Won			✓				✓
DMU21	Won	✓						✓
DMU22	Lost	✓		✓				✓
DMU23	Lost			✓				✓
DMU24	-	-	-	-	-	-	-	-
DMU25	Won	✓						✓
DMU26	Won			✓				✓
DMU27	Lost			✓				✓
DMU28	Won	✓						✓
DMU29	Lost			✓				✓
DMU30	Won			✓				✓
Number of indices repetition		13	1	16	0	0	0	29

The results of the games have been obtained based on Table (5). The results of the game for Tables 8-12 are based on a comparison of other DMUs with the reference DMU. According to Table (8), it can be seen that DMU₂₄ has lost 16 games in 29 games with other units and has won 13 games. In the games played in the "Inputs" section, the "Total costs" index, and in the "Outputs" section, the "Average number of important currency services per day" index has the highest number of repetitions. In other words, they have the highest weight. Accordingly, to be efficient, the branches management needs to focus on these two indicators such that in the "Outputs" section, the "the number of important currency services provided per day" index should be increased. In the "Inputs" section, the "Total costs" index should be decreased. Thus, in this way, their efficiency score is grown, and they can reach an efficient boundary. Results of the game of DMU₈, DMU₁₁, DMU₁₈, and DMU₂₃ with other DMUs are represented as tables (9-12) in section "Appendix". Similarly, DMU₈, DMU₁₁, DMU₁₈ and DMU₂₃ have lost 1, 4, 29, and 24 games in 29 games. For this reason, DMU₁₈ is ranked last (see Table (7)). Also, in each table, the inputs and outputs that determine the result of the game are specified. As can be seen from the tables, "The average number of essential currency services provided per day", "Total costs" and "Number of staff" are important indicators in games. But in some games, such as DMU₁₃ with DMU₁₁, "Total training hours" and "The average number of essential currency services provided per day" play a key role (see Table (10) in appendix).

VIII. CONCLUSION AND FUTURE SUGGESTIONS

In this study, a model was developed by applying the Nash bargaining game in cross efficiency, on the other hand, in terms of functional aspect, after evaluating performance indicators using the BSC method. We assess the effectiveness and positioning of currency units in the competitive-cooperative environment. To evaluate DMUs in a competitive-cooperative climate, the proposed Nash bargaining model in cross-capability is used. In the proposed model, players are the same decision-making units (DMUs) or banking units that compete. Then, using the proposed model, the banking units are compared in a paired competitive environment. Finally, using the scores obtained and averaging for each DMU, we rank the groups in the competitive-cooperative space. The proposed model of using Nash bargaining in cross-performance has not been presented so far and, for the first time, is applied to assess the efficiency of bank currency units.

To evaluate efficiency, its indices must first be examined and finally determined. Finally, using the foreign exchange experts' opinions and consulting; finally, seven of the most important indices for evaluating the efficiency of Melli Bank Currency Units in four BSC perspectives were selected, which include: The number of personnel and operating profit and total costs in financial terms, Number of Currency and Rial Accounts (Opened in 2017) in Customer Perspective, Average

Number of Important Currency Services Per Day (Important Currency Services: Includes Imported Documents, Imported Documents and Issued Foreign Currency Remittances, etc.) in internal Processes And total staff training hours in terms of learning and growth.

Solving the research problem, or in other words, evaluating the currency units, using the proposed Nash bargaining model of cross-competitiveness in the competitive environment, was performed by Gams, Matlab, and Excel software, which were the first written model in Gams. Then MATLAB solves the problem from Gams and then compiles the results in Excel. The developed model helps to discriminate more effectively among DMUs. Also, because decision-making units are competing in pairs, the efficiency score is obtained to figure out which player wins and which one loses. If the performance score is higher than one, the rival player wins over the base player. If a performance score is less than one, it means defeat, and if it is equal to one, it means two players draw in the game. According to the results of the developed model solution, it can be concluded that the number of games played according to the weights of the indices and the result of the games Inputs in the total costs index and Outputs averages weighted average daily services provided have the most masses. Moreover, the branches management needs to focus on these two indices to be effective. How to increase the output index and reduce the input to increase their efficiency score and reach an efficient boundary. The findings suggest that bank management should seek to reduce branch costs and improve service and profitability.

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APPENDIX

Results of the game of DMU₈, DMU₁₁, DMU₁₈, and DMU₂₃ with other DMUs are represented as Tables (9-12).

Table IX. Results of the game of DMU8 with other DMUs

Currency unit name	Result of game	Number of staff	Total training hours	Total costs	Operating profit	Number of currency accounts	Number of Rial accounts	The average number of essential currency services provided per day
DMU1	Lost	✓		✓				✓
DMU2	Lost							✓
DMU3	Lost	✓						✓
DMU4	Lost			✓				✓
DMU5	Lost							✓
DMU6	Lost			✓				✓
DMU7	Lost							✓
DMU8	-	-	-	-	-	-	-	-
DMU9	Lost			✓				✓
DMU10	Lost			✓				✓
DMU11	Lost	✓						✓
DMU12	Lost			✓				✓
DMU13	Won		✓					✓
DMU14	Lost	✓						✓
DMU15	Lost			✓				✓
DMU16	Lost							✓
DMU17	Lost	✓						✓
DMU18	Lost			✓				✓
DMU19	Lost	✓						✓
DMU20	Lost			✓				✓
DMU21	Lost	✓						✓
DMU22	Lost			✓				✓
DMU23	Lost			✓				✓
DMU24	Lost			✓				✓
DMU25	Lost	✓						✓
DMU26	Lost			✓				✓
DMU27	Lost			✓				✓
DMU28	Lost	✓						✓
DMU29	Lost			✓				✓
DMU30	Lost			✓				✓
Number of indices repetition		8	1	16	0	0	0	29

