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Hybrid optimization of production scheduling and maintenance using mathematical programming and NSGA-II meta-heuristic method

Mohamad Sharifzadegan [1], Tahmoores Sohrabi [2*], Ahmad Jafarnejad Chaghoshi [3]

[1] *Department of Management, Central Tehran Branch, Islamic Azad University, Tehran, Iran*

[2] *Department of Management, Central Tehran Branch, Islamic Azad University, Tehran, Iran*

[3] *Department of Management, University of Tehran, Tehran, Iran*

**** Corresponding Author:** Tahmoores Sohrabi (Email: dr.tsohrabi@gmail.com)*

***Abstract** – This paper presents an integrated hybrid optimization problem for production and maintenance scheduling within a comprehensive system using overall cost and reliability. The total cost consists of three parts: production costs, inventory costs, and workforce costs. This integration aims to simultaneously find the optimal value of the function in a period. Using mixed-integer linear programming, the optimal values are minimized over a limited horizon in the various samples considered for different numbers of workers and machines. In order to evaluate the model in larger dimensions, the NSGA-II metaheuristic method has been used. Given that the error rate of the developed mathematical model with the results of the meta-heuristic method in small dimensions can be neglected, so this meta-heuristic method has been used to perform sensitivity analysis in larger dimensions of the problem. In general, the results of this paper provide valuable information about changes in the number of workers and machines simultaneously to prevent interruptions and save on production to managers and analysts in the field of production planning.*

***Keywords**– Production Planning, Scheduling, maintenance, Hybrid Model.*

I. INTRODUCTION

In most production units, practical information is at an undesirable level of coordination and exchange with other activities. The result of such activities is nothing but a waste of resources (cost, time, materials, etc.) and the emergence of an island culture in the organization. Obviously, all parts of a production unit are interdependent, and it is necessary to use an integrated format that provides vital factors to make management decisions. Many researchers and industry managers have long considered the need for integration (Khalili-Damghani et al., 2021). Therefore, some researchers, such as Nourelfath et al. (2016), explored coordinating and integrating production, maintenance, and repair and maintenance. He pointed out that production, maintenance, repair, and quality control are strongly related to each other, so these issues should be integrated well (Aghezzaf and Najid, 2008). Other researchers, such as Hlioui et al., emphasized the importance of integrating supply and maintenance (Behnezhad and Khoshnevis 1988, Babaeinesami and Ghasemi, 2020), and another group, such as Bouslah et al., sought to integrate production, quality control, and maintenance (Berrichi et al., 2009, Khalilzadeh et al., 2021). Therefore, it can be seen that many researchers have considered the integration of vital information of different production sectors because, over the past few decades, there have always been many concerns about the integration of different areas related to production activities. These areas are

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the beating heart of production units and should be used in various decisions. Due to the interaction of each of these areas, they cannot be considered islands, and it is reasonable to plan a mechanism that can take all the important factors into account. Many efforts have been made to integrate production planning, maintenance and repair, and the supply of raw materials. As it is known, this system's components are connected to each other in a chain, and the decision in each part shows its effects on other parts.

Recent advances in the integration of production, maintenance, and preventive maintenance have included Economic Production Quantity (EPQ) and maintenance and Preventive Maintenance (PM) policies (Berrichi et al., 2010, Shafipour-omran et al., 2020, Shirazi et al., 2021), simultaneous production control of maintenance rates, and preventive maintenance (Fitouhi and Nourelfath, 2014, Ghasemi and Babaeinesami, 2019). On the other hand, research focused on integrating production, maintenance, and repairs dating back to the 1970s and 1980s. Research in this period focused on several critical effects, such as product complexity and technology, operation rate, triggering planning, and design of tolerances considering the deterioration of production equipment (Gajpal and Nourelfath, 2015, Ghasemi and Talebi Brijani, 2014) or the program's impact. Maintenance inspections have focused on production flow (Jafari and Makis, 2015, Ghasemi et al., 2021), as well as some studies focused on the integration of production and maintenance and repairs about system depreciation (Berrichi et al., 2009, Khanchehzarrin et al., 2021). In fact, few published studies have considered the production, maintenance, repair, and planning aspects of human resources. Maintenance planning is one of the most common and important issues that industrial and production systems always face. Production planning is often interrupted due to equipment failure, and timely preventive maintenance can be an excellent tool to solve this problem. However, the recommended maintenance intervals are often delayed in order to speed up production. Maintenance activities are time-consuming if this time can be devoted to production, but delays in preventive repairs may increase the probability of machine failure. Therefore, there are always contradictions between maintenance planning and production planning. It is believed that the efficiency of the production system can be improved by integrating these decisions. As mentioned, preventive maintenance planning is an activity that has priority. Today, in strategic industrial units, because of production importance, requirements of reducing costs, and in-line production, there is the significance for equipment availability and the prevention of unexpected breaks.

In addition to updating additional costs to the system, accidental breakdowns result in public dissatisfaction. Therefore, efforts are made to reduce accidental breakdowns by preventive maintenance and repair activities. These activities are not limited to a particular industry and are important in most industries such as power plants, airplanes, and manufacturing centers where accidental breakdowns result in financial and health risks. Hence, preventive repairs are a broad term that includes a set of activities that improve a system's reliability and overall accessibility. All systems, from conveyors to machines, have particular repair schedules provided by the manufacturer to reduce the risk of failure and overall system maintenance costs. The importance of preventive maintenance planning as part of a company's profitability strategy cannot be ignored. Preventive repairs are crucial in any organization, especially in manufacturing systems. Preventive repairs generally include inspection, cleaning, lubrication, adjustment, and replacement of subsystems. Regardless of the type of system, maintenance and repair can be divided into two general groups of component repair and replacement. Therefore, according to the importance of our subject in this research, a new mathematical model for optimizing production planning, maintenance, and preventive repairs by considering production resources will be presented. Also, to answer the problem on a larger scale, the appropriate meta-heuristic method has been used. Considering the concepts of maintenance and repair without evaluating the maintenance effect of production equipment causes inefficiency of production planning. So, developing a hybrid framework of production and maintenance and equipment repair causes dynamism in the production system by providing a model. Hybrid mathematics and limitations related to human resources cause a new model to be presented in the cellular production system. In addition, to consider the system's reliability for production, this parameter is considered fuzzy to bring the modeling closer to the real-world production environment. Because hybrid models have large dimensions and are called NP-Hard, the evaluation of mathematical models in large dimensions also requires innovative or meta-heuristic algorithms. Therefore, the selection of an efficient algorithm makes the estimation conditions of the mathematical model much better. Therefore, in this study, according to the evaluations, the NSGA-II metaheuristic method is considered. The main challenge of this research is to manage the production of parts by considering inventory costs. Another challenge of this research is the management of maintenance costs in each period. Finally, maximizing the reliability of the system is another challenge of this research. Therefore, the balance between costs and system reliability is the most important goal of this research.

