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A Game-theoretic Approach with Cooperative Advertisement and Quality Level in Two-level Supply Chain with Considering Complementary Products and Uncertainty in Demand

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Abstract *– Vertical cooperative advertisement is one of the market strategies in which the manufacturer pays for a part of the supplier's local advertisements. Considering the quality level of products in the model and demand function is necessary and exciting. In this paper, a two-level supply chain with one manufacturer and one supplier, and one complementary manufacturer is developed, which advertisement costs share among manufacturer and retailer. It means that we have a cooperative advertisement in this model. The demand in this paper has uncertainty, and retailers order products from the manufacturer with uncertainty. This supply chain and distribution channel with uncertainty in retailers' demand, products quality level, and considering complementary price and qualities is one of the most practical models in this field. The main aim of these models is to give insights to managers and decision-makers by investigating the effect of the participation rate and some of the important parameters on other essential variables and profit functions and display the relationship between this rate and the trend of the variables. There are two non-cooperative games, including manufacturer Stackelberg and supplier Stackelberg. The results are obtained from these games, and the trends are demonstrated.*

Keywords– *Cooperative advertising, Game theory, supply chain, uncertainty, complementary product.*

I. INTRODUCTION

In today's world, the production and distribution of goods have increased by growth in customers' awareness, and members and supply chains do more to satisfy customers. By expanding internet use, customers have become more aware of the quality of products and their price. Therefore, retailers are obliged to provide suitable goods at reasonable prices to attract customers. On the other hand, this pressure comes from retailers to manufacturers or wholesalers and obliges them to provide the right quality and cost to buy their products. Overall, this increase in cooperation awareness enhances the profits of all production and distribution chain members and the quality level of the product.

Another issue dealt with in this paper is the type of advertising. In this new century, modern methods and advances have been made in advertising to impact the buyer's mind at a lower cost significantly. In the literature review, one of these methods is named cooperative advertising. The tremendous and main benefit of this kind of high-impact advertising is its low cost. Today, television or billboard advertisements cost a lot to companies and factories.

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On the other hand, many middle-class companies are not able to pay for such costs. And, as a result, companies are more interested in using cooperative advertising than before. In today's highly competitive market, effective advertising and reasonable spending on it, together with the understanding of making optimal advertising and cost decisions of manufacturers and retailers, depending on customers' buying behavior. The critical effect of goods quality on customers' views and goods advertising is another issue to be dealt with in this paper. We usually examine the impact and role of the price of the products. Our goal is not to deny the importance of the pricing debate, but our goal is to discuss proper pricing in this paper. Besides price, we will pay much attention to quality. In today's world, where people can find and buy their intended goods through a simple search in cyberspace, it is unwise not to attend to goods quality. Extensive research around the world has proven that quality will play a key role in family consumption.

Mathematical modeling of the contribution of cooperative advertising on the supply chain, modeling the effect of quality on the demand function of products from customers, optimization coupled with decisions about optimal price, participatory advertising of producer and consumer and product quality level, evaluation and comparison of decision-making conditions in the monopolistic and complementary markets and an assessment of the comparison of decision-making conditions of members in different markets are the essential purposes of this study.

Effective supply chain management requires cooperation among supply chain members. By coordinating activities across the supply chain, it is expected that there will be many benefits for the supply chain members. There are many different types of advertising in the world today. For example, mouth to mouth, TV advertisements, online advertisements, etc., are some of the most common advertising methods. The significance of attention to harmonization of supply chain members results from the idea that generally, the supply chains consist of independent decision-makers at different levels of a supply chain. None of these decision-makers have complete control over the entire supply chain, and consequently, none will have the ability to optimize the whole supply chain. It is reasonable to assume that any of these decision-makers are seeking to optimize their decisions. Therefore, their profitable decisions cannot lead to optimal policies for the entire supply chain. Researchers in the supply chain literature have proposed synchronization as a solution to this problem. The main issue in the supply chain is developing mechanisms to regulate supply chain decision makers' goals and coordinating their activities and their performance to achieve optimal performance throughout the system. Individual decision-making by these members, when their decisions are affected by and at the same time interdependent of each other, can result in optimal local performance at each of the different levels of the supply chain, and consequently, the inappropriate performance of the whole chain. Therefore, one of the most fundamental issues in supply chain management is developing strategies and mechanisms for regulating the chain members and coordinating their activities to optimize the overall system performance. Various research published in cooperative advertising separated by year of publication is presented in *Fig 1*.

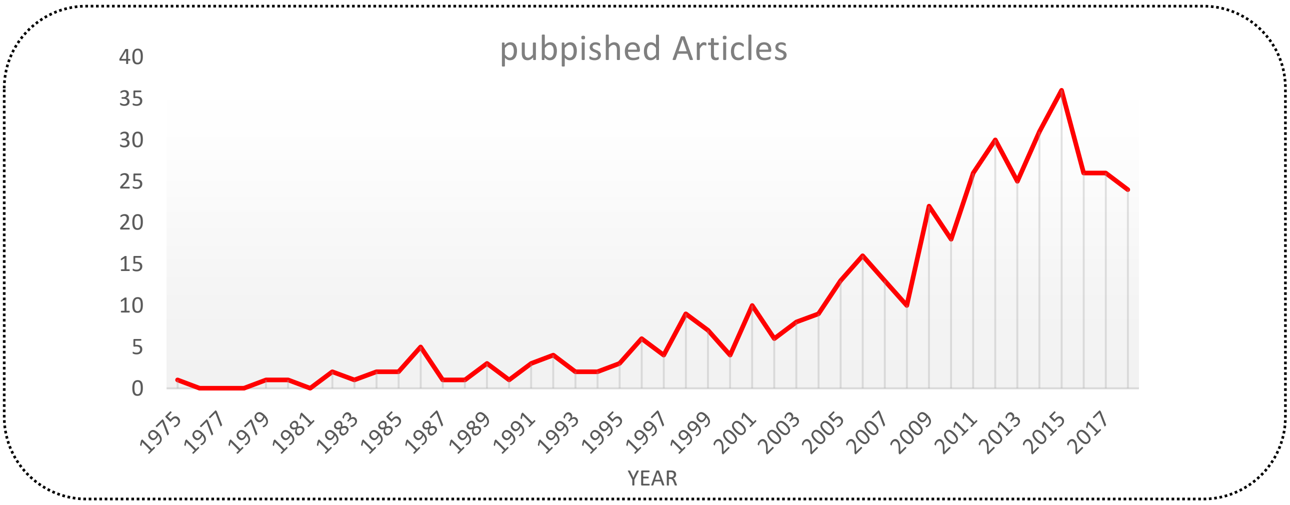

Figure. 1. Distribution of published articles in different years

Research cited for the year 2018 indicates papers published till now, and their number may increase until the end of the following year. As shown in the figure, the increase in the number of studies done in this area s promising. (Jorgensen and Zaccour, 2014) and (Aust and Buscher, 2014) made a thorough review of the literature on cooperative advertising. Furthermore, (Spais and Papakonstantinidis, 2014) examined the statistical analysis and prediction of the research process in the field of cooperative advertising. These studies indicate the significance and novelty of this topic and many research gaps to be studied by future researchers. A summary of studies in this field separated by the variables considered, the conditions studied (Static, dynamic, certainty, or uncertainty), market conditions (exclusive or competitive), and the function of interdependence are shown in Table I. In this table, the purpose of certainty or uncertainty is whether or not the demand function is specific. In most of these studies, the demand function is assumed to be specific. The literature on the field of uncertainty is extensive and comprehensive. (Pishvaee and Torabi, 2010) designed a model for the uncertain demand of a multi-cycle chain. Their model was an integer programming model to reduce costs. (Geng, and Mallik, 2011), in their paper, presented their model for multi-product applications. In the model presented by Chambers et al., 2006), quality in the demand function is also taken into account. (Wang et al., 2013) offered one of the best studies in this literature. In their model, which dynamically looked at the problem, important variables of price and quality are simultaneously seen. Another point is about the type of market. If the researcher has used a manufacturer and a retailer in his research, there exists a monopoly. Most studies are done in this way, and this will reduce the complexity of the problem. If at least one of them is more than one, and competition has been formed between retailers and wholesalers, the market will be defined as competitive. Also, if the researcher has paid attention to the complementary producers and the role of their decisions, in fact, the market is called complementary. Toothbrush and toothpaste, car, and gasoline are typical examples of complementary products. Demand function in fixed or static models given that collaborative advertising is the subject of the survey; in all mathematical models done in previous research, advertising is considered one of the decision variables. The advertiser variable in the mentioned mathematical models is considered in two ways: advertising cost or advertising level. Customer demand is also calculated as a function of the cost of advertising. If the variable is defined as ad level, it must first be converted by function to the cost of advertising, and then, according to the calculated price, customer demand should be obtained. In most mathematical models, a second-level function has been used to convert advertising levels to advertising costs. Table I explains the literature review.

