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Designing a multivariate exponentially weighted moving average control chart with measurement errors

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Abstract *– Control chart is one of the useful tools of statistical process control, which monitor the processes over time. In most of the designed control charts, it is assumed that the measurement errors do not exist in the measurement system, while this assumption is usually violated in practice as well. The presence of measurement errors leads to poor performance of the control charts. In this paper, a multivariate exponentially weighted moving average control chart is designed by considering measurement errors in Phase II. To decline the effect of measurement errors on the performance of the proposed control chart, multiple measurements method is applied. Also, sensitivity analysis about the effect of the number of measurements on the ARL performance of the proposed control chart is conducted. Note that different scenarios for the variance-covariance matrix are considered in simulation studies, including Case 1. Uncorrelated case with equal variances. Case 2. Negatively correlated case with equal variances. Case 3. Uncorrelated case with unequal variances. Case 4. Positively correlated case with unequal variances. Moreover, the performance of the proposed control chart is compared with the performance of Hotelling's* T^2 *control chart. Results show the admissible performance of the proposed method in decreasing the effect of measurement errors.*

Keywords*– Average run length, measurement errors, multiple measurements, multivariate exponentially weighted moving average control chart.*

I. INTRODUCTION

In some processes, the measurement system is not accurate enough and causes measurement errors in the collected observations. The measurement errors affect the performance of control charts as well. Control charts are widely used to monitor processes and play an essential role in improving the quality of processes and products. The most common ones are Shewhart control charts that are applied to monitor a production process. Today, fewer processes that have a quality characteristic can be found, so multivariate control charts should be applied. Among the multivariate control charts, the multivariate exponentially weighted moving average (MEWMA) can detect small and medium shifts in the process mean better than Hotelling's T^2 chart.

In most researches, it is assumed that there are no measurement errors in the measurement system. Though, to be realistic, measurement errors should be considered in the measurement systems. The effect of measurement errors on $\bar{X}$ chart is assessed by Bennet (1954). The model used in this study is $Y = X + \varepsilon$, where the real values of quality

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characteristics are shown by X and the measured value by Y, and ε is the random measurement error. Linna and Woodall (2001) employed a linear model $Y = A + BX + \varepsilon$ between the measured value Y and the real value X, where A and B are both constants and known. They studied the effect of measurement errors on $\bar{X}$ control and showed that the power of the control chart decreases. Lina et al. (2001) investigated the effect of measurement errors on the performance multivariate chi-squared control chart and showed that the performance of the chi-squared control chart in the presence of measurement errors deteriorates. Maravelakis et al. (2004) also studied the effect of measurement errors on the performance of exponentially weighted moving average (EWMA) control chart and represented that the effect of measurement errors on the observed values affects the control limits. They also proposed multiple measurements strategy to neutralize the effect of the measurement errors.

The effect of two- component measurement errors on the performance of the Shewhart control chart is examined by Cocchi and Scagliarin (2007). Abbasi (2010) investigated the effect of the two-component measurement errors on the EWMA control chart performance and used multiple measurements method in each sample to decrease the effect of measurement errors. Maravelakis (2012) examined the effect of the measurement errors on the performance of the Cumulative Sum (CUSUM) control chart. Noorossana and Zerehsaz (2015) studied the effect of the measurement errors on the performance of the EWMA-3 control chart for monitoring simple linear profiles. Ding and Zeng (2015) examined the effect of measurement errors in multi-stage processes. They showed that the measurement errors affect the estimated parameters of the regression model. Amiri et al. (2016) evaluated the performance of the EWMA control chart for simultaneous monitoring of mean and variability of multivariate normal processes in the presence of measurement errors. Ghasghaei et al. (2016) investigated the performance of the control chart for simultaneous monitoring of mean and variability of the process by using ranked set sampling (RSS) in the presence of measurement errors. Maleki et al. (2017) prepared a review paper on statistical process monitoring (SPM) in the presence of measurement errors. Sabahno et al. (2018) examined the performance of variable sample size Hotelling's T^2 control chart in the presence of measurement errors. Sabahno et al. (2019a) evaluated the effect of measurement errors on the variable sampling intervals (VSI) Hotelling's T^2 control chart performance. Sabahno et al. (2019b) investigated the performance of variable parameters (VP) $\bar{X}$ control chart in the presence of measurement errors. Sabahno et al. (2020a) studied the performance of variable parameters (VP) multivariate control chart, which simultaneous monitoring of the process mean and variability. Sabahno et al. (2020b) proposed an adaptive VP chart for monitoring the mean and variability of the process simultaneously in auto-correlated multivariate normal processes. Nguyen et al. (2020) evaluated the performance of the EWMA median control chart in the presence of measurement errors. Noor-ul-Amin (2020) examined the effect of measurement errors on the performance of mixed EWMA-CUSUM control chart. Zaidi et al. (2020) studied the effect of measurement errors on the performance of MEWMA-compositional data. They extended the research by Tran et al. (2015) to monitor compositional data using a multivariate exponentially weighted moving average (MEWMA- compositional data chart) by considering potential measurement errors.

By reviewing the literature in the field, the effect of measurement errors on the performance of control charts, evaluating the performance of the multivariate exponentially weighted moving average (MEWMA) control chart has not yet been performed in the presence of measurement errors. Therefore, it is considered as the research gap in this paper, and the importance of simultaneous monitoring of several correlated quality characteristics and the necessity of detecting small and medium shifts in the mean process is investigated. We discuss a multivariate normal process and investigate the effect of the measurement errors on the in-control (IC) and out-of-control (OC) ARL performance of chi-squared control charts in this paper. In other words, the effect of measurement errors on MEWMA control chart performance for monitoring a multivariate normal process is examined. Hence, in this paper, a multivariate exponential weighted moving average (MEWMA) is designed by considering measurement errors in Phase II. To reduce the effect of measurement errors, multiple measurements method has been used. Sensitivity analyses have been performed on different parameters in simulation studies.

The structure of the paper is such that in Section II, the effect of measurement errors on the performance of the MEWMA control chart is discussed. In Section III, a numerical example is provided for simulation. In Section IV, simulation studies are conducted to evaluate the performance of the MEWMA control chart under multiple measurements in the presence of measurement errors. In Section V, the performance of the proposed control chart is compared with Hotelling's T^2 control chart in the presence of measurement errors. Conclusion and some suggestions for future research are given in the final section.

