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Improving Envelopment in Data Envelopment Analysis by Means of Unobserved DMUs: An Application of Banking Industry

Mohammad Reza Alirezaee, Fatemeh Rakhshan
Iran University of Science and Technology, Faculty of Mathematics, Tehran, Iran

***Corresponding Author:** Fatemeh Rakhshan (E-mail: rakhshan@mathdep.iust.ac.ir)

Abstract- *In data envelopment analysis, the relative efficiency of a decision-making unit (DMU) is defined as the ratio of the sum of its weighted outputs to the sum of its weighted inputs allowing the DMUs to freely allocate weights to their inputs/outputs. However, this measure may not reflect a true efficiency of a DMU because some of its inputs/outputs may not contribute reasonably in computing the efficiency measure. Traditionally, to overcome this problem weights restrictions have been imposed. But, an approach for solving this problem by inclusion of some unobserved DMUs, obtained via a process with four steps, has been proposed in 2004. These unobserved DMUs are created by adjusting the output levels of certain observed relatively efficient DMUs. The method used in this research is regarded for DMUs that are operating under a constant return to scale (CRS) technology with a single input multi-output context. This method is implemented for 47 branches of bank Maskan in northeast of Tehran and the results will be analyzed.*

Keywords: *Data envelopment analysis, Linear programming applications, Value judgments, Banking Industry*

I. INTRODUCTION

Data envelopment analysis (DEA) is a method for assessing the relative efficiency of decision-making units (DMUs) that converts some inputs to some outputs. This method is first introduced by (Charnes et al., 1978) that freely allow the weights to be selected in order to maximize the efficiency rating of assessed DMU subject to only the constraint that all weights should be greater than a value of ε regarded as a very small positive number ('non-Archimedian infinitestimal'). However, it can be proved that this amount of flexibility of weights can lead to unacceptable results. For this reason, several methods have been developed that restricted the input/output weights. Some of them could be found in (Roll & Golany, 1993), (Roll et al., 1991), (Wong & Beasley, 1990), (Dyson & Thanassoulis, 1988), (Charnes et al., 1990) and (Thompson et al., 1990). For a review of such DEA methods, readers can see Allen et al. (1997).

DEA weight restrictions are some constraints that are imposed to model and they implicitly modify the production possibility set of DEA assessment. For a formal definition of production possibility set under CRS, readers can see Banker et al. (1984). (Thanassoulis & Allen, 1998) demonstrated the equivalence between DEA weight restrictions and incorporation of unobserved DMUs (UDMUs). UDMUs are virtual DMUs (branches in the current application) that we specify the value of their inputs and outputs for some purposes. In this study, we want to decrease the weak efficient frontier of the production possibility set. (Allen & Thanassoulis, 2004) employed this equivalence and represented an approach where UDMUs are used directly to capture the value judgments in DEA rather than weight restrictions.

A DMU is properly enveloped if all its input/output weights will be greater than ε (Allen & Thanassoulis, 2004). This approach is based on extending the DEA efficient frontier by means of suitably constructed UDMUs. It has been developed for the case where:

- The technology under which DMUs are operating is CRS.

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- DMUs use single input to produce multiple outputs (the modification of this method for multiple inputs multiple outputs and VRS case is developed in (Thanassoulis et al., 2012) study that is not the subject of this paper).
- At least some of the ε-weights are not due to projection on non-full dimensional efficient facets (NFDEFs).

Full dimensional efficient facets (FDEFs) are efficient production possibility set facets whose dimension is $m+s-1$, where m and n are the number of input and output variables in the DEA assessment respectively (Olesen and Petersen, 1996). For other studies dealing with unobserved DMUs, we can mention (Sowlati & Paradi, 2004), (Jahanshahloo & Soleimani-Damaneh, 2005) and (Diallo et al., 2007). More recently unobserved DMUs have been utilized in generalizing production possibility set under weak disposability assumption (Kuosmanen, 2005), (Kuosmanen & podinovski, 2009) and (Podinovski & Kuosmanen, 2011).

In this paper, we verify the approach of incorporating UDMUs in order to improving envelopment on a real-life application, 47 branches of a bank in Tehran and the results of this procedure will be analyzed. Our aim is to enhance the number of properly enveloped branches. Section II describes the method mentioned above for increasing the number of observed DMUs, which are fully enveloped. Section III implements the algorithm on the bank branches.

