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Design of single-sampling inspection-plan approach by mathematical programming and linear assignment method

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Abstract *– This study proposes a new approach for single-sampling plan by determining sample size and acceptance number. The proposed approach is based on a two-step methodology. In the first step: quality management step, different single sampling inspection plans were generated by running an optimization model for different possible acceptance numbers. While, in the second step: Multi-Attribute Decision Making (MADM) step, Shannon Entropy Approach (SEA) and Linear Assignment Method (LAM) were applied for ranking the inspection plans, generated in the previous step and selecting an appropriate plan. In the MADM step, single-sampling inspection plans defined as alternatives and Expected Non-conforming Cost (ENC), Inspection Cost (IC), and Average Outgoing Quality (AOQ) were introduced as main criteria. The proposed approach is able to determine acceptance number or maximum allowable defective number, besides the sample size for inspection lot in manufacturing lines. An example is given for illustration. The results reveal that the proposed approach could provide insightful implications for quality management.*

Keywords *– Single-Sampling Inspection, Sample Size, Acceptance Number, Linear Assignment.*

I. INTRODUCTION

Sampling based on acceptance number is a tool to analyze and survey the quality of materials and products, present in a production system. The sampling based on acceptance number randomly selects a sample from each lot and determines the possibility of acceptance or rejection of that lot, based on the acquired inspection results for a specific sample. Inspection a lot completely is often not desirable, in case it is expensive, time-consuming, or destructive in nature. So, this process needs a special type of acceptance sampling to be used to differentiate between the good lots from the bad ones, which can reduce the chance of entering the bad lots into a production system (Qin et al., 2015).

One of the most conventional and handy plans for incoming inspections is single-sampling policy (Qin et al., 2015). This policy accepts a lot, only if the number of defects, identified in the inspected sample is equal or less than the number of maximum allowable-defects (also called acceptance number).

When a single sampling is exploited, the lot quality depends on the sample size and also acceptance number. This shows the importance of determining the sample size and the acceptance number. Economic modeling approach is capable of supporting the optimization of sample-size and acceptance-number. This approach quantifies the components of quality-related cost and tends to minimize the total cost (Qin et al., 2015).

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Specifying a single sampling plan means determining sample size and acceptance number, which is a very important task for quality managers that has not been mentioned in the literature. For filling some of these gaps, this paper proposes an approach in two steps. The first step models the single- sampling inspection and offer a plan for each acceptance number. The second step compares the different single- sampling plans, by SEA and LAM to rank and select the most suitable one among the plans produced in the first step. LAM is one of the MADM approaches that determines the ranking order of alternatives based on linear programming technique. LAM refers to a linear process for the interaction of attributes and the combination, which makes it practical and easy to apply (Antonio et al., 2018).

The remainder of this paper is as follows. The relevant literature has been summarized in section 2. Section 3 presents the problem formulation for single- sampling inspection. Section 4 proposes solution approach in two steps: 1) optimal sampling plan for each acceptance number, with mathematical programming, 2) comparing different single-sampling plan by MADM techniques. An example is solved and discussed in Section 5. Section 6 represents a comparative analysis and the results of proposed sampling plan are compared with the literature. Finally, conclusions of the paper are discussed in Section 7.

II. LITERATURE

The review of literature for this paper is divided into two categories based on the subject of the paper; one focused on the single sampling inspection and the other on the MADM techniques used in this paper, means SEA and LAM.

Recently, single- sampling inspection has been growing in manufacturing and also in academic research because of making the inspection easier and simplifying the sampling plans. Under this policy, a lot is accepted only if the total defects number in the inspected sample is equal or less than a specific number that is the maximum allowable number of defects or acceptance number. While single-sampling policy is extremely important as a research need, there has not been given specific attention to it in the literature.