II. EFFECT OF MEASUREMENT ERRORS ON THE PERFORMANCE OF MEWMA CHART

Suppose that at time i (i=1,2,…), there are observations $\left\{x_{i,1,k}, x_{i,2,k}, \ldots, x_{i,n,k}\right\}$ for kth (k=1,2,...,p) quality characteristic. The measured values correspond to the real value of $x_{i,j,k}$ with m replicates are as the set of $\{y_{i,j,k,1}, y_{i,j,k,2}, \ldots, y_{i,j,k,m}\}$, $m \geq 1$. Using the covariate error model, the observed values are modeled through Equation (1):

$$\mathbf{y}_{i,j,h} = \mathbf{A} + \mathbf{B}\mathbf{x}_{i,j} + \mathbf{e}_{i,j,h}. \tag{1}$$

$\mathbf{e}_{i,j,h}$ is the error vector with the mean vector zero and known covariance matrix $\mathbf{\Sigma}_m$. $\mathbf{y}_{i,j,h}$ is the vector of h^{th} replication for the j^{th} observation at time i. The sample mean vector of observations at the time of i is:

$$\bar{\mathbf{y}}_i = \frac{1}{mn}\sum_{j=1}^{n}\sum_{h=1}^{m}\mathbf{y}_{i,j,h} = \mathbf{A} + \frac{1}{n}\left(\mathbf{B}\sum_{j=1}^{n}\mathbf{x}_{i,j} + \frac{1}{m}\sum_{j=1}^{n}\sum_{h=1}^{m}\mathbf{e}_{i,j,h}\right), \tag{2}$$

The proposed statistic for monitoring variability of the mean process is as follows:

$$\mathbf{z}_i = \lambda\bar{\mathbf{y}}_i + (1-\lambda)\mathbf{z}_{i-1}, \mathbf{z}_0 = \mathbf{A} + \mathbf{B}\boldsymbol{\mu}. \tag{3}$$

$$T_i^2 = \frac{2-\lambda}{\lambda}\mathbf{z}_i^T(\mathbf{\Sigma}_{\mathbf{z}_i})^{-1}\mathbf{z}_i, \tag{4}$$

$$T_i^2 = \frac{2-\lambda}{n(\lambda)}\mathbf{z}_i^T(\mathbf{B}\mathbf{\Sigma}_{\mathbf{x}}\mathbf{B}^T + \frac{\mathbf{\Sigma}_m}{m})^{-1}\mathbf{Z}_i, \tag{5}$$

The upper control limit (UCL) of the proposed statistic is specified by simulation such that ARL_0 of the control chart is obtained equal to 200. The performance of the MEWMA control chart gets worse in detecting shifts in the process mean under measurement errors. To neutralize the negative effect of measurement errors, multiple measurements method is proposed, and in the following section, in simulation studies, it can be represented that by increasing (m) the performance of the proposed control chart in detecting shifts in the process mean gets better.

III. NUMERICAL EXAMPLE

To evaluate the effect of measurement errors on the performance of the proposed control chart, a sample with five observations is used, which follows a bivariate normal process with a known mean and variance-covariance matrix. The mean vector is equal to [2 2], and the variance-covariance matrix is equal to $\begin{bmatrix} 2 & 0.5 \\ 0.5 & 2 \end{bmatrix}$. The error vector $\mathbf{e}_{i,j,h}$ for each observation, which is a bivariate normal random variable, is generated with the mean vector [0 0] and with the known variance-covariance matrix. The number of simulation runs is 10000. It is assumed that at each sampling time, five consecutive samples with 1,2,3, or 4 replications on the product is conducted. The values of the variance-covariance matrix of the measurement errors are considered as $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0.75 & 0.5 \\ 0.5 & 0.75 \end{bmatrix}$, $\begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$ and $\begin{bmatrix} 2 & 0.5 \\ 0.5 & 2 \end{bmatrix}$.

IV. PERFORMANCE EVALUATION OF MEWMA CONTROL CHART IN THE PRESENCE OF MEASUREMENT ERRORS

In this paper, the aim is to evaluate the effect of measurement errors on the performance of the MEWMA control chart, so ARL is used to figure out its efficiency. ARL is the average number of samples taken before an out- of-control situation happens. Monte Carlo simulation is conducted for evaluating the performance of the MEWMA control chart in the presence of measurement errors in detecting shifts in the process mean. Also, the performance of the MEWMA control chart with and without measurement errors is compared. It should be noted that simulation was performed in MATLAB software using 10,000 runs.

According to Table I, which is conducted for a control chart with two quality characteristics, we have compared values of ARL_1 in terms of different measurement errors covariance matrices for different shifts in the process mean. When there are no measurement errors, the UCL is equal to 9.6, which is obtained such that the ARL_0 equals 200. According to the results, increasing the value of the diagonal variance terms in the measurement errors covariance matrix will increase ARL_1. Hence, in the presence of measurement errors, the proposed control chart tends to detect mean shifts not as fast as in the case of no measurement errors. Hence, the power of the MEWMA control chart in detecting shifts in the mean process has deteriorated.

Table I: The values of ARL_1 simulated under changes in the mean of the quality characteristics under different error variances (ARL_0 =200)

$\mathbf{B}=\mathbf{I}, m=1, p=2$								
$\boldsymbol{\Sigma}_m$	UCL/ Shift in μ_1, μ_2	0	0.5	1	1.5	2	2.5	3
$\boldsymbol{\Sigma}_m = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	9.6	200.22	6.0080	2.5650	1.8130	1.2660	1.0180	1.0000
$\boldsymbol{\Sigma}_m = \begin{bmatrix} 0.75 & 0.5 \\ 0.5 & 0.75 \end{bmatrix}$	9.62	200.61	11.341	4.0360	2.5600	1.9810	1.6140	1.2730
$\boldsymbol{\Sigma}_m = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$	9.64	200.45	12.613	4.3110	2.6900	2.0480	1.7200	1.3620
$\boldsymbol{\Sigma}_m = \begin{bmatrix} 2 & 0.5 \\ 0.5 & 2 \end{bmatrix}$	9.66	200.20	16.84	5.4460	3.2400	2.3690	1.9950	1.7260

According to Table II, which is conducted for a control chart with two quality characteristics, the effect of multiple measurements on the ARL_1 is evaluated for $m \in \{1, 2, 3, 4\}$ under changes in the mean of both quality characteristics. The UCL is equal to 9.66, which is set in such a way that the ARL_0 equals 200. By increasing the number of measurements (replications), the UCL decreases, and the power of the control chart in detecting shifts in the mean process improves. The results show that by increasing the number of replicates, ARL_1 values decrease, leading to a better performance of the proposed control chart in the presence of measurement errors.

