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Monitoring binary response profiles in multistage processes

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Abstract *– Monitoring Binomial regression profiles in Phase II is examined in this study for multistage manufacturing processes where the quality characteristic is binary. In these kinds of processes, the quality of the final product depends on the quality characteristic of the previous stages, which is referred to as the cascade property. The U statistic was used to diminish the effect of this property. Then, four approaches, such as T2 and MEWMA control chart, LRT, and LRT/EWMA method, have been used, and the performance of these methods have been evaluated using simulation and a numerical example by means of ARL. An actual case study was also used to investigate the effectiveness of monitoring methods in further depth. Studies reveal that the proposed schemes perform well.*

Keywords*– Binomial regression profile, Cause selecting control charts, Cascade Property, Multistage Processes, Profile Monitoring.*

I. INTRODUCTION

Statistical control charts were adopted to monitor processes due to today's competitive industries and advances in technology. The majority of arbitrary control methods are intended to monitor a quality characteristic at the end of a multistage process. As we now know, the quality of a product at a given stage in these types of processes is determined not just by the quality of that stage but also by the quality of previous stages. Many research including Jin and Shi (1999), Shi (2007), Jearkpaporn *et al.* (2007), Tsung *et al*. (2008), Shi and Zhou (2009), Niaki and Davoodi (2009), Jiao and Djurdjanovic (2010), Noorossana and Shekary (2012), Shang *et al*. (2013), Asgari *et al*. (2014), Asadzadeh *et al*. (2015), Du *et al.* (2015), Zolfaghari and Amiri (2016), Goodarzi *et al.* (2016) have investigated multistage processes monitoring. Any changes in the quality characteristics of the current stage would have an impact on the quality characteristics of the succeeding stages in these processes. This characteristic is known as the cascade property, and adequate corrective actions are needed to diminish or remove it, making control chart interpretations simple. Using Zhang's control chart, also known as cause selecting control charts, is one method of addressing this issue (Zhang, 1984). Zhang's studies were reviewed by Wade and Woodall (1993), who presented a cause selecting control chart with prediction limits as a modification of the standard CSC. Furthermore, Shu *et al*. (2004) proposed multiple causes selecting the chart, while Asadzadeh *et al*. (2008) studied a robust cause selecting control chart. Also, other research such as Yang and Su (2007), Yang and Yeh (2011), Noorossana and Shekary (2012) investigated CSCs and their applications.

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A profile is a relationship between dependent and independent variables that can be used to describe the quality of a process or product. Researchers like Kang and Albin (2000), Mahmoud and Woodall (2004), Soleimani *et al.* (2009), Noorossana *et al.* (2010), Shang *et al.* (2011), Dai *et al.* (2014), Amiri *et al.* (2015), Hadiddoust *et al.* (2015), Zhang *et al.* (2015), Khedmati and Niaki (2016), Chen *et al.* (2016), Qi *et al.* (2016) and Maleki *et al.* (2018) investigated the applications of profile monitoring.

Despite the considerable literature on both multistage process monitoring and profile monitoring, there are few studies in the area of profile monitoring in multistage processes. According to a review paper by Maleki *et al.* (2019), researches in the area of monitoring profiles in multistage processes can be summarized as follows:

Table I: researches in the area of monitoring profiles in multistage processes Maleki et al. (2019)

Year	***Researcher***	***Area***	***Profile Type***	***Method***	***Phase***	***Performance criterion***	***Practical application***
2014	Eghbali Ghahyazi *et al.*	Statisticsal Design	Simple Linear	T^2 MEWMA	II	ARL	-
2016a	Khedmati and Niaki	Statistical Design	Simple Linear	Max-EWMA	II	ARL and Correct classification	-
2016b	Khedmati and Niaki	Statistical Design/ Diagnosis	Simple Linear	Max-EWMA-3	II	ARL and Correct classification	-
2017	Esmaeili *et al.*	Statistical Design/ Diagnosis	Simple Linear	EWMA/R	II	ARL, Correct Diagnosis	-
2016	Kalaei *et al.*	Statistical Design	Simple Linear	Kang and Albin (2000), Stover and Brill (1998) and Williams *et al.* (2007)	I	Signal probability	Piston Manufacturing Line
2017	Khedmati and Niaki	Statistical Design	Linear	T^2, LRT	I		
2019	Bahrami *et al.*	Statistical Design	Multivariate	MEWMA, MEWMA/ T^2 Max-MEWMA	II	ARL	-
2020	Derakhshani *et al.*	Statistical Design	Poisson Regression	T^2, LRT, MEWMA and EWMA/R	II	ARL	Manufacturing automobile Glasses

As shown in Table I, most of the body of literature on profile monitoring in multistage processes contains linear profiles .However, in some real situations, this relationship is not a linear function (see Derakhshani *et al.* 2020).

A phase II monitoring of the Binomial regression profile in multistage processes is studied in this paper. Four methods are proposed for monitoring the process. Simulation studies in terms of the ARL, an illustrative and real example, are used to compare the performance of the schemes.

The remainder of the study is as follows:

The following section presents profile modeling in the multistage process and parameter estimation. In section III, proposed control schemes based on the U statistic are investigated. Simulation studies are presented in section IV. An illustrative example and a case study are given in sections V and VI. Section VII concludes this paper.

II. PROFILE MODELING IN MULTISTAGE PROCESSES AND PARAMETER ESTIMATION

A. Profile Modeling

Assume that the *j*th sample collected overtime at the *s*th step of a multistage process in Phase II while the process is in control, (x_{ijs}, y_{ijs}) are available in which $i = 1, 2, 3, ..., n$ represents the *i*th observation in each profile, $j = 1, 2, 3, ...$ represents the number of each profile, and $s = 1, 2, 3, ..., S$ represents the stage of the process. *n* is the sample size, and *S* indicates the number of stages. According to the definition of Binomial regression profiles, the profile model in a multistage process is as follows (Agresti, 2002):

$$\pi_{ij1} = \frac{\exp\left(\mathbf{X}_{i1}^T \boldsymbol{\beta}_{j1}\right)}{1+\exp\left(\mathbf{X}_{i1}^T \boldsymbol{\beta}_{j1}\right)}, \tag{1}$$

$$\pi_{ijs} = \frac{\exp\left(\mathbf{X}_{is}^T \boldsymbol{\beta}_{js} + \varphi \pi_{ij(s-1)}\right)}{1+\exp\left(\mathbf{X}_{is}^T \boldsymbol{\beta}_{js} + \varphi \pi_{ij(s-1)}\right)}. \tag{2}$$

