Extension of the Coverage Region of Multiple Access Channels by Using a Relay

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***Abstract*— From practical and theoretical viewpoints, performance analysis of communication systems by using information-theoretic results is important. In this paper, based on our previous work on Multiple Access Channel (MAC) and Multiple Access Relay Channel (MARC), we analyze the impact of a relay on the fundamental wireless communications concept, i.e., coverage region of MARC, as a basic model for uplink communications between a base station and users with the help of a relay. This analysis includes the coverage region for the MAC as a special case. Considering rate regions for the Gaussian MARC and fading Gaussian MARC, it is proved that in a fixed transmission rate, the relay extends coverage region of the MARC. Numerical illustrations confirm our theoretical results.**

Index Terms— Multiple Access Relay Channel, Coverage region, Fixed desired transmission rates, Cellular network.

I. INTRODUCTION

In the relay channel, first introduced by Van der Meulen [1], the relay can increase transmission rate and extend coverage area. Fundamental coding strategies and the capacity for some special classes of relay channel and a more general class were studied in [2-6].

In some practical models, such as uplink mobile cellular networks, a relay can facilitate communication between mobile users and base station. This model known as Multiple-Access Relay Channel (MARC) was first introduced by Kramer in [7] and is a combination of Multiple-Access Channel (MAC) and the relay channel. MARC with common message was investigated, and its achievable regions and bounds were derived in [9, 10]. Furthermore the results of discrete memoryless MARC were extended to Gaussian in [9]. The bounds of MARC with non-causal CSI at the relay were derived in [11]. Capacity bounds of MARC in Gaussian and fading environment with full-duplex and half-duplex relay constraints via Decode and Forward (DF) and Compress and forward (CF) strategies have been derived in [8, 12]. Two special classes of Multiple-Access relay Channel which include a non-interfering finite-capacity link from relay to decoder were studied in [13]. The inner and outer bounds of the capacity region of multiple-access channel with multiple

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relays (MACMR) were investigated in [14]. The authors in [15] investigated Ultra-wideband (UWB) multiple access relay channel with correlated noises at the relay and the receiver. Bounds for multiple access relay channels with receiver-source feedback via two-way relay channel was investigated in [16].

The existing results in the literature are that they all consider classic perspective of maximizing the achievable rates for given locations of the transmitters, relay and the destination. However, in many cases, the problem is to maximize coverage for the fixed desired transmission rates. The authors in [17] have studied Gaussian relay channel considering fixed transmission rate and optimal relay location to maximize the coverage region. In [18] the authors analyzed coverage region in MIMO relay channel over Rayleigh fading environment. The authors in [19] investigated the coverage region and energy efficiency for Gaussian relay channel for a specific network geometry. In [20] the coverage region for MIMO relay channels in Rayleigh and Rician fading environments were analyzed.

A. Motivation and our work

Although the coverage region in relay channels have been investigated, it has not been analyzed in Multiple Access Relay channel so far. In this paper we analyze the coverage region, as a fundamental wireless communications concept, in MARC considering fixed transmission rates and fixed relay location, based on our previous work [9]. First, we investigate the coverage area in the Gaussian MARC while the destination (Base Station) and the relay are located in the fixed locations at the cellular network. Next, we analyze the coverage region in the Rayleigh Fading environment.

B. Paper organisation

The rest of the paper is organized as follows: in Section II, we have definitions and background. In Section III, the main theorems are discussed. In section IV, numerical examples are provided for Gaussian and Rayleigh models. Finally, the conclusion is drawn in Section V.

II. DEFINITIONS AND BACKGROUND

First, in this section we describe the channel model. Next, the coverage definition in MARC is introduced. Finally, the achievable rate regions for Gaussian MARC are expressed.

A. Definitions

Channel Model

Consider multiple access relay channel as shown in Fig. 1. The sources (mobile users) s1 and s2 transmit the messages X_1 and X_2 respectively. Let us consider Y_R as the received signal at the relay

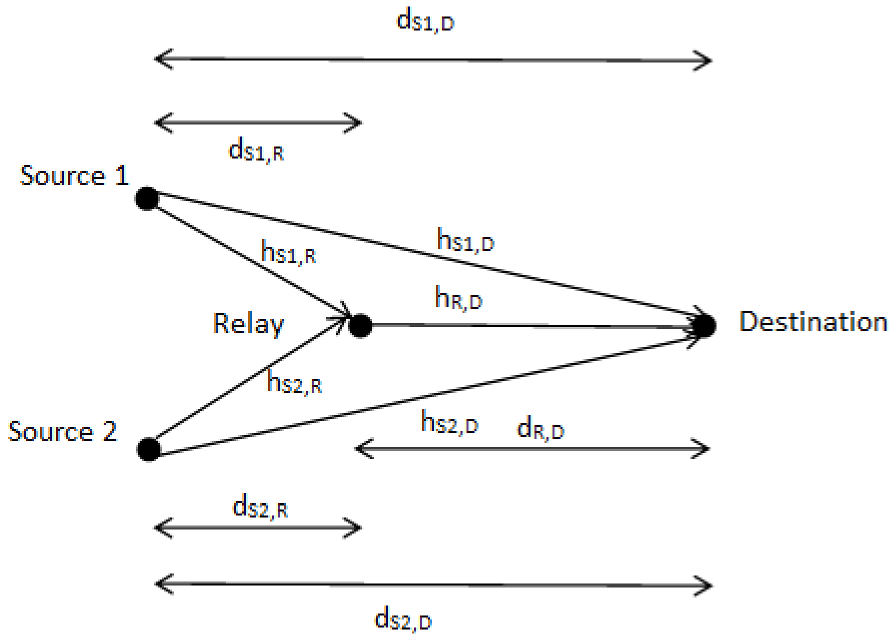

Figure 1. A two-user multiple-access relay channel

station. Based on prior received signals, the relay then transmits a message X_R that is intended to facilitate the sources transmission to the destination (Base station). Thus, the received signals at the base station and the relay can be expressed as:

$$Y_D = h_{S1,D}X_1 + h_{S2,D}X_2 + h_{R,D}X_R + Z_D \tag{1}$$
$$Y_R = h_{S1,R}X_1 + h_{S2,R}X_2 + Z_R \tag{2}$$

Where the destination (Base station) and the relay are placed in fixed locations and the sources (mobile users) can move in the cellular network. The channel gains are modeled as independent Z_R and Z_D zero-mean Gaussian noise at the relay and the destination respectively which are independent of each other and transmitted signals. Also we suppose α as the distance-based path-loss power attenuation exponent. $h_{S1,D}$, $h_{S2,D}$ is the channel gain between the source1, source2 and the destination, $h_{S1,R}$, $h_{S2,R}$ is the channel gain between the source1, source2 and the relay station and $h_{R,D}$ is the channel gain between the relay and the destination. Also we denote $d_{S1,D}$, $d_{S2,D}$ as the distance between the source1, source2 and the destination, $d_{S1,R}$, $d_{S2,R}$ as the distance between the source1, source2 and the relay and $d_{R,D}$ as the distance between the relay and the destination.

Coverage definition

In this paper the definition of the coverage has a closed relation to the concept of the outage capacity in [19] and to what defined in [15] for relay channel, and we consider it for MARC as a geographic region at which the rates of at least $R_1, R_2 > 0$ is guaranteed, i.e.,

$$A(d_{R,D}) \stackrel{\text{def}}{=} \{d_{S1,R}, d_{S1,D}: C(d_{R,D}, d_{S1,R}, d_{S1,D}) \geq R_1;\ d_{S2,R}, d_{S2,D}: C(d_{R,D}, d_{S2,R}, d_{S2,D}) \geq R_2;\ d_{S1,R}, d_{S2,R}, d_{S1,D}, d_{S2,D}: C(d_{R,D}, d_{S1,R}, d_{S2,R}, d_{S1,D}, d_{S2,D}) \geq R_1 + R_2\}, \quad (3)$$

where R_1, R_2 denotes the desired transmission rates for source 1 and source 2 in bps/Hz, respectively, $C(d_{R,D}, d_{S1,R}, d_{S1,D}), C(d_{R,D}, d_{S2,R}, d_{S2,D}), C(d_{R,D}, d_{S1,R}, d_{S2,R}, d_{S1,D}, d_{S2,D})$ are the channel capacity region terms when there is a fixed distance between the destination and the relay($d_{R,D}$), also we denote $d_{S1,D}$, $d_{S2,D}$ as the distance between the source1, source2 and the destination respectively, $d_{S1,R}$, $d_{S2,R}$ as the distance between the source1, source2 and the relay respectively as shown in Fig. 1.