II. IMPROVING ENVELOPMENT BY MEANS OF UDMUs: A SUMMARY OF THE APPROACH

The concept of UDMUs could be described in the following figure that demonstrates the production possibility set of 7 DMUs with one input two outputs. By introducing the UDMU8 and UDMU9, we can extend efficient frontier. Clearly, DMUs such as those of numbers 1, 5, 7 would be properly enveloped if we introduce the underlying unobserved DMUs. We could refer to DMUs whose efficiency rating is computed using at least on ε-weight as non-enveloped DMUs.

Each step of proposed algorithm of (Allen & Thanassoulis, 2004) will be verified in the following sub-sections:

A. Step1- Assessing the DMUs

Consider a set of N DMUs, each use one input to produce s outputs. Let y_{rj} be the level of output r per unit of input secured by DMU j. Normalising the weighted input to 1 the DEA model yielding the DEA-efficiency rating of DMU j_0 under CRS is (Charnes et al., 1978)

$$\begin{aligned} h_{j0}^* &= max_{u_r} \textstyle\sum_{r=1}^{s} u_r y_{rj} \\ s.t. \textstyle\sum_{r=1}^{s} u_r y_{rj} + D_j &= 1,\ j = 1,\dots,N \\ -u_r &\le \varepsilon,\ r = 1,\dots,s \\ D_j &\ge 0,\ j = 1,\dots,N \end{aligned} \qquad \text{(M1)}$$

where ε is a non-Archimedian infinitestimal and D_j represents a slack value.

- DMU j_0 is called DEA efficient if and only if $D_{j0} = 0$ at the optimal solution to (M1).

Assess all observed DMUs via model (M1). If all the inefficient DMUs are properly enveloped in the sense that non accords any output a ε-weight then stop. Otherwise, go to step 2.

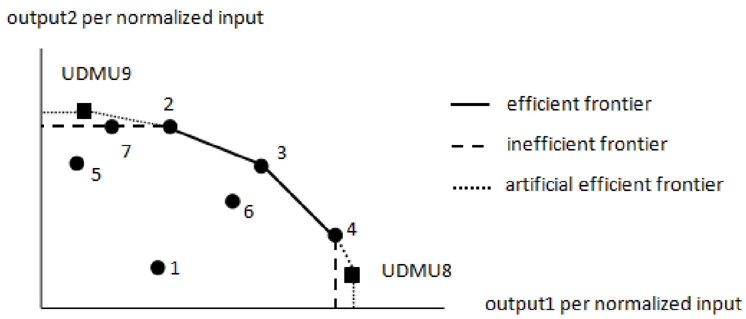

FIG. 1. Extended production possibility set

B. Step2- Identify anchor DMUs

This step identifies suitable observed DMUs, referred to as anchor DMUs (ADMUs) whose input-output levels can be modified to reduce the DEA inefficient part of the production possibility set. In respect of the classification of DMUs, as introduced by (Charnes et al.,1991), we can say that in the single input multi-output CRS case, ADMUs are those of DEA-efficient DMUs which with reference to the rest of DEA efficient DMUs can be rendered class F by contracting readily their output levels, while keeping their input level constant.

ADMUs can be identified by the concept of super-efficiency (SE) introduced by (Andersen & Petersen, 1993) as follows. Let the set JE consist of the DEA-efficient DMUs identified by model (M1) and let JE_{j0} be the set JE excluding DMU $j_0 \in JE$. In respect of each $j_0 \in JE$ solve the following envelopment model:

$$
\begin{aligned}
& h'_{j0} = \min_{\varphi_0,\lambda_j,G_r} \varphi_0 - \varepsilon \textstyle\sum_{r=1}^{s} G_r \\
& s.t. \textstyle\sum_{j\epsilon JE_{j0}} \lambda_j = \varphi_0 \\
& \textstyle\sum_{r=1}^{s} \lambda_j y_{rj} - G_r - y_{rj0} = 0, r = 1,\dots,s \\
& G_r, \lambda_j \geq 0, r = 1,\dots,s, j \in JE_{j0}.
\end{aligned}
\qquad \text{(M2)}
$$

DMU j_0 is an ADMU if the optimal value of φ_0 satisfies $\varphi_0 > 1$ and at least one positive slack variable. For the proof is available in Allen et al. (2004).