Where $\boldsymbol{\beta}_{js} = \left(\beta_{0js}, \beta_{1js}, ..., \beta_{qjs}\right)$ is the parameters vector of the *j*th profile in stage *s*, $\mathbf{x}_{ijs} = \left(x_{ij1s}, x_{ij2s}, ..., x_{ijqs}\right)$ represents independent variables, and π_{ij} is the mean of the response variable for the *i*th observation in the *j*th profile. $k = 1, 2, 3, ..., q$ shows the number of independent variables in each stage of the process and φ is the auto-correlation coefficient. Furthermore, because this research aims to track profiles in Phase II, the parameters' in-control values φ are expected to be known. The **U** statistic of Hauck *et al*. (1999) is used to eliminate the effect of the cascade property across stages. So for the jth profile, we will have the following in the first stage of a multistage process:

$$\mathbf{U}_{j1} = \hat{\boldsymbol{\beta}}_{j1} \tag{3}$$

$$\mathbf{U}_{js} = \hat{\boldsymbol{\beta}}_{js} - \Sigma_{s(s-1)} \Sigma_{(s-1)(s-1)}^{-1} \hat{\boldsymbol{\beta}}_{j(s-1)} . \tag{4}$$

$\hat{\boldsymbol{\beta}}_{js}$ is the estimate of the *j*th profile parameters vector for stage *s*. $\sum_{s(s-1)}$ is the covariance matrix between $\hat{\boldsymbol{\beta}}_{js}$ and $\hat{\boldsymbol{\beta}}_{j(s-1)}$ and $\sum_{(s-1)(s-1)}$ is the covariance matrix of $\hat{\boldsymbol{\beta}}_{j(s-1)}$. The values of $\mathbf{U}_j$ in different stages are independent of each other (Jearkpaporn *et al*. 2007).

The mean vector and variance matrix of the **U** statistic are:

$$\begin{aligned} \mu_{\mathbf{U}_{j1}} &= \mu_{\hat{\boldsymbol{\beta}}_{j1}}, \\ \mu_{\mathbf{U}_{js}} &= \mu_{\hat{\boldsymbol{\beta}}_{js}} - \Sigma_{s(s-1)} \Sigma_{(s-1)(s-1)}^{-1} \mu_{\hat{\boldsymbol{\beta}}_{j(s-1)}}, \\ \Sigma_{\mathbf{U}_{j1}} &= \Sigma_{11}, \\ \Sigma_{\mathbf{U}_{js}} &= \Sigma_{ss} - \Sigma_{s(s-1)} \Sigma_{(s-1)(s-1)}^{-1} \Sigma_{(s-1)s}. \end{aligned} \tag{5}$$

B. Parameter estimation

Generalized linear models are used to model profiles with discrete response variables. However, logistic regression is one of the most widely used methods for this subject when the response variable takes only the values of zero and one. Suppose that there are *n* independent observations and *q* independent variables which are represented as $\mathbf{x}_i = x_{i1}, x_{i2}, ..., x_{iq}$. Besides, assume that the variable z_i is a Bernoulli variable with a probability of success π_i ($i = 1, 2, ..., n$). Mean and variance of z_i , are equal to $E(z_i) = \pi_i$ and $Var(z_i) = \pi_i(1-\pi_i)$, respectively. Considering the logit function as a link function, we have:

$$g(\pi_i) = \log\left(\frac{\pi_i}{1-\pi_i}\right) = \mathbf{x}_i^T \beta = \beta_1 \mathbf{x}_{i1} + \beta_2 \mathbf{x}_{i2} + ... + \beta_q \mathbf{x}_{iq}. \quad (6)$$

In which $\boldsymbol{\beta} = \beta_1, \beta_2, ..., \beta_q$ represents the vector parameters of the model. Generally, $\mathbf{x}_{i1} = 1$ and as a result, β_1 is the intercept of the model. The probability of success is equal to:

$$\pi_i = \frac{\exp(\mathbf{x}_i^T \boldsymbol{\beta})}{1+\exp(\mathbf{x}_i^T \boldsymbol{\beta})} \quad (7)$$

Suppose that for each independent variable, there are m_i independent observations such that $M = \sum_{i=1}^{n} m_i$ is the total number of observations. $y_i = \sum_{j=1}^{m_i} z_{ij}$ represents the total number of successes in the *i*th sample which follows a binomial distribution with (m_i, π_i). As a result:

$$\begin{aligned} &E(y_i) = m_i \pi_i \\ &Var(y_i) = m_i \pi_i (1-\pi_i) \\ &= m_i \times \frac{\exp(\mathbf{x}_i^T \boldsymbol{\beta})}{1+\exp(\mathbf{x}_i^T \boldsymbol{\beta})} \times \frac{1}{1+\exp(\mathbf{x}_i^T \boldsymbol{\beta})} \end{aligned} \quad (8)$$

The maximum likelihood function for $y_1, y_2, ..., y_n$ is as follows:

$$L\ \pi, y\ = \prod_{i=1}^{n} \binom{m_i}{y_i} \pi_i^{y_i}\ 1-\pi_i\ ^{m_i - y_i} \quad (9)$$

By taking logarithm from the above equation, we will have:

$$\log L\ \pi, y\ = \sum_{i=1}^{n} \log\binom{m_i}{y_i} + \sum_{i=1}^{n} y_i\ \mathbf{x}_i^T \boldsymbol{\beta}\ - \sum_{i=1}^{n} m_i \log\ 1+\exp\ \mathbf{x}_i^T \boldsymbol{\beta} \quad (10)$$

In which $\boldsymbol{\pi} = (\pi_1, ..., \pi_n)^T$ and $\mathbf{y} = (y_1, ..., y_n)^T$. By deriving Eq. (10) with respect to β , we will have:

$$\frac{\partial \log L(\boldsymbol{\pi}, y)}{\partial \boldsymbol{\beta}} = \sum_{i=1}^{n} y_i \mathbf{X}_i^T - \sum_{i=1}^{n} m_i \frac{\exp(\mathbf{x}_i^T \boldsymbol{\beta})}{1+\exp(\mathbf{x}_i^T \boldsymbol{\beta})} \mathbf{X}_i^T$$

$$= \mathbf{X}_i^T \mathbf{y} - \sum_{i=1}^{n} m_i \pi_i \mathbf{X}_i^T = \mathbf{X}^T (\mathbf{y} - \boldsymbol{\mu}) \tag{11}$$