B. Achievable rate regions for two-user Gaussian MARC

Theorem 1. [Theorem 1,[9]] The inner bound rate region for discrete memoryless (DM) MARC with common message via partial decode and forward strategy is the union set of rate tuples (R_0, R_1, R_2) satisfying:

$$\left\{\begin{array}{c} R_1 \leq \min\big(I(X_1; Y_R|X_R, X_2, U_0, V_0, V_1, V_2), I(X_1, X_R; Y_D|X_2, U_0, V_0, V_2)\big) \\ R_2 \leq \min\big(I(X_2; Y_R|X_R, X_1, U_0, V_0, V_1, V_2), I(X_2, X_R; Y_D|X_1, U_0, V_0, V_1)\big) \\ R_1 + R_2 \leq \min\big(I(X_1, X_2; Y_R|X_R, U_0, V_0, V_1, V_2), I(X_1, X_2, X_R; Y_D|U_0, V_0)\big) \\ R_0 + R_1 + R_2 \leq \min(I(X_1, X_2; Y_R|X_R, V_0, V_1, V_2), I(X_1, X_2, X_R; Y_D)) \end{array}\right\} \quad (4)$$

where R_0, R_1, R_2 are sources common message rate, source 1 private message rate and source 2 private message rate, respectively. U_0 is the common message sent by the relay, V_0 is the common message sent by the sources which can be decoded at the relay. V_1, V_2 are the private messages which can be decoded at the relay sent via source 1 and source 2, respectively.

Proof: refer to [9].

Theorem 2. [Theorem 3,[9]] An achievable rate region for Gaussian MARC with common message is given as follows:

$$\left\{\begin{array}{c} R_1 \leq min\left\{\begin{array}{c}\frac{1}{2}\log\left(1+\frac{\gamma_1 p_1}{N_R}\right), \\ \frac{1}{2}\log\left(1+\frac{(\gamma_1+\beta_1)p_1+\alpha_R p_R+2\sqrt{\alpha_R\beta_1 p_1 p_R}}{N_D}\right)\end{array}\right\} \\ R_2 \leq min\left\{\begin{array}{c}\frac{1}{2}\log\left(1+\frac{\gamma_2 p_2}{N_R}\right), \\ \frac{1}{2}\log\left(1+\frac{(\gamma_2+\beta_2)p_2+\beta_R p_R+2\sqrt{\beta_R\beta_2 p_2 p_R}}{N_D}\right)\end{array}\right\} \\ R_1+R_2 \leq \ min\left\{\begin{array}{c}\frac{1}{2}\log\left(1+\frac{\gamma_1 p_1+\gamma_2 p_2}{N_R}\right), \\ \frac{1}{2}\log\left(1+\frac{\gamma_1 p_1+\gamma_2 p_2+\left(\sqrt{\alpha_R p_R}+\sqrt{\beta_1 p_1}\right)^2+\left(\sqrt{\beta_R p_R}+\sqrt{\beta_2 p_2}\right)^2}{N_D}\right)\end{array}\right\} \\ R_0+R_1+R_2 \leq min \\ \left\{\begin{array}{c}\frac{1}{2}\log\left(1+\frac{\gamma_1 p_1+\gamma_2 p_2+\left(\sqrt{1-\alpha_1-\beta_1-\gamma_1}\sqrt{p_1}+\sqrt{1-\alpha_2-\beta_2-\gamma_2}\sqrt{p_2}\right)^2}{N_R}\right), \\ \frac{1}{2}\log\left(\begin{array}{c}1+\frac{p_1+p_2+p_R+2\sqrt{p_1 p_R}\left(\sqrt{\alpha_R\beta_1}+\sqrt{\alpha_1(1-\alpha_R-\beta_R)}\right)+2\sqrt{p_2 p_R}\left(\sqrt{\beta_R\beta_2}+\sqrt{\alpha_2(1-\alpha_R-\beta_R)}\right)}{N_D}+ \\ \frac{2\sqrt{p_1 p_2}\left(\sqrt{\alpha_1\alpha_2}+\sqrt{(1-\alpha_1-\beta_1-\gamma_1)(1-\alpha_2-\beta_2-\gamma_2)}\right)}{N_D}\end{array}\right)\end{array}\right\}\end{array}\right\} \quad (5)$$

Proof: refer to [9].

Explanatory note for the proof in Theorem 2: Theorem 2 is the extended version of Theorem 1 to the continuous alphabet Gaussian channel. As shown in Fig. 1, we consider independent AWGN Z_D, Z_R with the variance N_D, N_R respectively. The sources and the relay have individual allocated powers. Each node allocates to each message a portion of its available transmit power. Denote W_0 as the common message sent by the sources which cannot be decoded at the relay, T_1, T_2 as the private message which can be decoded at the relay sent by the sources 1,2 respectively. W_1, W_2 are the private message which cannot be decoded at the relay sent by the sources 1,2, respectively. Consider the independent, zero mean and unit variance Gaussian random variables $\mathrm{V_0}, \mathrm{W_0}, \mathrm{T_k}, \mathrm{W_k}, \mathrm{k} = 1{,}2$, The transmit signal can be written as:

$$\begin{array}{l} X_1 = \sqrt{P_1}\left(\sqrt{\alpha_1}V_0+\sqrt{\beta_1}T_1+\sqrt{1-\alpha_1-\beta_1-\gamma_1}W_0+\sqrt{\gamma_1}W_1\right) \\ X_2 = \sqrt{P_2}\left(\sqrt{\alpha_2}V_0+\sqrt{\beta_2}T_2+\sqrt{1-\alpha_2-\beta_2-\gamma_2}W_0+\sqrt{\gamma_2}W_2\right) \\ X_R = \sqrt{P_R}\left(\sqrt{\alpha_R}T_1+\sqrt{\beta_R}T_2+\sqrt{1-\alpha_R-\beta_R}V_0\right)\end{array} \quad (6)$$

where $\alpha_k, \beta_k and\ \gamma_k \in [0,1], k = 1{,}2$, such that $\alpha_k+\beta_k+\gamma_k \leq 1$ and $\alpha_R+\beta_R \leq 1$. Denote α_k as the portion of source power, P_k, allocated to the part of the message which is in common with the other source message's and can be decoded at the relay. Similarly, β_k is the portion of the source power, P_k, allocated to the part of the message which is private and can be decoded at the relay. γ_k is the portion of the source power, P_k, allocated to the private message which cannot be decoded at the relay. α_R *and* β_R are the portions of the relay power P_R allocated to the private message of the source 1 and source 2 respectively.

Considering the path loss and channel gains in Gaussian multiple access relay channel, the achievable rates in equation (5) would change as shown in the next Section.

III. Main Theorems

In this section, considering fixed desired transmission rates, we obtain coverage region in Gaussian MARC in cellular network based on an exact expression for the ergodic capacity and next, we extend the results to the Rayleigh fading Gaussian MARC. We consider there is no common message between the sources and we try to obtain the coverage region based on maximizing $R_1, R_1 + R_2$.

A- Coverage region in Gaussian MARC

In this subsection, we derive the coverage region of Gaussian Multiple- Access Relay Channel.

Theorem 3. The coverage region in Gaussian MARC is proved to be (as explained above, the coverage region is studied by considering the rates $R_1, R_1 + R_2$):

$$\left\{\begin{array}{ll} d_{S1,R} \le \left(\frac{\gamma_1 p_1}{N_R\left(e^{2R_1}-1\right)}\right)^{\frac{1}{\alpha}} & (7-a) \\ d_{S2,R} \le \left(\frac{\gamma_2 p_2}{A}\right)^{\frac{1}{\alpha}} & (7-b) \\ d_{S1,D} \le \left(\frac{(\gamma_1+\beta_1)p_1}{\sqrt{\frac{\alpha_R\beta_1 p_1 p_R}{d_{R,D}^{\alpha}}+(\gamma_1+\beta_1)p_1 B}-\sqrt{\frac{\alpha_R\beta_1 p_1 p_R}{d_{R,D}^{\alpha}}}}\right)^{\frac{2}{\alpha}} & (7-c) \\ d_{S2,D} \le \left(\frac{C}{\sqrt{A'^2+B'C}-A'}\right)^{\frac{2}{\alpha}} & (7-d) \end{array}\right\}$$

where: $A = N_R\left(e^{2(R_1+R_2)}-1\right) - \frac{\gamma_1 p_1}{d_{S1,R}^{\alpha}}$, $B = N_D(e^{2R_1}-1) - \frac{\alpha_R p_R}{d_{R,D}^{\alpha}}$, $C = (\gamma_2+\beta_2)p_2$, $A' = \sqrt{\frac{\beta_R\beta_2 p_2 p_R}{d_{R,D}^{\alpha}}}$, $B' = N_D\left(e^{2(R_1+R_2)}-1\right) - \frac{\gamma_1 p_1}{d_{S1,D}^{\alpha}} - \left(\sqrt{\frac{\alpha_R p_R}{d_{R,D}^{\alpha}}} + \sqrt{\frac{\beta_1 p_1}{d_{S1,D}^{\alpha}}}\right)^2 - \frac{\beta_R p_R}{d_{R,D}^{\alpha}}$ and also $d_{S1,D}$, $d_{S2,D}$ is the distance between the source1, source2 and the destination, $d_{S1,R}$, $d_{S2,R}$ is the distance between the source1, source2 and the relay, respectively. And $d_{R,D}$ is the distance between the relay and the destination.