C. Step 3- Determining which output levels of the ADMUs to adjust

As already noted, when model (M2) identifies DMU j_0 as ADMU at least one of the slack variables is positive at the optimal solution to the model. Let K_{j0} be the set of outputs for which the instance of model (M2) corresponding to ADMU j_0 yields positive slack. For each $k \in K_{j0}$, solve the following model:

$$
\begin{aligned}
& h'_{j0} = max\, G_k \\
& s.t. \textstyle\sum_{j\epsilon JE_{j0}} \lambda_j = \varphi_0^* \\
& \textstyle\sum_{r=1}^{s} \lambda_j y_{rj} - G_r - y_{rj0} = 0, r = 1,\dots,s \\
& G_r, \lambda_j \geq 0, r = 1,\dots,s, j \in JE_{j0}.
\end{aligned}
\qquad \text{(M3)}
$$

φ_0^* is the optimal value of φ_0 in (M2). If there exists an optimal solution to the instance of (M2) corresponding to ADMU j_0 in which output k has a positive slack value; then, G_k would be positive at the optimal solution to (M3).

UDMUs are constructed by allowing each output level of ADMU j_0 that for which model M3 yields a positive slack value at its optimal solution to be equal to a lowest permissible level of all observed DMUs. This reduction in the level of an output of an ADMU will need to be compensated for by a rise in the level of one or more of its other outputs or alternatively a reduce in the input level if we want the UDMU to be DEA-efficient.

D. Step4- Specifying suitable UDMUs

In this stage, DM is asked to adjust the levels of $s - 1$ outputs of ADMU so that the ADMU and the UDMU created could be deemed equally efficient.

E. Step5- Assessing all observed and unobserved DMUs

Once the UDMUs were created, then the observed DMUs can be assessed using model (M1); but, permitting observed as well as UDMUs to be referent DMUs. The number of observed DMUs that are properly enveloped should be greater than in the absence of the UDMUs.

The increase in the properly enveloped DMUs after step4 will depend largely on how successful the DM has been in giving trade-offs between the output levels of ADMUs which lead to efficient UDMUs. However, we state again that all of these are providing that there exist no DMU which is not properly enveloped by virtue of reference to NFDEFs.

We remind that the algorithm would be stopped if one of the following conditions holds:

- All of the DMUs had been properly enveloped,
- The DM had been consented with the current number of properly enveloped DMUs.
-

TABLE I. The input and outputs used to assess 47 bank branches

Input	Outputs
Staff Costs	Resources Expenditures Services

III. AN APPLICATION OF THE USE OF UDMUS TO IMPROVING ENVELOPMENT IN THE BANKING INDUSTRY

Consider a set of 47 bank branches with the input and output variables on Table I. The data of these branches were obtained from Northeast Branch Management of Bank Maskan in Tehran. Our purpose is assessing the bank branch performance from cost efficiency aspect using the algorithm mentioned.

A. Preparing data before starting the procedure

Similar to the (Farrell, 1957)'s study, , we first normalize all outputs by dividing them to their corresponding input. Let the value of ε in above models be equal to $10e-6$. The models are being solved using GAMS 23.4 software.

B. Initial assessment of the branches

The initial assessment of the branches, carried out by solving model (M1), has revealed that only 5 were DEA efficient. Fig. (2) summarizes the number of outputs assigned an ε-weight by each of the remaining 42 DEA-inefficient branches. A branch assigning even one ε-weight to a variable is not properly enveloped.

Only about 33% of the DEA-inefficient branches are properly enveloped. As shown in Fig. 2, Most branches assign one output variable an ε-weight, and so, effectively ignore that variable in the assessment. Hence, it could be concluded that attained efficiency scores do not reflect the true performance of the majority of the branches. The branches' efficiency scores along with their output weights can be seen in Appendix A.

C. Identify the anchor branches

Anchor branches (ADMUs) were identified by solving model (M2). It was found that each of the five DEA-efficient DMUs were recognized in previous step are anchor DMUs. They are DMUs with numbers 10, 17, 26, 27 and 37. These are anchor because based on the results of model (M2) for them we have $\varphi_0 > 1$ and at least one slack variable is positive. For the results of the model (M2), see Appendix B.

