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A New Uncertain Modeling of Production Project Time and Cost Based on Atanassov Fuzzy Sets

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Abstract- *Uncertainty plays a major role in any project evaluation and management process. One of the trickiest parts of any production project work is its cost and time forecasting. Since in the initial phases of production projects uncertainty is at its highest level, a reliable method of project scheduling and cash flow generation is vital to help the managers reach successful implementation of the project. In the recent years, some scholars have tried to address uncertainty of projects in time and cost by using basic uncertainty modeling tools such as fuzzy sets theory. In this paper, a new approach is introduced to model project cash flow under uncertain environments using Atanassov fuzzy sets or intuitionistic fuzzy sets (IFSs). The IFSs are presented to calculate project scheduling and cash flow generation. This modern approach enhances the ability of managers to use their intuition and lack of knowledge in their decision-makings. Moreover, unlike the recent studies in this area, this model uses a more sophisticated tool of uncertain modeling which is highly practical in real production project environments. Furthermore, a new effective IFS-ranking method is introduced. The methodology is exemplified by estimating the working capital requirements in an activity network. The proposed model could be useful for both project proposal evaluation during feasibility studies and for performing earned value analysis for project monitoring and control.*

Keywords: *Production projects, Atanassov fuzzy sets, Intuitionistic fuzzy project scheduling, Intuitionistic fuzzy cost flow.*

I. INTRODUCTION

Project managers rely on accurate prediction of cash flow time series over the life cycle of the project to be in a position to predict potential problems and to create appropriate strategies to minimize the negative impacts of such on successful project implementation (Hwee & Tong, 2002). Accurately forecasting production and estimating its required reserves have been more urgent and important than ever before (Duong, 2011). Moreover, in production projects the products' characteristics create production constraints in different aspects like scheduling and cash flows. Future position of production firms highly depends on research and development (R&D) activities. Their survival is imperiled by ever-growing complexity of technologies. The importance of R&D in highly industrialized economies is undeniable (Santamaría et al., 2010). To put differently, innovation is considered by many firms as a central requirement to their survival programs. New product development (NPD) is a highly effective way to provide production firms with a leading edge over rivals and provide new opportunities (Lawson et al., 2006). To consider R&D projects, production organizations have to identify new projects to make their costs lower, bring new products to their markets, and make their quality higher. Properly addressing risk and uncertainty in R&D environment is essential since knowledge of the proposed projects are often vague and uncertain. This uncertainty puts firm's plans and strategies in danger. Future events and opportunities have undeniable impact on R&D decisions. Consequently, most of the information used in project decision-makings is at the best uncertain and at the worst condition very unreliable (Bhattacharyya, 2015). On the other hand, the job-shops or smaller firms simply cannot match the dominance or resources that the larger firms

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enjoy, allowing them to be inflexible along their supply chains (Chen et al., 2014). Therefore, they cannot afford to be stuck in unexpected situations.

Project cash flow consists of complete record of all cash related events like expenses, insufficiency, loans and borrowings. Cash flow fluctuations increase the level of expected external financing costs and as a result, project faces high monetary costs. Cash flow forecasting is almost the main objective of project cash flow related researches. Project cash flow forecasting should be based on a method that is both effective and reliable. Consequently, an effective and reliable approach for forecasting cash flow is necessary (Jiang et al., 2011).

In the conventional project management approach, the stability of the project plan is regarded as a critical success factor. However, the ever-increasing level of complexity and uncertainty in today's business environment has made it almost impossible to ignore the great need to a high level of adaptability to expected and unexpected changes (Caron & Comandulli, 2014). One of the main sources of complexity in any project related analysis and planning process is uncertainty. Uncertainty can be caused by different sources like information that due to nature of the project is unquantifiable, incomplete or even non-obtainable (Mousavi et al., 2013).

The main purpose of this paper is to propose a model of project cost and time planning that addresses project uncertainty by using an uncertainty-modeling tool that is capable of expressing knowledge as well as lack of knowledge. In other words, this paper proposes a model of project scheduling and a model of cost forecasting that is based on Atanassov fuzzy sets or intuitionistic fuzzy sets (IFSs). This approach enables the decision maker (DM) to express membership, non-membership and hesitation degrees when expressing data that is based on judgment and due to lack of enough historical data cannot be expressed by statistical tools.

Furthermore, the main characteristics of this IFS based project cost and time planning that distinguish it from existing studies in the literature are as follows: (1) introducing a new approach in modeling project time and cost uncertainty based on the IFS, (2) proposing a new project scheduling method based on the concept of uncertain critical path method (CPM) and Atanassov fuzzy sets, (3) modeling a novel approach of IFS ranking and ordering, and (4) proposing a new model of cash forecasting that uses IFS as an uncertainty modeling tool.

This paper is organized as follows: Section II reviews the existing literature on the project cost and time analysis. Some preliminary introductions to IFSs are provided in section III. Section IV presents a novel ranking method for IFS and introduces IF-project scheduling. In section V, the project cost and cash flow analysis model is proposed. Section VI includes a practical example in construction projects and its corresponding results and finally, Section VII provides the conclusion remarks of this paper.

II. Literature Review

Using S-curves to predict project cash flow is very popular among project managers since it is the simplest way of projection (Touran et al., 2004). Project costs and durations are two primary quantities of total relevant quantity that form the S-curve (Cioffi, 2005). The term "S-Curve" is used due to the shape of cost-time curve that is quite like letter "S" (Cooke & Jepson, 1979). Accuracy of any S-curve based method is determined by the effectiveness of assumptions of conditions representing the real-world situation of the project (Boussabaine & Kaka, 1998).

Projects involvement in uncertainties and complexities is notable and managers have to make decisions under uncertain environments (Mohagheghi et al., 2015). The complex nature of production project environment imposes a high degree of uncertainty in different aspects of project management including project cash flow analysis and project scheduling. In project environments, facing situations with few data, extreme values, emerging changes, uncertainties and uniqueness is very common. In fact, the aforementioned conditions are a part of project nature (San Cristóbal et al., 2015).

As a matter of fact, cash is the main bloodline of any organization. It is possible for a firm to survive without profit for some time, but without cash no firm can operate properly and all firms, no matter in which sector they are, risk facing bankruptcy after a while (Tangsucheeva & Prabhu, 2014). Several sources cause the financial risks, which highlights the need for a reliable cash flow assessing method that overcomes the existing difficulties by providing a comprehensive cash flow forecasting (Barbosa & Pimentel, 2001). Fuzzy sets theory has been widely applied as a tool to model uncertainty in different problems. Due to uncertain nature of projects and vague and unknown information of projects, fuzzy sets theory has been applied as an appropriate tool to handle project uncertainty. (Kumar et al., 2000) introduced a model of working capital requirements prediction under fuzzy uncertainty. (Lam et al., 2001) proposed a fuzzy optimization model that finds the optimum cash flow using the minimum amount of resources. (Hsu, 2003) introduced statistical models based on S-curve to predict or evaluate construction project cash flow. (Blyth & Kaka, 2006) proposed a multiple linear regression model to predict S-curves of individual projects. They focused on standardizing the activities, and predicting the duration, cost and end dates of the activities. (Gormley & Meade, 2007) employed time series to introduce a model to predict expectations of further cash flows. (Jarrah et al., 2007) introduced a model based on a fourth degree polynomial regression analysis approach in order to forecast and predict cash flows and trends. (Khosrowshahi & Kaka, 2007) introduced a model based on the concept of decision support systems to manage cash flows of construction projects. (Cheng & Roy, 2009) introduced an evolutionary fuzzy decision model that applied S-curves to foresee project cash flow.(Maravas & Pantouvakis, 2012) developed a cash flow assessment model under fuzzy environment. (Rostami et al., 2013) introduced a fuzzy statistical expert system for cash flow analysis, which was designed to handle the uncertain environment of projects. (Chen et al., 2013) introduced a multivariate model in order to evaluate how key variables in project initiation and planning phases. (Ungureanu & Vernic, 2014) introduced a fuzzy cash flow model with applications in risk mitigation. (Cheng et al., 2015) proposed a model of cash flow prediction that was based on a hybrid artificial intelligence model. Their model was solely depending on data gathered from a specific project. In other words, the model was unable to address different sort of projects.