In which $\boldsymbol{\mu} = (\mu_1,...,\mu_n)^T = E(\mathbf{y})$. $\mathbf{X} = (\mathbf{X}_1, \mathbf{X}_2, ..., \mathbf{X}_n)^T$ is also a $n \times q$ matrix. Maximum likelihood estimator of β is obtained by $\mathbf{X}^T(\mathbf{y}-\boldsymbol{\mu}) = 0_q$. This estimation has a normal q-variate distribution (Yeh *et al.,* 2009). As a result, $\hat{\boldsymbol{\beta}} \sim N_q\left(\boldsymbol{\beta}, \left(\mathbf{X}^T\mathbf{W}\mathbf{X}\right)^{-1}\right)$ in which $\mathbf{W} = diag\left(\mu_1, \mu_2, ..., \mu_n\right)$.

III. PROPOSED MONITORING SCHEMES

A. Hotelling T^2 Control chart

If the process under study has more than one correlated quality characteristic, multivariate control charts, including T^2 ,are used to monitor these kinds of products and processes (Niaki *et al*., 2007). The control chart presented by Kang and Albin (2000) provides the foundation for this chart. According to Lowry *et al*. (1992), this chart is useful when the shift size is large. In this modified monitoring chart, the U statistic of the *j*th profile is first calculated through equations (3) and (4), then the modified T^2 statistic is obtained from the following equation:

$$T^2_{\mathbf{U}_{js}} = (\mathbf{U}_{js} - \mu_{\mathbf{U}_s}) \Sigma^{-1}_{\mathbf{U}_s} (\mathbf{U}_{js} - \mu_{\mathbf{U}_s})^T \tag{12}$$

$T^2_{\mathbf{U}_{js}}$ follows a chi-square distribution with q degrees of freedom if the normality assumption of the estimated Binomial regression parameter holds, which means $UCL = \chi^2_{q,\alpha}$ (Kang and Albin 2000). The value of the α parameter is determined so that in control, ARL= $1/\alpha$ is achieved.

B. MEWMA control chart

However, Zou *et al*. (2007) used this chart for generalized linear profiles, but this chart was also used by Soleymanian *et al*. (2013) to monitor Binomial regression profiles. In this method, first, for each profile, the following variable is calculated:

$$Q_{\mathbf{U}_{js}} = (\textstyle\sum_{\mathbf{U}_{js}}{}^{-1})^{1/2} \left(\mathbf{U}_{js} - \boldsymbol{\mu}_{\mathbf{U}_{js}}\right). \tag{13}$$

The MEWMA statistic is then calculated using the following equation:

$$V_{js} = \theta Q_{\mathbf{U}_{js}} + \left(1-\theta\right) V_{(j-1)s}. \tag{14}$$

In equation above θ is the smoothing parameter and assumed to be between 0 and 1. Also, it is assumed that $V_{0s} = 0$. The control chart signals when:

$$F_{js} = V_{js}^T V_{js} > L_1 \frac{\theta}{2-\theta} \tag{15}$$

In the above equation, L_1 will be calculated by simulation runs in order to achieve the predetermined in-control ARL. Notice that the smaller the , the rapid smaller shift detection in the parameters. (Lucas and Saccucci 1990; Prabhu and Runger 1997).

C. The Likelihood Ratio Test

In this section, a method for monitoring Binomial regression profiles in multistage processes in Phase II is provided, based on the likelihood ratio test method proposed by Niaki *et al.* (2007). Since the profile parameters in Phase II are known, the goal of the test is to assume the following:

$$\begin{aligned} &H_0 : U = U_0 \\ &H_1 : U \neq U_0 \end{aligned} \tag{16}$$

Soleymanian *et al.* (2013) used this approach to monitor logistics profiles. As a result, the hypothesis test in Equation (16) can be rewritten as follows:

$$\begin{aligned} &H_0 : U_1 = U_{01}, U_2 = U_{02}, \ldots, U_q = U_{0q} \\ &H_1 : otherwise \end{aligned} \tag{17}$$

$\mathbf{U}_k$ is the vector of the logistic regression model parameters and U_{0k} is the known values of the model parameters in an in-control state. According to the above:

$$Dev\ \ U_F\ \ = -2\left[\sum_{i=1}^{n} \log\binom{m_i}{y_i} + \sum_{i=1}^{n} y_i\ \ \mathbf{X}_i^T \mathbf{U}\ \ - \sum_{i=1}^{n} m_i \log\ \ 1 + \exp\ \ \mathbf{X}_i^T \mathbf{U}\ \ \ \ \right], \tag{18}$$

$$Dev\ \ U_F\ \ = -2\left[\sum_{i=1}^{n} \log\binom{m_i}{y_i} + \sum_{i=1}^{n} y_i\ \ \mathbf{X}_i^T \mathbf{U}_0\ \ - \sum_{i=1}^{n} m_i \log\ \ 1 + \exp\ \ \mathbf{X}_i^T \mathbf{U}_0\ \ \ \ \right], \tag{19}$$

$$\begin{aligned} &Dev_j\ \ U_R \mid U_F\ \ = Dev_j\ \ U_R\ \ - Dev_j\ \ U_F\ \ , \\ &j = 1, 2, \ldots . \end{aligned} \tag{20}$$

When the null hypothesis is accepted, and large sample sizes are used, the partial deviation follows a chi-square distribution with q degrees of freedom. As a result, if $Dev_j\ \ U_R \mid U_F\ \ \leq \chi^2_{1-\alpha,q}$ then the null hypothesis is accepted, and the jth profile is in control (Soleymanian *et al.*, 2013). The value of the α parameter is determined so that in control, ARL= $1/\alpha$ is achieved.