Table II: The values of ARL_1 simulated under changes in the mean of the quality characteristics under different numbers of measurements (ARL_0 =200)

$\mathbf{B} = \mathbf{I}, m = 1, \mathbf{\Sigma}_m = \begin{bmatrix} 2 & 0.5 \\ 0.5 & 2 \end{bmatrix}$								
m	UCL/ Shift in μ_1, μ_2	0	0.5	1	1.5	2	2.5	3
m=1	9.66	200.20	16.84	5.4460	3.2400	2.3690	1.9950	1.7260
m=2	9.65	200.101	11.2560	4.0280	2.5710	2.0210	1.6400	1.2640
m=3	9.62	200.457	9.4030	3.6100	2.2960	1.8440	1.4530	1.1320
m=4	9.59	200.703	8.2550	3.3190	2.2010	1.7110	1.3380	1.0590

According to Table III, which is conducted for a control chart with two quality characteristics, when $\mathbf{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, the UCL is equal to 9.66, which leads to the ARL_0 equal to 200. We have examined the effect of matrix **B** in the linear covariate error model for **B** = **I**, 2**I**, 3**I**, 4**I** on the ARL_1 values of the proposed control chart. By increasing diagonal terms of matrix **B**, the UCL increases, and the power of the control chart in detecting shifts in the mean process improves.

Table III: The values of ARL_1 simulated under changes in the mean of the quality characteristics under different values of B (ARL_0 =200)

$m = 1, p = 2, \mathbf{\Sigma}_m = \begin{bmatrix} 2 & 0.5 \\ 0.5 & 2 \end{bmatrix}$								
B	UCL/ Shift in μ_1, μ_2	0	0.5	1	1.5	2	2.5	3
$\mathbf{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	9.66	200.20	16.84	5.4460	3.2400	2.3690	1.9950	1.7260
$\mathbf{B} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$	9.71	200.33	8.5920	3.2970	2.2030	1.7610	1.3290	1.0500
$\mathbf{B} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$	9.74	200.541	7.0850	2.8740	2.0190	1.5280	1.0970	1.0080

$\mathbf{B}=\begin{bmatrix}4 & 0\\0 & 4\end{bmatrix}$	9.76	200.5920	6.3060	2.6280	1.8370	1.3310	1.0370	1.0020

In Table IV, ARL_1 values for different measurement errors covariance matrices under different shifts in the process mean of four quality characteristics are reported. If there are no measurement errors, the UCL is equal to 13.7, which leads to the ARL_0 equal to 200. By increasing the diagonal variance terms in the measurement errors covariance, the UCL increases, and the power of the control chart in detecting shifts in the mean process decreases. Also, the proposed control chart cannot detect mean shifts as quickly as in the case of no measurement errors.

Table IV: The values of ARL_1 simulated under changes in the mean of the quality characteristics under different error variances (ARL_0 =200)

$\mathbf{B}=\mathbf{I}, m=1, p=4$								
$\mathbf{\Sigma}_m$	UCL/ Shift in μ_1, μ_2	0	0.5	1	1.5	2	2.5	3
$\mathbf{\Sigma}_m=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}$	13.7	200.266	4.4040	2.1030	1.4650	1.0210	1.0000	1.0000
$\mathbf{\Sigma}_m=\begin{bmatrix}0.75&0.5&0.5&0.5\\0.5&0.75&0.5&0.5\\0.5&0.5&0.75&0.5\\0.5&0.5&0.5&0.75\end{bmatrix}$	13.8	200.18	10.7120	3.9110	2.4890	1.9930	1.6700	1.2620
$\mathbf{\Sigma}_m=\begin{bmatrix}1&0.5&0.5&0.5\\0.5&1&0.5&0.5\\0.5&0.5&1&0.5\\0.5&0.5&0.5&1\end{bmatrix}$	13.9	200.438	10.9680	4.0930	2.6230	2.0470	1.7480	1.3290
$\mathbf{\Sigma}_m=\begin{bmatrix}2&0.5&0.5&0.5\\0.5&2&0.5&0.5\\0.5&0.5&2&0.5\\0.5&0.5&0.5&2\end{bmatrix}$	13.95	200.8370	14.2140	4.8530	2.9990	2.2130	1.9270	1.6320

In Table V, the effect of multiple measurements on the ARL_1 is evaluated for $m \in \{1,2,3,4\}$ under changes in the mean of the process with four quality characteristics is investigated. When one measurement is conducted, the UCL is equal to 13.95, leading to the ARL_0 equal to 200. By increasing the number of measurement operations (replications), the UCL decreases, and the power of the control chart in detecting shifts in the mean process is getting better.

According to Table VI, when $\mathbf{B}=\mathbf{I}_{4\times 4}$ for a process with four quality characteristics, the UCL is set equal to 9.66, which leads to the ARL_0 of 200. The effect of matrix **B** in the model for **B** = **I**, 2**I**, 3**I**, 4**I** on the ARL_1 values of the proposed control chart is examined. As shown in Table VII, increasing the diagonal terms of matrix **B** causes decreasing in ARL_1 values and increasing the UCL. Hence, the power of the control chart in detecting shifts in the mean process improves.

Table VII reports ARL_1 in terms of different measurement errors covariance matrices under changes in the mean of the first one out of two quality characteristics. In the case that there are no measurement errors, the UCL is equal to 9.59 to obtain the ARL_0 of 200. By increasing the diagonal variance terms in the measurement errors covariance, the UCL increases, and the power of the control chart in detecting shifts in the mean process decreases. Moreover, the proposed control chart does not detect mean shifts as quickly as in the case of no measurement errors. In comparison to Table II, the power of the control chart decreases under changes in the mean of one of the quality characteristics.