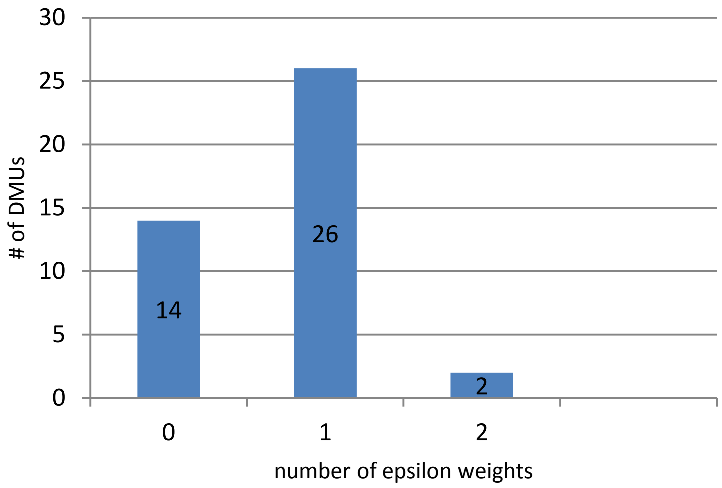

FIG. 2. Frequency of branches with ε-weighted output variables.

TABLE II. Minimum output levels observed per unit input

Resources	Expenditures	Services
0.97	0.23	0.77

TABLE III. Input-output leves of the unobserved branches based on each ADMU

	Resources	Expenditures	Services
DMU10	4.258163751	9.654139133	2.985349136
UDMU48	5.2	9.7	0.79
DMU17	2.511097713	9.749587768	3.371287167
UDMU49	0.97	9.8	4.5
DMU26	5.994493531	0.781528979	2.771252844
UDMU50	6.2	0.23	5.0
UDMU51	6.1	2.0	0.77
DMU27	3.42697184	8.38979327	5.98389973
UDMU52	1	10	6.1
DMU37	5.912231099	1.109653942	4.405808987
UDMU53	6.2	0.3	5.5

D. Identifying output levels of the anchor branches to adjust

Outputs of an anchor branch that require adjustment in order to improve envelopment were identified by solving models (M2) and (M3) and noting the outputs which had a positive slack at any optimal solution to either model. With respect to the results of model (M2), only DMU26 has two positive slack variables; thus, we run model (M3) only for DMU26. In other words, the output levels of other ADMUs which should be adjusted are clear. For example, output 3 of DMU10 and output1 of DMU17 should be changed.

By applying model (M3) for DMU26 revealed that both of output 2 and output 3 should be adjusted by the DM. Note that for each output level that must be changed, we will introduce an unobserved DMU. The DM decided to decrease the level of foregoing output almost near the minimum level observed for that factor in all branches. Table II displays these minimum observed output levels.

E. Specifying suitable unobserved branches

This is the stage where the value judgments of the DM are captured. The UDMUs created from each ADMU is listed in Table III. Notice that from ADMU26, we have obtained two UDMUs, namely UDMU50 and UDMU51.

F. Assessment of branches permitting referent unobserved branches

The observed branches were assessed using (M1), permitting both the observed as well as the unobserved branches to be referents. The inclusion of the unobserved branches led, as expected, to a general reduction in the relative efficiency scores of the inefficient branches. However, the main aim of this procedure was to improve envelopment of the observed branches. By inclusion the six UDMUs obtained in the previous stage; now, we have 53 DMUs. Recall that we had 42 inefficient branches and now, by adding these 5 UDMUs to the primary set of 47 branches, we expect that the number of ε weights can be reduced. Fig. (3) summarizes the envelopment of the branches. The details such as efficiency scores and output weights are mentioned in Appendix C.

Clearly the number of properly enveloped branches in comparison with Fig. (2) has been vastly increased by the introduction of UDMUs. The number of fully enveloped branches has risen from 14 to 23, and there is no branch with two ε-weight. Furthermore, we should consider that this improvement is after one repetition of the algorithm and surely applying the procedure in several times would lead to better results.

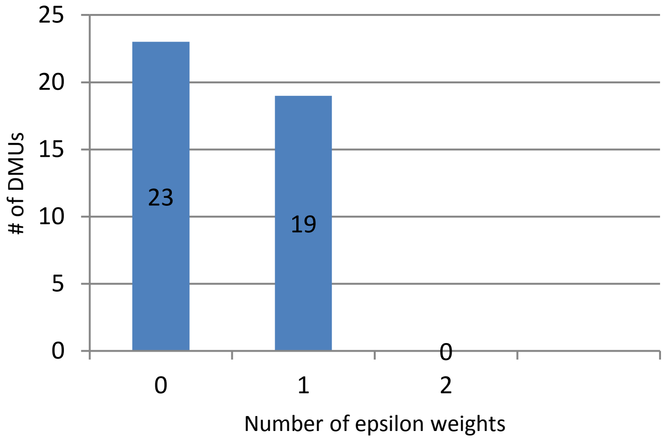

FIG. 3. Frequency of branches with $\boldsymbol{\varepsilon}$ weighted output variables when unobserved branches used.