Table X. Results of the game of DMU11 with other DMUs

Currency unit name	Result of game	Number of staff	Total training hours	Total costs	Operating profit	Number of currency accounts	Number of Rial accounts	The average number of essential currency services provided per day
DMU1	Lost	✓		✓				✓
DMU2	Won	✓						✓
DMU3	Lost	✓						✓
DMU4	Lost	✓		✓				✓
DMU5	Lost	✓						✓
DMU6	Lost			✓				✓
DMU7	Lost							✓
DMU8	Won	✓						✓
DMU9	Lost	✓		✓				✓
DMU10	Lost			✓				✓
DMU11	-	-	-	-	-	-	-	-
DMU12	Lost			✓				✓
DMU13	Won		✓					✓
DMU14	Lost	✓						✓
DMU15	Lost	✓		✓				✓
DMU16	Won							✓
DMU17	Lost	✓						✓
DMU18	Lost	✓		✓				✓
DMU19	Lost	✓						✓
DMU20	Lost			✓				✓
DMU21	Lost	✓						✓
DMU22	Lost			✓				✓
DMU23	Lost	✓		✓				✓
DMU24	Lost	✓						✓
DMU25	Lost	✓		✓				✓
DMU26	Lost	✓		✓				✓
DMU27	Lost			✓				✓
DMU28	Lost	✓						✓
DMU29	Lost			✓				✓
DMU30	Lost	✓		✓				✓
Number of indices repetition		19	1	17	0		0 0	29

Table XI. Results of the game of DMU18 with other DMUs

<i>Currency unit name</i>	<i>Result of game</i>	<i>Number of staff</i>	<i>Total training hours</i>	<i>Total costs</i>	<i>Operating profit</i>	<i>Number of currency accounts</i>	<i>Number of Rial accounts</i>	<i>The average number of essential currency services provided per day</i>
DMU1	Won		✓					✓
DMU2	Won		✓					✓
DMU3	Won	✓						✓
DMU4	Won			✓				✓
DMU5	Won	✓						✓
DMU6	Won			✓				✓
DMU7	Won			✓				✓
DMU8	Won			✓				✓
DMU9	Won		✓					✓
DMU10	Won		✓					✓
DMU11	Won	✓		✓				✓
DMU12	Won			✓				✓
DMU13	Won		✓					✓
DMU14	Won	✓						✓
DMU15	Won	✓						✓
DMU16	Won		✓					✓
DMU17	Won	✓						✓
DMU18	Won	-	-	-	-	-	-	-
DMU19	Won	✓						✓
DMU20	Won			✓				✓
DMU21	Won	✓						✓
DMU22	Won		✓	✓		✓		✓
DMU23	Won			✓				✓
DMU24	Won			✓				✓
DMU25	Won	✓						✓
DMU26	Won			✓				✓
DMU27	Won			✓		✓		✓
DMU28	Won	✓						✓
DMU29	Won			✓				✓
DMU30	Won			✓				✓
<i>Number of indices repetition</i>		10	7	14	0	2	0	26

Table XII. Results of the game of DMU23 with other DMUs

<i>Currency unit name</i>	<i>Result of game</i>	<i>Number of staff</i>	<i>Total training hours</i>	<i>Total costs</i>	<i>Operating profit</i>	<i>Number of currency accounts</i>	<i>Number of Rial accounts</i>	<i>The average number of essential currency services provided per day</i>
DMU1	Won	✓	✓	✓		✓		✓
DMU2	Won	✓	✓	✓		✓		✓
DMU3	Won	✓			✓			✓
DMU4	Won			✓				✓
DMU5	Lost	✓			✓			✓
DMU6	Won			✓				✓
DMU7	Won	✓		✓				✓
DMU8	Won			✓		✓		✓
DMU9	Won		✓			✓		✓
DMU10	Won	✓	✓			✓		✓
DMU11	Won	✓		✓				✓
DMU12	Won			✓				✓
DMU13	Won		✓	✓				✓
DMU14	Lost	✓		✓				✓
DMU15	Won	✓						✓
DMU16	Won		✓		✓			✓
DMU17	Won	✓			✓			✓
DMU18	Lost			✓				✓
DMU19	Lost	✓						✓
DMU20	Won	✓		✓	✓			✓
DMU21	Won	✓			✓			✓
DMU22	Won	✓	✓	✓		✓		✓
DMU23	-	-	-	-	-	-	-	-
DMU24	Won			✓				✓
DMU25	Won	✓			✓			✓
DMU26	Won	✓		✓	✓			✓
DMU27	Won		✓	✓	✓	✓		✓
DMU28	Won	✓	✓					✓
DMU29	Won	✓		✓				✓
DMU30	Won	✓		✓				✓
<i>Number of indices repetition</i>		19	9	18	9	7	0	30