Table V: The values of ARL_1 simulated under changes in the mean of the quality characteristics under different measurement times (ARL_0 =200)

$\mathbf{B}=\mathbf{I}, p=4, \mathbf{\Sigma}_m=\begin{bmatrix} 2 & 0.5 & 0.5 & 0.5 \\ 0.5 & 2 & 0.5 & 0.5 \\ 0.5 & 0.5 & 2 & 0.5 \\ 0.5 & 0.5 & 0.5 & 2 \end{bmatrix}$								
m	UCL/ Shift in μ_1, μ_2	0	0.5	1	1.5	2	2.5	3
m=1	13.95	200.8370	14.2140	4.8530	2.9990	2.2130	1.9270	1.6320
m=2	13.8	200.5930	9.0320	3.5010	2.3010	1.8410	1.4520	1.1060
m=3	13.77	200.506	7.7190	3.0970	2.0910	1.7040	1.1830	1.0090
m=4	13.75	200.49	8.6184	4.9907	2.3302	1.9143	1.033	1.0023

Table VI: The values of ARL_1 simulated under changes in the mean of the quality characteristics under different values of B (ARL_0 =200)

$m=1, p=4, \mathbf{\Sigma}_m=\begin{bmatrix} 2 & 0.5 & 0.5 & 0.5 \\ 0.5 & 2 & 0.5 & 0.5 \\ 0.5 & 0.5 & 2 & 0.5 \\ 0.5 & 0.5 & 0.5 & 2 \end{bmatrix}$								
$\mathbf{B}$	UCL/ Shift in μ_1, μ_2	0	0.5	1	1.5	2	2.5	3
$\mathbf{B}=\mathbf{I}_{4\times4}$	13.95	200.8370	14.2140	4.8530	2.9990	2.2130	1.9270	1.6320
$\mathbf{B}=2\times\mathbf{I}_{4\times4}$	14	200.934	6.6700	2.8370	2.0050	1.5720	1.0850	1.0030
$\mathbf{B}=3\times\mathbf{I}_{4\times4}$	14.1	200.026	5.6870	2.4790	1.8140	1.2130	1.0090	1.0000
$\mathbf{B}=4\times\mathbf{I}_{4\times4}$	14.2	200.761	5.1210	2.3350	1.6890	1.1150	1.0010	1.0000

In Table VIII, the effect of multiple measurements on the ARL_1 is evaluated for $m \in \{1,2,3,4\}$ under changes in the mean of the first quality characteristic. The UCL is equal to 9.65, which is set in such a way that the ARL_0 equals 200. By increasing the number of measurements (replications), the UCL decreases, and the power of the control chart in detecting shifts in the mean process is getting better. The results show that by increasing the number of replicates, ARL_1

values decrease, leading to a better performance of the proposed control chart in the presence of measurement errors. In comparison to Table III, the power of the control chart decreases under changes in the mean of one of the quality characteristics.

TableVII: The values of ARL_1 simulated under changes in the mean of the first quality characteristic under different error covariance matrices (ARL_0=200)

$\mathbf{B}=\mathbf{I}, m=1, p=2$								
$\mathbf{\Sigma}_m$	UCL/ $shift-in-\mu_1$	0	0.5	1	1.5	2	2.5	3
$\mathbf{\Sigma}_m=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$	9.59	200.4060	10.218	3.7220	2.4360	1.9090	1.5360	1.1680
$\mathbf{\Sigma}_m=\begin{bmatrix}0.75 & 0.5\\ 0.5 & 0.75\end{bmatrix}$	9.61	200.6390	15.416	5.0580	3.0800	2.2750	1.9250	1.6290
$\mathbf{\Sigma}_m=\begin{bmatrix}1 & 0.5\\ 0.5 & 1\end{bmatrix}$	9.63	200.5840	16.807	5.5590	3.4280	2.4900	2.0260	1.7680
$\mathbf{\Sigma}_m=\begin{bmatrix}2 & 0.5\\ 0.5 & 2\end{bmatrix}$	9.65	200.16	25.810	7.9040	4.4850	3.1260	2.4500	2.1010

Table VIII: The values of ARL_1 simulated under changes in the mean of the first quality characteristic under different number of measurements (ARL_0=200)

$\mathbf{B}=\mathbf{I}, p=2, \mathbf{\Sigma}_m=\begin{bmatrix}2 & 0.5\\ 0.5 & 2\end{bmatrix}$								
m	UCL/ $shift-in-\mu_1$	0	0.5	1	1.5	2	2.5	3
m=1	9.65	200.16	25.810	7.9040	4.4850	3.1260	2.4500	2.1010
m=2	9.6	200.9840	18.6500	5.8010	3.5770	2.5280	2.0670	1.8050
m=3	9.59	200.6060	15.3520	5.1960	3.1270	2.3110	1.9350	1.6770
m=4	9.57	200.6220	14.3150	4.7790	3.0070	2.2350	1.8680	1.5520

According to Table IX, when $\mathbf{B}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ under changes in the mean of the first quality characteristic, the UCL is set equal to 9.65, which leads to the ARL_0 equal to 200. The effect of matrix **B** in the model for **B** = **I**, 2**I**, 3**I**, 4**I** on the ARL_1 values of the proposed control chart is examined. By increasing the diagonal term of matrix **B**, the UCL increases, and the power of the control chart in detecting shifts in the mean process is getting better. As shown in Table

X, increasing the diagonal terms of matrix **B** causes decreasing in ARL_1 values and increasing the UCL. Hence, the power of the control chart in detecting shifts in the mean process improves.

According to Table X, ARL_1 in terms of different measurement errors covariance matrices under changes in the mean of the first one out of two quality characteristics. In the case that there are no measurement errors, the UCL is equal to 9.58, which to obtaining the ARL_0 of 200. By increasing diagonal variance terms in the measurement errors covariance matrix, the UCL increases, and the power of the control chart in detecting shifts in the mean process deteriorates under changes in the mean of one of the quality characteristics.

Table IX: The values of ARL_1 simulated under changes in the mean of the first quality characteristic under different values of B (ARL_0=200)

$m=1, p=2, \mathbf{\Sigma}_m = \begin{bmatrix} 2 & 0.5 \\ 0.5 & 2 \end{bmatrix}$								
B	UCL/ $shift-in-\mu_1$	0	0.5	1	1.5	2	2.5	3
$\mathbf{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	9.65	200.16	25.810	7.9040	4.4850	3.1260	2.4500	2.1010
$\mathbf{B} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$	9.67	200.5190	14.1360	4.8960	2.9850	2.2220	1.8580	1.5520
$\mathbf{B} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$	9.68	200.3730	12.0660	4.1790	2.6080	2.0560	1.6940	1.3530
$\mathbf{B} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$	9.69	200.2020	11.1830	4.0120	2.5500	1.9750	1.6680	1.2810

Table X: The values of ARL_1 simulated under changes in the mean of the second quality characteristic under different error variances (ARL_0=200)