It is concluded from the above that several fuzzy techniques such as fuzzy averaging, fuzzy composition matrices, fuzzy reasoning, fuzzy optimization, fuzzy multi objective decision models, and neuro-fuzzy inference have been utilized to generate project cash flows. Despite this effort, all the mentioned studies were based on the concept of classical fuzzy sets theory, which has its own shortcomings. Although it is mainly accepted that uncertainty management is a vital requirement for effective project management, it can be discussed that it should get more sophisticated before obtaining practical results (Atkinson et al., 2006). One area of classical fuzzy sets theory inadequacies is where a DM is expected to give an exact opinion in a number in interval [0, 1]. As mentioned earlier in initial project phases the level of uncertainty is so high that using a crisp value to express the membership degree decreases the effectiveness of uncertainty modeling. Classical fuzzy sets are unable to express the expert's degree of hesitation. Moreover, they only consider the membership degree and do not address non-membership degree. In other words, in today's competitive business environment uncertainty needs to be fully addressed in any decision-making process. In production projects that often deal with new technological advancements and uncertain markets, uncertainty requires a sophisticated addressing tool. Since classical fuzzy sets lack the ability to fully express uncertain project elements, IFSs are applied in this paper. Using IFS gives the model the following advantages over the classical fuzzy based studies:

- IFS presents membership degree, non-membership degree and hesitancy degree by using three grades of membership function, respectively. Whereas the triangular and the trapezoidal fuzzy numbers lack this ability and each only can denote one crisp grade of membership in the unit interval [0,1] (Szmidt et al., 2014).
- Despite the similarities between IFS and interval-valued fuzzy set, a number of references in the literature on IFSs (around 1000 papers) suggest that many researchers find the advantages of IFSs over the equivalent interval-valued fuzzy sets (Atanassov, 2008) ; (Zhu & Liao, 2014).
- IFS unlike all of triangular fuzzy numbers, trapezoidal fuzzy numbers and interval-valued fuzzy numbers can reflect the "disagreement" of the DM in addition to the fuzziness of "agreement" (Zhu & Liao, 2014).

These sets despite their novelty have been successfully used in a wide range of real world problems. For instance, they were employed to select renewable energy technologies for electricity generation in Turkey (Boran et al., 2012). They were used in construction site layout planning (Ning et al., 2011). They were applied in supplier selection problems (Chai et al., 2012). They were employed to address software selection problems (Wang, 2012). Medical diagnosis were carried out by these sets (Szmidt & Kacprzyk, 2001) ; (Neog & Sut, 2011). Eventually, in project management one of its applications were presented by (Gerogiannis et al., 2011). They used IFSs in project evaluation and portfolio management.

III. PRELIMINARY KNOWLEDGE OF TRIANGULAR INTUITIONISTIC FUZZY SETS

In the following, in order to illustrate the basic concepts of the introduced model some basic concepts related to intuitionistic fuzzy sets are introduced.

(Zadeh, 1965) proposed fuzzy sets to illustrate imprecise or vague information. Through the years fuzzy sets theory has proven itself as a useful tool to handle uncertain situations by denoting a degree to which a certain object belongs to a set. In real situations, an object can belong to a set to a certain degree, but there can be hesitations. In other words, when there is hesitation or uncertainty about the membership degree, fuzzy set theory has no way to imply that hesitation in the membership degrees (Zimmermann, 2001). IFS introduced by A (tanassov, 1983) is a possible solution to overcome this problem. IFS uses a degree of truth membership function $u_A(x)$ and one of falsity membership function $\nu_A(X)$ to represent lower bound $(u_A(x))$and upper bound $\left(1-\nu_A(X)\right)$such that $u_A(x)+\nu_A(X)\leq 1$. By complementing the membership degree with a non-membership degree that expresses how the element is not in the IFS, the interval $[u_A(x), 1-\nu_A(X)]$ can develop the fuzzy set of membership function. The hesitation or uncertainty of *x* can be measured for each *x* by the size of the interval $\pi_A(x)=1-\nu_A(X)-u_A(x)$. If the $\pi_A(x)$ is small, it represents more certainty about *x*. As $\pi_A(x)$gets greater, it denotes more uncertainly about *x*. obviously, if $u_A(x)=1-\nu_A(X)$ for all elements of the universe, the IFS becomes traditional fuzzy set (Shu et al., 2006).

From definition of triangular intuitionistic fuzzy set, four arithmetic operations for triangle vague sets are defined as follows (Shu et al., 2006). *A* and *B* are two IFSs, as depicted in Fig. 1 (Lee, 1998). If two intuitionistic fuzzy sets $u_A(x)\neq u_B(x)$, and$\nu_A(x)\neq \nu_B(x)$, then the arithmetic operations are defined as:

$$A=\langle[(a_1^{'},b_1,c_1^{'});u_A],[(a_1,b_1,c_1);\nu_A]\rangle \tag{1}$$
$$B=\langle[(a_2^{'},b_2,c_2^{'});u_B],[(a_2,b_2,c_2);\nu_B]\rangle \tag{2}$$
$$\begin{aligned}A+B&=\langle[(a_1^{'},b_1,c_1^{'});u_A],[(a_1,b_1,c_1);\nu_A]\rangle+\langle[(a_2^{'},b_2,c_2^{'});u_B],[(a_2,b_2,c_2);\nu_B]\rangle\\&=\langle[(a_1^{'}+a_2^{'},b_1+b_2,c_1^{'}+c_2^{'});\min(\mu_A,\mu_B)],[(a_1+a_2,b_1+b_2,c_1\\&+c_2);\min(\nu_A,\nu_B)]\rangle\end{aligned} \tag{3}$$
$$\begin{aligned}A-B&=\langle[(a_1^{'},b_1,c_1^{'});\mu_A],[(a_1,b_1,c_1);\mu_B]\rangle-\langle[(a_2^{'},b_2,c_2^{'});\nu_A],[(a_2,b_2,c_2);\nu_B]\rangle\\&=\langle[(a_1^{'}+c_2^{'},b_1+b_2,c_1^{'}+a_2^{'});\min(\mu_A,\mu_B)],[(a_1+c_2,b_1+b_2,c_1\\&+a_2);\min(\nu_A,\nu_B)]\rangle\end{aligned} \tag{4}$$
$$\begin{aligned}A\times B&=\langle[(a_1^{'},b_1,c_1^{'});\mu_A],[(a_1,b_1,c_1);\mu_B]\rangle\times\langle[(a_2^{'},b_2,c_2^{'});\nu_A],[(a_2,b_2,c_2);\nu_B]\rangle\\&=\langle[(a_1^{'}a_2^{'},b_1b_2,c_1^{'}c_2^{'});\min(\nu_A,\nu_B)],[(a_1a_2,b_1b_2,c_1c_2);\min(\nu_A,\nu_B)]\rangle\end{aligned} \tag{5}$$
$$\begin{aligned}A\div B&=\langle[(a_1^{'},b_1,c_1^{'});\mu_A],[(a_1,b_1,c_1);\mu_B]\rangle\div\langle[(a_2^{'},b_2,c_2^{'});\nu_A],[(a_2,b_2,c_2);\nu_B]\rangle\\&=\langle[(a_1^{'}\div c_2^{'},b_1\div b_2,c_1^{'}\div a_2^{'});\min(u_A,u_B)],[(a_1\div c_2,b_1\div b_2,c_1\\&\div a_2);\min(\nu_A,\nu_B)]\rangle\end{aligned} \tag{6}$$