The rest of the article is organized as specified. In the second section, a literature review of previous research related to the topic is presented. The third section includes mathematical modeling of production and maintenance scheduling and the introduction of the meta-heuristic method. In the fourth section, the research results obtained from the solution method are presented, and finally, in the fifth section, a general conclusion is provided along with suggestions for future research.

II. LITERATURE REVIEW

The production category is susceptible due to its strong dependence on machinery and equipment and shows fluctuations in machinery changes. This goes so far that the breakdown or improper operation of machinery and equipment has an adverse effect on production hours, production costs, and production quality. Since production and maintenance integration was raised, there has always been a desire to determine the appropriate mechanism for these two fields' proper connection and coordination. In this section, related research is briefly introduced. For example, Tasaw (2012) examined the production and maintenance decisions of the system with the process of system degradation and how the system can be maintained in the best possible way. In the model designed, there is a restriction of producing a percentage of the defective product in the controlled and out-of-control areas. Liao (2013) generalized a model of economic production program by considering maintenance and production in an incomplete process associated with the deterioration and depreciation of the production system by considering the risk rate. In their research, Jiang et al. (2013) produced and planned maintenance and repairs by considering the cyclic review period, which includes random demand and returns. Their purpose is to integrate batch production and maintenance policies for systems where production, maintenance, backlog, and maintenance costs are integrated and minimized. Yalaoui et al. (2014) presented an extended linear programming model as a hybrid approach to calculate optimal production planning with the least cost. For this purpose, they used a model with two objective functions that proposed the production planning and maintenance of repairs. Nourelfath et al. (2016) examined the integration of production, maintenance, and repair and the quality of an incomplete process where failure occurs, and defective goods are produced in a multi-period multi-production system. In their model, the production system is considered as an incomplete device whose status is under control and non-control modes. Their main goal is to minimize all costs while meeting all demands. Ettaye et al. (2017) have studied the issue of integrated maintenance planning and production with a periodic replacement system with minimal repairs (maintenance and preventive repairs) against unplanned failures. This model was extended by total cost based on the relationship between the maintenance and the production schedule. In their model, the total cost includes two parts: triggering production costs, warehousing, and costs of disrupting demand satisfaction, preventive and corrective maintenance for multi-period and multi-product systems. Ettaye et al. (2017) modeled and optimized an integrated production and maintenance planning problem. This study continues with minimal repairs and periodic replacements. By mixed-integer linear programming, they minimized total cost values over a limited horizon. Bensmain et al. (2019) proposed a preventive reproduction planning for production equipment under operational and incomplete maintenance constraints based on a genetic algorithm-based method. This study investigates the opportunities for the reproduction of production equipment to produce a product to meet the definite and dynamic demands in a limited period. Bahria et al. (2020) developed a joint production, maintenance, and quality control strategy that uses mathematical modeling to include a preventive periodic maintenance policy. This paper focuses on finding the optimal values of the preventive maintenance period, the amount of confidence level, sample size, sampling distance, and control chart constraints to minimize the total cost of waiting per unit time. Saha et al. (2021) designed an integrated economic model of quality control and maintenance management and examined the implications of production process management. The findings of this study show that among all the considered cost components, process and equipment failures have a significant effect on the total costs of the optimized model.

Duffuaa et al. (2020) developed an integrated model of production scheduling, maintenance, and quality for a single machine. This paper develops a model that integrates and optimizes production, maintenance, and process control decisions simultaneously for a single machine. Both types of maintenance, preventive and corrective, are taken into consideration. The methodology starts by developing an optimal preventive maintenance schedule. Then an integrated model that determines the decision variables and optimizes the total cost per unit time, resulting from production scheduling, inventory holding and maintenance, and process control, is developed. The comparisons indicate that the developed model results in a saving that are ranged from 2.62 to 6.78 percent.

Kolus et al. (2020) provided an integrated mathematical model for production scheduling and preventive maintenance planning. This paper considers the simultaneous scheduling of production and maintenance activities with the objective of minimizing the expected total tardiness cost on a single machine. The proposed integrated model shows a high potential for significant improvements in performance with respect to the cost of tardiness in delivery and machine availability.

Zheng et al. (2020) developed a two-stage integrating optimization of production scheduling, maintenance, and quality. The aim of this article is to jointly optimize the job sequence, the preventive maintenance locations, the preventive maintenance interval, and decision variables of the control chart such that the expected cost per unit time and penalty cost due to schedule delay is minimized. A single machine system is developed a two-stage integrating optimization model of production scheduling, maintenance, and quality. Numerical examples and thorough sensitivity analyses are provided to illustrate the proposed integrated model. Finally, the efficiency of the proposed two-stage integration model is verified by the comparison of the stand-alone model under the same condition. Samimi and Sydow (2020) have studied human resource management and maintenance in project-based organizations. In this research, various researches in the field of human resource management have been reviewed. Then its effect on maintenance in project-based organizations has been studied and evaluated. The results show that it is important that manpower problems can potentially affect the performance of the organization in various aspects. Moussavi-Haidar et al. (2021) combine production, inventory, and maintenance. In this study, a mathematical model is presented with the aim of reducing total costs, and the exact solution method is used to optimize it. The results show that policies to increase product inventory increase machine failure rates and maintenance costs.