$\mathbf{B} = \mathbf{I}, m=1, p=2$								
$\mathbf{\Sigma}_m$	UCL/ $shift-in-\mu_2$	0	0.5	1	1.5	2	2.5	3
$\mathbf{\Sigma}_m = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	9.58	200.78	9.8990	3.7410	2.4310	1.8970	1.5260	1.1640
$\mathbf{\Sigma}_m = \begin{bmatrix} 0.75 & 0.5 \\ 0.5 & 0.75 \end{bmatrix}$	9.6	200.71	14.598	5.0470	3.0990	2.2810	1.9110	1.6260
$\mathbf{\Sigma}_m = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$	9.63	200.15	17.726	5.6290	3.4040	2.4590	2.0460	1.7650

$\mathbf{\Sigma}_m = \begin{bmatrix} 2 & 0.5 \\ 0.5 & 2 \end{bmatrix}$	9.65	200.35	26.978	7.7280	4.4050	3.1830	2.4730	2.1020

In Table XI, which is conducted for a control chart with two quality characteristics, the effect of multiple measurements on the ARL_1 is evaluated for $m \in \{1,2,3,4\}$ under changes in the mean of the second one out of two quality characteristics. The UCL is equal to 9.65, which is set such that the ARL_0 equals 200. By increasing the number of measurements (replications), the UCL decreases, and the power of the control chart in detecting shifts in the mean process improves. The results show that by increasing the number of replicates (*m*), ARL_1 values decrease, leading to a better performance of the proposed control chart in the presence of measurement errors.

Table XI: The values of ARL_1 simulated under changes in the mean of the second quality characteristic under different numbers of measurements (ARL_0 =200)

$\mathbf{B} = \mathbf{I}, p = 2, \mathbf{\Sigma}_m = \begin{bmatrix} 2 & 0.5 \\ 0.5 & 2 \end{bmatrix}$								
m	UCL/ $shift-in-\mu_2$	0	0.5	1	1.5	2	2.5	3
m=1	9.65	200.3560	26.9780	7.7280	4.4050	3.1830	2.4730	2.1020
m=2	9.62	200.7150	18.5890	5.8650	3.4330	2.5270	2.0710	1.8230
m=3	9.6	200.4120	15.5320	5.2480	3.1490	2.3410	1.9280	1.6550
m=4	9.58	200.2250	14.2570	4.8180	2.9610	2.2260	1.8570	1.5620

Table XII: The values of ARL_1 simulated under changes in the mean of the second quality characteristic under different values of B (ARL_0 =200)

$m = 1, p = 2, \mathbf{\Sigma}_m = \begin{bmatrix} 2 & 0.5 \\ 0.5 & 2 \end{bmatrix}$								
B	UCL/ $shift-in-\mu_2$	0	0.5	1	1.5	2	2.5	3
$\mathbf{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	9.65	200.3560	26.9780	7.7280	4.4050	3.1830	2.4730	2.1020
$\mathbf{B} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$	9.66	200.2040	14.5910	4.8190	2.9580	2.2370	1.8450	1.5820

$\mathbf{B}=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$	9.67	200.6100	12.1500	4.2960	2.6440	2.0630	1.7200	1.3710
$\mathbf{B}=\begin{bmatrix}4 & 0\\ 0 & 4\end{bmatrix}$	9.68	200.5710	11.6370	4.1080	2.5700	1.9960	1.6500	1.2840

Table XII reports when $\mathbf{B}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ under changes in the mean of the second quality characteristic, the UCL is set equal to 9.65, which leads to the ARL_0 of 200. The effect of matrix **B** in the model for **B** = **I**, 2**I**, 3**I**, 4**I** on the ARL_1 values of the proposed control chart is computed. By increasing **B**, the UCL increases, and the power of the control chart in detecting shifts in the mean process is getting better. In comparison to Table IV, the power of the control chart decreases when we have a shift in the mean of just one of the quality characteristics.

In Table XIII, which is conducted for a control chart with two quality characteristics, when $\mathbf{\Sigma}_x=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ and the measurement errors variance- covariance matrix equal to $\mathbf{\Sigma}_m=\begin{bmatrix}2 & 0.5\\ 0.5 & 2\end{bmatrix}$, the UCL is set equal to 9.66 which leads to the ARL_0 equal to 200. By increasing the non-diagonal terms of the covariance matrix of **x**, the UCL increases, and the power of the control chart in detecting shifts in the mean process deteriorates.

Table XIII: The values of ARL_1 simulated under changes in the mean of the quality characteristics under different covariance matrices of x (ARL_0=200)

$\mathbf{B}=\mathbf{I}, m=1, p=2, \mathbf{\Sigma}_\varepsilon=\begin{bmatrix}2 & 0.5\\ 0.5 & 2\end{bmatrix}$								
$\mathbf{\Sigma_x}$	UCL/ Shift in μ_1, μ_2	0	0.5	1	1.5	2	2.5	3
$\mathbf{\Sigma_x}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$	9.66	200.20	16.84	5.4460	3.2400	2.3690	1.9950	1.7260
$\mathbf{\Sigma_x}=\begin{bmatrix}1 & 0.25\\ 0.25 & 1\end{bmatrix}$	9.68	200.104	17.769	5.6990	3.4120	2.4550	2.0560	1.7820
$\mathbf{\Sigma_x}=\begin{bmatrix}1 & 0.5\\ 0.5 & 1\end{bmatrix}$	9.65	200.657	18.953	5.9130	3.5560	2.5420	2.1010	1.8120
$\mathbf{\Sigma_x}=\begin{bmatrix}1 & 0.75\\ 0.75 & 1\end{bmatrix}$	9.62	200.883	18.933	6.1260	3.5980	2.6260	2.1310	1.8300

The results of ARL_1 under different measurement errors covariance matrices for changes in the mean of both quality characteristics are reported in Table XIV. In this case, uncorrelated measurement errors covariance matrices with equal variances are considered. Also, the UCL of the proposed MEWMA control chart is set through simulation to obtain the ARL_0 of 200. Based on the obtained results, by increasing diagonal error variance terms in the measurement errors covariance matrix, the UCL increases, and the power of the control chart in detecting shifts in the process mean deteriorates under changes in the mean of the quality characteristics.

ARL_1 values under different negatively correlated measurement errors covariance matrices with equal variances for changes in the mean of both quality characteristics are reported in Table XV. In the case that $\mathbf{\Sigma}_m = \begin{bmatrix} 0.75 & -0.5 \\ -0.5 & 0.75 \end{bmatrix}$ the UCL is equal to 9.66, which leads to the ARL_0 of 200. By increasing diagonal error variance terms in the measurement errors covariance matrix, the UCL increases, and the power of the control chart in detecting shifts in the process mean deteriorates under changes in the mean of both quality characteristics.