TABLE IV. UDMUs btained after iteration 3 of the procedure

	UDMU54	**UDMU55**	**UDMU56**	**UDMU57**
Resources	5.5	6.3	0.9	6.3
Expenditures	9.8	2.5	10.1	0.2
services	0.7	0.69	6.2	5.8

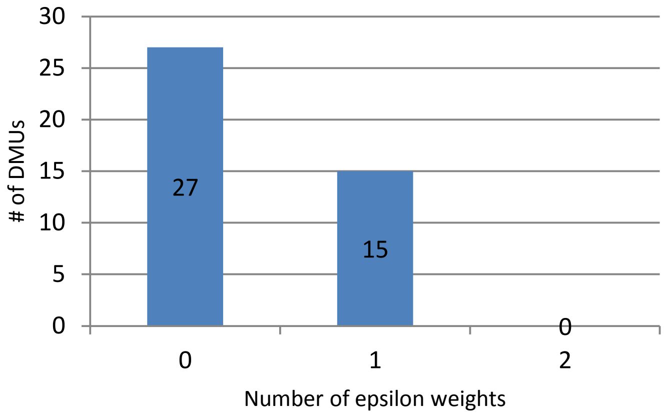

FIG. 4. Frequency of branches with ε weighted output variables after iteration 3 of the procedure.

After iteration 3, we will have the following results as shown in Table IV and Fig. (4).Based on Table IV, after repetition of number 3, we have 10 UDMUs, as shown in Fig. (4), which lead to 27 branches that are fully enveloped. It seems that some of the branches with one ε weight are not properly enveloped because of presence of NFDEFs in this application. The final scores of efficiency and output weights can be seen in Appendix D. Efficiency scores of all branches has reduced between 1 to 8 units and as it can be seen, only DMU10 and DMU27 are really efficient.

Increasing the discrimination power of DEA models is one of the advantages of this procedure. It means that before running this procedure, the scores of sample branches are obtained while 31 from 47 branches ignored the effects and roles of one or two outputs from the efficiency assessment. Thus, the acquired ranks at the stage two are not real. On the other hand, we know that increasing the number of DMUs will get more precisely evaluation results. Indeed, the efficiency rankings and hence the number of efficient DMUs would be decreased. It is remarkable that the number of iterations of the procedure depends on the DM's perspective. If the current number of properly enveloped DMUs are enough with respect to bank manager's idea, the procedure will be stopped.

IV.CONCLUSION

This paper has implemented an algorithm introduced by (Allen & Thanassoulis, 2004) on a real problem of assessing branches of Bank Maskan in Tehran. The results show that using UDMUs in improving envelopment can be very useful in heightening the discrimination power of DEA models. The use of UDMUs as a means to capture value judgments in DEA overcomes some of the drawbacks of weights restrictions, but introduces its own problems, notably the time required to specify many UDMUs.

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APPENDIX A.

Title Model M1

	Efficiency	output-weights		
DMU01	0.92	0.161865	0.031633	0.001796
DMU02	0.47	0.077818	0.000001	0.122547
DMU03	0.42	0.126610	0.032631	0.048855
DMU04	0.74	0.126610	0.032631	0.048855
DMU05	0.31	0.126610	0.032631	0.048855
DMU06	0.25	0.161865	0.031633	0.001796

	Efficiency	output-weights		
DMU07	0.42	0.126610	0.032631	0.048855
DMU08	0.30	0.126610	0.032631	0.048855
DMU09	0.53	0.126610	0.032631	0.048855
DMU10	1.00	0.126610	0.032631	0.048855
DMU11	0.40	0.126610	0.032631	0.048855
DMU12	0.31	0.126610	0.032631	0.048855