Table I- Literature review

Reference	***variables***				***Certain/ Uncertain Demand***	***Market***		
	Advertisement	***Price***	***Quality***	***Inventory***		***monopoly***	***Competitive***	***Complementary***
Wang (2006)		●			Uc			●
Mukhopadhyay et al. (2011)		●			C			●
He et al. (2013)	●	●			C		●	
Zhang & Yang (2013)	●	●			C		●	
He et al. (2014)	●	●		●	C		●	
Zhao et al. (2016)	●	●			C	●		
Zhou et al. (2016)	●				C	●		
Zhou et al. (2018)	●			●	Uc	●		
Martin-Herrán and Sigué (2017)	●	●			C	●		

Continue Table I- Literature review

Reference	*variables*				*Certain/ Uncertain Demand*	*Market*		
	Advertisement	*Price*	*Quality*	*Inventory*		*monopoly*	*Competitive*	*Complementary*
Li et al. (2017)	●				C	●		
Karay et al. (2017)	●	●			C		●	
Karay et al. (2017)	●	●			C		●	
Wang et al. (2018)	●	●			C	●		
Zhang et al. (2017)	●				C	●		
Zhang et al. (2017)	●				C	●		
Li & Chen (2018)		●	●		C		●	
Ezimadu (2018)	●	●			C	●		
Zhang and Zhang (2018)	●	●			Uc	●		
Farshbaf-Geranmayeh et al. (2017)	●	●			C	●		
Johari and Hosseini-Motlagh (2018)	●	●			C		●	
Sacco and De Giovanni (2019)	●	●			Uc	●		
De Giovanni et al. (2019)	●	●		●	C	●		
Zhou et al. (2018)	●	●			C		●	
Vasnani et al. (2018)	●	●			C		●	
Xu et al. (2018)	●	●			Uc	●		
Farshbaf-Geranmayeh et al. (2018)	●	●			C	●		
This paper	●	●	●		Uc			●

There are other papers in cooperative advertisement fields which are relative to this paper. (Lu et al., 2019) presented the model for cooperative advertisement with sticky price and prisoner's dilemma game model. The impacts of co-op advertising on channel members' behavior choices were analyzed in that paper. (Lu et al., 2019) developed the model about the contract in a supply chain considering dynamic advertising. They believed that supply chain contracts are widely regarded as valuable and necessary tools to guide and restrict channel members' behaviors. Comparing static and dynamic pricing is one of the most popular concepts among many researchers. Many papers explain a different kind of pricing in a supply chain with advertising. (Zhang et al., 2017), (Lu et al., 2016), (Liu et al., 2015); and (Feng et al., 2015) in their papers, discussed pricing in the supply chain with advertising. Productiveness and the effect of advertisement and cooperative advertisement in the supply chain is another concept that is common in many papers. (Nie and Zhang, 2017), (Lu et al., 2016) and (Zhang et al., 2015) are some of these papers. Asymmetric demand information is used in some researches. For example, (Zhang et al., 2019) investigate manufacturer encroachment with both endogenous quality decision and asymmetric demand information to examine the effects of encroachment and information structure on quality and profits for chain members. And they shared many results in different situations in that paper.

In recent years, so many articles published about this area. This fact clearly shows the importance of this area. (Sarkar et al., 2020) In their paper, the optimal results of the co-op advertisement ensured an increase in the revenue of the whole supply chain. (Liu and Liu, 2020) considered in-App advertising through the optimal control theory. That paper establishes a dynamic advertising strategy model of one platform and n Apps under the decentralized and integrated systems. Also (Li et al., 2020) have addressed pricing and cooperative advertising in the online supply chain. (Cao et al., 2020), (Alirezaee and Sadjadi, 2020), (Ezimadu, 2020) and (Alaei et al., 2020) are other examples of recent articles that have been examined.

According to the presented materials, the following points can be described as gaps in this literature:

Participatory advertising is one of the most effective and cheapest advertising methods and influencing customers, given its positive attributes. But one of the most critical points in this paper is the role of a quality variable. Considering the quality and presence of demand function, it is a significant research gap. Paying attention to the manufacturers of complementary goods and basically, the role of complementary goods in the demand for our product is another advance in research. Few researchers have modeled the problem with other competitors. However, there is no concern with the manufacturer of the complementary product. The synchronization literature in the supply chain indicates that the contribution to the supply chain members is profitable. The profitability of the various markets is different. The role of participatory advertising in a complete monopoly is different from when the person has rival or coordination with the manufacturer of a complementary product. And basically, we compare the effectiveness of this type of advertising in different markets. The standard assumption of demand is one of the other things that can be expressed. Modeling ads based on quality variables, prices, and model advertising leads the model one step closer to reality. But the uncertainty of demand can be another step forward in the pursuit of reality.

II. MODEL DESCRIPTION

For a comprehensive review of this topic and the topics discussed, we define two general models. As indicated in the following table, each of these two topics is divided into several different modes. Each of these different scenarios is in line with the research gap introduced in the previous chapter and will help solve the research questions. Fig. 2 will explain these steps.

A. Cooperative advertising and pricing, taking into account the quality of products in a two-level chain

The statement of the problem:

In a two-level chain, including a wholesaler and a retailer, wholesalers sell their products with a q-quality level at a wholesale price w to the retailer, and retailers sell them at retail prices p. In the case of a partnership, the participation rate is t. The first point that can be expressed for modeling this problem is the introduction of the problem function. In the first problem, the demand function is defined definitively. The form for the function of the first problem is as follows:

$$D = \alpha - \beta * p + \gamma * a + \mu * q \tag{1}$$

This demand function indicates that product demand depends on price, advertising level, and product quality. And as stated here, the demand is explicitly examined. The relationship between demand and price is inversely proportional to advertising and direct quality.

The higher the price of a product, the lower its demand, and the more local products are advertised on the product, or the quality is more favorable, the higher the demand is. The parameters β, γ, and μ, express the customer's sensitivity