Table XIV: The values of ARL_1 simulated under changes in the mean of both quality characteristics under different error variances (ARL_0 =200)-uncorrelated measurement errors covariance matrices with equal variances

$\mathbf{B}=\mathbf{I}, m=1, p=2$								
$\mathbf{\Sigma}_m$	UCL/ Shift in μ_1, μ_2	0	0.5	1	1.5	2	2.5	3
$\mathbf{\Sigma}_m = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	9.60	200.28	6.00	2.78	1.80	1.25	1.00	1.00
$\mathbf{\Sigma}_m = \begin{bmatrix} 0.75 & 0 \\ 0 & 0.75 \end{bmatrix}$	9.60	202.67	14.19	4.70	2.94	2.16	1.83	1.54
$\mathbf{\Sigma}_m = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	9.61	199.88	13.93	4.91	3.00	2.21	1.88	1.57
$\mathbf{\Sigma}_m = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$	9.63	200.58	16.00	5.34	3.22	2.40	1.98	1.70

Table XV: The values of ARL_1 simulated under changes in the mean of both quality characteristics under different error variances (ARL_0 =200)- negatively correlated measurement errors covariance matrices with equal variances

$\mathbf{B}=\mathbf{I}, m=1, p=2$								
$\mathbf{\Sigma}_m$	UCL/ Shift in μ_1, μ_2	0	0.5	1	1.5	2	2.5	3
$\mathbf{\Sigma}_m = \begin{bmatrix} 0.75 & -0.5 \\ -0.5 & 0.75 \end{bmatrix}$	9.66	200.87	12.98	4.37	2.72	2.07	1.78	1.40
$\mathbf{\Sigma}_m = \begin{bmatrix} 0.9 & -0.5 \\ -0.5 & 0.9 \end{bmatrix}$	9.66	200.05	13.13	4.63	2.84	2.11	1.82	1.48
$\mathbf{\Sigma}_m = \begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix}$	9.67	200.72	13.30	4.55	2.80	2.14	1.79	1.48
$\mathbf{\Sigma}_m = \begin{bmatrix} 2 & -0.5 \\ -0.5 & 2 \end{bmatrix}$	9.68	201.79	15.85	5.09	3.09	2.29	1.94	1.64

In Table XVI, which is reported for a control chart with two quality characteristics, we have compared values of ARL_1 under different measurement errors covariance matrices for different shifts in the process mean. In this case, uncorrelated measurement errors covariance matrices with unequal variances are considered. When $\mathbf{\Sigma}_m = \begin{bmatrix} 0 & 0 \\ 0 & 0.5 \end{bmatrix}$ the UCL is set equal to 9.66 to achieve the ARL_0 of 200, according to the results, increasing the value of the diagonal variance terms in the measurement errors covariance matrix will increase ARL_1. Hence, the power of the MEWMA control chart in detecting shifts in the process mean deteriorates.

Table XVI: The values of ARL_1 simulated under changes in the mean of both quality characteristics under different error variances (ARL_0 =200)- uncorrelated measurement errors covariance matrices with unequal variances

$\mathbf{B}=\mathbf{I}, m=1, p=2$								
$\mathbf{\Sigma}_m$	UCL/ Shift in μ_1,μ_2	0	0.5	1	1.5	2	2.5	3
$\mathbf{\Sigma}_m = \begin{bmatrix} 0 & 0 \\ 0 & 0.5 \end{bmatrix}$	9.66	200.58	12.66	4.46	2.73	2.11	1.76	1.42
$\mathbf{\Sigma}_m = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.75 \end{bmatrix}$	9.67	199.92	13.92	4.63	2.88	2.17	1.81	1.48
$\mathbf{\Sigma}_m = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$	9.68	201.39	15.81	5.04	3.15	2.33	1.90	1.62
$\mathbf{\Sigma}_m = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$	9.69	199.82	18.59	5.81	3.45	2.53	2.06	1.80

Table XVII reports ARL_1 under different measurement errors covariance matrices for changes in the mean of both quality characteristics. In this case, positively correlated measurement errors covariance matrices with unequal variances are considered. When $\mathbf{\Sigma}_m = \begin{bmatrix} 0.6 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$, the UCL is equal to 9.66 to obtain the ARL_0 of 200. By increasing the diagonal variance terms in the measurement errors covariance, the UCL increases, and the power of the control chart in detecting shifts in the process mean decreases. Moreover, the proposed control chart does not detect mean shifts as quickly as the case of no measurement errors.

Table XVII: The values of ARL_1 simulated under changes in the mean of both quality characteristics under different error variances (ARL_0 =200)- positively correlated measurement errors covariance matrices with unequal variances

$\mathbf{B}=\mathbf{I}, m=1, p=2$								
$\mathbf{\Sigma}_m$	UCL/ Shift in μ_1,μ_2	0	0.5	1	1.5	2	2.5	3
$\mathbf{\Sigma}_m = \begin{bmatrix} 0.6 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$	9.661	201.17	14.66	5.00	3.02	2.23	1.87	1.59
$\mathbf{\Sigma}_m = \begin{bmatrix} 0.75 & 0.5 \\ 0.5 & 1 \end{bmatrix}$	9.665	198.48	15.60	5.08	3.08	2.33	1.93	1.62

$\mathbf{\Sigma}_m = \begin{bmatrix} 3 & 0.5 \\ 0.5 & 2 \end{bmatrix}$	9.668	199.97	18.96	5.90	3.43	2.55	2.08	1.80
$\mathbf{\Sigma}_m = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 3 \end{bmatrix}$	9.67	201.53	21.82	6.62	3.79	2.69	2.18	1.89

V. COMPARING THE PERFORMANCE OF THE PROPOSED CONTROL CHART WITH HOTELLING'S T^2 CONTROL CHART IN THE PRESENCE OF MEASUREMENT ERRORS

The purpose of this section is to compare the performance of the MEWMA control chart with Hotelling's T^2 control chart in the presence of measurement errors for detecting shifts in the process mean vector through Monte Carlo simulation studies in MATLAB software using 10,000 runs.