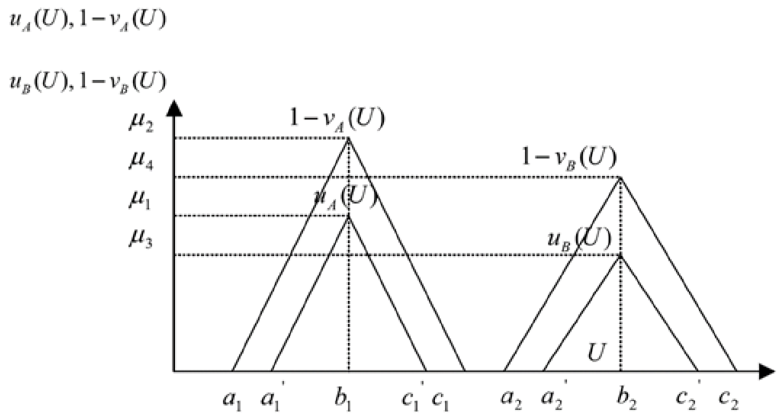

Fig. 1. Triangular IFS A and B

IV. THE INTRODUCED MODEL OF UNCERTAIN PRODUCTION PROJECT SCHEDULING

A. Proposed ranking method

In this paper, a distance-based similarity measure between two triangular intuitionistic fuzzy numbers is proposed, that is based on the concept of the model introduced by (Deng, 2014) for measuring the closeness between two IFNs. The step by step algorithm is introduced as follows:

1. Determine the triangular intuitionistic fuzzy positive ideal solution as $\tilde{x}_{max} = \left(\left[\tilde{x}_{max,1}, \tilde{x}_{max,2}, \tilde{x}_{max,3}\right]; u_{\tilde{x}_{max}}, v_{\tilde{x}_{max}}\right)$ and the negative ideal solution as $\tilde{x}_{min} = \left(\left[\tilde{x}_{min,1}, \tilde{x}_{min,2}, \tilde{x}_{min,3}\right]; u_{\tilde{x}_{min}}, v_{\tilde{x}_{min}}\right)$.
2. Determine the degree of similarity between each triangular intuitionistic fuzzy number $\tilde{A}_i = \left(\left[a_{i,1}, a_{i,2}, a_{i,3}\right]; u_{a_i}, v_{a_i}\right)$ $(i = 1,2,\dots,n)$and the positive triangular intuitionistic fuzzy ideal solution $(\tilde{x}_{max})$ by using the following (Liang et al., 2014):

$$d_i^+(\tilde{A}_i, \tilde{x}_{max}) = \frac{1}{6}\Big(\Big|(1 + u_{\tilde{A}_i} - v_{\tilde{A}_i})a_1 - \big((1 + u_{\tilde{x}_{max}} - v_{\tilde{x}_{max}})\tilde{x}_{max,1}\big)\Big| + \Big|(1 + u_{\tilde{A}_i} - v_{\tilde{A}_i})a_2 - \big((1 + u_{\tilde{x}_{max}} - v_{\tilde{x}_{max}})\tilde{x}_{max,2}\big)\Big| + \Big|(1 + u_{\tilde{A}_i} - v_{\tilde{A}_i})a_3 - \big((1 + u_{\tilde{x}_{max}} - v_{\tilde{x}_{max}})\tilde{x}_{max,3}\big)\Big|\Big) \tag{7}$$

3. Determine the degree of similarity between each triangular intuitionistic fuzzy number $\tilde{A}_i = \left(\left[a_{i,1}, a_{i,2}, a_{i,3}\right]; u_{a_i}, v_{a_i}\right)$ $(i = 1,2,\dots,n)$and the negative triangular intuitionistic fuzzy ideal solution $(\tilde{x}_{min})$ by using the following:

$$d_i^+(\tilde{A}_i, \tilde{X}_{min}) = d_i^+(\tilde{A}_i, \tilde{x}_{min}) = \frac{1}{6}\Big(\Big|(1 + u_{\tilde{A}_i} - v_{\tilde{A}_i})a_1 - \big((1 + u_{\tilde{x}_{min}} - v_{\tilde{x}_{min}})\tilde{x}_{min,1}\big)\Big| + \Big|(1 + u_{\tilde{A}_i} - v_{\tilde{A}_i})a_2 - \big((1 + u_{\tilde{x}_{min}} - v_{\tilde{x}_{min}})\tilde{x}_{min,2}\big)\Big| + \Big|(1 + u_{\tilde{A}_i} - v_{\tilde{A}_i})a_3 - \big((1 + u_{\tilde{x}_{min}} - v_{\tilde{x}_{min}})\tilde{x}_{min,3}\big)\Big|\Big) \tag{8}$$

4. Calculate the overall performance index (P_i) of each triangular intuitionistic fuzzy number $\tilde{A}_i$ $(i = 1,2,\dots,n)$ by using the following:

$$P_i = \frac{d_i^-}{d_i^+ + d_i^-}, i = 1,2,\dots,n \tag{9}$$

Step 5. Rank the triangular intuitionistic fuzzy number $\tilde{A}_i$ $(i = 1,2,\dots,n)$ in descending order of P_i.

This method provides the DM with more control over the process. Selecting the triangular intuitionistic fuzzy positive ideal solution as $\tilde{x}_{max}$ and the negative ideal solution as $\tilde{x}_{min}$ is a step that can be done based on the nature of the project. To enhance the ability of model in ranking numbers with very different values a normalization step could also be added to the process. This approach, in addition to its simplicity, is effective which makes the process more practical for project environments.

B. Proposed project scheduling model

Applying project scheduling in modeling real-world projects requires a method that can model uncertainty and hesitations. Therefore, in this section an IF-project scheduling model is presented. IF-project scheduling can model activities with uncertain durations by considering a degree of membership, non-membership and hesitation. By calculating early start and early finish time, forward pass can be obtained (Chanas & Kamburowski, 1981); (McCahon & Lee,1988); (Prade, 1979).

$$\tilde{E}S_{start} = (0) \tag{10}$$
$$\tilde{E}S = m\tilde{a}x\left(\tilde{E}F_p\right)_{p\in P} = \langle\left[\left(ef_{p_1}', ef_{p_2}, ef_{p_3}'\right); u_{ef}\right], \left[\left(ef_{p_1}, ef_{p_2}, ef_{p_3}\right); \nu_{ef}\right]\rangle \tag{11}$$
$$\tilde{d} = \langle[(d_1', d_2, d_3'); u_A], [(d_1, d_2, d_3); \nu_d]\rangle \tag{12}$$
$$\tilde{E}F = \tilde{E}S + \tilde{d} = \langle\left[\left(ef_{p_1}', ef_{p_2}, ef_{p_3}'\right); u_{ef}\right], \left[\left(ef_{p_1}, ef_{p_2}, ef_{p_3}\right); \nu_{ef}\right]\rangle + \langle[(d_1', d_2, d_3'); u_A], [(d_1, d_2, d_3); \nu_d]\rangle = \langle\left[\left(ef_{p_1}' + d_1', ef_{p_2} + d_2, ef_{p_3}' + d_3'\right); min\left(\mu_{ef}, \mu_d\right)\right], \left[\left(ef_{p_1} + d_1, ef_{p_2} + d_2, ef_{p_3} + d_3\right); min\left(\nu_{ef}, \nu_d\right)\right]\rangle. \tag{13}$$

where $\tilde{E}S$ is the IF-early start time, $\widetilde{E}F$ is the IF-early finish time, P is the set of proceeding activities and $\tilde{d}$ is the IF-activity duration.