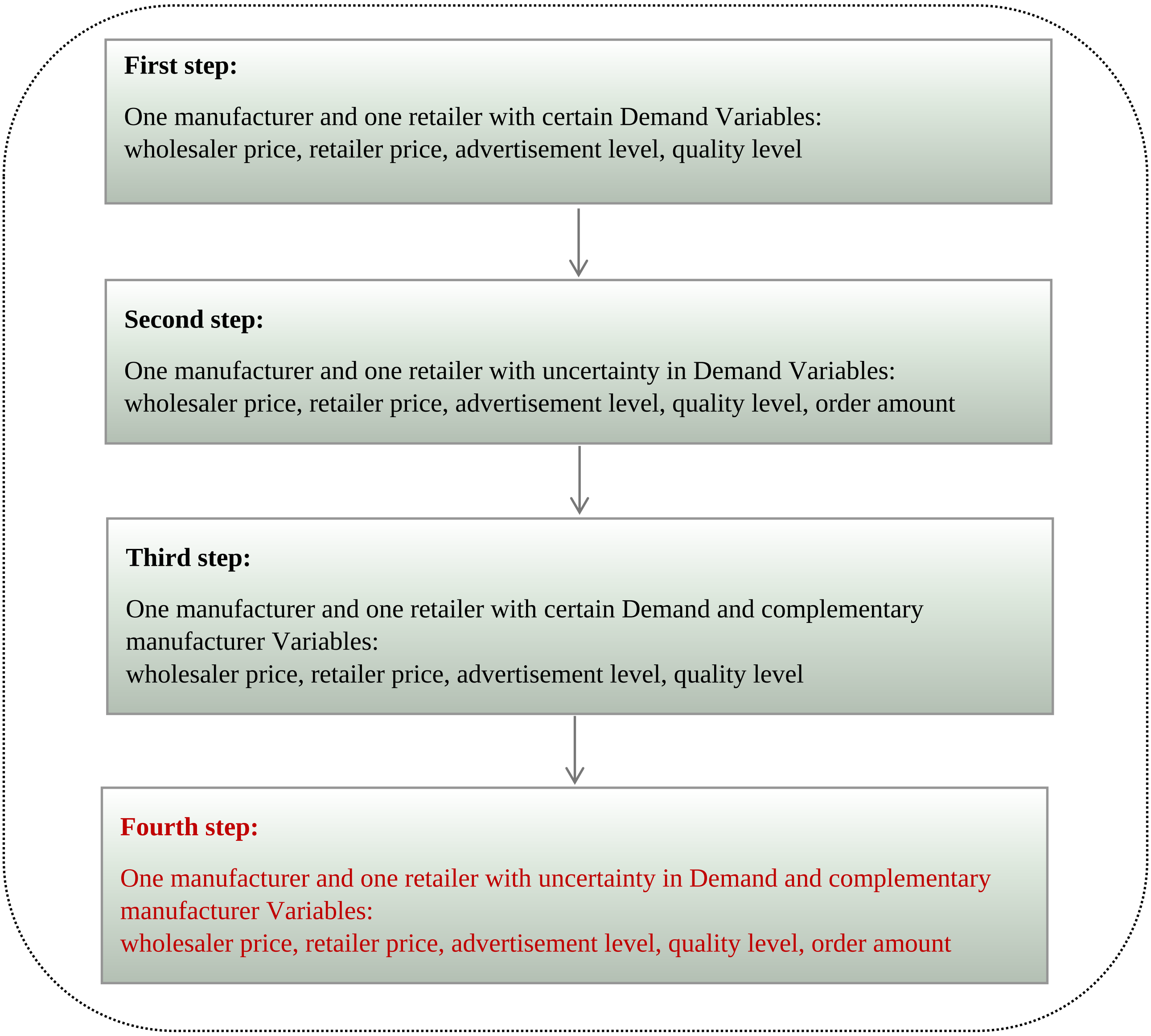

Figure. 2. Description of modeling steps

to price, level of advertising, and quality level. And as you can see, these three parameters point out the real numbers between zero and one. The value of these three parameters provides valuable and accurate information to decision-makers to pinpoint their decisions and strategies.

It is also worth mentioning that modeling the demand for this linear form is a common method in the literature on this subject.

After defining the demand function, it is now time to define the producer and retailer profit functions. As we have already stated, in the first problem, we follow two modeling modes. There is no cooperation between the producer and the retailer to use cooperative advertising in the first case. We have:

$$\pi_M = D * w - a^2 - s * q^2 \tag{2}$$

$$\pi_M = (\alpha - \beta * p + \gamma * a + \mu * q) * w - a^2 - s * q^2 \tag{3}$$

$$\pi_R = D * (p - w) \tag{4}$$

$$\pi_R = (\alpha - \beta * p + \gamma * a + \mu * q) * (p - w) \tag{5}$$

But if the decentralized supply chain is different, the definition of the profit functions of the members of this chain will be different from the two levels; this type of modeling is usual in cooperative advertising literature, for example (Karray et al., 2017a,b) used this type of modeling and proved the correctness of the statements, and will be as follows:

$$\pi_M = D * w - t * a^2 - s * q^2 \tag{6}$$

$$\pi_M = (\alpha - \beta * p + \gamma * a + \mu * q) * w - t * a^2 - s * q^2 \tag{7}$$

$$\pi_R = D * (p - w) - (1 - t) * a^2 \tag{8}$$

$$\pi_R = (\alpha - \beta * p + \gamma * a + \mu * q) * (p - w) - (1 - t) * a^2 \tag{9}$$

In the related literature and studies that use uncertainty, there are various ways to model this uncertainty. We also intend to use one of these methods to model demand. The uncertain demand in this issue depends on price, advertising level, and product quality level. Here we have:

$$D = \alpha - \beta * p + \gamma * a + \mu * q + \varepsilon \tag{10}$$

Where ε is a function of the uncertainty representation that has a uniform distribution function. In fact we have $\varepsilon \sim uniform[A, B]$. Now the change in the form of the demand function also affects the retail and producer profit functions.

There are two perspectives about the profit of the retailer. First, the retailer's order quantity from the producer is more than the market demand. In this case, the retailer sells as much as the demand, and the additional amount of the product has to sell in the secondary market at a lower price. But if the retail order is lower than demand, then the retailer faces a lost opportunity. In this case, retail sells less than market demand, does not have the product to offer to customers who are in greater demand, and does not offer the opportunity cost. If we show the retailer's order quantity with the Q, then the retail sales will be equal to Min (D, Q) in both cases.

In the case of uncertainty of demand, we must first calculate the profit function instead of the profit function of the retailer. Here we have:

$$E[\pi_R] = E[\min(D, Q) * p - Q * w - i * E[D - Q]^+ - j * E[Q - D]^+] - (1 - t) * a^2 \tag{11}$$

In this regard, i represents the opportunity cost of the loss and relates to when the demand is greater than the order, and j represents the sale in another market at a lower price and occurs when the demand for the order is lower.

To simplify the equation (3-16), we define a variable called Z:

$$Z = Q - (\alpha - \beta * p + \gamma * a + \mu * q) = Q - D + \varepsilon \tag{12}$$

$$Z - \varepsilon = Q - D \tag{13}$$

Taking this definition into account, we have:

$$Q = (\alpha - \beta * p + \gamma * a + \mu * q) + Z \tag{14}$$

Now for computing E [Min (D,Q)], we can say:

$$\mathrm{E}[\mathrm{Min}\ (\mathrm{D},\mathrm{Q})] = E[\alpha - \beta * p + \gamma * a + \mu * q + \min[\mathrm{Z}, \varepsilon]] \tag{15}$$

$$\mathrm{E}[\mathrm{Min}\ (\mathrm{D},\mathrm{Q})] = \alpha - \beta * p + \gamma * a + \mu * q + E[\min[Z, \varepsilon]] \tag{16}$$

As previously stated, ε has a uniform distribution function and is in range A to B. And with the same probability of number in this range. The value of Z is also in the range that the retailer determines.

Now to get $E[\min[Z, \varepsilon]$, proceed as follows:

$$E[\min[Z, \varepsilon] = \int_A^Z x f(x)\, dx + \int_Z^B z f(x)\, dx = \tau - \theta(z) \tag{17}$$

$$\mathrm{E}[D - Q]^+ = E[\varepsilon - Z]^- = \theta(z) \tag{18}$$

$$\mathrm{E}[Q - D]^+ = E[Z - \varepsilon] = Z - E[\min(Z, \varepsilon)] = Z - \tau + \theta(z) \tag{19}$$

Given the uniform distribution function presented, we calculate the values of τ and θ (z).

$$\tau = \frac{A + B}{2}$$

$$\theta(z) = \frac{B + Z}{2} - Z\frac{B - Z}{B - A}$$

As the values of all variables are known, we determine the profit-return function of the retailer and the producer's profit function.

$$\begin{aligned}\mathrm{E}[\pi_R] = &\left(\alpha - \beta * p + \gamma * a + \mu * q + \frac{A + B}{2} - \frac{B + Z}{2} + Z\frac{B - Z}{B - A}\right) * p - Q * w - i\left(\frac{B + Z}{2} - Z\frac{B - Z}{B - A}\right) \\ &- j\left(Z - \frac{A + B}{2} + \frac{B + Z}{2} - Z\frac{B - Z}{B - A}\right) - (1 - t) * a^2\end{aligned} \tag{20}$$

$$\mathrm{E}[\pi_M] = \big(Z + (\alpha - \beta * p + \gamma * a + \mu * q)\big) * w - t * a^2 - s * q^2 \tag{21}$$

With considering complementary manufacture, the demand function and profit functions become:

How to enter the complementary commodity variable was explained based on the concepts of microeconomics.

$$D = \alpha - \beta * p + \gamma * a + \mu * q - \beta' * \ p' + \ \mu' * q' + \varepsilon \tag{22}$$

$$\mathrm{E}[\pi_R] = \left(\alpha - \beta * p + \gamma * a + \mu * q + \frac{A+B}{2} - \frac{B+Z}{2} + Z\frac{B-Z}{B-A}\right) * p - Q * w - i\left(\frac{B+Z}{2} - Z\frac{B-Z}{B-A}\right) - j\left(Z - \frac{A+B}{2} + \frac{B+Z}{2} - Z\frac{B-Z}{B-A}\right) - (1-t) * a^2 \tag{23}$$

$$\mathrm{E}[\pi_M] = \big(Z + (\alpha - \beta * p + \gamma * a + \mu * q)\big) * w - t * a^2 - s * q^2 \tag{24}$$

For the convex functions of advertising and quality and the backing of the literature of this subject, we use second-order functions to use advertising and quality in profit functions.