Table XVIII: Comparing the values of ARL_1 simulated for both T^2 and proposed control chart under different error variances (ARL_0 =200)

$\mathbf{B} = \mathbf{I}, m = 1, p = 2$										
$\mathbf{\Sigma}_m$		Type of control chart	UCL/ Shift in μ_1, μ_2	0	0.5	1	1.5	2	2.5	3
a	$\mathbf{\Sigma}_m = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	T^2	10.64	201.24	50.87	9.48	2.90	1.46	1.08	1.00
		Proposed chart	9.6	200.22	6.0080	2.5650	1.8130	1.2660	1.0180	1.0000
b	$\mathbf{\Sigma}_m = \begin{bmatrix} 0.75 & 0.5 \\ 0.5 & 0.75 \end{bmatrix}$	T^2	10.61	201.53	64.20	13.48	4.08	1.86	1.22	1.00
		Proposed chart	9.62	200.61	11.341	4.0360	2.5600	1.9810	1.6140	1.2730
c	$\mathbf{\Sigma}_m = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$	T^2	10.62	201.74	64.91	14.71	4.43	1.95	1.2500	1.00
		Proposed chart	9.64	200.45	12.613	4.3110	2.6900	2.0480	1.7200	1.3620
d	$\mathbf{\Sigma}_m = \begin{bmatrix} 2 & 0.5 \\ 0.5 & 2 \end{bmatrix}$	T^2	10.62	201.61	71.07	17.00	5.05	2.24	1.43	1.10
		Proposed chart	9.66	200.20	16.84	5.4460	3.2400	2.3690	1.9950	1.7260

According to Table XVIII, which is conducted for a control chart with two quality characteristics, we have compared values of ARL_1 with different measurement errors covariance matrices under different shifts in the process mean. The UCLs of MEWMA and Hotelling's T^2 control chart under different cases are set such that the ARL_0 equals 200 is obtained. Based on the results, increasing the value of the diagonal variance terms in the measurement errors covariance matrix will increase ARL_1. Also, Fig.1 shows the results schematically for both MEWMA and Hotteling's T^2 control charts.

The effect of multiple measurements on the ARL_1 performance of both MEWMA and Hotelling's T^2 control chart $m \in \{1, 2, 3, 4\}$ is evaluated under changes in the mean of both quality characteristics. The UCLs of both MEWMA and Hotelling's T^2 control charts are set in a way that the ARL_0 equals to 200 is obtained. As the results show, by increasing the number of measurements (replications), the UCL decreases, and the power of the control chart in detecting shifts in the mean process improves. Also, by increasing the number of replicates, ARL_1 values decrease, leading to a better performance of the proposed control chart in the presence of measurement errors. Fig. 2 represents the comparison results schematically for both MEWMA and Hotelling's T^2 control chart.

As the results of Figures 1 and 2 show, the proposed MEWMA control chart outperforms Hotelling's T^2 control chart in the presence of measurement errors, especially under small and medium shifts in the process mean.

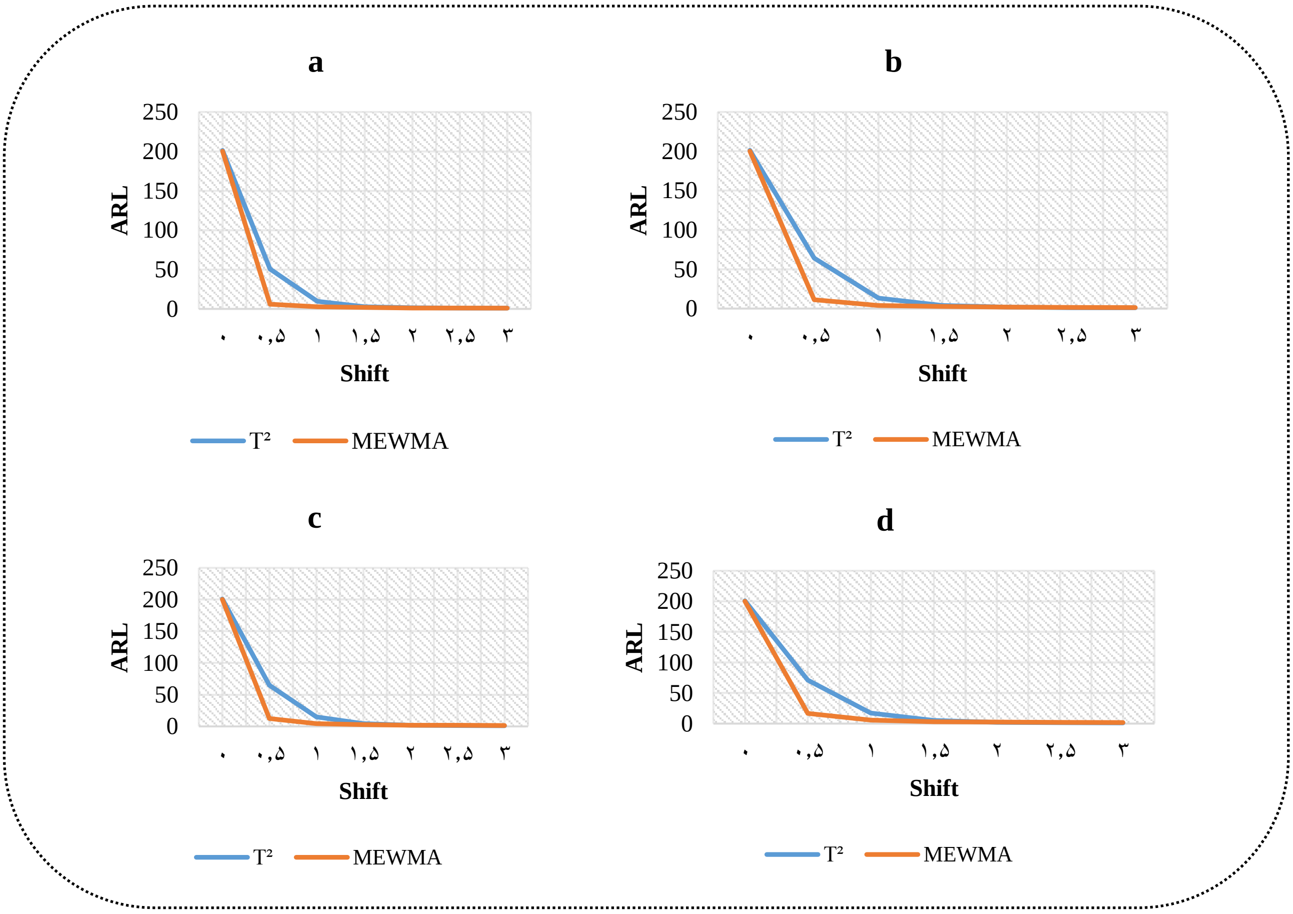

Fig 1. Comparing the values of ARL_1 simulated for both T^2 and proposed control chart under different error variances