Proof of $(\mathbf{7}-\boldsymbol{a})$***:*** The desired transmission rates R_1, R_1+R_2 by considering the sources-relay links in equation (5) and power attenuation due to path loss can be defined as follows:

$$\left\{\begin{array}{c} R_1 \leq \frac{1}{2}\log\left(1+\frac{\gamma_1 p_1}{N_R d_{S1,R}^{\alpha}}\right) \\ R_1+R_2 \leq \frac{1}{2}\log\left(1+\frac{\frac{\gamma_1 p_1}{d_{S1,R}^{\alpha}}+\frac{\gamma_2 p_2}{d_{S2,R}^{\alpha}}}{N_R}\right) \end{array}\right\} \tag{8}$$

In (8) we consider R_1 and R_1+R_2 as maximum achievable rates which the sources transmit and try to find maximum acceptable distance between the sources and the relay and the destination to satisfy equation (8). Then the distance between the source 1 and the relay ($d_{S1,R}$) can be expressed as follow:

$$d_{S1,R} \leq \left(\frac{\gamma_1 p_1}{N_R(e^{2R_1}-1)}\right)^{\frac{1}{\alpha}} \tag{9}$$

Proof of $(\mathbf{7}-\boldsymbol{b})$***:*** Regarding equations (8) and (9), the distance between the source2 and the relay station can be obtained as follows:

$$d_{S2,R} \leq \left(\frac{\gamma_2 p_2}{A}\right)^{\frac{1}{\alpha}} \tag{10}$$

where: $A = N_R\left(e^{2(R_1+R_2)}-1\right)-\frac{\gamma_1 p_1}{d_{S1,R}^{\alpha}}$.

Since R_1, R_2 cannot be maximized simultaneously, the coverage region of the source1 and source 2, regarding maximizing R_1, R_1+R_2, cannot be symmetric.

Proof of $(\mathbf{7}-\boldsymbol{c}),(\mathbf{7}-\boldsymbol{d})$ ***:*** Considering the sources-destination links in equation (5), the desired transmission rates for computing coverage area considering path loss can be expressed as:

$$\left\{\begin{array}{c} R_1 \leq \frac{1}{2}\log\left(1+\frac{(\gamma_1+\beta_1)\frac{p_1}{d_{S1,D}^{\alpha}}+\alpha_R\frac{p_R}{d_{R,D}^{\alpha}}+2\sqrt{\alpha_R\beta_1\frac{p_1}{d_{S1,D}^{\alpha}}\frac{p_R}{d_{R,D}^{\alpha}}}}{N_D}\right) \\ R_1+R_2 \leq \frac{1}{2}\log\left(1+\frac{\gamma_1\frac{p_1}{d_{S1,D}^{\alpha}}+\gamma_2\frac{p_2}{d_{S2,D}^{\alpha}}+\left(\sqrt{\alpha_R\frac{p_R}{d_{R,D}^{\alpha}}}+\sqrt{\beta_1\frac{p_1}{d_{S1,D}^{\alpha}}}\right)^2+\left(\sqrt{\beta_R\frac{p_R}{d_{R,D}^{\alpha}}}+\sqrt{\beta_2\frac{p_2}{d_{S2,D}^{\alpha}}}\right)^2}{N_D}\right) \end{array}\right\} \tag{11}$$

Similarly, by considering (11), the sources to destination distance can be obtained after computing some mathematical equations as follows:

$$d_{S1,D} \leq \left(\frac{(\gamma_1+\beta_1)p_1}{\sqrt{\frac{\alpha_R\beta_1 p_1 p_R}{d_{R,D}^{\alpha}}+(\gamma_1+\beta_1)p_1 B}-\sqrt{\frac{\alpha_R\beta_1 p_1 p_R}{d_{R,D}^{\alpha}}}} \right)^{\frac{2}{\alpha}} \tag{12}$$

where $B = N_D(e^{2R_1}-1) - \frac{\alpha_R p_R}{d_{R,D}^{\alpha}}$, and

$$d_{2D} \leq \left(\frac{C}{\sqrt{A'^2+B'C}-A'} \right)^{\frac{2}{\alpha}} \tag{13}$$

where: $A' = \sqrt{\frac{\beta_R\beta_2 p_2 p_R}{d_{R,D}^{\alpha}}}$, $C = (\gamma_2+\beta_2)p_2$, $B' = N_D\left(e^{2(R_1+R_2)}-1\right) - \frac{\gamma_1 p_1}{d_{S1,D}^{\alpha}} - \left(\sqrt{\frac{\alpha_R p_R}{d_{R,D}^{\alpha}}} + \sqrt{\frac{\beta_1 p_1}{d_{S1,D}^{\alpha}}}\right)^2 - \frac{\beta_R p_R}{d_{R,D}^{\alpha}}$.

Considering equations (9, 10, 12, 13), the geographical location of the sources can be obtained in the cellular network due to fixed location of destination (Base station) and relay.

B- Coverage region in Rayleigh fading Gaussian MARC

In this sub-section we consider wireless Rayleigh fading environment and extend the coverage region computation in this area.

Theorem 4. The coverage region in Rayleigh fading Gaussian MARC is proved to be:

$$\begin{cases} d_{S1,R} \leq \left(\frac{\gamma_1 p_1 \sigma_{1R}^2}{N_R} \frac{\pi^2}{\pi^2-16} \mathrm{Ln}\left(\frac{4R_1}{\sqrt{\pi}}\right) \right)^{\frac{1}{\alpha}} & (14-a) \\ d_{S1,D} \leq \left(\frac{2(\gamma_1+\beta_1)p_1\sigma_{S1,D}^2}{\sqrt{\frac{\pi^2\sigma_{R,D}^2\alpha_R\beta_1 p_1 p_R}{4 d_{R,D}^{\alpha}}+4(\gamma_1+\beta_1)p_1\sigma_{S1,D}^2\left(2R_1N_D-\frac{\alpha_R p_R\sigma_{R,D}^2}{d_{R,D}^{\alpha}}\right)}-\frac{\pi}{2}\sqrt{\frac{\sigma_{R,D}^2\alpha_R\beta_1 p_1 p_R}{d_{R,D}^{\alpha}}}} \right)^{\frac{2}{\alpha}} & (14-b) \\ d_{S2,D} \leq \left(\frac{2(\gamma_2+\beta_2)\sigma_{S2,D}^2 p_2}{\sqrt{B^2+4C(\gamma_2+\beta_2)\sigma_{S2,D}^2 p_2}-B} \right)^{\frac{2}{\alpha}} & (14-c) \end{cases}$$

where: $R_1 < \frac{\sqrt{\pi}}{4}$, $\mathrm{B} = \frac{\pi}{2}\sqrt{\frac{\beta_R\beta_2 p_2 p_R \sigma^2_{S2,D}\sigma^2_{R,D}}{d^{\alpha}_{R,D}}}$, $\mathrm{C} = 2(R_1 + R_2)N_D - \frac{(\gamma_1+\beta_1)\sigma^2_{S1,D}p_1}{d^{\alpha}_{S1,D}} - \frac{(\alpha_R+\beta_R)\sigma^2_{R,D}p_R}{d^{\alpha}_{R,D}} - \frac{\pi}{2}\sqrt{\frac{\alpha_R\beta_1 p_1 p_R \sigma^2_{S1,D}\sigma^2_{R,D}}{d^{\alpha}_{S1,D}d^{\alpha}_{R,D}}}$

and also $d_{S1,D}$, $d_{S2,D}$ is the distance between the source1, source2 and the destination, $d_{S1,R}$, $d_{S2,R}$ is the distance between the source1, source2 and the relay and $d_{R,D}$ is the distance between the relay and the destination. Note that an analytical closed form bound of $d_{S2,R}$ is not mentioned in Theorem 4. However, $d_{S2,R}$ is obtained numerically in numerical results.

Proof:

Part 1: As mentioned before, because of being an open problem of MARC capacity, the lower bound of channel is considered. Consider the wireless multiple access relay channel, the achievable rate regions in equation (4) regarding the equations (1), (2) are used to obtain Rayleigh fading Gaussian MARC achievable rates. Since in the Gaussian MARC the channel gains in equations (1), (2) are assumed to be 1, the achievable rate regions for $R_1, R_1 + R_2$ in the Rayleigh fading Gaussian MARC, assuming that the receiver and the relay are coherent, can be expressed by extending the Gaussian MARC achievable rates in equation (5) as follow:

$$\left\{\begin{array}{l} R_1 \le min\left\{\begin{array}{c} \frac{1}{2}\log\left(1+\frac{\gamma_1 p_1 |h_{S1,R}|^2}{N_R d^{\alpha}_{S1,R}}\right), \\ \frac{1}{2}\log\left(1+\frac{(\gamma_1+\beta_1)\frac{p_1|h_{S1,D}|^2}{d^{\alpha}_{S1,D}}+\alpha_R\frac{p_R|h_{R,D}|^2}{d^{\alpha}_{R,D}}+2\sqrt{\alpha_R\beta_1\frac{p_1p_R|h_{S1,D}|^2|h_{R,D}|^2}{d^{\alpha}_{S1,D}d^{\alpha}_{R,D}}}}{N_D}\right)\end{array}\right\} \\ R_1 + R_2 \le\ min\left\{\begin{array}{c} \frac{1}{2}\log\left(1+\frac{\frac{\gamma_1 p_1|h_{S1,R}|^2}{d^{\alpha}_{S1,R}}+\frac{\gamma_2 p_2|h_{S2,R}|^2}{d^{\alpha}_{S2,R}}}{N_R}\right), \\ \frac{1}{2}\log\left(1+\frac{\gamma_1\frac{p_1|h_{S1,D}|^2}{d^{\alpha}_{S1,D}}+\gamma_2\frac{p_2|h_{S2,D}|^2}{d^{\alpha}_{S2,D}}+\left(\sqrt{\alpha_R\frac{p_R|h_{R,D}|^2}{d^{\alpha}_{R,D}}}+\sqrt{\beta_1\frac{p_1|h_{S1,D}|^2}{d^{\alpha}_{S1,D}}}\right)^2+\left(\sqrt{\beta_R\frac{p_R|h_{R,D}|^2}{d^{\alpha}_{R,D}}}+\sqrt{\beta_2\frac{p_2|h_{S2,D}|^2}{d^{\alpha}_{S2,D}}}\right)^2}{N_D}\right)\end{array}\right\}\end{array}\right\} \quad (15)$$