Vijayaraghavan et al. (2008) presented a design analysis and selection of parameters of single sampling plans for specific requirements (strengths) when distribution of sampling is gamma prior and Poisson distribution. They compared relative efficiency of gamma-Poisson single sampling plans and conventional plans with the help of empirical illustrations. Rajagopal et al. (2009) presented an iterative procedure for studying the parameters of a single sampling plan by attributes in Polya distribution satisfying conditions relative to the producer’s and consumers’s risks in tabloid form for selecting the sampling parameters. Fallah Nezhad and Hosseini Nasab (2011) introduced a novel control policy for accepting sample problem. A decision can be made according to the number of defectives items in the group under inspection. The purpose of their model was to investigate about a constant control level that can minimize the total costs, including the cost of batch rejection, the inspection and defective item costs. The optimization is done by approximating the negative binomial distribution with Poisson distribution and binomial distribution. Baloui Jamkhaneh et al. (2011) studied the single acceptance sampling plan, in which, the proportion of nonconforming products is a fuzzy number. They have observed that the operating characteristic curve of the plan is a band with high and low bounds. While, for fixed sample size and acceptance number, the bandwidth depends on the ambiguity proportion parameter in a lot. Baloui Jamkhaneh et al. (2011) designed an acceptance single sampling plan with inspection errors when the fraction of defective items is a fuzzy number. They have shown that the wrong classification of a good item decreases the fuzzy probability of acceptance and incorrect classification of defective item results in a higher fuzzy probability of acceptance. Dumičić and Žmuk (2012) used statistical methods of difference in proportion to test if there is some difference statistically in probabilities of lot fraction defects between a single and a double sampling plan at the same levels of acceptance probability. Their results have shown that, in some cases, a statistically significant difference can be present. Narayanan and Rajarathinam (2013) designed a single sampling plan by variables when a Pareto distribution was considered for the quality characteristic. They determined the plan parameters by considering both the producer and consumer. Qin et al. (2015) proposed a nonlinear program to determine an optimal plan of zero-defect and single-sampling. Their model inspect sample size for each part, coming to an assembly line in a resource-constrained condition where a product’s nonconforming risk is not a linear combination of nonconforming risks of the individual parts. Klufa

(2015) dealt with the LTPD (Lot Tolerance Percent Defective) single sampling plans when the remainder of rejected lots is inspected. He has shown that under the same protection of consumer the LTPD plans for the inspection by variables are in many situations more economical than the corresponding Dodge-Romig attribute sampling plans. Govindaraju (2016) introduced a new method for the single sampling attributes plan, ensuring that the decision of acceptance or rejection is consistent for both current and future lot inspection. Subramani and Balamurali (2016) proposed a single sampling plan for the inspection of products in which the nonconforming items were classified in two categories; namely, critical and non-critical. Ahmadi Yazdi and Fallah Nezhad (2016) introduced a new sampling system, based on cumulative conforming control charts' concept. They compared their proposed sampling system with traditional sampling methods like Dodge-Romig single sampling plan, based on lot tolerance percent defective (LTPD) and Dodge-Romig single sampling plan, based on average outgoing quality limit (AOQL). Klufa (2016) referred to the AOQL single sampling plans when the remainder of the rejected lots was inspected. They have shown that under the same protection of consumer, the AOQL plans for inspection by variables are in many situations more economical than the corresponding Dodge-Romig AOQL attribute sampling plans. Rao and Aslam (2017) focused on the design of resubmitted lots plan. They determined the parameters of sampling plan through the nonlinear optimization solution. The advantage of their proposed plan, was in terms of average sample number. Huang et al. (2018) compared the quality economical design with traditional single sampling plan under the total quality cost. Traditional quality inspection plans determine the sample size and rejection rule, based on the lot size, consumer’s and producer’s risk and AQL, but they do not consider internal and external quality costs. Alturki et al. (2019) modeled and optimized an economical single sampling plan, which was independent of the supplier's process level, where the loss caused by accepting low quality lots was treated as a Taguchi's loss function; the model also considered inspection cost, and replacement cost. Luca et al. (2019) proposed a web-based tool to study single and double-stage sampling plans. Their tool was an interactive applet, freely available in contrast to existing solutions. They derived analytic properties to support the development of search strategies for the design of double-stage sampling plans. They also presented a number of case studies.