Concerning cooperative advertising, variable a represents the level of cooperative advertising. To convert this level into cost and use it in the earnings function, we must use its second power (g (a) = a^2). For quality, the story is the same, with the difference that the quality, when it comes out of the surface and reaches power, becomes a variable between zero and one, which, with the coefficient multiplied, becomes the genus of other variables.

In this case, the decision variables are producer price, retail price, cooperative advertising, order amount, and product quality. In this study, we will consider the participation rate as a parameter to have a closer look at the rate of participation, and we examine the sensitivity analysis for its various quantities, producer and retailer behavior.

B. Solving the initial model

The non-cooperative game of manufacture Steckelberg:

To solve the first problem when dealing with cooperative advertising, we use the Steckelberg equilibrium. To do this, we first examine the balance of the leading manufacturer of Steckelberg. In this case, the manufacturer is a leading one, and the retailer is a follower.

As a result, a retailer decides first. The manufacturer will make the optimal decision based on the optimal answer of the retailer.

$$\frac{\partial \pi_R}{\partial p} = (-2p + w) * \beta + \gamma * a + \mu * q + \alpha \tag{25}$$

$$\frac{\partial \pi_R}{\partial a} = \gamma * (p - w) + (2 * (t - 1)) * a \tag{26}$$

To obtain the optimal values of these variables, we use two equations of two unknowns.

Equations (25) and (26) are simultaneously equal to zero, and in the form of a two-dimensional equation, the optimal values are obtained.

The concave function of the objective function is proved to show that the solutions obtained from this system are optimal. This proof is also explained in the appendix.

The optimal values obtained for the price and local advertising from the above device are respectively $p =$

$\frac{2w(t-1)\beta+(2\mu q+2\alpha)t+\gamma^2 w-2\mu q-2\alpha}{(4t-4)\beta+\gamma^2}$ and $a = \frac{-\gamma(-\beta w+\mu q+\alpha)}{(4t-4)\beta+\gamma^2}$.

$$\frac{\partial \pi_M}{\partial q} = \frac{4w\mu(\left(\gamma-\frac{1}{2}\right)(t-1)\beta+\frac{1}{4}\gamma^2(\gamma-1)}{4\beta(t-1)+\gamma^2} + t(2qs + \frac{\gamma\mu}{(4\beta t+\gamma^2-4\beta)^2} \tag{27}$$

$$\frac{\partial \pi_M}{\partial w} = \frac{8\beta(-2w(t-1)^2\beta^2 + \left((\mu q+\alpha)t^2 + \left(-\frac{3}{4}\gamma^2 w - 2\mu q - 2\alpha\right)t + \frac{1}{2}\gamma^2 w + \mu q + \alpha\right)\beta + \frac{1}{2}(\alpha+\mu q)\gamma^2\left(t-\frac{1}{2}\right))}{(4\beta t+\gamma^2-4\beta)^2} \tag{28}$$

Now, the producer decides on the optimal values for these two variables and maximizes his profit.

Again we have partial derivatives.

This time, to obtain optimal solutions, we solve two equations of two unknowns and the concave function of this section is described below.

The following equation is used to prove the concatenation of a function. Of course, there are other ways, such as the Hessian method, which is similar to the method below and does not require multiple methods.

$$H = [p \quad a].\begin{bmatrix} \frac{\partial^2 \pi_R}{\partial p \partial p} & \frac{\partial^2 \pi_R}{\partial p \partial a} \\ \frac{\partial^2 \pi_R}{\partial a \partial p} & \frac{\partial^2 \pi_R}{\partial a \partial a} \end{bmatrix}.\begin{bmatrix} p \\ a \end{bmatrix} \tag{29}$$

If the value of H is less than zero, then our function is concave. In this case, we have:

$$H = a(\gamma p + 2a(t-1)) + p(a\gamma - 2\beta p) < 0 \tag{30}$$

By simplifying the above statement, we get the following:

$$a^2 > \frac{a\gamma p - \beta p^2}{1-t} \tag{31}$$

Among the parameters of the problem, a, s are much larger parameters than others. Therefore, the above statement will permanently be established. Thus, this mathematical condition is written for the correct scientific presentation.

The optimal values for these variables are:

$$q = \frac{(t-1)^2\alpha\mu}{(8\beta t^2 + (\gamma^2 - 16\beta)t - \gamma^2 + 8\beta)s - \mu^2 t^2 + 2\mu^2 t - \mu^2}$$

$$w = \frac{(\gamma^2 t + 4\beta t^2 - .3 - 8\beta t + 4\beta)\alpha s}{(\gamma^2 st - \gamma^2 s - \mu^2 t^2 + 2\mu^2 t - \mu^2 + 8\beta st^2 - 16\beta st + 8\beta s)\beta}$$

Because of the large number of parameter variables, we present function values of the profit of the chain members in the numerical resolution section.

The non-cooperative game of retailer Steckelberg:

The second part of this situation occurs when the retailer is a leading character, and the manufacturer is following it. The above steps are repeated with the difference that the manufacturer is initially a primary decision-maker, and the retailer is maximizing its profits using its optimal solutions. It is important to point out that instead of the retailer's price, we have the price of the wholesaler, which is definite in amount. That is, p = w + m. Not using this change will make problems in solving methods and in derivative variables. Before solving the problem, we first consider the concavity of the target function. To this end, we use the method presented earlier.

$$H = [w \quad q].\begin{bmatrix} \frac{\partial^2 \pi_m}{\partial w \partial w} & \frac{\partial^2 \pi_m}{\partial w \partial q} \\ \frac{\partial^2 \pi_m}{\partial q \partial w} & \frac{\partial^2 \pi_m}{\partial q \partial q} \end{bmatrix}.\begin{bmatrix} w \\ q \end{bmatrix} \tag{32}$$

Consequently, we simplify it as:

$$H = -w(2\beta w - \mu q) - q(2qs - \mu w) < 0 \tag{33}$$

$$q\mu w < w^2\beta + q^2 s \tag{34}$$

As noted above, due to the largeness of the parameters and the binary parameters of the above expressions, the above expression will always be present.

After partial derivation, obtaining optimal values, inserting into the retail profit function, and re-dividing it from the profit function, the optimal values for the variables are obtained as follows in Table II:

Table II- The number of variables in the retailer Stackelberg initial model

Variable	***Optimum Amount***
q	$\frac{(-\boldsymbol{\beta m}+.5\boldsymbol{a}+\boldsymbol{\alpha})\boldsymbol{\mu}}{4\boldsymbol{\beta s}-\boldsymbol{\mu}^2}$
w	$\frac{(1.1\boldsymbol{a}-2\boldsymbol{\beta m}+2\boldsymbol{\alpha})\boldsymbol{s}}{4\boldsymbol{\beta s}-\boldsymbol{\mu}^2}$
a	$\frac{-0.5\boldsymbol{\alpha s}}{(0.3+(\boldsymbol{t}-1)8\boldsymbol{\beta})\boldsymbol{s}+(1-\boldsymbol{t})2\boldsymbol{\mu}^2}$
m	$\frac{(4\boldsymbol{\beta s}-\boldsymbol{\mu}^2)(\boldsymbol{t}-1)\boldsymbol{\alpha}}{(.3\boldsymbol{s}+(2-2\boldsymbol{t})\boldsymbol{\mu}^2)\boldsymbol{\beta}+(\boldsymbol{t}-1)8\boldsymbol{s\beta}^2}$

C. Solving the main model

The non-cooperative game of Manufacturer Steckelberg:

We assume in the first part that the producer is the leader. Therefore, the retailer is first to make a decision. In the uncertain mode of demand, the z variable, representing the retailer's sales, is added to our process variable.