V. THE PROPOSED COST FORECAST MODEL

Proposing IF-project scheduling will involve the impacts of activities durations vagueness in cost forecast and cash flow analysis. In order to get an insight of durations under optimistic and pessimistic situations, activities beginning in the earliest time and requiring the least duration (Min D_α) and activities beginning in the latest time and finishing at the longest duration (Max D_α) should be considered in cost forecasting calculations (Maravas & Pantouvakis, 2012). Duration calculation for activities with IF times and durations is presented. These durations for activities with early start $\widetilde{\mathrm{E}}\mathrm{S} = \langle[(es_1', es_2, es_3'); u_{es}], [(es_1, es_2, es_3); \nu_{es}]\rangle$ and early finish $\widetilde{\mathrm{E}}\mathrm{F} = \langle\left[(ef_1', ef_2, ef_3'); u_{ef}\right], \left[(ef_1, ef_2, ef_3); \nu_{ef}\right]\rangle$ are presented as follows*:*

$$\text{Min } D_\alpha = [(minD_\alpha); u_D, (minD_\alpha); (1 - \nu_D)] \tag{14}$$
$$Min\, D_\alpha\,; u_D = [(inf\,ES_\alpha); u_{es}, (inf\,ES_\alpha + inf\,D_\alpha); \min u_{es}, u_D] = [\frac{\alpha}{u_{es}}(es_2 - es_1) + es_1, \frac{\alpha}{u_{ef}}(ef_2 - ef_1) + ef_1] \tag{15}$$
$$Min\, D_\alpha\,; (1 - \nu_D) = [(inf\,ES_\alpha); (1 - \nu_{es}), (inf\,ES_\alpha + inf\,D_\alpha); \min \nu_{es}, \nu_D] = [\frac{\alpha}{(1 - \nu_{es})}(es_2 - es'_1) + es'_1, \frac{\alpha}{\left(1 - \nu_{ef}\right)}\left(ef_2 - ef'_1\right) + ef'_1] \tag{16}$$
$$\text{Max } D_\alpha = [(maxD_\alpha); u_D, (maxD_\alpha); (1 - \nu_D)] \tag{17}$$
$$Max\, D_\alpha\,; u_D = [(\sup ES_\alpha); u_{es}, (\sup ES_\alpha + \sup D_\alpha); \min u_{es}, u_D] = [\frac{\alpha}{u_{es}}(es_2 - es_3) + es_3, \frac{\alpha}{u_{ef}}(ef_2 - ef_3) + ef_3] \tag{18}$$
$$Max\, D_\alpha\,; (1 - \nu_D) = [(\sup ES_\alpha); (1 - \nu_{es}), (\sup ES_\alpha + \sup D_\alpha); \min \nu_{es}, \nu_D] = [\frac{\alpha}{(1 - \nu_{es})}(es_2 - es'_3) + es'_3, \frac{\alpha}{\left(1 - \nu_{ef}\right)}\left(ef_2 - ef'_3\right) + ef'_3] \tag{19}$$

where $Min\, D_\alpha\,; u_D$ and $Min\, D_\alpha\,; (1 - \nu_D)$ represent α-cut of upper and lower bounds of minimum duration in different levels of α, $Max\, D_\alpha\,; u_D$ and $Max\, D_\alpha\,; (1 - \nu_D)$ denote α-cut of upper and lower bounds of maximum durations under different levels of α, respectively. Sup denotes supremum and inf denotes infimum. It should be noted the trust degree of triangular intuitionistic fuzzy number is between $[u, 1 - \nu]$.

The resulting IF-CPM not only enables the managers to express membership degree, non-membership degree and hesitation in calculations but also provides a more thorough understanding of the activities durations under different levels of knowledge by using the concept of α-cuts.

Project cost fluctuations are caused by uncertainty and vagueness of cost and duration. Correlation of lack of knowledge and vagueness in time and cost can follow different patterns. In real production project environment, they are most of the times positively correlated. Therefore, the upper and lower bounds of the best and the worst conditions of cash distribution activity i with $Cost_i = \langle[(ci_1^{'}, ci_2, ci_3^{'}); u_{ci}], [(ci_1, ci_2, ci_3); v_{ci}]\rangle$ and duration of $d_i = \langle[(d_{i1}^{'}, d_{i2}, d_{i3}^{'}); u_{di}], [(d_{i1}, d_{i2}, d_{i3}); v_{di}]\rangle$ per unit of time t at level α is introduced as the following.

$$\min Cash\ Distribution_{i,t_\alpha}; u_{cd} = \frac{\inf cost_{i,\alpha}\ ; u_{ci}}{\sup Durationi_\alpha\ \ ; u_{ci}} = \frac{\frac{\alpha_c}{u_c}(ci_2 - ci_1) + ci_1}{\frac{\alpha_{di}}{u_{di}}(di_2 - di_3) + di_3} \tag{20}$$

$$\min Cash\ Distribution_{i,t_\alpha}; (1 - v_{cd}) = \frac{\inf cost_{i,\alpha}\ ; (1 - v_{ci})}{\sup Durationi_\alpha\ \ ; (1 - v_{ci})} = \frac{\frac{\alpha_{ci}}{(1-v_{ci})}(ci_2 - ci'_1) + ci'_1}{\frac{\alpha_{di}}{(1-v_{ci})}(di_2 - di'_3) + di'_3} \tag{21}$$

$$\max Cash\ Distribution_{i,t_\alpha}; u_{cd} = \frac{\sup cost_{i,\alpha}\ ; u_{ci}}{\inf Durationi_\alpha\ \ ; u_{ci}} = \frac{\frac{\alpha_c}{u_{ci}}(ci_2 - ci_3) + ci_3}{\frac{\alpha_d}{u_{ci}}(di_2 - di_1) + di_1} \tag{22}$$

$$\max Cash\ Distribution_{i,t_\alpha}; (1 - v_{cd}) = \frac{\sup cost_{i,\alpha}\ ; (1 - v_{ci})}{\inf Durationi_\alpha\ \ ; (1 - v_{ci})} = \frac{\frac{\alpha_c}{(1-v_{ci})}(ci_2 - ci'_3) + ci'_3}{\frac{\alpha_d}{(1-v_{ci})}(di_2 - di'_1) + di'_1} \tag{23}$$

In order to calculate the cost in the entire activity network, sum of direct cash distribution of all activities in each period can be obtained by the following.