SEA and LAM are the less-known methods of MADM approach that was mentioned by few researcher reports. This research used SEA and LAM for comparing inspection plans of single- sampling inspection that has not been analyzed in the literature. A brief literature review on SEA and LAM for MADM problems can be presented as follows.

Behboudi Asl et al. (2012) identified the most important factors of ERP selection that organizations should consider. They identified cost, software quality, vendor and software capability as the main criteria for consideration by organizations. They ranked these criteria by Shannon Entropy approach and identified the vendor as the most important criterion. Chen (2013) developed a new LAM to produce an optimal preference ranking of the alternatives in accordance with a set of criterion-wise rankings and a set of criterion importance within the context of interval type-2 trapezoidal fuzzy numbers. They have shown that the proposed method produces actionable results by applying it in a case. Chen (2014) extended the traditional LAM for solving multiple criteria evaluation problems in the interval-valued intuitionistic fuzzy context. He presented a ranking procedure, consisting of score functions, accuracy functions, membership uncertainty indices, and hesitation uncertainty indices to determine a criterion-wise preference of the alternatives. Hafezalkotob and Hafezalkotob (2015) extended MULTIMOORA method, based on Shannon entropy theory for material selection procedure. They considered entropy concept to assign relative importance to decision-making attributes. Abdolazimi et al. (2015) used ELECTRE and LAM to rank Shahroud–Bastam watershed. They compared the results of two methods and showed that the results of LAM are more consistent with reality and are more accurate. Baykasoglu et al. (2016) proposed a fuzzy linear assignment protocol for multi-attribute group decision-making problems. They applied their method to a multi-criteria spare part inventory classification problem to present the validity and practicality of the proposed method. Wei et al. (2016) proposed the LAM to obtain optimal preference ranking of the alternatives, according to a set of criteria-based rankings and a set of criteria importance within the context of hesitant fuzzy elements on the basis of the Hesitant Euclidean distance. Antonio et al. (2018) proposed a new multi-objective evolutionary algorithm. They transformed the multi-objective optimization problem into a linear assignment problem, which was solved by the Kuhn–Munkres’ (Hungarian) algorithm. Haghighi et al. (2019) proposed a new group decision approach with LAM. They constructed weight of each evaluation factor, according to subjective and objective data, based on a new developed version of LAM. In their proposed method, weights of decision makers are computed based on the concept of ideal solutions.

A comparison of literature differences can be seen on Table I. This table explains the differences between this paper and the literature.

Table I. Literature comparison

Feature / Author	Optimization model	Using MADM techniques	ENC	IC	AOQ
Fallah Nezhad and Hosseini Nasab (2011)	*		*	*	
Dumičić and Žmuk (2012)			*		
Qin et al. (2015)	*		*	*	
Rao and Aslam (2017)	*			*	
Govindaraju (2016)			*	*	*
Subramani and Balamurali (2016)			*	*	
Ahmadi Yazdi and Fallah Nezhad (2016)	*			*	*
Klufa (2016)				*	*
Huang et al. (2018)	*		*	*	*
Alturki et al. (2019)	*		*	*	
Luca et al. (2019)	*				*
This paper	*	*	*	*	*

As the literature review shows, specifying single-sampling plan means determining the size of sampling and the specific number for acceptance which have not been mentioned in the literature. In this paper, an approach is proposed in two steps for planning single-sampling inspection. In the first step an optimization model is developed to offer different plans for single-sampling inspection. In the second step different single- sampling plans are compared by proposed procedure based on SEA and LAM. The novelty of this paper is the developed optimization model in the first step. Also, the procedure based on SEA and LAM in the second step for comparing inspection plans is the other contribution of this study that to the best of the knowledge no literature has made this attempt.