Part 2: Since, the channel gains are random variables; in order to achieve certain rates, expectation should be computed. In Fig. 1, the links are independent with Rayleigh distributions and the square of a Rayleigh distribution ($|h|^2$) is an exponential distribution:

$$\mathrm{P}_{|h|^2}(|h|^2) = \frac{1}{\sigma^2}\mathrm{e}^{-\frac{h^2}{\sigma^2}} \tag{16}$$

where σ is scaling parameter in Rayleigh distribution.

Proof of $(\mathbf{14}-\boldsymbol{a})$***:*** Noting to the distribution of the $|h|^2$ and the sources-relay links in equation (15), the coverage region is obtained as follows:

$$R_1 \le E_{|h_{S1,R}|^2}\left[\frac{1}{2}\log\left(1+\frac{\gamma_1 p_1 |h_{S1,R}|^2}{N_R d_{S1,R}^{\alpha}}\right)\right] = \int_0^{\infty}(\frac{1}{2}\log\left(1+\frac{\gamma_1 p_1 |h_{S1,R}|^2}{N_R d_{S1,R}^{\alpha}}\right))(\frac{1}{\sigma_{S1,R}^2}e^{-\frac{h_{S1,R}^2}{\sigma_{S1,R}^2}})\, dh_{S1,R}^2$$
$$= \frac{1}{2\sigma_{S1,R}^2}\int_0^{\infty}\log(1+\frac{\gamma_1 p_1 |h_{S1,R}|^2}{N_R d_{S1,R}^{\alpha}})e^{-\frac{h_{S1,R}^2}{\sigma_{S1,R}^2}} dh_{S1,R}^2 \tag{17}$$

By computing the above expectation, the distance between source1 and the relay $(d_{S1,R})$ is obtained as follows:

$$\mathrm{d}_{\mathrm{S1,R}} \le \left(\frac{\gamma_1 \mathrm{p}_1 \sigma_{\mathrm{S1,R}}^2}{\mathrm{N_R}}\frac{\pi^2}{\pi^2-16}\mathrm{Ln}\left(\frac{4\mathrm{R}_1}{\sqrt{\pi}}\right)\right)^{\frac{1}{\alpha}} \tag{18}$$

The details of the proof can be found in Appendix A. it is notable that in the above equation $R_1 < \frac{\sqrt{\pi}}{4}$

Regarding equation (18) and the sources-relay links in equations (15), the distance between the source 2 and the relay can be obtained by computing the expectation over $|h_{S1,R}|^2$and $|h_{S2,R}|^2$ as follows:

$$R_1 + R_2 \le E_{|h_{S1,R}|^2}E_{|h_{S2,R}|^2}\left[\frac{1}{2}\log\left(1+\frac{\frac{\gamma_1 p_1 |h_{S1,R}|^2}{d_{S1,R}^{\alpha}}+\frac{\gamma_2 p_2 |h_{S2,R}|^2}{d_{S2,R}^{\alpha}}}{N_R}\right)\right] \tag{19}$$

$$\mathrm{P}_{|h_{S1,R}|^2}\left(|h_{S1,R}|^2\right) = \frac{1}{\sigma_{\mathrm{S1,R}}^2}\mathrm{e}^{-\frac{\mathrm{h}_{\mathrm{S1,R}}^2}{\sigma_{\mathrm{S1,R}}^2}} \tag{20}$$

$$\mathrm{P}_{|h_{S2,R}|^2}\left(|h_{S2,R}|^2\right) = \frac{1}{\sigma_{\mathrm{S2,R}}^2}\mathrm{e}^{-\frac{\mathrm{h}_{\mathrm{S2,R}}^2}{\sigma_{\mathrm{S2,R}}^2}} \tag{21}$$

By computing the above expectations, the sum of desired rates are as follows:

$$\mathrm{R}_1+\mathrm{R}_2 \le \frac{\sqrt{\pi}}{4}\left[\mathrm{e}^{\left(1-\frac{16}{\pi^2}\right)\left(\frac{\mathrm{d}_{\mathrm{S2,R}}^{\alpha}\mathrm{N_R}}{\gamma_2 \mathrm{p}_2 \sigma_{\mathrm{S2,R}}^2}\right)}\left(1-\frac{1}{1-\frac{\gamma_2 \mathrm{p}_2 \sigma_{\mathrm{S2,R}}^2 \mathrm{d}_{\mathrm{S1,R}}^{\alpha}}{\gamma_1 \mathrm{p}_1 \sigma_{\mathrm{S1,R}}^2 \mathrm{d}_{\mathrm{S2,R}}^{\alpha}}}\right)+\frac{1}{1-\frac{\gamma_2 \mathrm{p}_2 \sigma_{\mathrm{S2,R}}^2 \mathrm{d}_{\mathrm{S1,R}}^{\alpha}}{\gamma_1 \mathrm{p}_1 \sigma_{\mathrm{S1,R}}^2 \mathrm{d}_{\mathrm{S2,R}}^{\alpha}}}\mathrm{e}^{\left(1-\frac{16}{\pi^2}\right)\left(\frac{\mathrm{d}_{\mathrm{S1,R}}^{\alpha}\mathrm{N_R}}{\gamma_1 \mathrm{p}_1 \sigma_{\mathrm{S1,R}}^2}\right)}\right] \tag{22}$$

The maximum distance between the sources 2 and the relay ($d_{S2,R}$) can be obtained from (22). The details of the proof can be found in Appendix B.

Proof of $(\boldsymbol{14-b})$***:*** The sources to destination distances would be obtained by considering the source-destination links in equation (15). First the expected values are computed as follows:

$$R_1 \leq \frac{1}{2}\log\left(1+\frac{(\gamma_1+\beta_1)\frac{p_1|h_{S1,D}|^2}{d_{S1,D}^{\alpha}}+\alpha_R\frac{p_R|h_{R,D}|^2}{d_{R,D}^{\alpha}}+2\sqrt{\alpha_R\beta_1\frac{p_1p_R|h_{S1,D}|^2|h_{R,D}|^2}{d_{S1,D}^{\alpha}d_{R,D}^{\alpha}}}}{N_D}\right) \tag{23}$$

$$\mathrm{P}_{|h_{S1,D}|^2}\left(|h_{S1,D}|^2\right) = \frac{1}{\sigma_{S1,D}^2}\mathrm{e}^{-\frac{h_{S1,D}^2}{\sigma_{S1,D}^2}} \tag{24}$$

$$\mathrm{P}_{|h_{R,D}|^2}\left(|h_{R,D}|^2\right) = \frac{1}{\sigma_{RD}^2}\mathrm{e}^{-\frac{h_{R,D}^2}{\sigma_{R,D}^2}} \tag{25}$$

$$R_1 \leq E_{|h_{S1,D}|^2}E_{|h_{R,D}|^2}\left[\frac{1}{2}\log\left(1+\frac{(\gamma_1+\beta_1)\frac{p_1|h_{S1,D}|^2}{d_{S1,D}^{\alpha}}+\alpha_R\frac{p_R|h_{R,D}|^2}{d_{R,D}^{\alpha}}+2\sqrt{\alpha_R\beta_1\frac{p_1p_R|h_{S1,D}|^2|h_{R,D}|^2}{d_{S1,D}^{\alpha}d_{R,D}^{\alpha}}}}{N_D}\right)\right] \tag{26}$$

$$\mathrm{R}_1 \leq \frac{1}{2}\left[(\gamma_1+\beta_1)\frac{\mathrm{p}_1\sigma_{S1,D}^2}{\mathrm{N_D}\mathrm{d}_{S1,D}^{\alpha}}+\frac{\alpha_R\mathrm{p_R}\sigma_{R,D}^2}{\mathrm{N_D}\mathrm{d}_{R,D}^{\alpha}}+\frac{\pi}{2\mathrm{N_D}}\sqrt{\frac{\sigma_{R,D}^2\alpha_R\beta_1\mathrm{p}_1\mathrm{p_R}}{\mathrm{d}_{S1,D}^{\alpha}\mathrm{d}_{R,D}^{\alpha}}}\right] \tag{27}$$