LaRoche-Boisvert et al. (2021) presented stochastic mathematical modeling to integrate mine-to-port transportation. Uncertainty is intended to supply products. Considering production scheduling of rail-connected mining complexes has been one of their research innovations. The case study is in an iron ore mining complex. Zhang et al. (2021) presented a multi-objective mathematical model for scheduling the production and maintenance of machines simultaneously. Considering the learning-forgetting effects and the multi-degradation effects have been one of their research innovations. They presented a case study of automobile engine manufacturing. The results show that our proposed model can reduce the total maintenance costs by 20%. Yang et al. (2021) presented multi-state maintenance and production scheduling based on reinforcement learning. Considering job types and machine states have been one of their research innovations. They presented a problem as a Markov decision process framework. The results show the effectiveness of the proposed approach. Lu et al. (2021) presented a mathematical model for high-end equipment production scheduling. Considering machine failures and preventive maintenance has been one of their research innovations. They presented a hybrid discrete black hole algorithm and variable neighborhood search to solve the proposed mathematical model. The results indicate the optimal performance of the proposed hybrid model.

Based on the above and previous studies, there is a lack of a hybrid optimization model for production and maintenance scheduling and repairs. Therefore, this research presents mathematical modeling and solving through exact and meta-heuristic methods to improve production and maintenance technology efficiency in production systems. In Table I, the studied articles are classified according to the type of planning, solution method, and other limitations that have been considered.

Besides, the contributions of this paper are as follows:

- Considering a hybrid approach to solving the proposed model
- Consider production and maintenance scheduling and repairs decisions simultaneously
- Customizing a Meta-heuristic solution method to solve and validate the proposed model
- Considering cost and reliability objectives simultaneously in the proposed multi-period model

III. SOLUTION METHOD

The primary method of this research is mathematical modeling and includes two parts: mathematical modeling and problem-solving. In the problem-solving method, two exact and meta-heuristic methods have been used. In the exact method, by forming a linear programming problem using the weighted sum method, the objectives of the problem were changed into a single-objective problem. Then, the optimal answer was obtained by using the BARON tool in GAMS software. Also, the NSGA-II meta-heuristic method has been used to solve the problem on a larger scale. For a detailed understanding of the mathematical modeling, the following hypotheses, sets, indices, parameters, variables, objective functions, and constraints are introduced separately. Figure 1 shows the details of the research method.

Table I- Classification of articles

Author and year	*Planning mode*				*Solution method*			*Other limitations*					
	Production	*Maintenance and repair*	*Hybrid*	*Linear*	*Exact*	*Meta-heuristic*	*Computational*	*Reliability*	*The least cost*	*Single-period*	*Multi-period*	*Depreciation*	*Defective goods*
Tsao (2013)	*	*					*						*
Liao (2013)	*	*					*					*	
Xiang et al. (2014)	*	*					*				*		
Yalaoui et al. (2014)	*	*	*	*	*		*		*				
Nourelfath et al. (2016)		*					*		*		*		
Ettaye et al. (2017)	*	*					*				*		
Bensmain et al. (2019)	*	*				*	*			*			
Bahria et al. (2020)	*	*					*						
Saha et al. (2021)	*	*					*						
LaRoche-Boisvert et al. (2021)	*	*					*			*			
Zhang et al. (2021)	*	*			*						*		
Yang et al. (2021)	*	*		*	*					*			
Lu et al. (2021)	*	*				*			*	*			
This study	*	*	*	*	*	*		*	*		*		

A. MATHEMATICAL MODEL ASSUMPTIONS

In this research, we assume that:

- Machines are parallel but different.
- Each period has work shifts (maximum three shifts).
- Workforce with a special skill level can drive the desired machines
- Each part is processed by a definite skill level and a machine.
- It is not necessary to process the definite part in each shift, but the model determines it. It has been avoided due to the simplification of modeling. Therefore, variable x_{imkjt} calculates this value. If workforce k is applied for producing part i by machine m in shift j and period t, this variable is 1; otherwise, it is 0.
- Workforce can be active or inactive in any period (take leave or get sick, etc.).
- The more machines are used, the higher the breakdown rate.
- In each period, only one workforce is allocated to a machine; in other words, each machine does not have two human resources.
- The machine failure rate is calculated based on the share of processing time.
- Playing the maintenance and repair will reduce this breakdown rate.
- Scheduling maintenance and repair are done in one shift.
- A time capacity was considered for each human resource.
- Total costs, demands, preparation time, and failure rate are considered based on fuzzy theory.

Also, in this study, we have assumed that the parameters of demand, failure rate, preparation time face uncertainty, which is considered triangular fuzzy. In this method, a definite representative is placed in the model instead of the fuzzy parameter. For example, if $\tilde{\lambda}_{tm}^{0} = (\lambda p_{tm}^{0}, \lambda m_{tm}^{0}, \lambda o_{tm}^{0})$, its definite representative is $\dfrac{\lambda p_{tm}^{0} + \lambda m_{tm}^{0} + \lambda o_{tm}^{0}}{3}$.