Hierarchical programming is a mathematical framework for Stackelberg games, in which several optimization problems are considered at different levels simultaneously. In a special case, if the problem has two levels, it is referred to as two-level planning. The problem of two-level programming is a particular case of mathematical programming models in which an optimization problem falls within the constraints of another optimization problem. The overall model of surface planning is shown in equations (35) to (37). As can be seen, the range of the first-level response is obtained from the solution of the second problem.

$$Max_{x,y}\ f(x,y) \tag{35}$$

$$St\!: (x,y) \in X \tag{36}$$

$$y \in S(x) \tag{37}$$

Here we have:

$$S(x) = argmax_y\ g(x,y) \tag{38}$$

$$St\!: (x,y) \in Y \tag{39}$$

The first and second-level decision-makers in the Stackelberg (Leading Producer) games are producer and retailer, respectively. In the subject literature, because of the NP-hard nature of the two-level issues (Ben-Ayed and Blair, 1990), the meta-methodologies for problem-solving have been used. For example, (Sadigh et al., 2012) used the method of colonial competition algorithm (ICA) to solve the first-level (or leading) producer's problem. In this way, the retailer (secondary or sequential) problem is solved using the GAMS software for each of them produced by the meta-algorithm. "

In this regard, because (1) decision variables are included in the first level only in price and quality, and their response space is between zero and one and the other has a certain range, (2) the purpose of the issue is to extract the policy of the members of the chain Supply as it is the best seller's answer for each manufacturer's answer (not just Stackelberg's equilibrium), without the use of the meta-binary algorithm, by solving the retailer problem using the GAMS software (for all quantities and (with many distances) Small) calculates the best retailer's response and therefore the Stackelberg balance (leading manufacturer). It's worth noting that the value of our numerical quality is between zero and one. And obviously, it cannot get any more. Regarding the wholesale price, we also need to calculate the lowest value that is obtained in all cases of the previous problem and consider a lower number for the lower limit. For the upper limit, it works the same way, and we consider its upper limit. Since the value of the wholesale price is always lower than the price of the sale, we can view the highest retail price calculated in the previous section for its upper limit. The steps used to calculate the Stackelberg equilibrium (leading manufacturer) are shown below.

Step 1 - The first level problem (manufacturer's question)

Production of 2000 in the intervals specified for w, q as the producer's (leading) answers.

Step 2: Second Level Problem (Retail Problem)

Optimizing the response of the retailer (sequence) for each (w, t) and returning (a *, p *, z *) to the producer model (leading).

Step 3. First Level Problem (Manufacturer Question)

Estimates of the producer's profits for each (w, q, a *, p *, z *) and Stackelberg's equilibrium based on the producer's maximum profit.

The non-cooperative game of retailer Stackelberg:

The second part of this situation occurs when the retailer is a leading character, and the manufacturer is following it. The steps are run with the difference that the manufacturer is initially a primary decision-maker, and the retailer is maximizing its profits using its optimal solutions. It is important to point out that instead of the retailer's price, we have the price of the wholesaler, which is definite in amount. That is, p = w + m. Not using this change will make problems in solving methods and in derivative variables. Before solving the problem, we first consider the concavity of the target function. To this end, we use the method presented earlier.

$$H = [w \quad q].\begin{bmatrix} \frac{\partial^2 \pi_m}{\partial w \partial w} & \frac{\partial^2 \pi_m}{\partial w \partial q} \\ \frac{\partial^2 \pi_m}{\partial q \partial w} & \frac{\partial^2 \pi_m}{\partial q \partial q} \end{bmatrix}.\begin{bmatrix} w \\ q \end{bmatrix} \tag{40}$$

The concavity of this function will prove with numerical examples.

As a result, a manufacturer decides first. The retailer will make the optimal decision based on the optimal answer of the manufacturer.

$$\frac{\partial \pi_M}{\partial q} = \frac{\mu(\alpha + z + a\gamma - \beta' p' - \beta m + \mu' q')}{-\mu^2 + 4\beta s} \tag{41}$$

$$\frac{\partial \pi_M}{\partial w} = \frac{2(\alpha s + sz + a\gamma s - \beta' p' s - \beta m s + \mu' q' s)}{-\mu^2 + 4\beta s} \tag{42}$$

As noted above, due to the largeness of the parameters and the binary parameters of the above expressions, the above expression will always be present.

After partial derivation, obtaining optimal values, inserting into the retail profit function, and re-dividing it from the profit function, the optimal values for the variables are obtained.

In this paper, the models are solved by MATLAB\R2016a and Maple 2016 software.

III. DISCUSSION OF THE RESULTS

The main model is the complete form of the problem dealt with in this paper. In fact, in these four models, we tried to add a part to the problem each day to bring us closer to the real world. Therefore, this model is acceptable with an acceptable approximation in the industry. The most important reason for the completeness of this problem is that there is also a complimentary item in the model here and an uncertain demand. In the third question, we witnessed a complementary product, but the demand was definitive. In the second issue, we faced an uncertain demand, and there was no complimentary item. This is an aggregation of the second and third issues. In the previous question, the number of retail orders from the producer was constant and based on market demand.

In this case, it is assumed that the order value can become more outstanding. If this amount exceeds the amount of the retail sales, then the cost of keeping the goods is more than that of the retailer, and if this amount is lower than the demand of the customers, the retailer will face the loss of profit. Therefore, the amount of demand is indeterminate, and thus a variable will be added to our decision variables. The existence of a complementary producer also adds to the number of problem parameters. Since we do not have the mastery of complementary producers, we cannot add any variables to the problem. The complementary product variables are outside the boundaries of our model, and we use them as input and parameter. And in sensitivity analysis, we show the effects of their possible changes on our model.

Sensitivity analysis on this issue has similarities and differences with previous cases, which we will continue to discuss. In general, the problem parameters specify the following values (Table III). When analyzing the sensitivity of the parameters, their values vary according to the defined step. Table III setting the parameters of this problem

Table III- parameter setting table

α	β	γ	μ	s	t	A	B	i	j	β'	μ'	p'	q'
1000	0.4	0.3	0.3	100000	0.4	0	50	1	1.5	0.1	0.1	100	**0.2**

A. Manufacturer Stackelberg

The solution method for the fourth problem is similar to that of the leading producer of the second problem. The answer space count is a method to solve this problem. The steps and the choice interval for a second decision-maker are like the second one. That means the model chooses the optimal answer from the 2000 resolution space. To avoid repetition, in Fig 3, we show the effect of the change in the rate of cooperation on profit functions and the level of advertising simultaneously. The optimal cooperation rate to the third problem has slightly increased, which is due to increasing demand.

For this reason, the partnership is more profitable for the producer, and retailers spend more on local advertising costs. The distribution of profit among retailers and wholesalers is also visible. Due to the progression of the producer, the producer will make a more significant contribution to the profit.

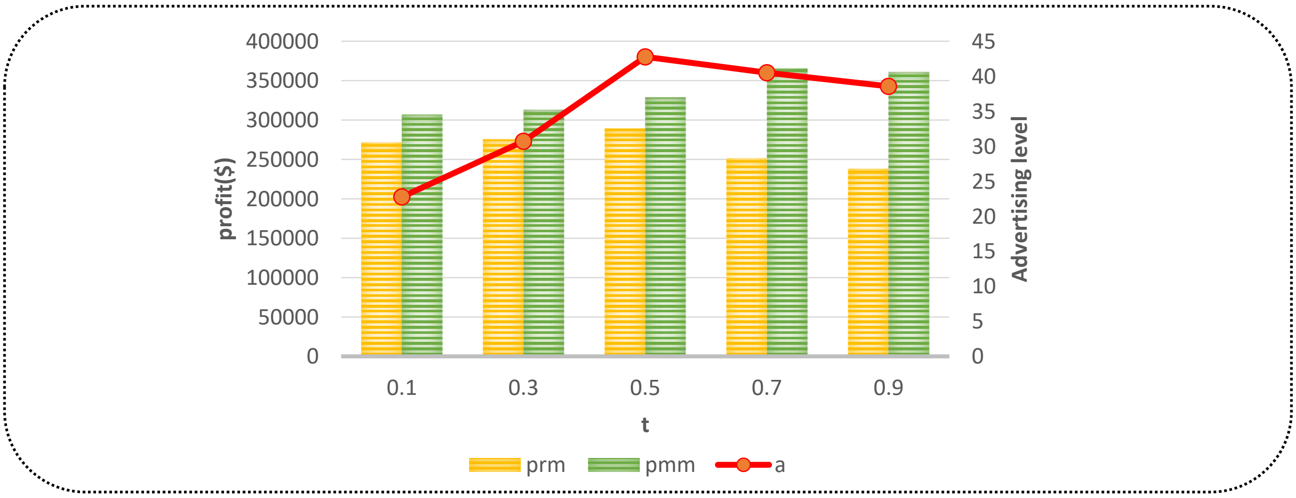

Figure. 3. Changes in the profit function and advertising level due to changes in the participation rate

Changes related to β and μ also provide a similar trend for related variables relative to previous states. The relation β is inverse with prices, and the relation μ is directly related to the quality level. Also, the linear behavior of the quality point is due to the solving method. Fig 4, 5 represent this behavior.