DMU13	0.32	0.077818	0.000001	0.122547
DMU14	0.30	0.126610	0.032631	0.048855
DMU15	0.51	0.126610	0.032631	0.048855
DMU16	0.38	0.126610	0.032631	0.048855
DMU17	1.00	0.013439	0.085372	0.039722
DMU18	0.36	0.077818	0.000001	0.122547
DMU19	0.57	0.077818	0.000001	0.122547
DMU20	0.51	0.077818	0.000001	0.122547
DMU21	0.72	0.077818	0.000001	0.122547
DMU22	0.70	0.077818	0.000001	0.122547
DMU23	0.88	0.163027	0.000001	0.008204
DMU24	0.72	0.163027	0.000001	0.008204
DMU25	0.49	0.077818	0.000001	0.122547
DMU26	1.00	0.163027	0.000001	0.008204
DMU27	1.00	0.077818	0.000001	0.122547
DMU28	0.50	0.126610	0.032631	0.048855
DMU29	0.41	0.077818	0.000001	0.122547
DMU30	0.63	0.077818	0.000001	0.122547
DMU31	0.34	0.077818	0.000001	0.122547
DMU32	0.40	0.077818	0.000001	0.122547
DMU33	0.36	0.077818	0.000001	0.122547
DMU34	0.39	0.077818	0.000001	0.122547
DMU35	0.83	0.077818	0.000001	0.122547
DMU36	0.61	0.077818	0.000001	0.122547
DMU37	1.00	0.077818	0.000001	0.122547
DMU38	0.56	0.077818	0.000001	0.122547
DMU39	0.70	0.077818	0.000001	0.122547
DMU40	0.77	0.077818	0.000001	0.122547
DMU41	0.41	0.077818	0.000001	0.122547
DMU42	0.38	0.077818	0.000001	0.122547
DMU43	0.59	0.000001	0.000001	0.167113
DMU44	0.59	0.077818	0.000001	0.122547
DMU45	0.74	0.000001	0.086924	0.045241
DMU46	0.36	0.000001	0.000001	0.167113
DMU47	0.47	0.000001	0.000001	0.167113

APPENDIX B.

Title Model M2

Efficiency		Reference-Set			phi0		G(r)	
DMU10	1.20	DMU27	DMU37		1.20	0.00	0.00	4.11
DMU17	1.02	DMU10	DMU27		1.02	1.76	0.00	0.00
DMU26	1.01	DMU37			1.01	0.00	0.34	1.70
DMU27	1.54	DMU17	DMU37		1.54	3.05	0.00	0.00
DMU37	1.15	DMU26	DMU27	DMU37	1.15	0.00	2.68	0.00

APPENDIX C.

Title Model M1

Efficiency		output-weights		
DMU01	0.88	0.157487	0.018399	0.003283
DMU02	0.43	0.116098	0.036869	0.048933
DMU03	0.41	0.116098	0.036869	0.048933
DMU04	0.72	0.116098	0.036869	0.048933
DMU05	0.30	0.116098	0.036869	0.048933
DMU06	0.25	0.108463	0.041088	0.047391
DMU07	0.40	0.116098	0.036869	0.048933
DMU08	0.30	0.116098	0.036869	0.048933
DMU09	0.53	0.116098	0.036869	0.048933
DMU10	1.00	0.108463	0.041088	0.047391
DMU11	0.39	0.116098	0.036869	0.048933
DMU12	0.31	0.116098	0.036869	0.048933
DMU13	0.30	0.116098	0.036869	0.048933
DMU14	0.30	0.116098	0.036869	0.048933
DMU15	0.50	0.116098	0.036869	0.048933

Efficiency		output-weights		
DMU16	0.37	0.116098	0.036869	0.048933
DMU17	0.99	0.007093	0.099290	0.000001
DMU18	0.35	0.116098	0.036869	0.048933
DMU19	0.52	0.026515	0.000001	0.151929
DMU20	0.48	0.116098	0.036869	0.048933
DMU21	0.68	0.026515	0.000001	0.151929
DMU22	0.65	0.026515	0.000001	0.151929
DMU23	0.85	0.157487	0.018399	0.003283
DMU24	0.69	0.160832	0.009463	0.000001
DMU25	0.46	0.116098	0.036869	0.048933
DMU26	0.97	0.160832	0.009463	0.000001
DMU27	1.00	0.116098	0.036869	0.048933
DMU28	0.48	0.116098	0.036869	0.048933
DMU29	0.39	0.026515	0.000001	0.151929
DMU30	0.58	0.116098	0.036869	0.048933