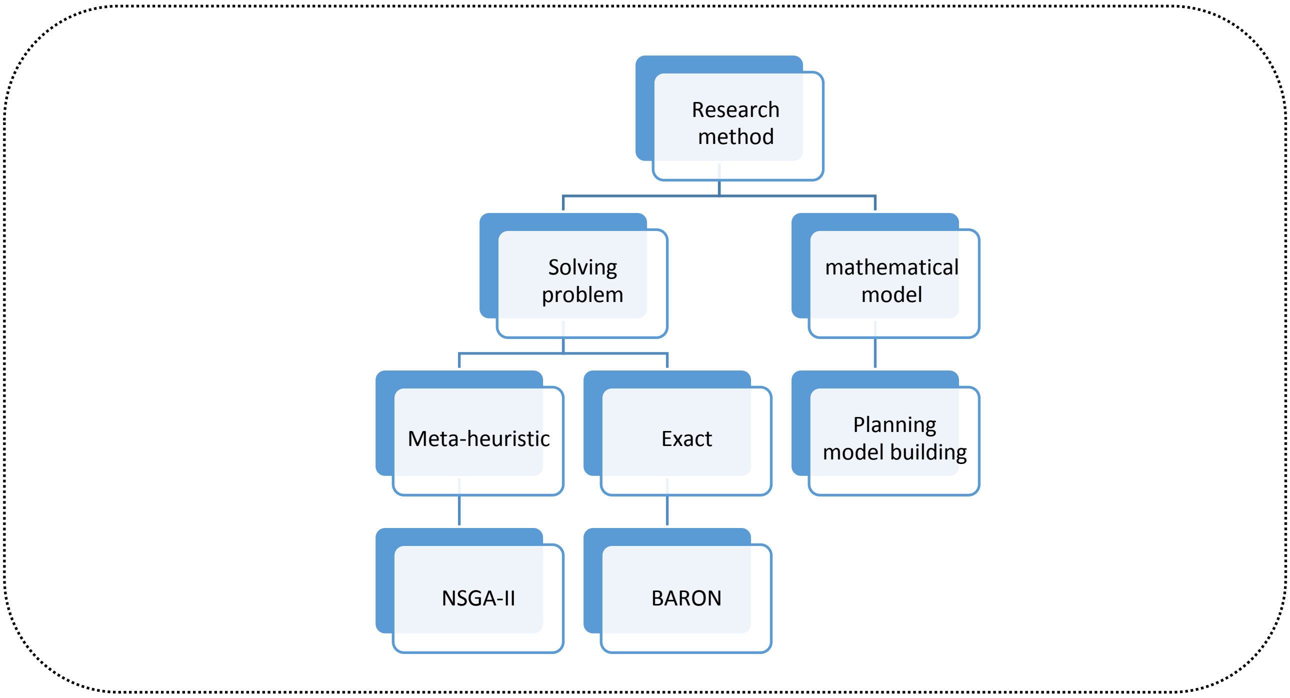


Figure 1- Conceptual model of research method

B. Sets and indices of the mathematical model

The sets considered in the mathematical model of this research are:

I	The set of parts
M	The set of machines
K	The set of human resources
J	The set of shifts
T	The set of periods
U_k	The set of machines that are processed by the human resource *k*.
U'_t	The set of human resources *k* who are available in the period *t*.

The indices of the proposed model are as follow:

I	The indices related to the set of parts
M	The indices related to the set of machines
K	The indices related to the set of human resources
J	The indices related to the set of shifts
T	The indices related to the set of periods

C. Parameters and variables of the mathematical model

The parameters considered for the mathematical model are:

MM	A big number
Cxx_{imjt}	The production cost of part a by machine *m* in *j* th shift and *t* th period
Cx_{imkjt}	The cost of human resource *k* to produce part *i* by machine *m* in *j* th shift and *t* th period
cz_{mjt}	Maintenance and repair cost by machine *m* in *j* th shift and *t* th period

$\tilde{d}_{it}$	fuzzy demand of part *i* in *t* th period
cap_{tk}	The maximized time for each workforce *k* by machine *m* in *t* th period
t_{imk}	processing time by workforce *k*, machine *m* in *t* th period
η_{mt}	The reduced percentage of breakdown rate based on maintenance and repair for machine *m* in *t* th period
$\tilde{\lambda}^{0}_{tm}$	The first breakdown rate of machine *m* in *t* th period
CI_{it}	Inventory cost of product *i* in *t* th period
$\tilde{s}_{imk}$	preparing time for part *i* by workforce *k* and machine *m*

The variables of the model are as follow:

I_{it}	The inventory of product *i* in period *t*
xx_{imjt}	The production amount of part *i* by machine *m* in shift *j* and period *t*
y_{tmk}	the processing time by workforce *k* and machine *m* in period *t*
λ_{tm}	the breakdown rate of machine *m* after repetitive usage in period *t*
λ'_{tm}	breakdown rate after maintenance and repair in period t for machine *m*
λ'_{tm}	breakdown rate before maintenance and repair in period t for machine *m*
$\lambda\lambda_{tm}$	breakdown rate before and after maintenance
R	reliability of the system

The binary variables:

x_{imkjt}	if workforce *k* is applied for producing part *i* by machine m in shift j and period *t*, this is 1, otherwise is 0.
z_{mjt}	if machine *m* is applied in shift *j* and period *t*, this is 1, otherwise is 0.
$z'_{mjtt'}$	if machine *m* is applied in shift *j* and period *t*, and maintenance is done in the other periods after *t*', this is 1; otherwise, it is 0.

$zz'_{mjtt'}$ if machine *m* is applied in shift *j* and period *t*, and maintenance is not done in the other periods before *t*', this is 1; otherwise, it is 0.

D. MATHEMATICAL MODEL

Mathematical modeling of the problem is as follows:

$$A=\sum_{t\in T}\sum_{j\in J}\sum_{m\in M}\sum_{i\in I}xx_{imjt}.Cx_{imjt}+\sum_{t\in T}\sum_{i\in I}I_{it}CI_{it}+\sum_{t\in T}\sum_{j\in J}\sum_{m\in M}\sum_{i\in I}\sum_{k\in K}x_{imkjt}Cx_{imkjt}$$

$$B=\sum_{t\in T}\sum_{j\in J}\sum_{m\in M}z_{mjt}.cz_{mjt}$$

The first objective function, the set of equations A, shows parts production, using workforces and inventory costs. The set of equations *B*, the second objective function, shows maintenance and repair costs in each period. The third objective, function *R* aims to maximize the reliability of the system. In order to turn a problem into a problem with two goals, we add the first objective function to the second one, which is homogeneous and identical, and consider the objective function *R* as the second one. The final objective functions are considered as follows.