$$Min\ TC_\alpha; u_{tc} = \sum_{i=1}^{n} \frac{\frac{\alpha_c}{u_c}(ci_2 - ci_1) + ci_1}{\frac{\alpha_{di}}{u_{di}}(di_2 - di_3) + di_3} \tag{24}$$

$$Min\ TC_\alpha; (1 - v_{tc}) = \sum_{i=1}^{n} \frac{\frac{\alpha_{ci}}{(1-v_{ci})}(ci_2 - ci'_1) + ci'_1}{\frac{\alpha_{di}}{(1-v_{ci})}(di_2 - di'_3) + di'_3} \tag{25}$$

$$Max\ TC_\alpha; u_{tc} = \sum_{i=1}^{n} \frac{\frac{\alpha_c}{u_{ci}}(ci_2 - ci_3) + ci_3}{\frac{\alpha_d}{u_{ci}}(di_2 - di_1) + di_1} \tag{26}$$

$$Max\ TC_\alpha; (1 - v_{tc}) = \sum_{i=1}^{n} \frac{\frac{\alpha_c}{(1-v_{ci})}(ci_2 - ci'_3) + ci'_3}{\frac{\alpha_d}{(1-v_{ci})}(di_2 - di'_1) + di'_1} \tag{27}$$

where TC is the total direct cash distribution of activities $(i = 0,1,2, \dots, n)$being implemented in the t time period. Total cash flow of project can be calculated by using the following:

$$\text{Min}\ CFT_\alpha, u_{cft} = \sum_{t=0}^{T} \sum_{i=1}^{n} \frac{\frac{\alpha_c}{u_c}\left(ci_{t_2} - ci_{t_1}\right) + ci_{t_1}}{\frac{\alpha_{di}}{u_{di}}\left(di_{t_2} - di_{t_3}\right) + di_{t_3}} \tag{28}$$

$$\text{Min}\ CFT_\alpha, \left(1 - v_{cft}\right) = \sum_{t=0}^{T} \sum_{i=1}^{n} \frac{\frac{\alpha_{ci}}{(1-v_{ci})}\left(ci_{t_2} - ci'_{t_1}\right) + ci'_{t_1}}{\frac{\alpha_{di}}{(1-v_{ci})}\left(di_{t_2} - di'_{t_3}\right) + di'_{t_3}} \tag{29}$$

$$\text{Max}\ CFT_\alpha, u_{cft} = \sum_{t=0}^{T} \sum_{i=1}^{n} \frac{\frac{\alpha_c}{u_c}\left(ci_{t_2} - ci_{t_3}\right) + ci_{t_3}}{\frac{\alpha_{di}}{u_{di}}\left(di_{t_2} - di_{t_1}\right) + di_{t_1}} \tag{30}$$

$$\text{Max}\ CFT_\alpha, \left(1 - v_{cft}\right) = \sum_{t=0}^{T} \sum_{i=1}^{n} \frac{\frac{\alpha_{ci}}{(1-v_{ci})}\left(ci_{t_2} - ci'_{t_3}\right) + ci'_{t_3}}{\frac{\alpha_{di}}{(1-v_{ci})}\left(di_{t_2} - di'_{t_1}\right) + di'_{t_1}} \tag{31}$$

The minimum and maximum values with their corresponding upper and lower bounds depict the limits of predicted cost in the case of the best and the worst events. This method would enable the DM to have a thorough understanding of uncertain and vague data in different stages of the project life cycle and would help the manager take the proper decisions. Upper and lower bound of uncertainty in cash flow in different α levels can be obtained by the following.

$$CFU_\alpha = [CFU_\alpha; u_{cfu}\,, CFU_\alpha; 1 - v_{cfu}] \tag{32}$$

$$CFU_\alpha; u_{cfu} = \text{Max}\, CFT_\alpha, u_{cft} - \text{Min}\, CFT_\alpha, u_{cft} = \sum_{t=0}^{T}\sum_{i=1}^{n} \frac{\frac{\alpha_c}{u_c}\left(ci_{t_2} - ci_{t_3}\right) + ci_{t_3}}{\frac{\alpha_{di}}{u_{di}}\left(di_{t_2} - di_{t_1}\right) + di_{t_1}} - \sum_{t=0}^{T}\sum_{i=1}^{n} \frac{\frac{\alpha_c}{u_c}\left(ci_{t_2} - ci_{t_1}\right) + ci_{t_1}}{\frac{\alpha_{di}}{u_{di}}\left(di_{t_2} - di_{t_3}\right) + di_{t_3}}, \tag{33}$$

$$CFU_\alpha; (1 - v_{cfu}) = \text{Max}\, CFT_\alpha, \left(1 - v_{cft}\right) - \text{Min}\, CFT_\alpha, \left(1 - v_{cft}\right) = \sum_{t=0}^{T}\sum_{i=1}^{n} \frac{\frac{\alpha_{ci}}{(1-v_{ci})}\left(ci_{t_2} - ci'_{t_3}\right) + ci'_{t_3}}{\frac{\alpha_{di}}{(1-v_{ci})}\left(di_{t_2} - di'_{t_1}\right) + di'_{t_1}} - \sum_{t=0}^{T}\sum_{i=1}^{n} \frac{\frac{\alpha_{ci}}{(1-v_{ci})}\left(ci_{t_2} - ci'_{t_1}\right) + ci'_{t_1}}{\frac{\alpha_{di}}{(1-v_{ci})}\left(di_{t_2} - di'_{t_3}\right) + di'_{t_3}} \tag{34}$$

VI. APPLICATION EXAMPLE

In order to illustrate the proposed model, a network of main activities in a production project is presented and its cost and dates are analyzed by the proposed model. The activity network is displayed in Fig. 2, and the adopted information is reported in Table I regarding the vague and uncertain activities durations and costs.

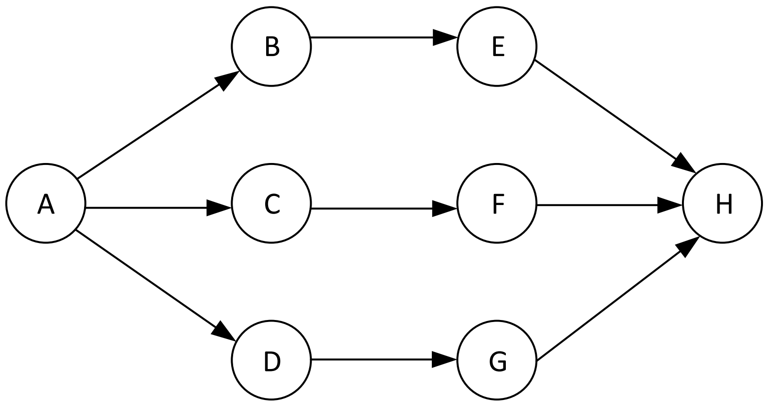

Fig. 2. Sample activity network

TABLE I. Activity network data

Activity	Predecessors	IF-Duration (days)	IF-cost (k$)
A	-	⟨[(3,4,5); 0.6], [(1,4,7); 0.3]⟩	⟨[(7,10,13); 0.6], [(5,10,16); 0.3]⟩
B	A	⟨[(4,5,6); 0.6], [(2,5,8); 0.3]⟩	⟨[(11,15,18); 0.6], [(8,15,22); 0.3]⟩
C	A	⟨[(6,8,10); 0.6], [(2,8,14); 0.3]⟩	⟨[(30,35,40); 0.6], [(25,35,45); 0.3]⟩
D	A	⟨[(3,4,5); 0.6], [(1,4,7); 0.3]⟩	⟨[(12,15,17); 0.6], [(10,8,21); 0.3]⟩
E	B	⟨[(9,12,15); 0.6], [(3,12,21); 0.3]⟩	⟨[(16,20,26); 0.6], [(14,20,36); 0.3]⟩
F	C	⟨[(15,20,25); 0.6], [(5,20,35); 0.3]⟩	⟨[(38,40,44); 0.6], [(32,40,50); 0.3]⟩
G	D	⟨[(5,7,9); 0.6], [(1,7,13); 0.3]⟩	⟨[(27,30,34); 0.6], [(22,30,38); 0.3]⟩
H	E,F,G	⟨[(8,11,14); 0.6], [(2,11,20); 0.3]⟩	⟨[(6,10,14); 0.6], [(4,10,18); 0.3]⟩

A. Computational results

Early start time and early finish time under vague environment is calculated by applying IF-project scheduling. Table II displays the corresponding results. For the purpose of illustration, the calculation for activity *C* is displayed in the following.

$$\widetilde{EF}_C = \widetilde{ES}_C + \tilde{d}_C = \langle[(3,4,5);0.6],[(1,4,7);0.3]\rangle + \langle[(6,8,10);0.6],[(2,8,14);0.3]\rangle \\ = \langle[(9,12,15);0.6],[(3,12,21);0.3]\rangle \tag{35}$$