D. LRT/EWMA method

In this section, we combine the LRT method with the EWMA in order to improve the LRT method's ability to detect small and large shifts. As a result, the normalization of partial deviation is produced by:

$$b_j = \frac{Dev_j\left(U_R | U_F\right) - \chi^2_{0.5,q}}{\sigma_{Dev}}, \tag{21}$$

Note that the partial deviance of the hypothesis testing follows a χ^2_q distribution. Accordingly, partial deviance only assumes positive values. The statistic of the EWMA control chart is calculated as:

$$\begin{aligned} & w_j = \theta b_j + \left(1-\theta\right) w_{j-1} \\ & j = 1,2,... \end{aligned} \tag{22}$$

Where θ is the smoothing parameter and $w_0 = 0$? This chart signals when $w_j > L_2 \sqrt{\frac{\theta}{2-\theta}}$. Simulation runs will be used to calculate L_2 in order to obtain the predetermined in-control ARL.

IV. PERFORMANCE EVALUATION OF THE PROPOSED METHODS

Consider the following Binomial regression profiles for the first and second stages of a two-stage process:

$$\pi_{ij1} = \frac{\exp\left(\beta_{01} + \beta_{11} x_{ij1}\right)}{1+\exp\left(\beta_{01} + \beta_{11} x_{ij1}\right)},$$

$$\pi_{ij2} = \frac{\exp\left(\left(\beta_{02} + \varphi\pi_{ij1}\right) + \left(\beta_{12} + \varphi\pi_{ij1}\right) x_{ij2}\right)}{1+\exp\left(\left(\beta_{02} + \varphi\pi_{ij1}\right) + \left(\beta_{12} + \varphi\pi_{ij1}\right) x_{ij2}\right)},$$

where $i = 1,2,3,...,n$ and $j = 1,2,3,...$

In order to evaluate the performance of the proposed methods, the numerical example investigated by Derakhshani *et al.* (2020) is borrowed. The number and the levels of independent variables at each stage and the in-control values of the model parameters are assumed as follows:

$$\mathbf{x}_{j1} = \left(0.1, 0.2, 0.3, ..., 0.9\right), \ \mathbf{x}_{j2} = \left(1,2,3,...,9\right), \ \beta_{01} = 3, \ \beta_{11} = 4, \ \beta_{02} = 2, \ \beta_{12} = 1.$$

Also φ is assumed equal to be 0.5.

Ghahyazi *et al.* (2014) found that disregarding the cascade property impacts the overall in-control ARL and the ARL of the second stage of the process. In other words, even though the second stage was in control, changes in the parameters of the first stage profile resulted in a considerable decrease in the overall ARL as well as the in-control ARL of the second stage when this effect was ignored.

When the upper control limits for the T^2 chart for the first and second stages are 11.0174 and 15.5932, respectively, the performance of the proposed charts is evaluated in terms of out-of-control ARL to obtain an overall in control ARL of around 200. Furthermore, the LRT chart's UCL is adjusted to 12.541 and 16.756, respectively, to obtain an in-control ARL of 400 for each stage and an overall in-control ARL of around 200. *L1* of the MEWMA chart was calculated as 16.947 and 44.076 in the first and second stages, while *L2* of the LRT/EWMA chart was calculated as 10.321 and 21.473 in the first and second stages. Also, the value of θ is considered as 0.2.

The values of out-of-control ARL are calculated based on 10,000 simulation runs under different shifts in the model's parameters $\varphi = 0.5$. Tables II-VII summarize the performance of the proposed methods.

TABLE II. ARL values (shifts in β_{01} to $\beta_{01} + \gamma_1 \sigma_{\hat{\beta}_{01}}$)

Method	γ_1									
	0.1	***0.2***	***0.3***	***0.4***	***0.5***	***0.6***	***0.7***	***0.8***	***0.9***	***1***
T^2	166.01	150.27	117.92	99.11	40.12	22.24	10.7	5.11	3.12	2.43
MEWMA	**123.45**	**68.78**	**39.56**	**25.36**	15.21	11.43	8.47	6.9	5.2	4.57
LRT	153.87	79.24	59.01	33.18	**13.9**	**7.41**	**5.5**	**3.92**	2.91	1.96
LRT/EWMA	144.57	77.12	43.18	36.19	14.45	10.19	6.78	4.01	**1.99**	**1.02**

TABLE III. ARL values (shifts in β_{11} to $\beta_{11} + \gamma_2 \sigma_{\hat{\beta}_{11}}$)

Method	γ_2									
	0.1	***0.2***	***0.3***	***0.4***	***0.5***	***0.6***	***0.7***	***0.8***	***0.9***	***1***
T^2	164.6	118.9	93.21	70.1	40.29	20.8	9.98	4.2	2.23	1.04
MEWMA	**125.4**	**71.32**	**42.57**	**20.6**	**14.13**	**11.09**	**7.54**	6.42	5.68	3.48
LRT	139.11	119.18	77.42	39.45	29.13	14.11	7.98	**3.22**	**2.12**	**1.01**
LRT/EWMA	133.49	100.43	84.16	42.39	28.9	16.65	6.88	4.87	3.29	2.2

TABLE IV. ARL values (shifts in β_{02} to $\beta_{02} + \gamma_3 \sigma_{\hat{\beta}_{02}}$)

Method	γ_3									
	0.1	***0.2***	***0.3***	***0.4***	***0.5***	***0.6***	***0.7***	***0.8***	***0.9***	***1***
T^2	149.62	70.05	40.21	21.6	10.45	8.95	4.65	**2.54**	**1.54**	**1**
MEWMA	**70.98**	**35.45**	**20.14**	**10.01**	**8.45**	**5.17**	**4.32**	3.21	3.01	2.04
LRT	110.19	80.54	69.35	30.47	19.84	9.23	5.13	2.74	2.11	1.07
LRT/EWMA	98.43	79.16	50.09	25.18	12.17	7.32	4.45	2.59	2.18	1

TABLE V. ARL values (shifts in β_{12} to $\beta_{12}+\gamma_4\sigma_{\hat{\beta}_{12}}$)

Method	γ_4									
	0.1	***0.2***	***0.3***	***0.4***	***0.5***	***0.6***	***0.7***	***0.8***	***0.9***	***1***
T^2	111.49	87.78	45.11	24.81	10.8	**6.56**	4.19	2.21	**1.22**	**1.09**
MEWMA	**62.64**	**32.11**	**15.17**	**10.91**	**8.23**	7.4	4.62	3.99	2.75	1.99
LRT	144.26	52.32	33.79	28.45	14.33	9.98	4.57	2.64	1.53	1.45
LRT/EWMA	88.84	66.59	44.11	30.58	15.18	8.82	**3.97**	**2.11**	1.32	1.29