$$z1=Min(A+B)$$

$$z2=Max(R)$$

$$I_{it}=I_{it-1}+\sum_{j\in J}\sum_{m\in M}xx_{imjt}-\tilde{d}_{it} \qquad \forall i\in I,t\in T \tag{1}$$

$$\sum_{m\in U_k}x_{imkjt}\le 1 \qquad \forall i\in I,t\in T,j\in J,k\in K\cap U'_t \tag{2}$$

$$xx_{imjt}\le\sum_{k\in K\cap U'_t}x_{imkjt}.MM \qquad \forall i\in I,t\in T,j\in J \tag{3}$$

$$xx_{imjt}\ge\sum_{k\in K\cap U'_t}x_{imkjt} \qquad \forall i\in I,t\in T,j\in J \tag{4}$$

$$y_{tmk}=\sum_{i\in I}\sum_{j\in J}(t_{imk}+\tilde{s}_{imk}).x_{imkjt} \qquad \forall m\in U_k,t\in T,k\in K\cap U'_t \tag{5}$$

$$y_{tmk}\le cap_{tk} \qquad \forall m\in U_k,t\in T,k\in K\cap U'_t \tag{6}$$

$$\lambda_{tm} = (1+\sum_{t'\in T}\sum_{k\in K\cap U'_t}\frac{y_{t'mk}}{\sum_{i\in I} t_{imk}}).\tilde{\lambda}^0_{tm} \qquad \forall m\in M, t\in T \tag{7}$$

$$\sum_{t\in T}\sum_{j\in J} z_{mjt} \le 1 \qquad j\in J \tag{8}$$

$$xx_{imjt} \le (1-z_{mjt}).M \qquad \forall i\in I, t\in T, j\in J, k\in K\cap U'_t, m\in U_k \tag{9}$$

$$R = 1-e^{-\sum_{t\in T}\sum_{m\in M}\lambda\lambda'_{tm}} \qquad \forall m\in M, t\in T \tag{10}$$

$$\sum_{t'\ge t} zz_{mjtt'} = z_{mjt}.(|T|-t+1) \qquad \forall m\in M, t\in T \tag{11}$$

$$\sum_{t'\ge t} zz_{mjtt'} \le z_{mjt}.MM \qquad \forall m\in M, t\in T \tag{12}$$

$$\sum_{t'<t} zz'_{mjtt'} = z_{mjt}.(t-1) \qquad \forall m\in M, t\in T \tag{13}$$

$$\sum_{t'<t} zz'_{mjtt'} \le z_{mjt}.MM \qquad \forall m\in M, t\in T \tag{14}$$

$$\lambda'_{t'm} \ge \lambda_{tm} - \sum_{j\in J} zz_{mjtt'}.\eta_{mt} - MM.(1-\sum_{j\in J} zz_{mjtt'}) \qquad \forall m\in M, t, t'\in T,, t'\ge t \tag{15}$$

$$\lambda''_{t'm} \ge \lambda_{tm} - MM.(1-\sum_{j\in J} zz_{mjtt'}) \qquad \forall m\in M, t, t'\in T,, t'<t \tag{16}$$

$$\lambda\lambda_{tm} = \lambda'_{tm} + \lambda''_{tm} \qquad \forall m\in M, t\in T \tag{17}$$

$$x_{imkjt},\ z_{mjt}, z'_{mjtt'}, zz'_{mjtt'} \in \{0,1\} \qquad \forall i\in I, t\in T, m\in M, j\in J, k\in K\cap U'_t \tag{18}$$

The first constraint determines the demand for each product and the amount of inventory of each product based on shifts and machines. The second constraint allocates the workforce based on the level of expertise and per shift in each period. The third and fourth constraints determine the relationships between the two variables x_{imkjt} and xx_{imjt}. The fifth constraint ensures the calculation of the total processing time per machine in each shift based on the skill level of the workforce. The sixth constraint is the maximum time in each shift that must be observed. The seventh constraint is the calculation of the breakdown rate based on the use of each machine. The eighth constraint determines the time of the maintenance process. The ninth constraint guarantees that maintenance is done when we should not produce in the machine and shift. The tenth constraint calculates the reliability of the system. The eleventh and twelfth constraints specify only when the breakdown rate should decrease after the maintenance. The thirteenth and fourteenth constraints specify only the pre-maintenance times that the breakdown rate is the same as λ_{tm}. Constraint 15 calculates the failure rate after the maintenance is performed, constraint 16 calculates the failure rate before the maintenance is done, and constraint 17 calculates the failure rate before and after the maintenance is performed. Constraint (18) shows types of decision variables.

E. META-HEURISTIC ALGORITHM NSGA-II

The genetic algorithm type-2 is one of the most popular optimization algorithms in multi-objective optimization. This algorithm was introduced by Deb in 2002. In addition to all the functionality of NSGA II, it can be considered a model to form many multi-objective optimization algorithms. This algorithm and its unique approach for multi-objective optimization problems have been used to build newer multi-objective optimization algorithms.

Double-point crossover has been used, as seen in the figure. The mechanism of this operator is that the two points are selected randomly in the chromosome, and the strings of each chromosome are displaced (See figure 2).