Another parameter important to managers and decision-makers is a constant observation of the behavior of the producer of the complementary product. In Fig 6, 7, parameters related to the price and quality of the complementary product have changed, and changes in wholesale prices, retail prices, and product quality levels are displayed. In the leading manufacturer mode, both prices have dropped as commodity prices rise. Like when the price of tea is increasing, sugar sellers reduce their costs to keep customers satisfied.

Concerning the quality, as already stated, increasing the quality level of the complementary product causes the manufacturer to improve its quality. Otherwise, it will be challenging to attract customers. In general, the increase in the price of the complementary product will lead to lower demand for our product, and the producer will reduce its costs to cope with this decline in demand, which also leads to lower retail prices. On the other hand, increasing the quality of the complementary product will increase our demand, and the producer will invest in quality and increase the quality level to sustain this demand.

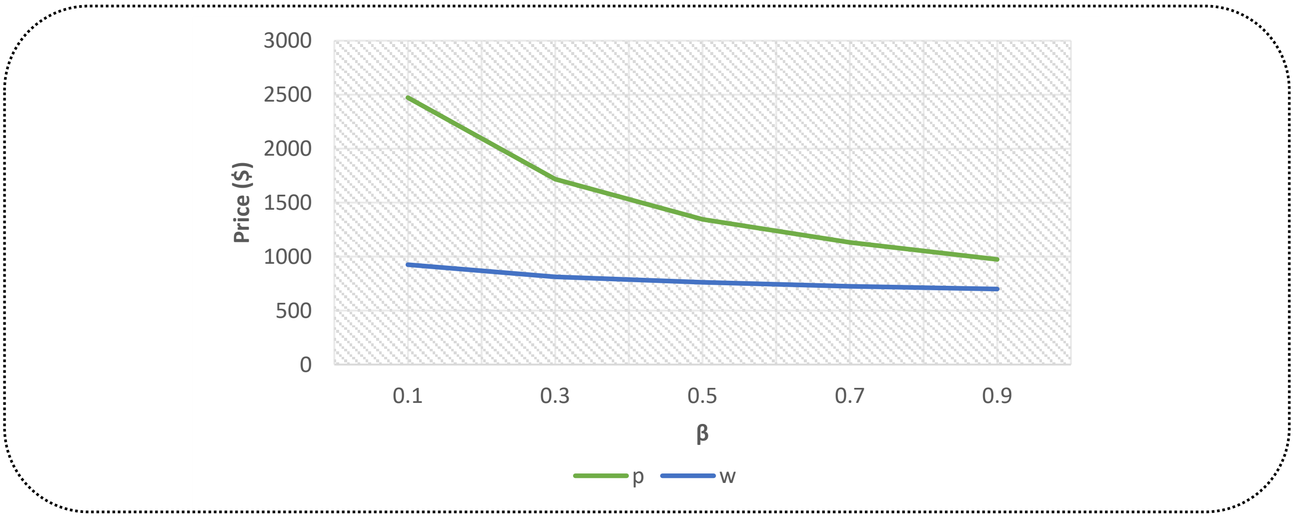

Figure. 4. Changes in the wholesale price and retailer price due to changes in the β

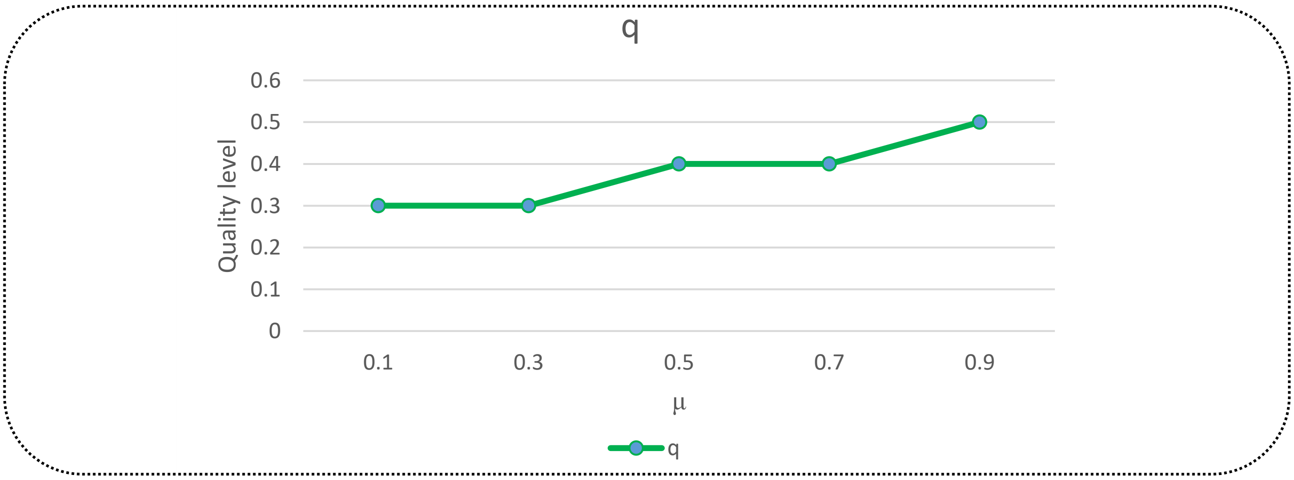

Figure. 5. Changes in the quality level due to changes in the μ

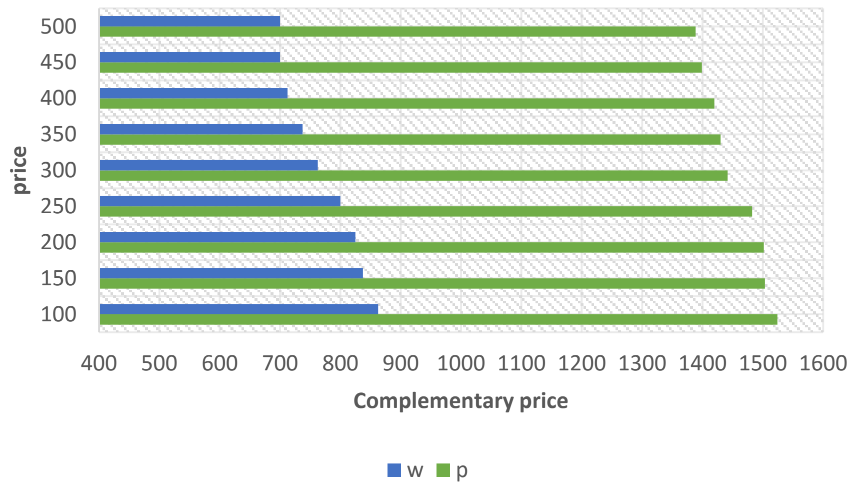

Figure. 6. Changes in the wholesale price and retailer price due to changes in the complementary price

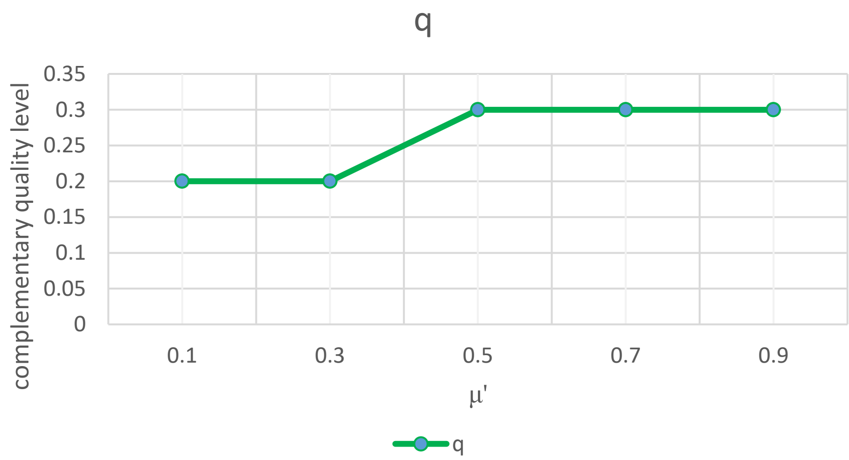

Figure. 7. Changes in the quality due to changes in the complementary quality level

If the range allowed for more orders of retailers changes, then the retailer gains more, and the producer's profits are also rising. When a retailer can order more, it's natural that the producer's profit increases (Table IV).