TABLE VI. ARL values (shifts in β_{01} to $\beta_{01}+\gamma_5\sigma_{\hat{\beta}_{01}}$ and β_{11} to $\beta_{11}+\gamma_5\sigma_{\hat{\beta}_{11}}$)

Method	γ_5									
	0.1	***0.2***	***0.3***	***0.4***	***0.5***	***0.6***	***0.7***	***0.8***	***0.9***	***1***
T^2	68.52	30.03	13.49	6.19	3.09	**1**	**1**	**1**	**1**	**1**
MEWMA	**32.14**	15.19	7.14	3.25	2.41	1.13	**1**	**1**	**1**	**1**
LRT	41.23	**12.19**	**5.87**	**2.99**	**1.82**	**1**	**1**	**1**	**1**	**1**
LRT/EWMA	39.18	14.23	7.89	4.33	2.01	1.18	**1**	**1**	**1**	**1**

TABLE VII. ARL values (shifts β_{02} to $\beta_{02}+\gamma_6\sigma_{\hat{\beta}_{02}}$ and β_{12} to $\beta_{12}+\gamma_6\sigma_{\hat{\beta}_{12}}$)

Method	γ_6									
	0.1	***0.2***	***0.3***	***0.4***	***0.5***	***0.6***	***0.7***	***0.8***	***0.9***	***1***
T^2	80.54	25.49	9.11	3.22	2.17	1.09	**1**	**1**	**1**	**1**
MEWMA	**28.29**	11.13	6.32	4.02	2.23	1.95	1.18	**1**	**1**	**1**
LRT	39.93	**10.17**	**5.32**	3.88	**1.23**	**1**	**1**	**1**	**1**	**1**
LRT/EWMA	40.11	20.54	10.09	**3.23**	**1.97**	**1.18**	**1.02**	**1**	**1**	**1**

Table II shows that the LRT performs better than the other methods for moderate to large shifts, and the MEWMA performs the best for small to moderate shifts in β_{01}. Also, the LRT/EWMA approach performs better than the T^2 control chart in detecting different shifts in β_{01}. Table III shows that the best scheme between proposed methods for detecting moderate to large shifts in β_{11} is the LRT and for small to moderate shifts is the MEWMA. Also, the

LRT/EWMA method outperforms the T^2 control chart in detecting different shifts in the slope of the first stage of the process. Table IV shows that the T^2 control chart outperforms the other methods for moderate to large shifts, and the MEWMA perform the best for small to moderate shifts in the intercept of the second stage of the process. Also, the LRT/EWMA approach performs better than the LRT method in detecting different shifts in β_{02}. Table V shows that the best scheme between proposed methods for detecting moderate to large shifts in β_{12} is the T^2 control chart and for small to moderate shifts is the MEWMA. Also, the LRT/EWMA method outperforms the T^2 control chart in detecting different shifts in the slope of the second stage of the process.

Tables VI and VII show that for the simultaneous shift in $\boldsymbol{\beta}_1$ and $\boldsymbol{\beta}_2$, the LRT method performs better. After the LRT method, the MEWMA control chart performs better than the other control scheme for detecting a simultaneous shift in the profile parameters. As a result, all the proposed methods perform well in detecting shifts in the Binomial regression profile parameters in the multistage process. The LRT method outperforms the other approaches for moderate to large shifts, and the MEWMA control chart outperforms the other schemes for small to moderate shifts in the Binomial regression profile parameters in the multistage process. Also, the LRT method detects simultaneous shifts in the profile parameters of the multistage process faster than the other proposed methods.

V. NUMERICAL EXAMPLE

The application of the provided approaches is demonstrated in this section using a numerical example in a two-stage process. The Binomial regression profile determines the quality of each stage of the process under investigation, and the process has a cascade property. The profiles in each stage of the process are described as follows, according to the assumptions presented in section 4:

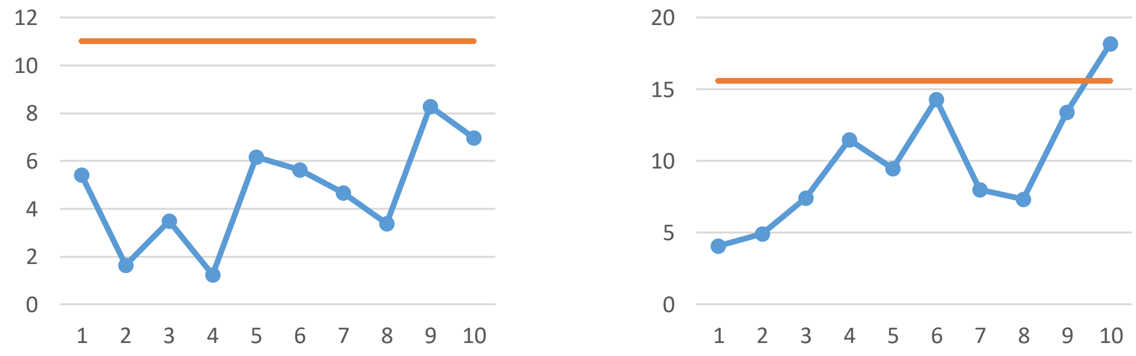
$$\pi_{ij1} = \frac{\exp(3+4x_{ij1})}{1+\exp(3+4x_{ij1})},\ \pi_{ij2} = \frac{\exp\left((2+\varphi\pi_{ij1}) + (1+\varphi\pi_{ij1})x_{ij2}\right)}{1+\exp\left((2+\varphi\pi_{ij1}) + (1+\varphi\pi_{ij1})x_{ij2}\right)},\ i=1,2,3,\dots,9.$$

The values of the independent variables are also defined as:

$$\mathbf{x}_{j1} = (0.1, 0.2, 0.3, \dots, 0.9)\ , \mathbf{x}_{ij2} = (1, 2, 3, \dots, 9)\ .$$