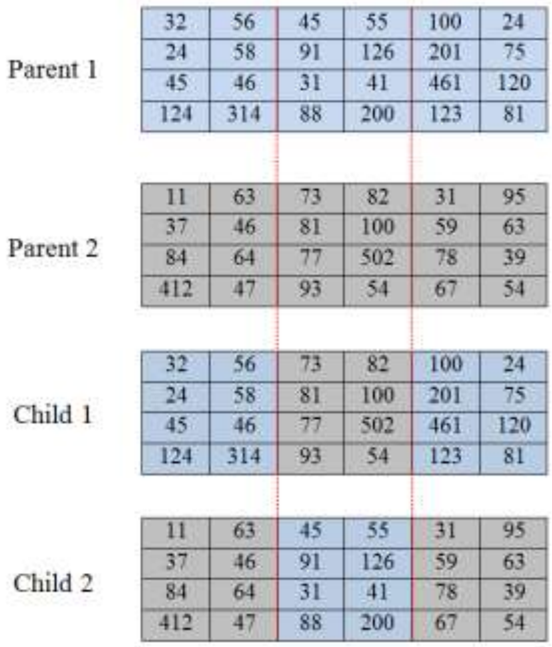
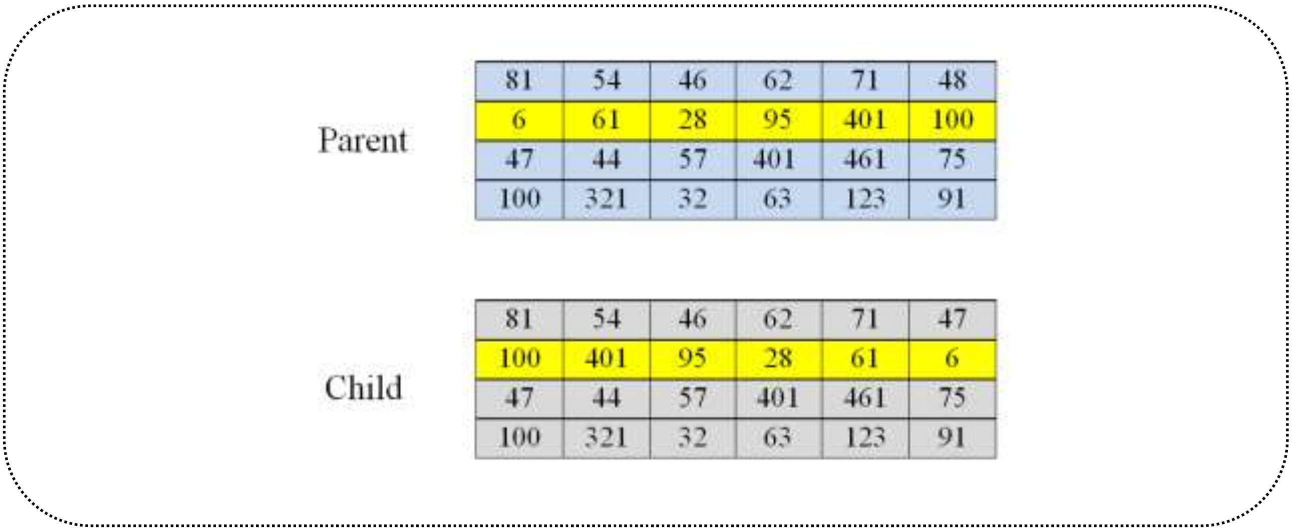


Figure 2. Double-point crossover operator display

In this research, mutation reversion has been used. The mechanism of this operator is that the two points are selected randomly, and each string in this row is reversed (See figure 3).



Parent

81	54	46	62	71	48
6	61	28	95	401	100
47	44	57	401	461	75
100	321	32	63	123	91

Child

81	54	46	62	71	47
100	401	95	28	61	6
47	44	57	401	461	75
100	321	32	63	123	91

Figure 3. Mutation operator display

To run the NSGAII meta-heuristic algorithm, it is necessary to pre-define parameters. The predetermined parameters in this study were adjusted using the Taguchi method, shown in Table II.

Table II- Predefined parameters

Parameter	***Description***	***Value***
nPop	Number of population	200
MaxIt	number of repetitions	150
Pc	Intersection rate	0.7
Pm	Mutation rate	0.3

IV. MODEL SOLUTION RESULT

To solve the model, we first need to determine the value of predefined random parameters for the model. In Table III, the random and decoded values of the fuzzy parameters are shown.

In order to solve the multi-objective mathematical model in this research, the objective weighting method has been used. According to this method, the decision-maker assigns weight to different objectives and then multiplies the objective functions by the corresponding weights. Then, the unique objective function is prepared. In this method, it is necessary to consider the following three points, which are:

- The weight of each objective w_i is a value between zero and one, and the sum of the weights must be one,
- All objective functions should be Max or Min,
- The coefficients of the decision variables in each objective function must be equal to the other objective function. Otherwise, it is necessary to scale the coefficients of each objective function.

Table III- Random parameters of the mathematical model

parameter	*Parameter type*				*Statistical distribution*
	Fuzzy			*random*	
	o	*m*	*p*		
Cxx_{imjt}	-	-	-	*	Uniform [200; 400]
Cx_{imkjt}	-	-	-	*	Uniform [300; 350]
cz_{mjt}	-	-	-	*	Uniform [200; 350]
$\tilde{d}_{it}$	Uniform [1; 3]+2	Uniform [1; 3]+1	Uniform [1; 3]	-	$P+m+o/3$
cap_{tk}	-	-	-	*	Uniform [8; 10]
t_{imk}	-	-	-	*	Uniform [8; 10]
η_{mt}	-	-	-	*	Uniform [0.1; 0.3]
$\tilde{\lambda}^{0}_{tm}$	Uniform [1; 3] +2	Uniform [1; 3] +1	Uniform [1; 3]	-	$P+m+o/3$
CI_{it}	-	-	-	*	Uniform [100; 300]
$\tilde{s}_{imk}$	Uniform [1; 3]+2	Uniform [1; 3]+1	Uniform [1; 3]	-	$P+m+o/3$

Therefore, Table III shows the total number of cases related to small samples per maximum number of workers three and the number of machines maximum 2. Also, the computational results of solving the NSGA-II mathematical and meta-heuristic model for small-scale problems are shown. In Table IV, the first column shows the number of samples for workers and machines for each run. The weighted objective function value is calculated from the weighted sum of the first and second objective functions in the second column. The third column shows the computational time value of solving the definite model to reach the optimal value. In the fourth column, the optimal value of the function in the NSGA-II method is shown. The fifth column shows the computational time to solve the meta-heuristic model. Also, in the last column, the relative absolute error between the value of the definite objective function and the meta-heuristic objective function is shown.

Table IV shows the results of solving the model in small dimensions. In the sample column, the total number of possible samples are shown for a maximum of 3 workers and a maximum of 2 machines. In Table 3, the exact solution results are compared with the results of the NSGA-II method. Also, the solution time of each method and the error percentage values, which represent the absolute relative error (ARE) between exact and meta-heuristic, are given in the last column. The absolute relative error method is a popular method that Duffuaa et al. (2020) and Kolus et al. (2020) have recently used to show the degree of error of the estimated models compared to the simulated results with meta-heuristic methods.