The details of the proof can be found in Appendix C. From equation (27), the distance between source 1 and destination can be obtained as follows:

$$\mathrm{d}_{S1,D} \leq \left(\frac{2(\gamma_1+\beta_1)\mathrm{p}_1\sigma_{S1,D}^2}{\sqrt{\frac{\pi^2\sigma_{R,D}^2\alpha_R\beta_1\mathrm{p}_1\mathrm{p_R}}{4\mathrm{d}_{R,D}^{\alpha}}+4(\gamma_1+\beta_1)\mathrm{p}_1\sigma_{S1,D}^2(2\mathrm{R}_1\mathrm{N_D}-\frac{\alpha_R\mathrm{p_R}\sigma_{R,D}^2}{\mathrm{d}_{R,D}^{\alpha}})}-\frac{\pi}{2}\sqrt{\frac{\sigma_{R,D}^2\alpha_R\beta_1\mathrm{p}_1\mathrm{p_R}}{\mathrm{d}_{R,D}^{\alpha}}}}\right)^{\frac{2}{\alpha}} \tag{28}$$

Proof of $(\boldsymbol{14-c})$***:*** Similarly, by considering the source-destination links in equation (15) and the source1- destination distance in (28), the source2- destination distance can be obtained by computing the same expectations as follows:

$$R_1 + R_2 \leq \frac{1}{2}\log(1 + \frac{\gamma_1 \frac{p_1|h_{S1,D}|^2}{d_{S1,D}^{\alpha}} + \gamma_2 \frac{p_2|h_{S2,D}|^2}{d_{S2,D}^{\alpha}} + (\sqrt{\alpha_R \frac{p_R|h_{R,D}|^2}{d_{R,D}^{\alpha}}} + \sqrt{\beta_1 \frac{p_1|h_{S1,D}|^2}{d_{S1,D}^{\alpha}}})^2 + (\sqrt{\beta_R \frac{p_R|h_{R,D}|^2}{d_{R,D}^{\alpha}}} + \sqrt{\beta_2 \frac{p_2|h_{S2,D}|^2}{d_{S2,D}^{\alpha}}})^2}{N_D}) \qquad (29)$$

$$\mathrm{P}_{|h_{S2,D}|^2}\left(|h_{S2,D}|^2\right) = \frac{1}{\sigma_{S2,D}^2} e^{-\frac{h_{S2,D}^2}{\sigma_{S2,D}^2}} \qquad (30)$$

$$R_1 + R_2 \leq E_{|h_{S1,D}|^2} E_{|h_{S2,D}|^2} E_{|h_{R,D}|^2} \left[\frac{1}{2}\log(1 + \frac{\gamma_1 \frac{p_1|h_{S1,D}|^2}{d_{S1,D}^{\alpha}} + \gamma_2 \frac{p_2|h_{S2,D}|^2}{d_{S2,D}^{\alpha}} + (\sqrt{\alpha_R \frac{p_R|h_{R,D}|^2}{d_{R,D}^{\alpha}}} + \sqrt{\beta_1 \frac{p_1|h_{S1,D}|^2}{d_{S1,D}^{\alpha}}})^2 + (\sqrt{\beta_R \frac{p_R|h_{R,D}|^2}{d_{R,D}^{\alpha}}} + \sqrt{\beta_2 \frac{p_2|h_{S2,D}|^2}{d_{S2,D}^{\alpha}}})^2}{N_D})\right] \qquad (31)$$

$$\mathrm{R}_1 + \mathrm{R}_2 \leq \frac{1}{2}\left\{\frac{(\gamma_1+\beta_1)\sigma_{S1,D}^2 p_1}{N_D d_{S1,D}^{\alpha}} + \frac{(\gamma_2+\beta_2)\sigma_{S2,D}^2 p_2}{N_D d_{S2,D}^{\alpha}} + \frac{(\alpha_R+\beta_R)\sigma_{R,D}^2 p_R}{N_D d_{R,D}^{\alpha}} + \frac{\pi}{2N_D}\left(\sqrt{\frac{\alpha_R \beta_1 p_1 p_R \sigma_{S1,D}^2 \sigma_{R,D}^2}{d_{S1,D}^{\alpha} d_{R,D}^{\alpha}}} + \sqrt{\frac{\beta_R \beta_2 p_2 p_R \sigma_{S2,D}^2 \sigma_{R,D}^2}{d_{S2,D}^{\alpha} d_{R,D}^{\alpha}}}\right)\right\} \qquad (32)$$

The details of proof can be found in Appendix D. Also from equation (32), the distance between source 2 and destination can be obtained as follows:

$$d_{S2,D} \leq \left(\frac{2(\gamma_2+\beta_2)\sigma_{S2,D}^2 p_2}{\sqrt{B^2+4C(\gamma_2+\beta_2)\sigma_{S2,D}^2 p_2} - B}\right)^{\frac{2}{\alpha}} \qquad (33)$$

where: $B = \frac{\pi}{2}\sqrt{\frac{\beta_R \beta_2 p_2 p_R \sigma_{S2,D}^2 \sigma_{R,D}^2}{d_{R,D}^{\alpha}}}$, $C = 2(R_1 + R_2)N_D - \frac{(\gamma_1+\beta_1)\sigma_{S1,D}^2 p_1}{d_{S1,D}^{\alpha}} - \frac{(\alpha_R+\beta_R)\sigma_{R,D}^2 p_R}{d_{R,D}^{\alpha}} - \frac{\pi}{2}\sqrt{\frac{\alpha_R \beta_1 p_1 p_R \sigma_{S1,D}^2 \sigma_{R,D}^2}{d_{S1,D}^{\alpha} d_{R,D}^{\alpha}}}$

Of equations (18, 22, 28, 33), the geographical location of the sources can be obtained in Rayleigh fading GMARC due to fixed location of destination (Base station) and relay in the cellular network. As mentioned before, since we try to maximize $R_1, R_1 + R_2$ simultaneously, the coverage region of the sources cannot be symmetric.

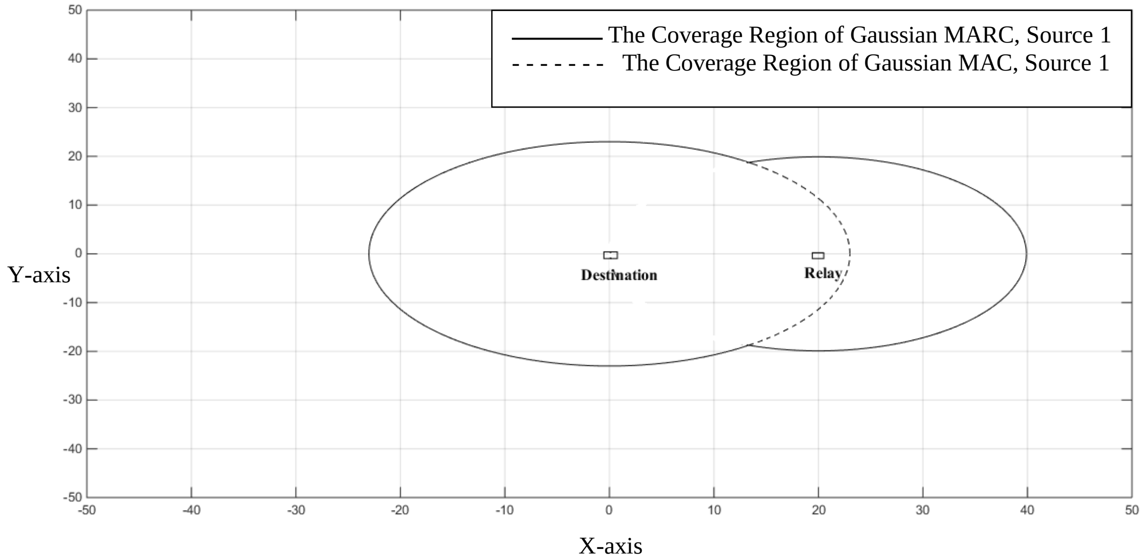

Figure 2. The Coverage region of Multiple-Access Channel, Source1, with and without the Relay.