Table XIX: Comparing the values of ARL_1 simulated for both T^2 and the proposed control chart under different numbers of measurements (ARL_0 =200)

$\mathbf{B}=\mathbf{I}, m=1, \boldsymbol{\Sigma}_m=\begin{bmatrix} 2 & 0.5 \\ 0.5 & 2 \end{bmatrix}$									
m	Type of control chart	UCL/ Shift in μ_1, μ_2	0	0.5	1	1.5	2	2.5	3
m=1	T^2	10.60	200.60	88.24	24.45	8.42	3.34	1.93	1.35
	Proposed chart	9.66	200.20	16.84	5.4460	3.2400	2.3690	1.9950	1.7260

m=2	T^2	10.61	201.24	76.75	18.20	5.37	2.31	1.41	1.1
	Proposed chart	9.65	200.101	11.2560	4.0280	2.5710	2.0210	1.6400	1.2640
m=3	T^2	10.62	200.90	63.57	15.12	4.30	1.98	1.28	1.06
	Proposed chart	9.62	200.457	9.4030	3.6100	2.2960	1.8440	1.4530	1.1320
m=4	T^2	10.62	200.95	62.52	13.49	3.88	1.83	1.22	1.04
	Proposed chart	9.59	200.703	8.2550	3.3190	2.2010	1.7110	1.3380	1.0590

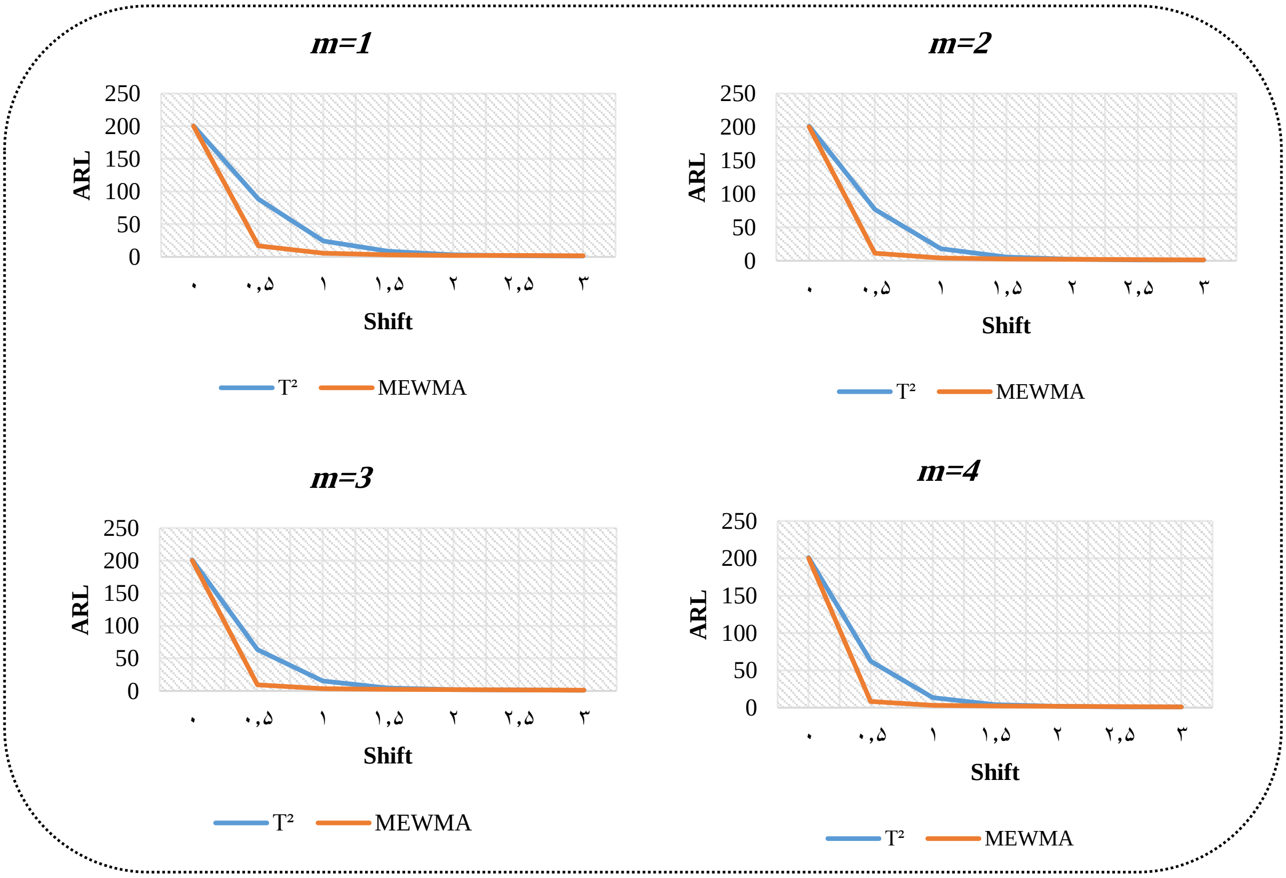

Fig 2. : Comparing the values of ARL_1 simulated for both T^2 and the proposed control chart under different number of measurements

VI. CONCLUSION AND SUGGESTIONS FOR FUTURE RESEARCH

In this paper, a multivariate normal process was considered, and the effect of measurement errors on the ARL_0 and ARL_1 performance of the MEWMA control chart was appraised. For this aim, different scenarios for the variance-covariance matrix are considered in simulation studies, including Case 1. Uncorrelated case with equal variances. Case 2. Negatively correlated case with equal variances. Case 3. Uncorrelated case with unequal variances. Case 4. Positively correlated case with unequal variances. The results of the simulation, in terms of ARL criterion, represented that the performance of the MEWMA control chart is affected by measurement errors in detecting shifts in the mean process. Multiple measurement strategies were used to improve the performance of the abovementioned control chart and reduce the effect of measurement errors. The results showed that by increasing the number of measurements, the performance of the control chart in detecting shifts improves. Furthermore, the performance of the proposed control chart is compared with the performance of Hotelling's T^2 control chart. As the results show, the performance of the proposed control chart is better than Hotelling's T^2 control chart in the presence of measurement errors in both small and medium shifts. For future research, one can propose an adaptive MEWMA control chart by considering measurement errors. Also, considering the effect of both parameters estimation on the performance of the MEWMA control chart with measurement errors is a challenging topic for further research.

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