III. THE PROBLEM FORMULATION

Single-sampling plan is the process that inspectors choose a random sample from the lot with no replacement. If the number of Non-Conforming (NC) object, found in the sample, means acceptance number, it is equal to or less than a specific number, the lot has accepted. Otherwise, the lot will be rejected. To plan a single-sampling inspection, the size of sample and also acceptance number for the lot waiting for inspection should be determined. In this paper, nonlinear integer programming is used for addressing this issue. This problem can be formulated as an optimization problem that minimizes the total quality-related cost by determining optimal sample size, as a decision variable, for each possible acceptance number. The trade-off among two types of costs, the inspection cost (IC) and the Expected Non- conforming Cost (ENC), are determined by solving the optimization problem (P) formulated as follows:

Variables

n: the inspection sample size;

Parameters

N: the lot size;

a: maximum number of defect permission in the lot to be accepted (acceptance number)

T: the amount of time available for inspection;

c: the NC cost pertaining to one NC item;

d: the number of NC items in the lot;

k: the number of NC items in the sample;

l: the unit cost of inspector labor per unit of time;

r: the NC rate in the lot;

t: average labor time to inspect one item;

b(*d*| *N*, *r*): the probability that the lot has *d* NC items given that the lot size is *N* and the NC rate is *r*;

h(*a*| *N*, *d*, *n*): the probability of detecting less than or equal of *a* number of NC item in the inspection, given that the lot size is *N*, the number of NC items is *d*, and the inspection sample size is *n*.

(*P*)

$$\min z = c \times \sum_{d=0}^{N} \{ b(d \mid N, r) \times \sum_{k=0}^{a} ((d-a) \times \frac{\sum_{k=0}^{a} (\binom{N-d}{n-k} \times \binom{d}{k})}{\binom{N}{n}}) \} + l \times t \times n \quad (1)$$

Subject to:

$$t \times n \le T \quad (2)$$

$$0 \le n \le N \quad (3)$$

n is an integer (4)

where in Eq. (1),

$$b(d \mid N, r) = \binom{N}{d} r^{d} (1-r)^{N-d} \quad (5)$$

Eq. 1 shows the objective function that minimizes the expected total cost, including the ENC and the IC. According to the fact that rejection cost in inspection planning does not significantly has an effect on optimal solution so it is not considered in this paper (Qin et al. 2015). Therefore, the objective function in Eq. 1 only includes ENC and the IC and excludes rejection cost. The first part of Eq. 1 is related to ENC and the second part is for IC. In fact, in the first part of the objective function, *c*, the NC cost pertaining to one NC item, multiplied to the number of expected NC items. In the second part of the objective function, *l*, the inspector labor cost per unit of time multiplied to the time needed for inspection. The constraint of inspection resource showed by (2), means the amount of time available for inspection is restricted by parameter *T*. (3) and (4) state that the decision variable of this problem, *n*, is a non-negative integer ranging from zero to *N*. The probability that *d* NC items be in the lot is defined by Binomial distribution, $b(d| N, r)$, in (5). The number of NC items in the lot, *d*, is a random parameter that can take different non-negative integer values from zero to *N*. Given that the lot size is *N*, the number of NC items is *d*, and the acceptance number is *a*, the probability of accepting the lot is a function of the sample size, *n*, and the acceptance number, *a*, as calculated in Eq. (6) and considered in the objective function (1):

$$\sum_{k=0}^{a}((d-a)\times\frac{\sum_{k=0}^{a}(\binom{N-d}{n-k}\times\binom{d}{k})}{\binom{N}{n}}) \tag{6}$$

IV. SOLUTION APPROACH

To plan a single-sampling inspection, the sample size, *n*, and also acceptance number, *a*, for the lot waiting for inspection should be determined. Therefore, this paper proposes a two-step methodology for this purpose. In the first step problem (*P*) that minimizes the total quality-related cost is formulated for each possible acceptance number to determine sample size. In the next step, SEA and LAM are used to compare different inspection plans obtained from the first step in order to the right one be selected. These two steps as depicted in Fig. 2, explained in the following steps:

4.1. Step 1: Determining an inspection plan for each possible acceptance number

By solving the proposed optimization problem for each possible acceptance number, *a*, different inspection plans obtained in this step. These generated single-sampling inspection plans should be compared to select the proper one in the next step of the proposed methodology. The optimization technique that used for problem (*P*) to find the value of *n* for each possible acceptance number, *a*, is direct search since the sample size, *n*, as decision variable is discrete and bounded by the lot size.