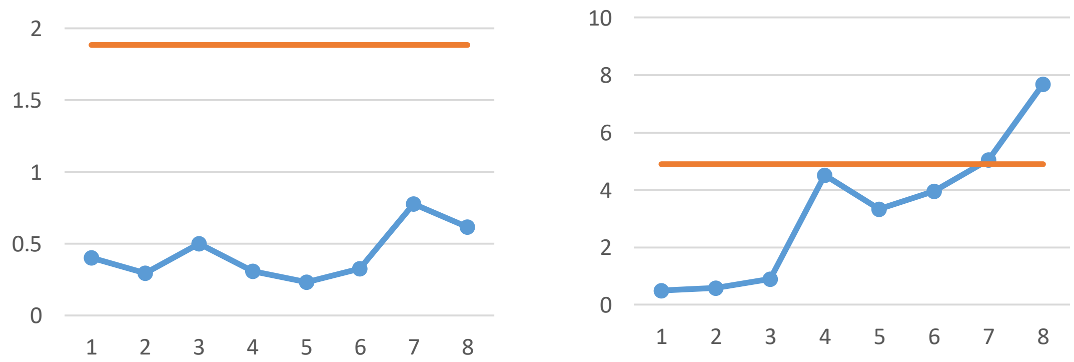
This two-stage process in Phase II is anticipated to be monitored by a shift of 0.3 in the profile slope of the second stage. As a result, data for the profiles of the second stage of the process with an intercept of 4 and a slope of $1+0.3\sigma_{\hat{\beta}_{12}}$ will be generated from the out-of-control process, and data for the first stage profiles of the process with an intercept of 3 and a slope of 4 will be generated. As previously stated, the suggested approaches' upper control limit is calculated by running 10,000 simulations, resulting in an overall in-control ARL of approximately 200.

The values of statistics related to each chart are then determined using the provided equations in section 3. The calculated statistics are plotted in the proposed charts until the charts generate an out-of-control signal. Figures 1 to 4 depict the outcomes.



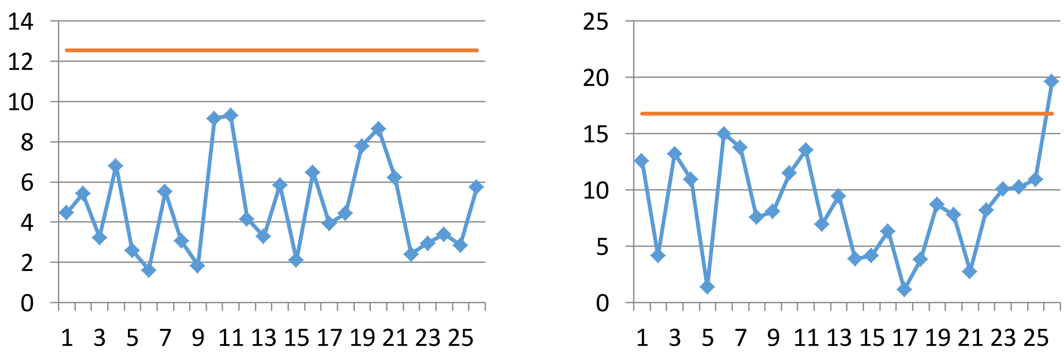
(a) First Stage (b) Second stage

Fig. 1. T^2 control chart (shift in the slope of the second stage from the 1 to $1+0.3\sigma_{\beta_{12}}$)



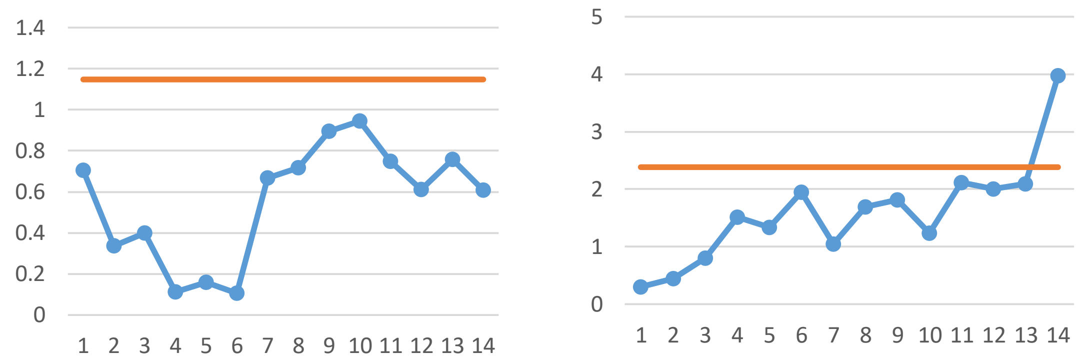
(a) First Stage (b) Second Stage

Fig. 2. MEWMA control chart (shift in the slope of the second stage from the 1 to $1+0.3\sigma_{\beta_{12}}$)



(a) First stage (b) Second Stage

Fig. 3. LRT control chart (shift in the slope of the second stage from the 1 to $1+0.3\sigma_{\beta_{12}}$)



(a) First stage (b) Second Stage

Fig. 4. LRT/EWMA control chart (shift in the slope of the second stage from the 1 to $1+0.3\sigma_{\beta_{12}}$)

The T^2 control chart signals at sample 10 according to Fig. (1), while the MEWMA chart identifies the shift at sample 8 according to Fig. (2). This suggests that the MEWMA outperforms the T^2 control chart in terms of detecting changes faster. The LRT approach, as shown in Fig. (3), detects the shift at sample 26. According to Fig. (4), the LRT/EWMA method detects the shift at sample 14, indicating that the LRT/EWMA method outperforms the LRT method in detecting profile parameter shifts. In addition, the T^2 control chart outperforms the LRT/EWMA approach. As a result, MEWMA detects the change in this example faster than the other proposed approaches.

VI. REAL CASE STUDY

The case study of this research is about the production process of car rear windows. After printing the heating lines on the rear window of the car, the quality control department, during a two-stage process, controls the quality of the produced windows. In the first stage of this process, the electrical power of the printed heating lines is monitored as a desired quality characteristic. The desired quality characteristic depends on the thickness of the heating lines printed on the window. If the measured electrical power is less than 0.2, the part is accepted and enters the second stage of the process; otherwise, the part is considered defective and is removed from the process as waste. Since the quality characteristic to be monitored is related to the thickness of the print, and its acceptance criterion is accepted as acceptance/refusal, in the first stage of the process, we have a Binomial regression profile quality characteristic.

In the second stage of the process, the electrical resistance of the heating lines is monitored as a quality characteristic of the second stage. The electrical resistance depends on the color density that prints the heating lines on the windshield. If the resistance is in the acceptable range between 0.75 to 0.9, the glass produced will be accepted; otherwise, the part is considered defective and is considered as process waste. Since the acceptance or refusal of the window is related to the density of the color, the quality characteristic of the second stage is also of the profile type, and since the acceptance/ refusal of the part is expressed discretely, it can be considered as a Binominal regression profile. On the other hand, the electrical resistance of the heating lines and the electrical power of the heating lines in each of the process stages are also related to each other, which indicates the cascading property between the process stages under study.