Table IV- Computational results of the model in small dimensions

Experiment number	*Sample*		*Weighed function method*	*Computational time (second)*	*NSGA-II*	*Computational time*	*Error (ARE)*
	Machine	*labor*	f^{exact}		f^{meta}		
1	1	1	550	180	551	15	0.001
2	2	1	570	200	572	35	0.003
3	1	2	600	253	601	65	0.001

Continue Table IV- Computational results of the model in small dimensions

Experiment number	*Sample*		*Weighed function method*	*Computational time (second)*	*NSGA-II*	*Computational time*	*Error (ARE)*
	Machine	*labor*	f^{exact}		f^{meta}		
4	2	2	541	332	542	110	0.001
5	1	3	1668	382	1670	122	0.001
6	2	3	1700	402	1702	163	0.001

ARE is calculated by $\frac{f^{meta}-f^{exact}}{f^{meta}}$ where f^{meta} is the value of the objective functions in the metaheuristic method and f^{exact} is the objective function in the exact method. If ARE is less than 0.1, the error rate is within an acceptable interval. As it is known, the average error rate of the values of the objective function in all samples is less than one percent. Due to the low error difference between the two methods, the performance and efficiency of the NSGA-II algorithm against the exact model are proved. In fact, the meta-heuristic model is able to cover the objective functions of the problem. Therefore, NSGA-II can also be assumed to solve large-scale problems. The solution results indicate that as the size of the problem increases, the solution time of both methods increases, although the speed of increasing the solution time of the e-constraint method is much higher than NSGA-II. Figure 4 shows a comparison of small-dimension solution times based on exact and meta-heuristic methods.

Figure 4 shows the solution times based on the exact and meta-heuristic method. According to Figure 4, as the dimensions of the problem increase from the second example, the solution time based on the exact method quickly increases. Therefore, it can be concluded that the problem has a degree of complexity of NP-HARD due to the ascending solution time.

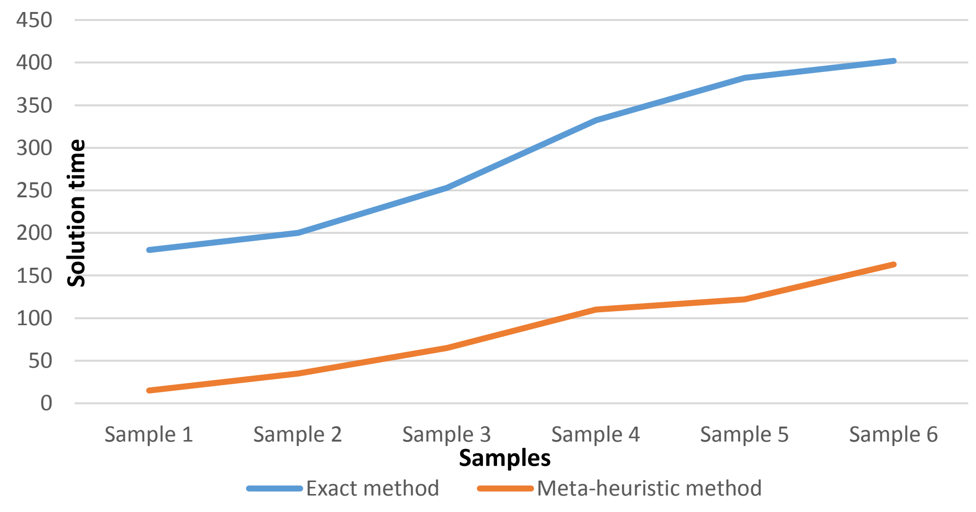


Figure 4- Computational time of solving mathematical model with exact and meta-heuristic method

The effect of production costs per change in the number of workers and machines is shown in Figures 5and 3, respectively. As shown in Figure 5, costs increase significantly as the number of workers increases. However, according to Figure 6, if the number of machines increases, the costs will increase much more. Therefore, if a new worker is added, it will put a high cost on the production system because the addition of machines due to adaptation to new technology provokes more costs for the employer.

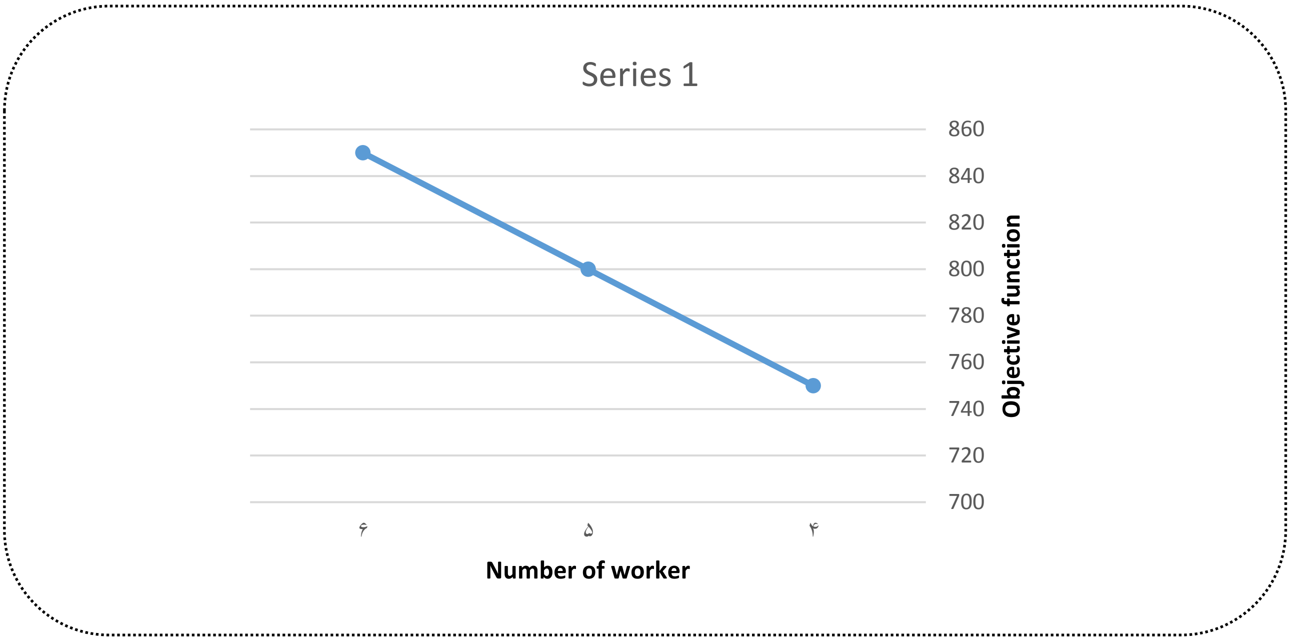


Figure 5- Sensitivity analysis of the objective function against changing the number of workers and keeping other factors constant