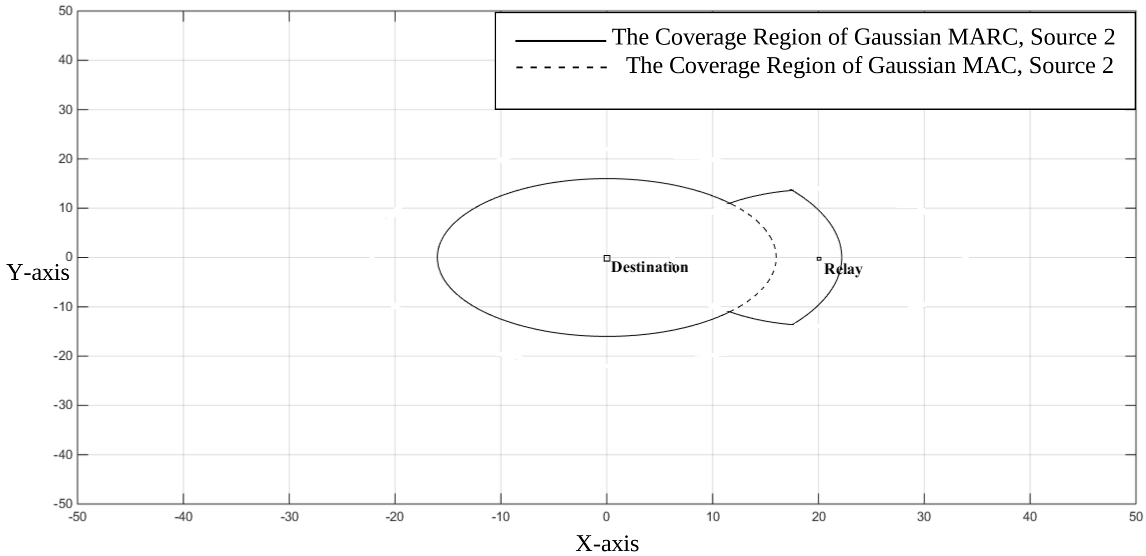

Figure 3. The Coverage region of Multiple-Access Channel, Source2, with and without the Relay.

IV. NUMERICAL RESULTS

In this section, with the help of computing we present the coverage area of Gaussian MARC and then extend it to Rayleigh GMARC and compare them with the coverage region of multiple access channel (MAC). For Gaussian MARC we assume that there is no common message between the sources and the channel parameters as: $\alpha = 3.52$, $\alpha_{1,2} = 0$, $\beta_1 = 0.4, \gamma_1 = 0.6$, $\beta_2 = 0.4, \gamma_2 = 0.6$, $\alpha_R = 0.35, \beta_R = 0.65, P_R = 0.2\, watt, P_{1,2} = 0.1\, watt, d_{R,D} = 20m, N_R = 10^{-7}$, $N_D = 10^{-7}, R_1 = 1.75, R_2 = 0.75$.

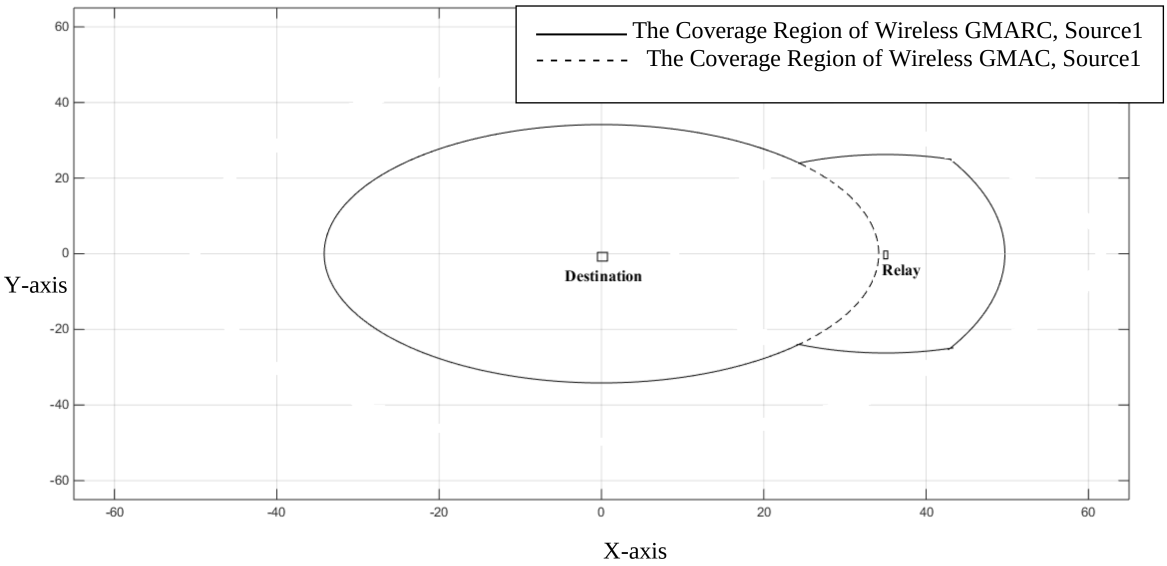

Figure 4.The Coverage region of Wireless Gaussian Multiple-Access Channel, Source1, with and without Relay.

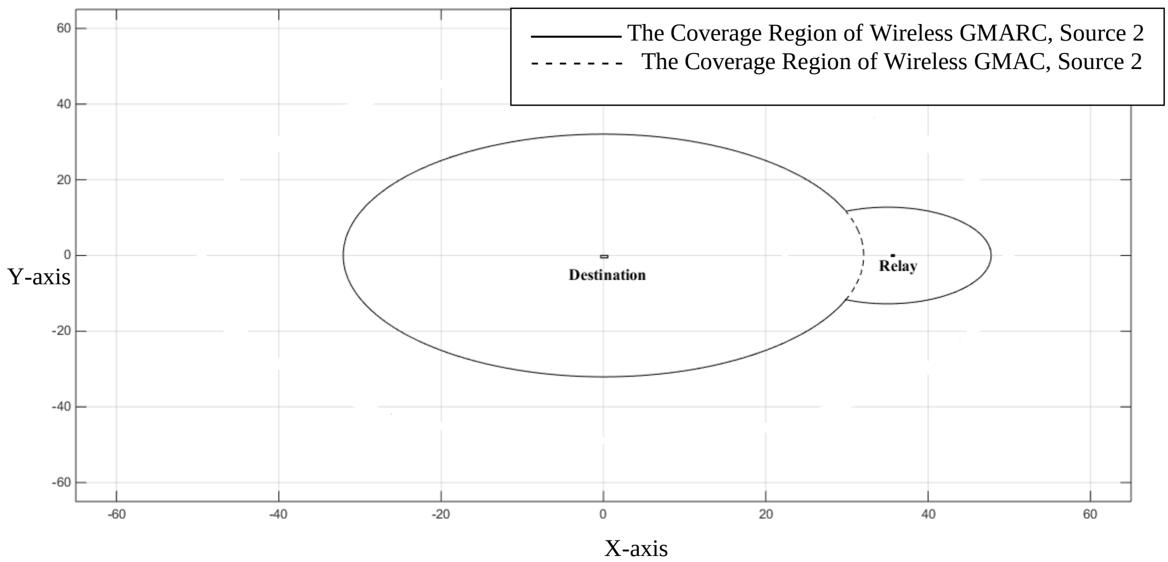

Figure 5. The Coverage region of Wireless Gaussian Multiple-Access Channel, Source2, with and without Relay.

Fig. 2 and Fig. 3 shows the coverage region for source 1 and source 2 based on the desired rates while the destination (Base Station) and the relay are located at fixed locations in (0,0) and ($d_{R,D}$,0) respectively. As shown in the figures by using the relay in the cellular network, the coverage area extends significantly compared with MAC. Furthermore, R_1, R_2 cannot be maximized simultaneously. As a result, the coverage region of the source1 and source2 regarding maximizing $R_1, R_1 + R_2$ cannot be symmetric.

The coverage area of Rayleigh GMARC is depicted in Fig. 4 and Fig. 5 while the destination (Base Station) and the relay are located at fixed locations in (0,0) and ($d_{R,D}$,0) respectively. In this situation the channel parameters are assumed to be: $\alpha = 3.52$, $\alpha_{1,2} = 0$, $\beta_1 = 0.4$, $\gamma_1 = 0.6$, $\beta_2 = 0.4, \gamma_2 = 0.6$, $\alpha_R = 0.35, \beta_R = 0.65, P_R = 0.2\ watt, P_{1,2} = 0.1\ watt, d_{R,D} = 35m, N_R = 10^{-7},\ N_D = 5 \times 10^{-7}, R_1 = 0.4, R_2 = 0.5$. As shown in the figures, the coverage region has been increased by using the relay station in the cellular network. Note that $d_{S2,R}$ is obtained numerically in numerical results from equation (22).

V. CONCLUSION

In this paper we investigated the influence of the relay on coverage region of cellular network in Multiple Access Channel. Considering the case of Gaussian MARC and Rayleigh Gaussian MARC, we defined the coverage region for the cellular network, and derived an exact analytical expression for desired transmission rates and fixed relay location at which the coverage region is maximized. Numerical results confirm the accuracy of our analysis, and show that using the relay station in Multiple Access Channel would improve the coverage area for fixed desired transmission rates, and also increases the transmission rates for a fixed coverage area in the cellular network.