The desired data were collected from the process in a specific period of time. The data contains 20 profiles, in each of which there are four observations. The levels for print thickness are considered equal to 30, 31, 32, and 33 and for density equal to 750, 760, 770, and 780. The acceptance or refusal of parts is counted in terms of power and electrical resistance for each of these levels. Each profile's parameter values are estimated using the parameter estimation approach outlined in section 3. As a result, the in-control values of the model parameters are as follows:

$\beta_{01} = -643.507$, $\beta_{11} = 22.218$, $\beta_{02} = -471.798$, $\beta_{12} = 0.617$.

First, the mean of response variables in each observation was obtained using the following equation to calculate the auto-correlation coefficient between different stages of the process:

$\pi_{is} = \dfrac{\sum_{j=1}^{20} y_{ijs}}{20}$, which i is the levels of the independent variable, s denotes the process stages, and 20 is the number of profiles in Phase I. Then the correlation coefficient between π_{k1} and π_{k2} is considered as the auto-correlation coefficient of the process stages, which has been calculated as 0.3.

According to Equations (1) and (2):

$$\pi_{ij1} = \frac{\exp\left(-643.507 + 22.218 x_{ij1}\right)}{1 + \exp\left(-643.507 + 22.218 x_{ij1}\right)},$$

$$\pi_{ij2} = \frac{\exp\left(\left(-471.798 + 0.3\pi_{ij1}\right) + \left(0.617 + 0.3\pi_{ij1}\right) x_{ij2}\right)}{1 + \exp\left(\left(-471.798 + 0.3\pi_{ij1}\right) + \left(0.617 + 0.3\pi_{ij1}\right) x_{ij2}\right)},$$

$$i = 1, 2, 3, 4,$$
$$j = 1, 2, 3, \ldots, 20.$$

The covariance matrix of the profile parameters in the first stage is calculated by $\left(\mathbf{X}_1^T \mathbf{W}_1 \mathbf{X}_1\right)^{-1}$ and in the second stage is calculated by $\left(\mathbf{X}_2^T \mathbf{W}_2 \mathbf{X}_2\right)^{-1}$ and is obtained as:

$$\Sigma_{1,1} = \begin{pmatrix} 198.7 & -6.3 \\ -6.3 & 0.2 \end{pmatrix} \text{ and } \Sigma_{2,2} = \begin{pmatrix} 1.17 \times 10^{-3} & -1.53 \\ -1.53 & 0.002 \end{pmatrix} \text{ respectively.}$$

The covariance matrix between the profile parameters in the first and second stages is also estimated by 10,000 simulation runs and estimated as:

$$\Sigma_{2,2} = \begin{pmatrix} 9.5 \times 10^{-3} & -8.4 \times 10^{-6} \\ -3.6 \times 10^{-6} & 3.24 \times 10^{-4} \end{pmatrix}.$$

The upper control limit for the T^2 control chart at the first stage is calculated as 13.2432, and second stage is calculated as 18.1411. The UCL of the MEWMA method at the first and second stage are calculated 2.8971 and 4.4714. Also, for the LRT method, the UCL of the first stage is equal to 15.6578 and for the second stage is equal to 17.0981. Finally, the upper control limit for the LRT/EWMA method at the first and second stages are equal to 2.2374 and

3.9726, respectively. Based on Equations (12), (15), (20), and (22), the statistic of each method is calculated and plotted on their relative chart. In Phase I, the process is considered stable because all of the plotted statistics are within their respective control limits. So, for monitoring this real case study, a size shift is imposed β_{12} . As a result, at the second stage of the process, the slope parameter has shifted from 0.617 to 1.947. Figures 5-8 show how the methods performed as a result of this change.

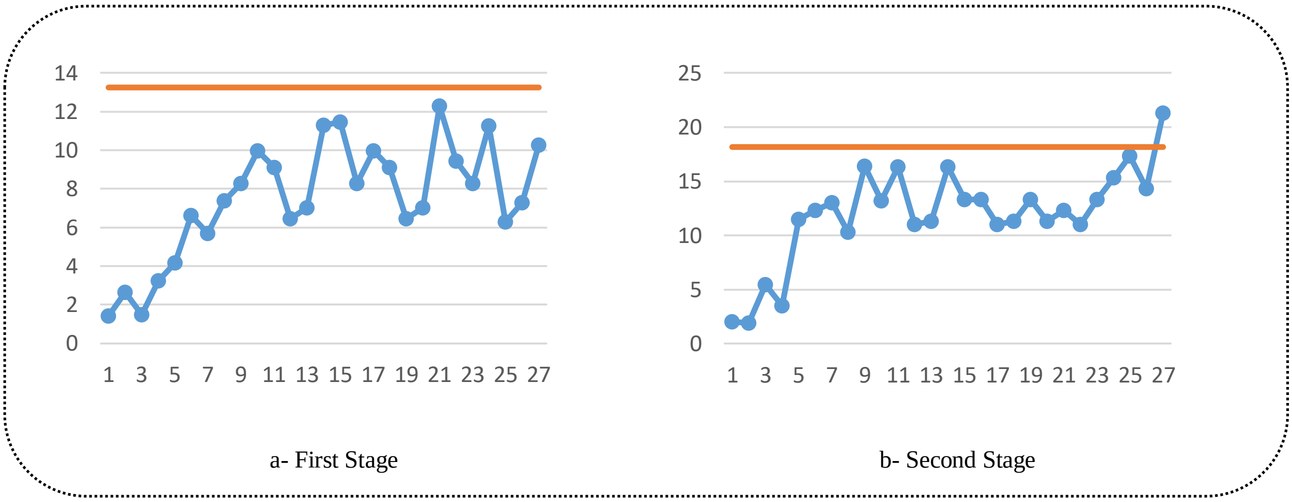


Fig. 5. T^2 control chart based on the case study

The T^2 control chart signals at sample number 27, implying that this method detects the out-of-control state of the process after seven samples when the shift occurs, whereas the MEWMA chart generates a signal at sample 24, implying that this method detects the out-of-control state of the process after four samples when the shift occurs according to Fig. (5) and Fig. (6). As a result, the MEWMA approach detects the shift faster and performs better than the T^2 control chart. According to Fig. (7), the LRT signals at sample number 32 indicate that this control chart detects the out-of-control state of the process after 12 samples when the shift happens. It means the LRT approach detects the shift slower than the T^2 control chart and the MEWMA. LRT/EWMA control chart generates a signal at sample 29, i.e., nine samples after the shift according to Fig. (8), which means the LRT/EWMA performs better than the LRT method. Generally, the MEWMA chart performs the best in this case study. After the MEWMA, the T^2 control chart performs better than the two other proposed methods (LRT and LRT/EWMA). Finally, the LRT/EWMA outperforms the LRT method in detecting a small shift in this real example.