4.2. Step2: Comparing inspection plans, obtained from the first step, by SEA and LAM

In this step, the right inspection plan is selected by ranking inspection plans obtained from the previous step, with SEA and LAM. The process of this step is as follows.

1- Decision matrix construction.

Decision matrix, which has *m* alternatives and *n* criteria is prepared as follows:

$$\begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \cdots\cdots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{pmatrix} \tag{7}$$

The intersection of every alternative and criteria is given as x_{ij}. The x_{ij} is performance ratings for each alternative A_i (i = 1, . . ., m) with respect to criteria C_j (j = 1, . . ., n).

As Fig. 1 shows, for comparing single-sampling inspection plans, alternatives are inspection plans obtained from the previous step, and three criteria included ENC, IC, and AOQ could be considered. Therefore, the decision matrix for ranking single- sampling inspection is as follows:

$$\begin{array}{c} \\ plan_1 \\ plan_2 \\ \vdots \\ plan_m \end{array} \begin{array}{c} \quad ENC \quad\quad IC \quad\quad AOQ \\ \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & x_{m3} \end{pmatrix} \end{array} \text{; where } Plan_i \text{ is the } i\text{th inspection plan} \tag{8}$$

In fact, the decision matrix is constructed based on the hierarchical process as shown in Fig. 1. This hierarchical process shows the framework of ranking inspection plans based on defined criteria.

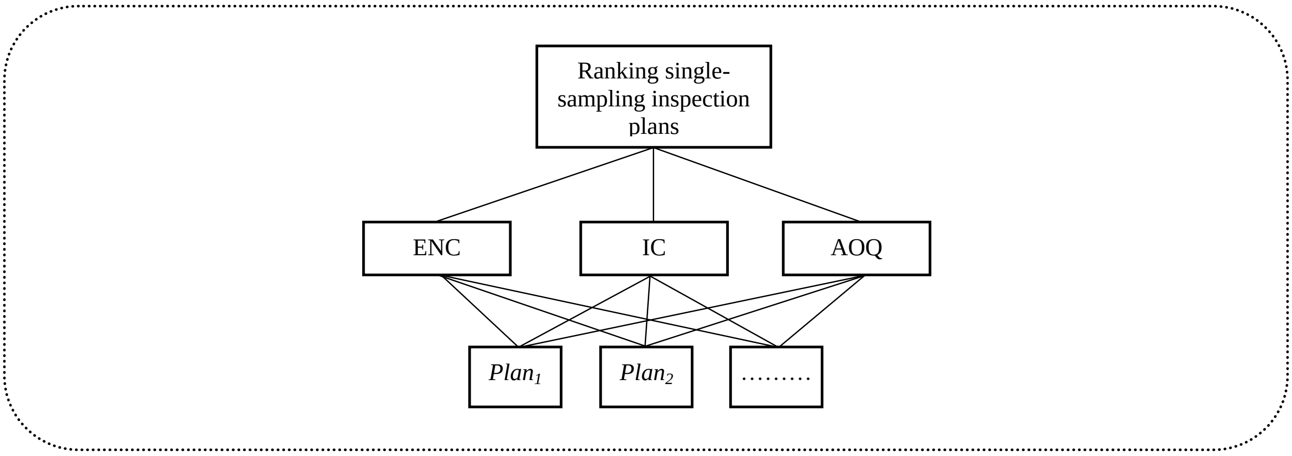

Figure 1. Framework for comparing single- sampling inspection plans

The ENC, IC, and AOQ are the most important criteria that could be considered for comparing inspection plans. Because in comparing inspection plans, two cases must be considered, decreasing expected total cost, and increasing the quality of the inspected lots. Expected total cost is taken into account by considering ENC and IC, and the quality of the inspected lots is taken into account by considering AOQ. The ENC and IC, the two parts of the objective function in Eq. 1, and AOQ are calculated as follows.