Table IV- the number of profit functions while B changes

A-B	*Z*	*Manufacturer profit*	*Retailer profit*
50	19.14~20	328911.44	289241.84
100	40.29~41	337158.03	295799.32
150	80.84~81	345148.25	299421.87
200	101.21~102	351584.57	31207.51

B. Retailer Stackelberg

In the context of the emergence of large retailers, a description has already been made. In today's world, attention to big retailers is becoming more and more critical. Many large retailers today play a leading role in a supply chain. The Wal-Mart department store, for example, is a shining example of this. In Iran, stores like Kourosh have a similar situation. In this case, as we ask for complementary and uncertain goods, how to change the variables of the problem provides general points. Fig 8 shows the distribution of profits and the dispersion of the advertising level due to changes in the cooperation rate of cooperative advertising.

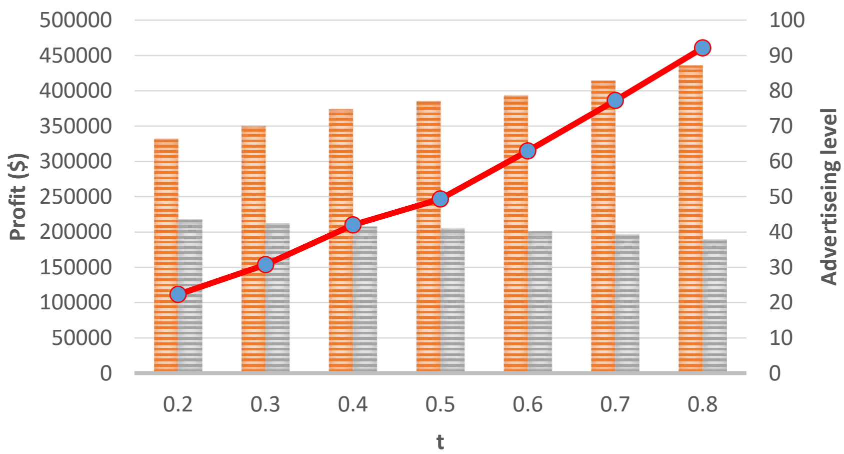

Figure. 8. Changes in the profit function and advertising level due to changes in the participation rate

Fig 9 and 10 show the way of changing wholesale and retail prices and quality levels, as before. β is in inverse relation with the price. By increasing the sensitivity of the demand function to the price, the members of the supply chain reduce their costs to satisfy the customer. Another aspect of μ has a positive relationship with the quality. Increasing attention to quality leads to increased demand.

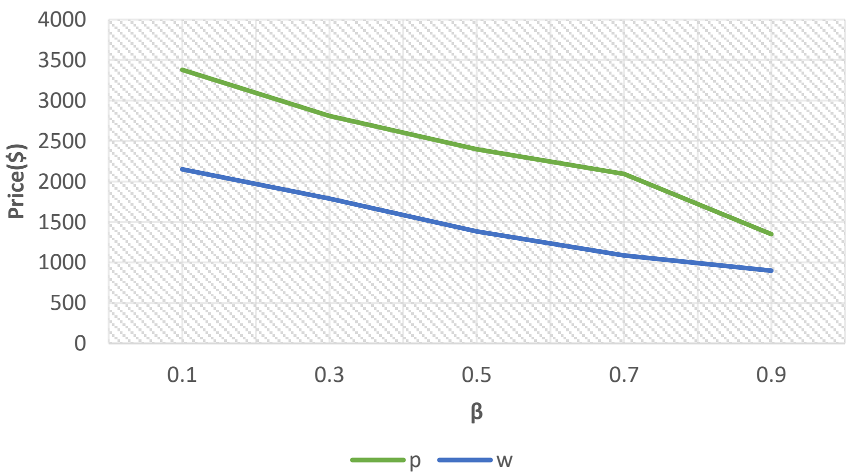

Figure. 9. Changes in the wholesale price and retailer price due to changes in the β

The next point about this is the changes in the parameters of the complementary product. Because the price of the complementary product along with its coefficient, the quality level of the complementary product, along with its coefficient, can mathematically demand an effect type on the function, the analysis of their behavioral parameters or their coefficients is not much different. Thus we are analyzing the sensitivity of the variation of the parameters on the variables problem and functions of our interest. Fig 11, 12 represent this sensitivity analysis. Here, with the increase in the price of the complementary item, the wholesale price will increase, and the retail price will drop slightly. In general, the rise in the cost of the complementary product leads to the closing of wholesale and retail prices.

The process of quality changes due to the increase in the quality of the complementary product is as it was in the previous question. The producer is also encouraged to raise the quality of his product by increasing the quality.

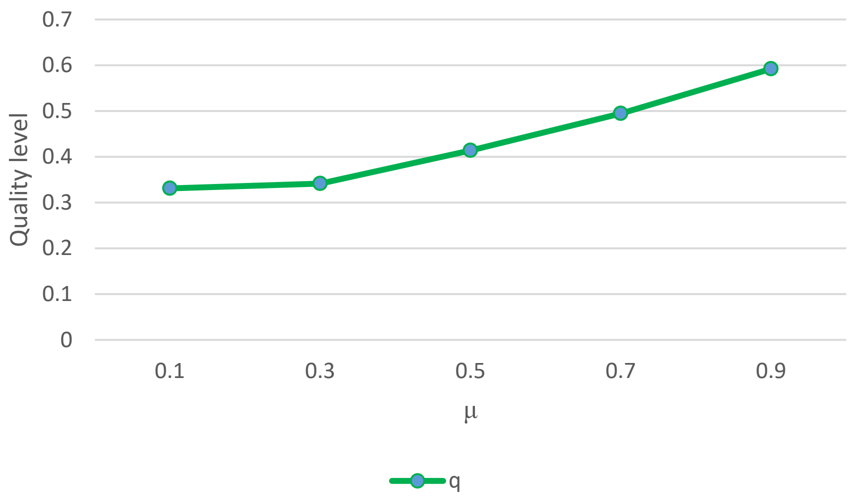

Figure. 10. Changes in the quality level due to changes in the μ

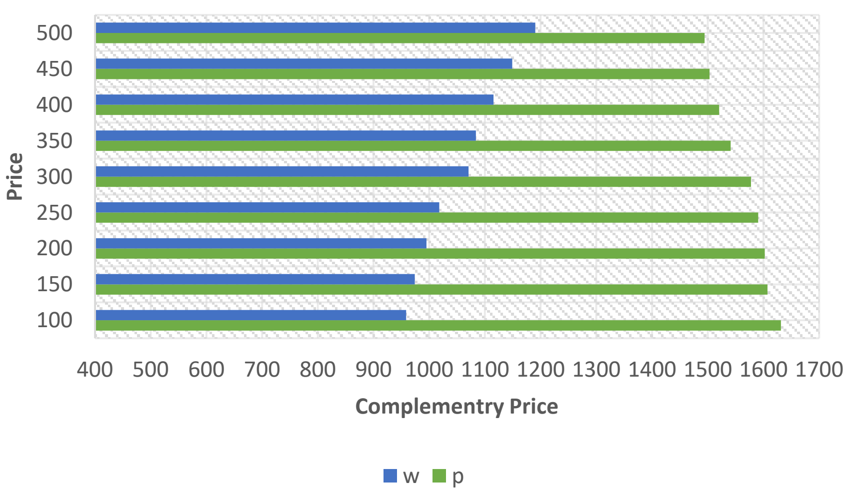

Figure. 11. Changes in the wholesale price and retailer price due to changes in the complementary price

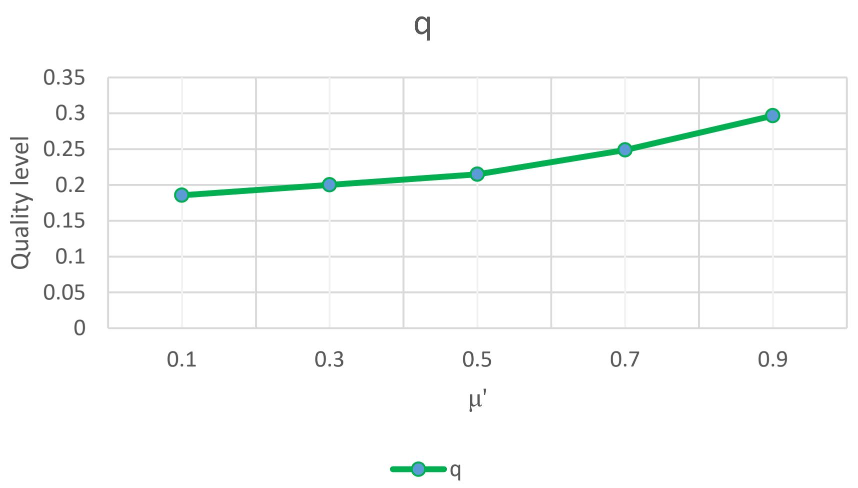

Figure. 12. Changes in the quality due to changes in the complementary quality level

C. Summing up the fourth research question

Fig 13 states that in most cases, the profit generated by the leading producer game is higher. Except at a high rate of participation, in which the mode of retail sales increases dramatically.