Cash distributions of activities under different levels of uncertainty are calculated, and the results are displayed in Tables III. For the purpose of illustration, the calculations for activity *A* are presented in the following.

$$\min Cash\ Distribution_{A,t_{0.3}};0.6_{cd} = \frac{\frac{0.3}{0.6}(10-7)+7}{\frac{0.3}{0.6}(4-3)+3} = 2.4 \tag{36}$$

$$\min Cash\ Distribution_{A,t_{0.3}};(1-0.3) = \frac{\frac{0.3}{0.7}(10-5)+5}{\frac{0.3}{0.7}(4-7)+7} = 0.86 \tag{37}$$

$$\max Cash\ Distribution_{A,t_{0.3}};0.6 = \frac{\frac{0.3}{0.6}(10-13)+13}{\frac{0.3}{0.6}(4-3)+3} = 3.28 \tag{38}$$

$$\max Cash\ Distribution_{A,t_{0.3}};(1-0.3) = \frac{\frac{0.3}{0.7}(10-16)+16}{\frac{0.3}{0.7}(4-1)+1} = 5.7 \tag{39}$$

TABLE II. IF-project scheduling (days)

Activity	Early start	Early finish
A	$\langle[(0,0,0);0.6],[(0,0,0);0.3]\rangle$	$\langle[(3,4,5);0.6],[(1,4,7);0.3]\rangle$
B	$\langle[(3,4,5);0.6],[(1,4,7);0.3]\rangle$	$\langle[(7,9,11);0.6],[(3,9,15);0.3]\rangle$
C	$\langle[(3,4,5);0.6],[(1,4,7);0.3]\rangle$	$\langle[(9,12,15);0.6],[(3,12,21);0.3]\rangle$
D	$\langle[(3,4,5);0.6],[(1,4,7);0.3]\rangle$	$\langle[(6,8,10);0.6],[(2,8,14);0.3]\rangle$
E	$\langle[(7,9,11);0.6],[(3,9,15);0.3]\rangle$	$\langle[(16,21,26);0.6],[(6,21,36);0.3]\rangle$
F	$\langle[(9,12,15);0.6],[(3,12,21);0.3]\rangle$	$\langle[(24,32,40);0.6],[(8,32,56);0.3]\rangle$
G	$\langle[(6,8,10);0.6],[(2,8,14);0.3]\rangle$	$\langle[(15,20,25);0.6],[(13,21,28);0.3]\rangle$
H	$\langle[(24,32,40);0.6],[(8,32,56);0.3]\rangle$	$\langle[(32,43,54);0.6],[(10,43,76);0.3]\rangle$

TABLE III. Cash distribution ($^{k\$}/_{day}$)

Activity	Max cash distribution			Min cash distribution		
	$(\mathbf{1}-v_{cfu})$	$\boldsymbol{u}$	**Maravas and Pantouvakis method (2012)**	$(\mathbf{1}-v_{cfu})$	$\boldsymbol{u}$	**Maravas and Pantouvakis method (2012)**
	0.5	0.4	0.4	0.5	0.4	0.4
A	3.73	3	3.47	1.76	2.08	1.78
B	4.1	3.43	3.8	2.22	2.56	2.25
C	6.02	5	5.5	3.31	3.85	3.47
D	3.73	4.27	4.76	1.76	3.23	2.86
E	2.61	2	2.3	1.25	1.44	1.27
F	2.73	2.25	2.49	1.55	1.82	1.68
G	2.6	2.85	3.17	2.09	2.23	2.04
H	1.46	1.13	1.34	0.61	0.72	0.59

The results presented in Table III show the upper and lower bounds of cash distribution of each activity under different levels of uncertainty. In order to verify the proposed model, illustrative example was solved with the recent method proposed by (Maravas & Pantouvakis, 2012). Applying the existing model yielded close results to the proposed model while it lacked some advantages of the proposed model under uncertainty. The difference is that while applying

IFSs, the model provided a better understanding of the situation by showing results based on membership and non-membership degrees. For instance, maximum cash distribution of activity A under classical fuzzy sets is 3.47 units whereas the IFS gives a result between 3 and 3.75. Furthermore, the ability of the IFS in expressing hesitation and non-membership degree, and consequently better expressing uncertainty should be added to the advantages of the introduced model.

Moreover, to give a better understanding of costs in different periods of the projects, shortest and longest activity durations should be calculated. Table IV displays maximum and minimum activity durations under different levels of uncertainty. For the purpose of illustration, the calculations for activity B are presented in the following.

$$Min\ D_{0.3}\ ;0.6 = \left[\frac{0.3}{0.6}(4-3)+3, \frac{0.3}{0.6}(9-7)+7\right] = [3.5,8] \tag{40}$$

$$Min\ D_{0.3}\ ;(1-0.3) = \left[\frac{0.3}{0.7}(4-1)+1, \frac{0.3}{0.7}(9-3)+3\right] = [2.3,5.6] \tag{41}$$

$$Max\ D_{0.3}\ ;0.6 = \left[\frac{0.3}{0.6}(4-5)+5, \frac{0.3}{0.6}(9-11)+11\right] = [5.5,12] \tag{42}$$

$$Min\ D_{0.3}\ ;(1-0.3) = \left[\frac{0.3}{0.7}(4-7)+7, \frac{0.3}{0.7}(9-15)+15\right] = [8.3,17.6] \tag{43}$$

In order to verify the proposed model, the activity duration calculations were done with classical triangular fuzzy numbers. Similarly, the comparison has provided the close results. The proposed model that is based on IFSs, presented a better understanding of the condition by displaying results based on a better method of uncertainty modeling. For example, minimum activity duration of activity A under classical fuzzy set is [0, 3.5] days whereas IFS gives results between [0, 3.8] and [0, 2.7]. This means that the introduced model helps the managers avoid unpleasant surprises by providing a better image of the activity.

B. Discussion of results

α-cut limits the degree of fuzziness and measures robustness of predictions. A higher levels of α denotes a higher confidence in the parameters (Li &Vincent, 1995). The risk level increases from "none" to "high" as the α-cut moves from 1 to 0. To demonstrate this risk analysis approach, calculations for 3 different α-levels of 0.1, 0.3 and 0.5 for u and 3 different α-levels of 0.2, 0.4 and 0.6 for $1-v_{cfu}$were made. Table V displays maximum and minimum cash distribution of each activity. The results show different amounts for different levels of knowledge and risk. Furthermore, the same approach is carried out for activity duration. Finally, Table VI displays maximum activity duration and Table VII shows minimum activity duration.