$$ENC = c \times \sum_{d=0}^{N} \{ b(d \mid N, r) \times \sum_{k=0}^{a} ((d-a) \times \frac{\sum_{k=0}^{a} (\binom{N-d}{n-k} \times \binom{d}{k})}{\binom{N}{n}}) \} \tag{9}$$

$$IC = l \times t \times n \tag{10}$$

$$AOQ = \frac{\sum_{d=0}^{N} \{ b(d \mid N, r) \times h(a \mid N, d, n) \} \times r \times (N-n)}{N} \tag{11}$$

‘,,,,,

The AOQ in Eq. (11) is described as the expected quality of the lot when an acceptance sampling plan is used for a given value of the lot’s quality; where $\sum_{d=0}^{N} \{ b(d \mid N, r) \times h(a \mid N, d, n) \}$ is the accepting probability of the lot and

$r \times (N-n)$ is the average of NC items in the lot.

2- Normalized decision matrix.

At this stage, the three criteria with different dimensions are converted to criteria with no dimension. There are different methods to make criteria dimensionless. In this paper the decision matrix is converted in to the normalized matrix, using the following equation:

$$p_{ij} = \frac{\left| x_{ij} - \mu_j \right|}{\delta_j} \qquad (12)$$

Where, μ_j is a mean value and δ_j is the standard deviation of jth criterion. μ_j and δ_j are calculated by below formula:

$$\mu_j = \frac{\sum_{i=1}^{m} x_{ij}}{m} \qquad (13)$$

$$\delta_j = \sqrt{\frac{\sum_{i=1}^{m} (x_{ij} - \mu_j)^2}{m-1}} \qquad (14)$$

3- Determining the criteria weights by SEA.

As the relative periority of three criteria is not the same and they have different weights, the weight (w_j) or the relative importance should be calculated. In this paper, w_j will be obtained based on SEA. Entropy idea can be used for determining the weights, because it distinguishes existent contrasts between sets of data and explains the average intrinsic information transferred to decision maker (Hafezalkotob and Hafezalkotob, 2015). On the other hand, the SEA is capable to create suitable weights of the criteria by decision matrix and without the paired comparison matrix. For determining criteria weight by SEA, the procedure could be used as follows:

First the entropy measure of each criterion is computed based on equation (15):

$$E_j = -k \sum_{j=1}^{m} p_{ij} \ln p_{ij}; \qquad j = 1,...,n \text{; in which } k=1/ln(m). \qquad (15)$$

Second the distance measure for each criterion is obtained as follows:

$$d_j = 1 - E_j; \qquad j = 1,...,n \qquad (16)$$

Finally, the criterion weight is obtained in equation (17):

$$w_j = \frac{d_j}{\sum_{k=1}^{n} d_k}; \qquad j = 1,...,n \qquad (17)$$

4- Ranking single- sampling inspection plans by LAM

The LAM is one of the MADM technique that ranks the alternatives. LAM can tackle with alternative ratings in a simple and efficient way. LAM determines the ranking order of alternatives by linear programming (Baykasoglu et al., 2016). The LAM mesures the alternative closeness to the ideal solution and an overall preference ranking of the alternatives provides according to a set of criterion wise rankings (Wei et al. 2016). So, this method is used in this paper for ranking the inspection plans as follows.