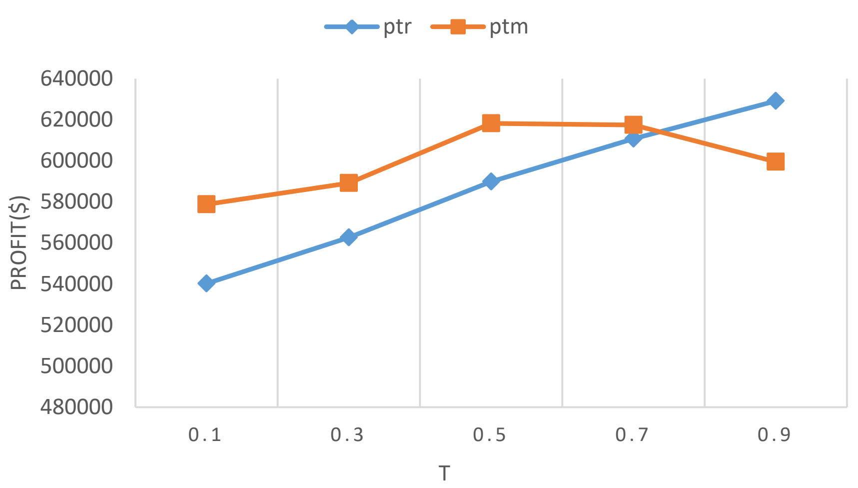

Figure. 13. Compering total profit functions

In Fig 14, the relationships and the number of earnings of members from the Stackelberg Games are displayed. The relationships and the way the profits are placed are like the preceding questions and are determined by each member's leadership in the Stackelberg game. prm is the retailer's profit in the retailer Stackelberg Game. pmm is the profit of the manufacturer in the manufacturer Stackelberg Game. prr and pmr are defined in the same way.

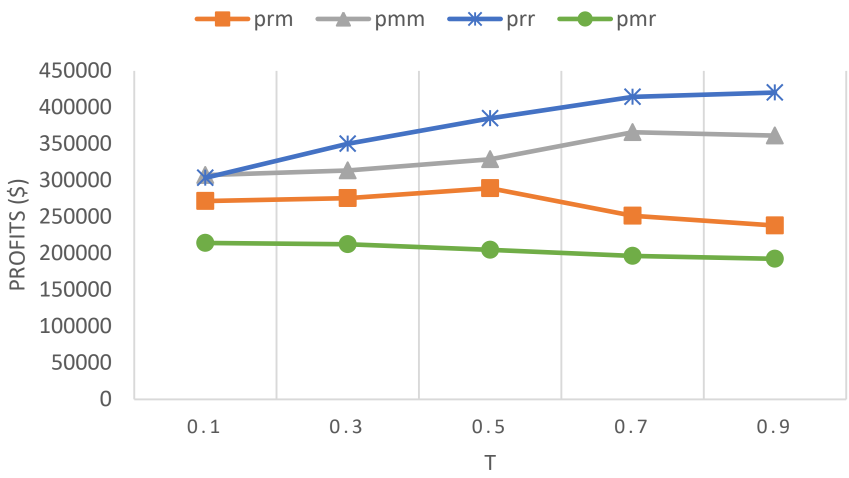

Figure. 14. Compering profit functions

IV. Managerial Insights

The most significant difference between manufacturer and retailer Stackelberg is the power of the leader in the supply chain. This difference help managers to know how they act in their supply chain according to their products. For example, big chain stores have a different relationship with their distributors or manufacturers compared with small supermarkets. Although both of these stores maybe sell the same products in their stores but their bargaining power in the supply chain is different. On the other hand, manufacturers should know how to deal with normal retailers or big retailers. This is very important for managers to earn maximum profit. Sometimes earning more profit is related to helping other supply chain members. This model and different parts give them appropriate insights.

According to their business, each of the four issues can give the manager good insight. Choosing the right strategy is entirely based on business and cannot be a general recommendation. The most crucial point is that we can increase our profits by recognizing the critical supply chain members and constructive interaction with them. This increase in profit in the optimal case can help improve all members' profit of a supply chain.

V. CONCLUSION

In the fourth chapter of the paper, we will summarize the results obtained from the models presented. The form of the demand function of the proposed model, as in the vast majority of studies in this field, is collective. Different variables and parameters are affected by the various coefficients on the demand function, and their changes change the amount of demand and profit of the supply chain members. The same discussion is the beginning of many of the results obtained in this paper. The most critical parameter considered with this sensitive analysis was the cooperation rate for cooperative advertising. In the Stackelberg games of the leading retailer, it increased to a value of one by increasing its rate, and the level of advertising dramatically has increased. The main reason was that when a retailer is a leader, he tends to increase the rate of cooperation, which puts the bulk of advertising costs on the other side and, as a leader, uses this opportunity to increase local advertising. But when the producer is the leader, the rate of cooperation becomes the optimal mode, in which the level of advertising is in the maximum state, and each rate below or higher than that rate will decrease the advertising level. The next point, the changes in the rate of cooperation, was the change in the members' profits. As seen in the previous chapter, the type of non-cooperative game chosen for the problem significantly impacts the dispersion and distribution of profits between the retailer and the producer. And naturally, the leading member will have a more significant share of the profit cake. In this form of our definition of the demand certainty lack, because of the rising order of the retailer from the producer, the producer usually generates more profit than its definitive status. The retailer cannot make a final statement; it depends on the retailer's fines. The effects of complementary product parameters on the primary model were other issues discussed in the previous sections. The quality of the competitor's product contributes to the increasing attention of the manufacturer to the quality and promotion of it. We discussed the price of complementary goods depending on the type of game played that usually affects our costs. The behavior of the main problem variables, such as retail prices, wholesale prices, and quality, also had a natural behavior due to changing their parameters. So that retail prices are correlated with demand, and an increase in the sensitivity of demand to prices leads to price reductions. Reducing retail costs will affect the wholesale price and reduce its value consequently. Concerning the quality, the situation is inverse, and the relation between demand and quality is direct. As the demand for quality increases, the quality level increases the quality of the product, and the members' profits will increase. In these four cases, the general behavior was described, but in each issue, according to the type and form of the problem and the gender of the demand, we were faced with differences in detail. The solution to these problems is also partially influential on the shape of the optimal points. Another variable in the primary model was the determination of the order value in the uncertain state. It was argued that the manufacturer would benefit more from receiving orders. Another essential thing to note here is that by increasing the allowed range for ordering retail, the order risk decreases slightly, and retailers and manufacturers receive more profits. The following suggestions are presented for future research to direct research processes in this area and extend the knowledge boundaries:

In future research, more producers and retailers can be used to get closer to the real world. There could even be a coalition between producers or use concepts such as the producer's class. The distribution process of the uncertainty function is uniform in this paper, based on this research's literature. Using other distributions like standard or fuzzy uncertainty also seems like an excellent way to expand this topic's literature. Although in future research, internet product distribution networks can be compared to traditional methods. Today, new product distribution methods are expanding, and cooperative advertising can be effective in improving these networks. Researchers can solve cooperative advertising models with Metaheuristic methods to solve more complex problems.

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