TABLE IV. Maximum and minimum activity durations

Activity	**Max activity duration**			**Min activity duration**		
	$(\mathbf{1}-v_{cfu})$	$\boldsymbol{u}$	**Maravas and Pantouvakis method (2012)**	$(\mathbf{1}-v_{cfu})$	$\boldsymbol{u}$	**Maravas and Pantouvakis method (2012)**
	0.4	0.5	0.5	0.4	0.5	0.5
A	[0,5.3]	[0,4.2]	[0,4.5]	[0,2.7]	[0,3.8]	[0,3.5]
B	[5.3,11.6]	[4.2,9.3]	[4.5,10]	[2.7,6.4]	[3.8,8.7]	[3.5,8]
C	[5.3,15.9]	[4.2,12.5]	[4.5,13.5]	[2.7,8.1]	[3.8,11.5]	[3.5,10.5]
D	[5.3,10.6]	[4.2,8.3]	[4.5,9]	[2.7,5.4]	[3.8,7.7]	[3.5,7]
E	[11.6,27.4]	[9.3,21.8]	[10,23.5]	[6.4,14.6]	[8.7,20.2]	[8,18.5]
F	[15.9,42.3]	[12.5,33.3]	[13.5,36]	[8.1,21.7]	[11.5,30.7]	[10.5,28]
G	[10.6,24]	[8.3,20.8]	[9,22.5]	[5.4,17.6]	[7.7,19.2]	[7,17.5]
H	[42.3,57.1]	[33.3,44.8]	[36,48.5]	[21.7,28.9]	[30.7,41.2]	[28,37.5]

TABLE V. Cash distribution under different levels of risk

Activity	Max cash distribution						Min cash distribution					
	$(1-v_{cfu})$			u			$(1-v_{cfu})$			u		
	0.1	0.3	0.5	0.2	0.4	0.6	0.1	0.3	0.5	0.2	0.4	0.6
A	10.6	5.88	3.73	3.6	3	2.5	0.87	1.25	1.76	1.71	2.08	2.5
B	8.65	5.78	4.1	3.92	3.43	3	1.19	1.64	2.22	2.18	2.56	3
C	15.25	8.91	6.02	5.75	5	4.38	2.01	2.56	3.31	3.39	3.85	4.38
D	13.4	6.75	3.73	4.9	4.27	3.75	1.48	1.6	1.76	2.79	3.23	3.75
E	7.87	4.25	2.61	2.4	2	1.67	0.75	0.97	1.25	1.24	1.44	1.67
F	6.8	4	2.73	2.56	2.25	2	1.01	1.24	1.55	1.66	1.82	2
G	3.27	2.92	2.6	3.27	2.85	2.5	1.67	1.87	2.09	2	2.23	2.5
H	5.13	2.49	1.46	1.41	1.13	0.91	0.26	0.41	0.61	0.56	0.72	0.91

TABLE VI. Maximum activity duration under different levels of risk

Activity	Max u			Max $(1-v_{cfu})$		
	0.1	**0.3**	**0.5**	**0.2**	**0.4**	**0.6**
A	[0,4.8]	[0,4.5]	[0,4.2]	[0,6.1]	[0,5.3]	[0,4.4]
B	[4.8,10.7]	[4.5,10]	[4.2,9.3]	[6.1,13.3]	[5.3,11.6]	[4.4,9.9]
C	[4.8,14.5]	[4.5,13.5]	[4.2,12.5]	[6.1,18.4]	[5.3,15.9]	[4.4,13.3]
D	[4.8,9.7]	[4.5,9]	[4.2,8.3]	[6.1,12.3]	[5.3,10.6]	[4.4,8.9]
E	[10.7,25.2]	[10,23.5]	[9.3,21.8]	[13.3,31.7]	[11.6,27.4]	[9.9,23.1]
F	[14.5,38.7]	[13.5,36]	[12.5,33.3]	[18.4,49.1]	[15.9,42.3]	[13.3,35.4]
G	[9.7,24.2]	[9,22.5]	[8.3,20.8]	[12.3,26]	[10.6,24]	[8.9,22]
H	[38.7,52.2]	[36,48.5]	[33.3,44.8]	[49.1,66.6]	[42.3,57.1]	[35.4,47.7]

TABLE VII. Minimum activity duration under different levels of risk

Activity	*Min* u			*Min* $(1-v_{cfu})$		
	0.1	**0.3**	**0.5**	**0.2**	**0.4**	**0.6**
A	[0,3.2]	[0,3.5]	[0,3.8]	[0,1.9]	[0,2.7]	[0,3.6]
B	[3.2,7.3]	[3.5,8]	[3.8,8.7]	[1.9,4.7]	[2.7,6.4]	[3.6,8.1]
C	[3.2,9.5]	[3.5,10.5]	[3.8,11.5]	[1.9,5.6]	[2.7,8.1]	[3.6,10.7]
D	[3.2,6.3]	[3.5,7]	[3.8,7.7]	[1.9,3.7]	[2.7,5.4]	[3.6,7.1]
E	[7.3,16.8]	[8,18.5]	[8.7,20.2]	[4.7,10.3]	[6.4,14.6]	[8.1,18.9]
F	[9.5,25.3]	[10.5,28]	[11.5,30.7]	[5.6,14.9]	[8.1,21.7]	[10.7,28.6]
G	[6.3,15.8]	[7,17.5]	[7.7,19.2]	[3.7,15.3]	[5.4,17.6]	[7.1,19.9]
H	[25.3,33.8]	[28,37.5]	[30.7,41.2]	[14.9,19.4]	[21.7,28.9]	[28.6,38.3]

VII. CONCLUSION

In this paper, a new approach in production project cash flow generation for activities with uncertain and vague duration and cost is proposed. This novel cost-forecasting model is based on IF-CPM that is proposed in this paper. This model gives DMs a comprehensive and thorough insight of project cost in different stages of project life cycle. This comprehensive insight improves the DM's knowledge of vagueness and lack of knowledge and as a result, upper and lower bounds of the required resources in different stages of project implementation are identified. Consequently, this model helps avoid unpleasant surprises in the worst-case scenarios. Applying IFSs enables the method to model uncertainty, vagueness and hesitation with more flexibility in addition to adding all the advantages of IFSs over other fuzzy sets. Using IFSs makes the model more suitable for projects like new product development (NPD) and research and development (R&D), in which the information is vague, unclear and with hesitation. Since the introduced model applies a sophisticated uncertainty modeling technique, it could be beneficial in feasibility study in addition to project implementation stage. To put differently, the results could provide reliable inputs in project evaluation methods such as net present value. For the purpose of illustration, the proposed model is applied in a practical example. In the example, uncertain early start times and early finish times are calculated under vague environment. Cash distribution of activities under different levels of uncertainty is also provided to demonstrate different cost conditions under different risk levels. Applying this method as an evaluation tool in earned value analysis could be a promising research direction.