Construct the rank frequency matrix π as follows. In this matrix the element π_{ik} (i= 1, 2, ..., m; k= 1, 2, …, m) displays the frequency that $plan_i$ is ranked as the kth ranking, by ranking the m single- sampling inspections plans with respect to each criterion.

$$\pi = \begin{matrix} \\ plan_1 \\ plan_2 \\ \vdots \\ plan_m \end{matrix} \begin{matrix} \begin{matrix} 1st & 2nd & & mth \end{matrix} \\ \begin{pmatrix} \pi_{11} & \pi_{12} & \cdots & \pi_{1m} \\ \pi_{21} & \pi_{22} & \cdots & \pi_{2m} \\ \vdots & \vdots & \cdots & \vdots \\ \pi_{m1} & \pi_{m2} & \cdots & \pi_{mm} \end{pmatrix} \end{matrix} \tag{18}$$

Consider the weighted rank frequency matrix Π, where the element Π_{ik} of the matrix is calculated as follows.

$$\Pi_{ik} = \sum_{j=1}^{n} f_{ikj} \times w_j \tag{19}$$

If inspections $plan_i$ is in the ranking k in the criterion j, then f_{ikj}=1. Therefore, the weighted rank frequency matrix Π is formed as follows:

$$\Pi = \begin{matrix} \\ plan_1 \\ plan_2 \\ \vdots \\ plan_m \end{matrix} \begin{matrix} \begin{matrix} 1st & 2nd & \cdots & mth \end{matrix} \\ \begin{pmatrix} \Pi_{11} & \Pi_{12} & \cdots & \Pi_{1m} \\ \Pi_{21} & \Pi_{22} & \cdots & \Pi_{2m} \\ \vdots & \vdots & \cdots & \vdots \\ \Pi_{m1} & \Pi_{m2} & \cdots & \Pi_{mm} \end{pmatrix} \end{matrix} \tag{20}$$

Establish the linear assignment model based on the Π_{ik} value. So, the model can be written in the following format:

$$\begin{aligned} & \max \sum_{i=1}^{m} \sum_{k=1}^{m} \Pi_{ik} \cdot h_{ik} \\ & Subject \quad to: \\ & \sum_{k=1}^{m} h_{ik} = 1, \quad i = 1, 2, \cdots, m; \\ & \sum_{i=1}^{m} h_{ik} = 1, \quad k = 1, 2, \cdots, m; \\ & h_{ik} = 0 \quad or \quad 1 \quad \forall i, k \end{aligned} \tag{21}$$

Solving the linear assignment model with Simplex method will gain the optimal permutation matrix H as a square ($m \times m$) matrix as follows.

$$H = \begin{matrix} & 1st & 2nd & \cdots & mth \\ plan_1 \\ plan_2 \\ \vdots \\ plan_m \end{matrix} \begin{pmatrix} h_{11} & h_{12} & \cdots & h_{1m} \\ h_{21} & h_{22} & \cdots & h_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ h_{m1} & h_{m2} & \cdots & h_{mm} \end{pmatrix} \tag{22}$$

Whereas h_{ik} is achived by the linear assignment model in (21). The optimal permutation matrix H determines the preferences of inspections plans.

The proposed procedure is summarized in Fig. 2.