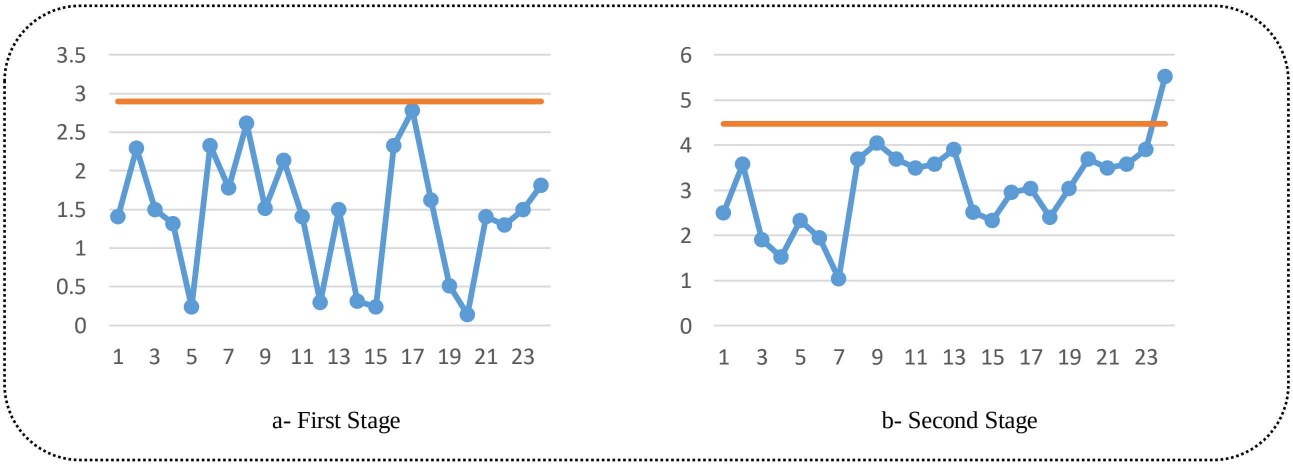
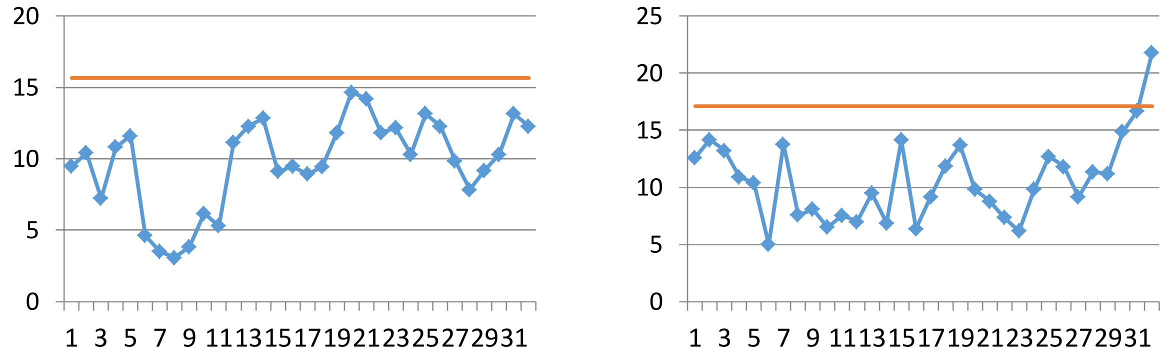


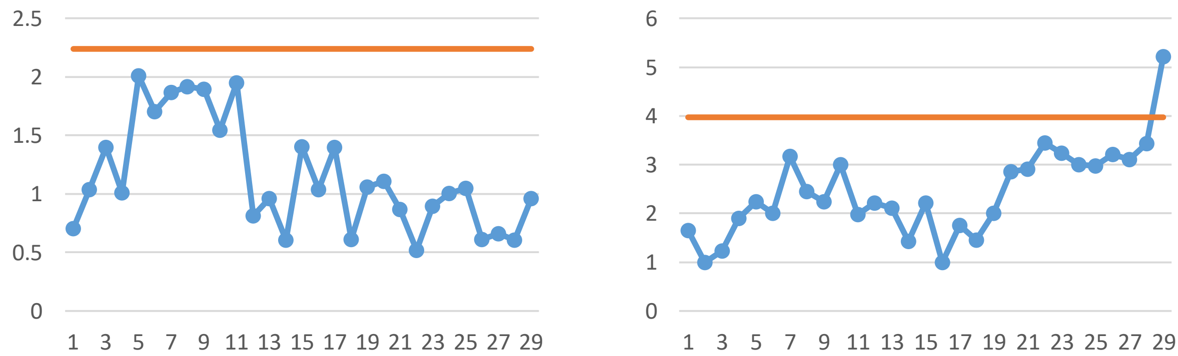
Fig. 6. MEWMA control chart based on the case study



a- First Stage

b- Second Stage

Fig. 7. LRT method based on the case study



a- First Stage

b-Second Stage

Fig. 8. LRT/EWMA method based on the case study

VII. CONCLUSION REMARKS

In order to monitor the Binomial regression profile as a quality characteristic at each stage of a multistage process in Phase II, four different monitoring approaches T^2, MEWMA, LRT, and LRT/EWMA, respectively, were proposed. ARL criterion was used to evaluate the methodologies. The results of simulations show that all of the strategies are effective at a reasonable level.

Evidently, The MEWMA control chart does better than the alternatives when it comes to detecting trivial changes in the parameters of the Binomial regression profile. This is when in finding out the large shifts, the T^2 control chart comes in first. On the other hand, the LRT approach was found to be the most prominent in detecting coincidental shifts.

In multistage processes, taking into account auto-correlation within or between profiles may be an intriguing recommendation for further research.

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