APPENDIX A: Proof of Equation (18)

$$R_1 \leq E_{|h_{S1,R}|^2}\left[\frac{1}{2}\log\left(1 + \frac{\gamma_1 p_1 |h_{S1,R}|^2}{N_R d_{S1,R}^{\alpha}}\right)\right] = \int_0^{\infty}(\frac{1}{2}\log\left(1 + \frac{\gamma_1 p_1 |h_{S1,R}|^2}{N_R d_{S1,R}^{\alpha}}\right))(\frac{1}{\sigma_{S1,R}^2} e^{-\frac{h_{S1,R}^2}{\sigma_{S1,R}^2}})\, dh_{S1,R}^2$$

$$= \frac{1}{2\sigma_{S1,R}^2}\int_0^{\infty}\log(1 + \frac{\gamma_1 p_1 |h_{S1,R}|^2}{N_R d_{S1,R}^{\alpha}}) e^{-\frac{h_{S1,R}^2}{\sigma_{S1,R}^2}} dh_{S1,R}^2 \quad (34)$$

Referring to Integral table [20] we have:

$$\int_0^{\infty} e^{-pt}\log(1 + at)\, dt = -\frac{e^{\frac{p}{a}}}{p} Ei(-\frac{p}{a}) \quad (35)$$

where: $Ei(-x) = -\int_x^{\infty}\frac{e^{-t}}{t}dt$

Considering (35), the equation (34) can be obtained as:

$$R_1 \leq -\frac{1}{2}\exp(\frac{N_R d_{S1,R}^{\alpha}}{\gamma_1 p_1 \sigma_{S1,R}^2}) Ei(-\frac{N_R d_{S1,R}^{\alpha}}{\gamma_1 p_1 \sigma_{S1,R}^2}) \quad (36)$$

The exponential integral (Ei) was approximated in [21] as follows:

$$Ei(-x) = -4\sqrt{2} a_N a_I \sum_{n=1}^{N+1}\sum_{i=1}^{I+1}\sqrt{b_n} e^{-4 b_n b_i x} \quad (37)$$

where: $\theta_0 = 0 < \theta_1 < \theta_2 < \cdots < \theta_{N+1} = \frac{\pi}{2}$, $a_n = \frac{(\theta_n - \theta_{n-1})}{\pi}$, $b_n = \frac{1}{2}\frac{cot(\theta_{n-1}) - \cot(\theta_n)}{(\theta_n - \theta_{n-1})}$

For $N = I = 1$ we have: $a_N = a_I = \frac{1}{4}, b_1 = \infty, b_2 = \frac{2}{\pi}$. So, $Ei(-x)$can be approximated as:

$$\mathrm{Ei}(-\mathrm{x}) \sim -\frac{\sqrt{\pi}}{2} \mathrm{e}^{-\frac{16}{\pi^2}\mathrm{x}} \tag{38}$$

The equation (36) can be approximated as follows:

$$\mathrm{R}_1 \le \frac{\sqrt{\pi}}{4} \mathrm{e}^{(1-\frac{16}{\pi^2})\frac{\mathrm{N_R} \mathrm{d}_{\mathrm{S1,R}}^{\alpha}}{\gamma_1 \mathrm{p}_1 \sigma_{\mathrm{S1,R}}^2}} \tag{39}$$

With having fixed desired transmission rate, the source1-relay distance can be obtained:

$$\mathrm{d}_{\mathrm{S1,R}} \le \left(\frac{\gamma_1 \mathrm{p}_1 \sigma_{\mathrm{S1,R}}^2}{\mathrm{N_R}} \frac{\pi^2}{\pi^2 - 16} \mathrm{Ln}\left(\frac{4\mathrm{R}_1}{\sqrt{\pi}}\right) \right)^{\frac{1}{\alpha}} \tag{40}$$

APPENDIX B: Proof of Equation (22)

$$R_1 + R_2 \le E_{|h_{S1,R}|^2} E_{\,|h_{S2,R}|^2} \left[\frac{1}{2} \log \left(1 + \frac{\frac{\gamma_1 p_1 |h_{S1,R}|^2}{d_{S1,R}^{\alpha}} + \frac{\gamma_2 p_2 \,|h_{S2,R}|^2}{d_{S2,R}^{\alpha}}}{N_R} \right) \right] \tag{41}$$

$$E_{|h_{S1,R}|^2} \left[\frac{1}{2} \log \left(1 + \frac{\frac{\gamma_1 p_1 |h_{S1,R}|^2}{d_{S1,R}^{\alpha}} + \frac{\gamma_2 p_2 \,|h_{S2,R}|^2}{d_{S2,R}^{\alpha}}}{N_R} \right) \right] = \int_0^{\infty} \frac{1}{2} \log \left(1 + \frac{\frac{\gamma_1 p_1 |h_{S1,R}|^2}{d_{S1,R}^{\alpha}} + \frac{\gamma_2 p_2 \,|h_{S2,R}|^2}{d_{S2,R}^{\alpha}}}{N_R} \right) \left(\frac{1}{\sigma_{S1,R}^2} e^{-\frac{h_{S1,R}^2}{\sigma_{S1,R}^2}} \right) d|h_{S1,R}|^2 \tag{42}$$

Referring to Integral table [20] we have:

$$\int \mathrm{e}^{-\mu\mathrm{x}} \mathrm{Ln}(\mathrm{B} + \mathrm{x}) \mathrm{dx} = \frac{1}{\mu} [\mathrm{LnB} - \mathrm{e}^{\mu\mathrm{B}} \mathrm{Ei}(-\mu\mathrm{B})] \tag{43}$$

Considering (35), (43), the equation (42) is obtained as:

$$\frac{1}{2\sigma_{S1,R}^2} \int_0^{\infty} \log \left(1 + \frac{\frac{\gamma_1 p_1 |h_{S1,R}|^2}{d_{S1,R}^{\alpha}} + \frac{\gamma_2 p_2 \,|h_{S2,R}|^2}{d_{S2,R}^{\alpha}}}{N_R} \right) e^{-\frac{h_{S1,R}^2}{\sigma_{S1,R}^2}} d|h_{S1,R}|^2 \tag{44}$$

$$= \frac{1}{2\sigma_{S1,R}^2} \left[\int_0^{\infty} log \frac{\gamma_1 p_1}{N_R d_{S1,R}^{\alpha}} e^{-\frac{h_{S1,R}^2}{\sigma_{S1,R}^2}} d|h_{S1,R}|^2 + \int_0^{\infty} \log\left(\frac{1 + \frac{\gamma_2 p_2 \,|h_{S2,R}|^2}{N_R d_{S2,R}^{\alpha}}}{\frac{\gamma_1 p_1}{N_R d_{S1,R}^{\alpha}}} + |h_{S1,R}|^2 \right) e^{-\frac{h_{S1,R}^2}{\sigma_{S1,R}^2}} d|h_{S1,R}|^2 \right] \tag{45}$$

$$=\frac{1}{2}\left[log\frac{\gamma_1 p_1}{N_R d_{S1,R}^{\alpha}}+\log\left(\frac{1+\frac{\gamma_2 p_2\,|h_{S2,R}|^2}{N_R d_{S2,R}^{\alpha}}}{\frac{\gamma_1 p_1}{N_R d_{S1,R}^{\alpha}}}\right)-e^{\left(\frac{1+\frac{\gamma_2 p_2\,|h_{S2,R}|^2}{N_R d_{S2,R}^{\alpha}}}{\frac{\gamma_1 p_1 \sigma_{S1,R}^2}{N_R d_{S1,R}^{\alpha}}}\right)}Ei(-\left(\frac{1+\frac{\gamma_2 p_2\,|h_{S2,R}|^2}{N_R d_{S2,R}^{\alpha}}}{\frac{\gamma_1 p_1 \sigma_{S1,R}^2}{N_R d_{S1,R}^{\alpha}}}\right))\right] \quad (46)$$

by $A=\frac{\gamma_1 p_1}{N_R d_{S1,R}^{\alpha}}, C=\frac{\gamma_2 p_2}{N_R d_{S2,R}^{\alpha}}, t=\ |h_{S2,R}|^2$:

$$E_{|h_{S1,R}|^2\,|h_{S2,R}|^2}[\dots]=\int_0^{\infty}\frac{1}{2}\left[logA+\log\left(\frac{1+C}{A}\right)-e^{\left(\frac{1+C}{A\sigma_{S1,R}^2}\right)}Ei(-\left(\frac{1+C}{A\sigma_{S1,R}^2}\right))\right](\frac{1}{\sigma_{S2,R}^2}e^{-\frac{t}{\sigma_{S2,R}^2}})dt \quad (47)$$

Referring to Integral table [20] we have:

$$\int_0^{x} e^{-\beta x}Ei(-ax)dx=\frac{-1}{\beta}\left[e^{-\beta x}Ei(-ax)+Ln\left(1+\frac{\beta}{a}\right)-Ei(-(a+\beta)x)\right] \quad (48)$$

$$\int_0^{\infty} Ei(-\beta x)e^{-\mu x}dx=\frac{-1}{\mu}Ln(1+\frac{\mu}{\beta}) \quad (49)$$

Of the above equations, the equation (47) becomes:

$$R_1+R_2\le \frac{1}{2\sigma_{S2,R}^2\mu}\left[\exp\left(\frac{1}{C\sigma_{S2,R}^2}\right)Ei\left(-\frac{1}{C\sigma_{S2,R}^2}\right)-\exp\left(\frac{1}{A\sigma_{S1,R}^2}\right)Ei\left(-\frac{1}{A\sigma_{S1,R}^2}\right)\right]-\frac{1}{2}\exp\left(\frac{1}{C\sigma_{S2,R}^2}\right)Ei\left(-\frac{1}{C\sigma_{S2,R}^2}\right) \quad (50)$$

where: $\mu=\frac{1}{\sigma_{S2,R}^2}-\frac{C}{A\sigma_{S1,R}^2}$.