DMU31	0.33	0.116098	0.036869	0.048933
DMU32	0.36	0.026515	0.000001	0.151929
DMU33	0.34	0.116098	0.036869	0.048933
DMU34	0.36	0.026515	0.000001	0.151929
DMU35	0.79	0.157487	0.018399	0.003283
DMU36	0.56	0.026515	0.000001	0.151929
DMU37	0.97	0.157487	0.018399	0.003283
DMU38	0.53	0.026515	0.000001	0.151929
DMU39	0.65	0.026515	0.000001	0.151929
DMU40	0.76	0.026515	0.000001	0.151929
DMU41	0.41	0.026515	0.000001	0.151929
DMU42	0.37	0.026515	0.000001	0.151929
DMU43	0.59	0.007782	0.000001	0.162657
DMU44	0.55	0.026515	0.000001	0.151929
DMU45	0.70	0.047991	0.069269	0.042511
DMU46	0.36	0.007782	0.000001	0.162657
DMU47	0.46	0.007782	0.000001	0.162657
DMU48	1.00	0.116098	0.036869	0.048933
DMU49	0.98	0.000001	0.099999	0.000001
DMU50	1.00	0.161289	0.000001	0.000001
DMU51	1.00	0.160832	0.009463	0.000001
DMU52	1.00	0.047991	0.069269	0.042511
DMU53	1.00	0.116098	0.036869	0.048933

APPENDIX D.

Title Model M1

Efficiency		output-weights		
DMU01	0.86	0.158730	0.000001	0.000000
DMU02	0.43	0.109054	0.037074	0.052680
DMU03	0.40	0.109054	0.037074	0.052680
DMU04	0.71	0.109054	0.037074	0.052680
DMU05	0.30	0.109054	0.037074	0.052680
DMU06	0.25	0.075735	0.056340	0.044750
DMU07	0.40	0.109054	0.037074	0.052680
DMU08	0.29	0.109054	0.037074	0.052680
DMU09	0.52	0.109054	0.037074	0.052680
DMU10	1.00	0.075735	0.056340	0.044750
DMU11	0.38	0.109054	0.037074	0.052680
DMU12	0.31	0.109054	0.037074	0.052680
DMU13	0.30	0.109054	0.037074	0.052680
DMU14	0.30	0.109054	0.037074	0.052680
DMU15	0.50	0.109054	0.037074	0.052680
DMU16	0.36	0.109054	0.037074	0.052680
DMU17	0.98	0.006421	0.098437	0.000001
DMU18	0.34	0.109054	0.037074	0.052680
DMU19	0.51	0.109054	0.037074	0.052680
DMU20	0.47	0.109054	0.037074	0.052680
DMU21	0.67	0.011822	0.000001	0.159573
DMU22	0.64	0.014447	0.001550	0.156668
DMU23	0.84	0.151336	0.016575	0.007460
DMU24	0.68	0.158730	0.000001	0.000000

Efficiency		output-weights		
DMU25	0.46	0.109054	0.037074	0.052680
DMU26	0.95	0.158730	0.000001	0.000000
DMU27	1.00	0.109054	0.037074	0.052680
DMU28	0.47	0.109054	0.037074	0.052680
DMU29	0.38	0.014447	0.001550	0.156668
DMU30	0.57	0.151336	0.016575	0.007460
DMU31	0.33	0.109054	0.037074	0.052680
DMU32	0.36	0.109054	0.037074	0.052680
DMU33	0.34	0.109054	0.037074	0.052680
DMU34	0.36	0.011822	0.000001	0.159573
DMU35	0.77	0.151336	0.016575	0.007460
DMU36	0.54	0.011822	0.000001	0.159573
DMU37	0.95	0.151336	0.016575	0.007460
DMU38	0.52	0.011822	0.000001	0.159573
DMU39	0.63	0.011822	0.000001	0.159573
DMU40	0.76	0.011822	0.000001	0.159573
DMU41	0.41	0.011822	0.000001	0.159573
DMU42	0.36	0.011822	0.000001	0.159573
DMU43	0.58	0.011822	0.000001	0.159573
DMU44	0.54	0.011822	0.000001	0.159573
DMU45	0.70	0.049920	0.068370	0.042666
DMU46	0.36	0.011822	0.000001	0.159573
DMU47	0.46	0.011822	0.000001	0.159573