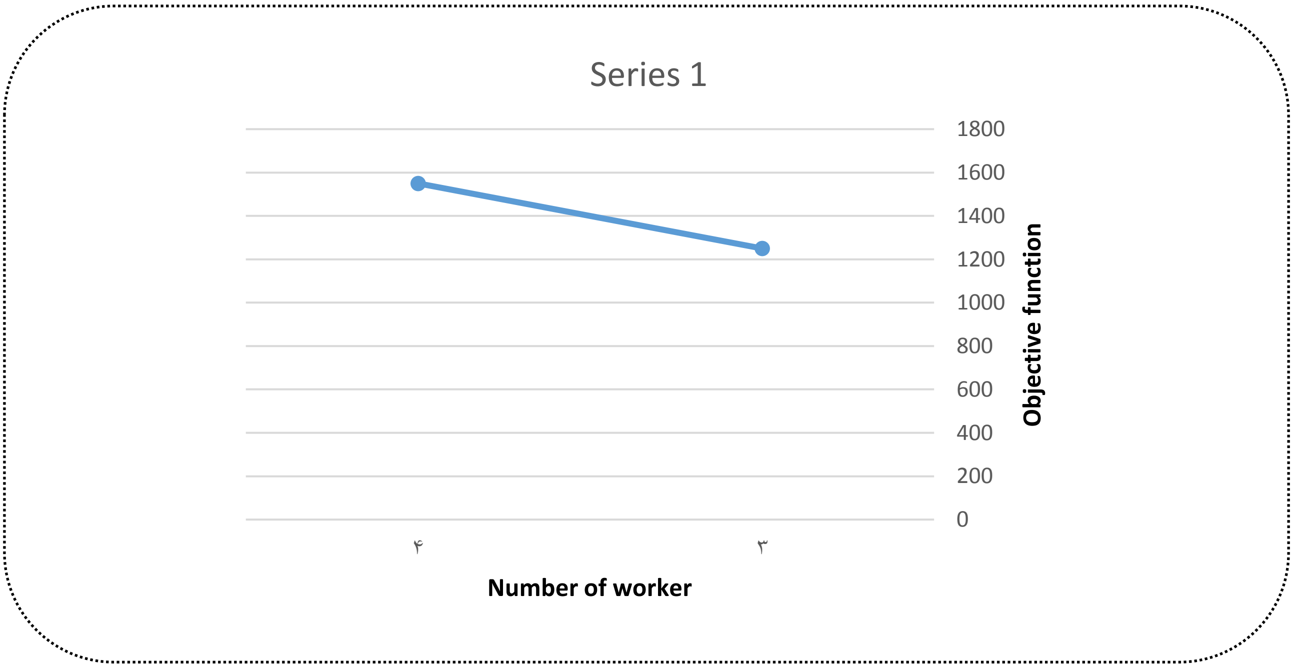


Figure 6- Sensitivity analysis of the objective function against changing the number of machines and keeping other factors constant

V. DISCUSSION

This paper presents a framework for the integrated hybrid optimization problem for production planning, maintenance, and repairs within a comprehensive system while considering overall cost and reliability. The total cost consists of three parts: production costs, inventory costs, and human resources cost. This integration aims to find the optimal value of the function considered in a period. By using mixed-integer linear programming, the optimal values are minimized over a limited horizon in the various samples considered for different numbers of workers and machines. In order to evaluate the model in larger dimensions, the NSGA-II meta-heuristic method has been used. In the solution method, two exact and meta-heuristic methods have been applied. The exact method forming a linear programming problem using the weighted sum has been changed to a single-objective problem. Then, the optimal solution has been obtained using the BARON tool in GAMS software. Also, the NSGA-II meta-heuristic method has been used to solve the problem on a larger dimension. The computational results of the error between the exact and meta-heuristic objective functions also show that the meta-heuristic objective function is able to estimate the response level of the exact objective function with a small error percentage. The results show that as the problem's dimensions increase from the second example, the solution time based on the exact method quickly increases. Therefore, it can be concluded that the problem has a degree of complexity of NP-HARD due to the ascending solution time.

VI. CONCLUSION

In this research, a hybrid optimization for production scheduling and maintenance is proposed by considering the limitation of access to production resources. This problem is solved using a meta-heuristic algorithm to increase the efficiency of the production system. In this research, a mathematical model has been developed while introducing the parameters and assumptions of the problem. First, this mathematical model is formulated in GAMS software and is solved using the BARON® tool. The optimal value of the objective functions is obtained. Then, the problem is solved in small dimensions for a maximum of 3 workers and a maximum of 2 machines. According to the exact and meta-heuristic NSGA-II, an error rate of less than 0.1% shows a good convergence in each sample. Therefore, we have concluded the efficiency of the meta-heuristic model to estimate the objective function in larger samples.

Then, we have proceeded to do sensitivity analysis on a larger scale using the meta-heuristic method. By increasing the number of workers to 4, 5, and 6 and keeping the other factors constant, we estimate the value of the objective function. It shows that changing the number of workers causes a significant increase in the objective function; in contrast, by changing the number of machines 3 and 4 and keeping other factors constant, there is a significant change in the amount of the objective function. We conclude that applying the model presented in different industrial sectors can help to make managerial decisions. Applications of this research can be useful for petrochemical industries, cement industry, non-perishable food industry, medical equipment, etc. Also, simultaneous consideration of production, scheduling, and maintenance decisions can be the timely delivery of products at the lowest cost. This can help decision-makers to schedule machines efficiently in shifts. Also, paying attention to the system's reliability can reduce the risk for decision-makers, which will reduce investment costs. Therefore, for future studies, there are several suggestions, which are:

1. Considering other meta-heuristics such as robust optimization and comparing its result with the meta-heuristic,
2. Considering other multi-objective solution methods such as e-constraint and comparing its results with existing results.
3. Considering other objective functions such as maximizing profits or minimizing the number of workers
4. Considering transportation time, transportation costs, or shortage costs
5. Considering fuzzy uncertainty or scenario-based uncertainty in the ordering cost parameter

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