Using approximation of $Ei(-x)$as mentioned in (38) we have:

$$R_1+R_2\le\frac{\sqrt{\pi}}{4}\left[e^{\left(1-\frac{16}{\pi^2}\right)\left(\frac{d_{S2,R}^{\alpha}N_R}{\gamma_2 p_2\sigma_{S2,R}^2}\right)}\left(1-\frac{1}{1-\frac{\gamma_2 p_2\sigma_{S2,R}^2 d_{S1,R}^{\alpha}}{\gamma_1 p_1\sigma_{S1,R}^2 d_{S2,R}^{\alpha}}}\right)+\frac{1}{1-\frac{\gamma_2 p_2\sigma_{S2,R}^2 d_{S1,R}^{\alpha}}{\gamma_1 p_1\sigma_{S1,R}^2 d_{S2,R}^{\alpha}}}e^{\left(1-\frac{16}{\pi^2}\right)\left(\frac{d_{S1,R}^{\alpha}N_R}{\gamma_1 p_1\sigma_{S1,R}^2}\right)}\right] \quad (51)$$

APPENDIX C: Proof of Equation (28)

$$R_1\le E_{|h_{S1,D}|^2}E_{|h_{R,D}|^2}\left[\frac{1}{2}\log\left(1+\frac{(\gamma_1+\beta_1)\frac{p_1|h_{S1,D}|^2}{d_{S1,D}^{\alpha}}+\alpha_R\frac{p_R|h_{R,D}|^2}{d_{R,D}^{\alpha}}+2\sqrt{\alpha_R\beta_1\frac{p_1 p_R|h_{S1,D}|^2|h_{R,D}|^2}{d_{S1,D}^{\alpha}d_{R,D}^{\alpha}}}}{N_D}\right)\right] \quad (52)$$

$$E_{|h_{S1,D}|^2}[\dots] = \int_0^\infty \left(\frac{1}{2}\log\left(1+\frac{(\gamma_1+\beta_1)\frac{p_1|h_{S1,D}|^2}{d_{S1,D}^{\alpha}}+\alpha_R\frac{p_R|h_{R,D}|^2}{d_{R,D}^{\alpha}}+2\sqrt{\alpha_R\beta_1\frac{p_1p_R|h_{S1,D}|^2|h_{R,D}|^2}{d_{S1,D}^{\alpha}d_{R,D}^{\alpha}}}}{N_D}\right)\right)\left(\frac{1}{\sigma_{S1,D}^2}e^{-\frac{h_{S1,D}^2}{\sigma_{S1,D}^2}}\right)d|h_{S1,D}|^2 \tag{53}$$

Using the inequality: $\log(1+x) \le x$

$$\mathrm{E}_{|h_{1D}|^2}[\dots] \le \int_0^\infty \frac{1}{2}\left(\frac{(\gamma_1+\beta_1)\frac{p_1|h_{S1,D}|^2}{d_{S1,D}^{\alpha}}+\alpha_R\frac{p_R|h_{R,D}|^2}{d_{R,D}^{\alpha}}+2\sqrt{\alpha_R\beta_1\frac{p_1p_R|h_{S1,D}|^2|h_{R,D}|^2}{d_{S1,D}^{\alpha}d_{R,D}^{\alpha}}}}{N_D}\right)\left(\frac{1}{\sigma_{S1,D}^2}e^{-\frac{h_{S1,D}^2}{\sigma_{S1,D}^2}}\right)d|h_{S1,D}|^2 \tag{54}$$

We have:

$$\int_0^\infty \mathrm{x}\mathrm{e}^{-\mathrm{ax}}\mathrm{dx} = \frac{1}{\mathrm{a}^2} \tag{55}$$

$$\int_0^\infty \sqrt{\mathrm{x}}\mathrm{e}^{-\mu\mathrm{x}}\mathrm{dx} = \frac{\sqrt{\pi}}{2}\mu^{\frac{-3}{2}} \tag{56}$$

After using above equations, (54) becomes:

$$\mathrm{E}_{|h_{S1,D}|^2}[\dots] \le \frac{1}{2}\left[\frac{\alpha_R p_R|h_{R,D}|^2}{N_D d_{R,D}^{\alpha}} + \frac{(\gamma_1+\beta_1)p_1\sigma_{S1,D}^2}{N_D d_{S1,D}^{\alpha}} + \sqrt{\frac{\beta_1\alpha_R p_R|h_{R,D}|^2 p_1\pi\sigma_{S1,D}^2}{{N_D}^2 d_{S1,D}^{\alpha}d_{R,D}^{\alpha}}}\right] \tag{57}$$

$$R_1 \le E_{|h_{S1,D}|^2}E_{|h_{R,D}|^2}[\dots] \le \int_0^\infty \left(\frac{1}{2}\left[\frac{\alpha_R p_R|h_{R,D}|^2}{N_D d_{R,D}^{\alpha}} + \frac{(\gamma_1+\beta_1)p_1\sigma_{S1,D}^2}{N_D d_{S1,D}^{\alpha}} + \sqrt{\frac{\beta_1\alpha_R p_R|h_{R,D}|^2 p_1\pi\sigma_{S1,D}^2}{{N_D}^2 d_{S1,D}^{\alpha}d_{R,D}^{\alpha}}}\right]\right)\left(\frac{1}{\sigma_{R,D}^2}e^{-\frac{h_{R,D}^2}{\sigma_{R,D}^2}}\right)|h_{R,D}|^2 \tag{58}$$

$$\mathrm{R}_1 \le \frac{1}{2}\left[(\gamma_1+\beta_1)\frac{p_1\sigma_{S1,D}^2}{N_D d_{S1,D}^{\alpha}} + \frac{\alpha_R p_R\sigma_{R,D}^2}{N_D d_{R,D}^{\alpha}} + \frac{\pi}{2N_D}\sqrt{\frac{\sigma_{R,D}^2\alpha_R\beta_1 p_1 p_R}{d_{S1,D}^{\alpha}d_{R,D}^{\alpha}}}\right] \tag{59}$$

With having a fixed desired rate, the distance between source1 and destination can be obtained as follows:

$$d_{S1,D} \leq \left(\frac{2(\gamma_1+\beta_1)p_1\sigma_{S1,D}^2}{\sqrt{\frac{\pi^2\sigma_{R,D}^2\alpha_R\beta_1 p_1 p_R}{4d_{R,D}^{\alpha}}+4(\gamma_1+\beta_1)p_1\sigma_{S1,D}^2(2R_1N_D-\frac{\alpha_R p_R\sigma_{R,D}^2}{d_{R,D}^{\alpha}})}-\frac{\pi}{2}\sqrt{\frac{\sigma_{R,D}^2\alpha_R\beta_1 p_1 p_R}{d_{R,D}^{\alpha}}}}\right)^{\frac{2}{\alpha}} \tag{60}$$

APPENDIX D: Proof of Equation (33)

$$R_1 + R_2 \leq E_{|h_{S1,D}|^2}E_{|h_{S2,D}|^2}E_{|h_{R,D}|^2}$$

$$\left[\frac{1}{2}\log(1+\frac{\gamma_1\frac{p_1|h_{S1,D}|^2}{d_{S1,D}^{\alpha}}+\gamma_2\frac{p_2|h_{S2,D}|^2}{d_{S2,D}^{\alpha}}+(\sqrt{\alpha_R\frac{p_R|h_{R,D}|^2}{d_{R,D}^{\alpha}}}+\sqrt{\beta_1\frac{p_1|h_{S1,D}|^2}{d_{S1,D}^{\alpha}}})^2+(\sqrt{\beta_R\frac{p_R|h_{R,D}|^2}{d_{R,D}^{\alpha}}}+\sqrt{\beta_2\frac{p_2|h_{S2,D}|^2}{d_{S2,D}^{\alpha}}})^2}{N_D})\right] \tag{61}$$

Using the inequality: $\log(1+x) \leq x$ the expectation in (61) will be computed the same as expectation in Appendix C.

$$R_1 + R_2 \leq \frac{1}{2}\left\{\frac{(\gamma_1+\beta_1)\sigma_{S1,D}^2 p_1}{N_D d_{S1,D}^{\alpha}} + \frac{(\gamma_2+\beta_2)\sigma_{S2,D}^2 p_2}{N_D d_{S2,D}^{\alpha}} + \frac{(\alpha_R+\beta_R)\sigma_{R,D}^2 p_R}{N_D d_{R,D}^{\alpha}} + \frac{\pi}{2N_D}\left(\sqrt{\frac{\alpha_R\beta_1 p_1 p_R\sigma_{S1,D}^2\sigma_{R,D}^2}{d_{S1,D}^{\alpha}d_{R,D}^{\alpha}}} + \sqrt{\frac{\beta_R\beta_2 p_2 p_R\sigma_{S2,D}^2\sigma_{R,D}^2}{d_{S2,D}^{\alpha}d_{R,D}^{\alpha}}}\right)\right\} \tag{62}$$

With having fixed desired rates, the distance between source2 and destination can be obtained and equation (33) will be obtained.

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