REFERENCES

1. Atanassov, K. T. (2008). My personal view on intuitionistic fuzzy sets theory. In *Fuzzy Sets and Their Extensions: Representation, Aggregation & Models* (pp. 23-43). Springer Berlin Heidelberg.
2. Atanassov, K. T. Intuitionistic fuzzy sets. Central Tech Library, Bulgarian Academy Science, Sofia, Bulgaria, 1983.
3. Atkinson, R., Crawford, L., & Ward, S. (2006). Fundamental uncertainties in projects and the scope of project management. *International journal of project management*, *24*(8), 687-698.
4. Barbosa, P. S., & Pimentel, P. R. (2001). A linear programming model for cash flow management in the Brazilian construction industry. *Construction management and Economics*, *19*(5), 469-479.
5. Bhattacharyya, R. (2015). A Grey Theory Based Multiple Attribute Approach for R&D Project Portfolio Selection. *Fuzzy Information and Engineering*, *7*(2), 211-225.
6. Blyth, K. & Kaka, A. (2006). A novel multiple linear regression model for forecasting S-curves, *Engineering, Construction and Architectural Management, 13*(1): 82–95.
7. Boran, F. E., Boran, K., & Menlik, T. (2012). The evaluation of renewable energy technologies for electricity generation in Turkey using intuitionistic fuzzy TOPSIS. *Energy Sources, Part B: Economics, Planning, and Policy*, *7*(1), 81-90.
8. Boussabaine A.H. & Kaka, A. (1998). A neural networks approach for cost-flow forecasting. *Construction Management and Economics Journal*, *16*, 471-479.
9. Caron, F., & Comandulli, M. (2014). A cash flow-based approach for assessing expansion options stemming from project modularity. *International Journal of Project Organization and Management*, *6*(1-2), 157-178.
10. Chai, J., Liu, J. N., & Xu, Z. (2012). A new rule-based SIR approach to supplier selection under intuitionistic fuzzy environments. *International Journal of Uncertainty, Fuzziness and Knowledge-based Systems*, *20*(3), 451-471.
11. Chanas, S., & Kamburowski, J. (1981). The use of fuzzy variables in PERT. *Fuzzy sets and systems*, *5*(1), 11-19.
12. Chen, C. C., & Zhang, Q. (2014). Applying quality function deployment techniques in lead production project selection and assignment. In *Advanced Materials Research* (Vol. 945, pp. 2954-2959).
13. Chen, H. L., Chen, C. I., Liu, C. H., & Wei, N. C. (2013). Estimating a project's profitability: A longitudinal approach. *International Journal of Project Management*, *31*(3), 400-410.
14. Cheng, M. Y., & Roy, A. F. (2011). Evolutionary fuzzy decision model for cash flow prediction using time-dependent support vector machines. *International Journal of Project Management*, *29*(1), 56-65.
15. Cheng, M. Y., Hoang, N. D., and Wu, Y. W. (2015). Cash flow prediction for construction project using a novel adaptive time-dependent least squares support vector machine inference model. *Journal of Civil Engineering and Management*, *21*(6), 679-688.
16. Cioffi, D.F., (2005). A tool for managing projects: an analytic parameterization of the S-curve. *International Journal of Project Management*, *23*(3), 215–222.
17. Cooke, B., & Jepson, W. B. (1979). *Cost and financial control for construction firms*. Macmillan.
18. Deng, H. (2014). Comparing and ranking fuzzy numbers using ideal solutions. *Applied Mathematical Modelling*, *38*(5), 1638-1646.
19. Duong, A. N. (2011). Rate-decline analysis for fracture-dominated shale reservoirs. *SPE Reservoir Evaluation and Engineering*, *14*(3), 377.

20. Gerogiannis, V. C., Fitsilis, P., & Kameas, A. D. (2011). Using a combined intuitionistic fuzzy set-TOPSIS method for evaluating project and portfolio management information systems. In *Artificial Intelligence Applications and Innovations* (pp. 67-81), Springer Berlin Heidelberg.

21. Gormley, F.M., & Meade, N., 2007. The utility of cash flow forecasts in the management of corporate cash balances. *European Journal of Operational Research* 182(2), 923–935

22. Hsu, K. (2003). Estimation of a double S-curve model, AACE International Transactions IT13.1– IT13.5.

23. Hwee, N. G. & Tiong, R. L. K., (2002). Model on cash flow forecasting and risk analysis for contracting firms, *International Journal of Project Management*, *20*, 351-363.

24. Jarrah, R., Kulkarni, D., & O'Connor, J.T., (2007). Cash flow projections for selected TxDoT highway projects. *Journal of Construction Engineering and Management*, *133*(3), 235–241.

25. Jiang, A., Issa, R. R., & Malek, M. (2011). Construction project cash flow planning using the Pareto optimality efficiency network model. *Journal of Civil Engineering and Management*, *17*(4), 510-519.

26. Khosrowshahi, F., & Kaka, A. P. (2007). A decision support model for construction cash flow management. *Computer-Aided Civil and Infrastructure Engineering*, *22*(7), 527-539.

27. Kumar, V. S., Hanna, A. S., & Adams, T. (2000). Assessment of working capital requirements by fuzzy set theory. *Engineering, Construction and Architectural Management*, *7*(1), 93-103.

28. Lam, K. C., et al. (2001). An integration of the fuzzy reasoning technique and the fuzzy optimization method in construction project management decision-making. *Construction Management and Economics*, *19*(1), 63-76.

29. Lawson, C. P., Longhurst, P. J., & Ivey, P. C. (2006). The application of a new research and development project selection model in SMEs. *Technovation*, *26*(2), 242-250.

30. Lee F. (1998). Fuzzy information processing system. Peking University Press Inc., 118–132.

31. Li, H., & Yen, V. C. (1995). *Fuzzy sets and fuzzy decision-making*. CRC press.

32. Liang, C., Zhao, S., & Zhang, J. (2014). Aggregation Operators on Triangular Intuitionistic Fuzzy Numbers and its Application to Multi-Criteria Decision Making Problems. *Foundations of Computing and Decision Sciences*, *39*(3), 189-208.

33. Maravas, A., & Pantouvakis, J. P. (2012). Project cash flow analysis in the presence of uncertainty in activity duration and cost. *International journal of project management*, *30*(3), 374-384

34. McCahon, C. S., & Lee, E. S. (1988). Project network analysis with fuzzy activity times. *Computers & Mathematics with applications*, *15*(10), 829-838.

35. Mohagheghi, V., Mousavi, S. M., & Vahdani, B. (2015). A new optimization model for project portfolio selection under interval-valued fuzzy environment. *Arabian Journal for Science and Engineering*, *40*, 3351–3361.

36. Mousavi, S. M., Jolai, F., & Tavakkoli-Moghaddam, R. (2013). A fuzzy stochastic multi-attribute group decision-making approach for selection problems. *Group Decision and Negotiation*, *22*(2), 207-233.

37. Neog, T. J., & Sut, D. K. (2011). An application of fuzzy soft sets in medical diagnosis using fuzzy soft complement. *International Journal of Computer Applications*, *33*(9).

38. Ning, X., Lam, K. C., & Lam, M. C. K. (2011). A decision-making system for construction site layout planning. *Automation in Construction*, *20*(4), 459-473.

39. Prade, H. (1979). Using fuzzy set theory in a scheduling problem: a case study. *Fuzzy sets and systems*, *2*(2), 153-165.

40. Rostamy, A. A., Takanlou, F., & AnvaryRostamy, A. (2013). A fuzzy statistical expert system for cash flow analysis and management under uncertainty. *Advances in Economics and Business*, *1*(2), 89-102.

41. Santamaría, L., Barge-Gil, A., & Modrego, A. (2010). Public selection and financing of R&D cooperative projects: Credit versus subsidy funding. *Research Policy*, *39*(4), 549-563.

42. Shu, M. H., Cheng, C. H., & Chang, J. R. (2006). Using intuitionistic fuzzy sets for fault-tree analysis on printed circuit board assembly. *Microelectronics Reliability*, *46*(12), 2139-2148.

43. Szmidt, E., & Kacprzyk, J. (2001). Intuitionistic fuzzy sets in some medical applications. In *Computational Intelligence. Theory and Applications* (pp. 148-151). Springer Berlin Heidelberg.

44. Szmidt, E., Kacprzyk, J., & Bujnowski, P. (2014). How to measure the amount of knowledge conveyed by Atanassov's intuitionistic fuzzy sets. *Information Sciences*, *257*, 276-285.

45. Touran, A., Atgun, M., & Bhurisith, I., (2004). Analysis of the United States department of transportation prompt pay provisions. *Journal of Construction Engineering and Management, 130*(5), 719–725.

46. Ungureanu, D., & Vernic, R. (2014). On a fuzzy cash flow model with insurance applications. *Decisions in Economics and Finance*, 1-16.

47. Wang, Y. (2012). An Approach to Software Selection with Triangular Intuitionistic Fuzzy Information. *International Journal of Advancements in Computing Technology*, *4*(2).

48. Xu, Z., & Liao, H. (2014). Intuitionistic Fuzzy Analytic Hierarchy Process, *IEEE Transactions on Fuzzy Systems, 22*(4),749-761.

49. Zadeh, L. A. (1965). Fuzzy sets. *Information and control*, *8*(3), 338-353.

50. Zimmermann, H. J. (2001). *Fuzzy set theory—and its applications*. Springer Science & Business Media.