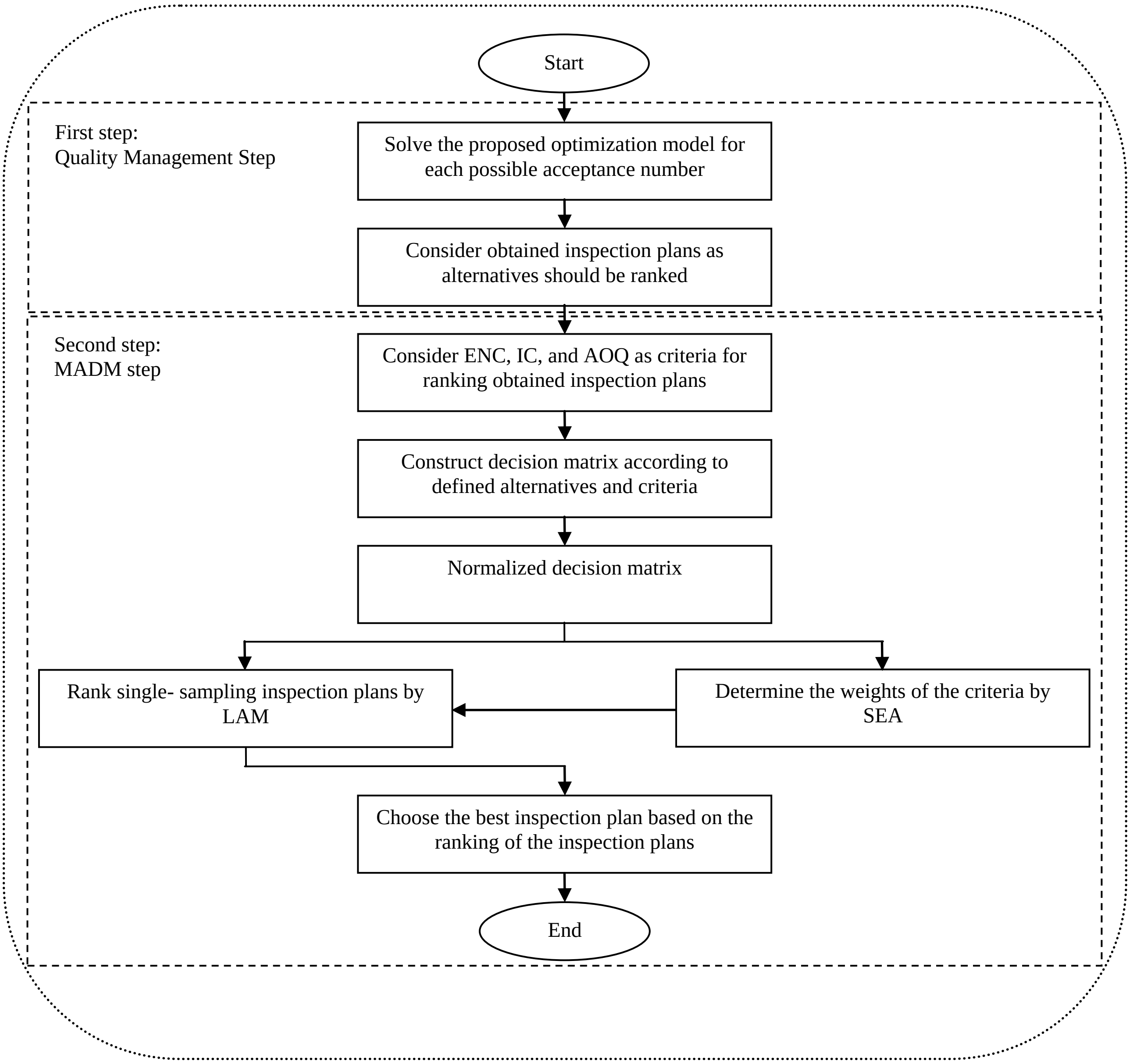

Figure 2. Procedure for the planning single- sampling inspection.

V. RESULTS AND ANALYSIS

This section takes an example to display the way that the proposed solution methodology could generate different single- sampling inspection plans and how the right one could be selected by comparing them with each other. In this example, the quality manager wants to set the sample size, *n*, and also acceptance number, *a*, for a lot waiting to inspect. The total available time for inspection, *T*, is 40 hours, because there are five inspectors that process the inspection during 8 hours. The other data for the example is as follows:

The salary rate per hour of inspector, *L*, is 40 dollars. The average inspection time per item, *t*, is 1 minute or (1/60) hour. The NC cost per item, *c*, is 87 dollars. The lot size, *N*, is 170, and the NC rate, *r*, is 0.05. For each possible acceptance number, *a*, the problem (*P*) is solved to achieve the optimal sample size, *n*. The possible acceptance numbers for the proposed example are 0 to 6. Therefore, different single- sampling inspection plans are generated for the acceptance number of 0 to 6 as displayed in Table II.

Table II. The results of problem p for different possible acceptance numbers

Results of problem *P*	**Acceptance number**						
	0	**1**	**2**	**3**	**4**	**5**	**6**
Sample size (*n*)	72	99	121	140	155	169	170
ENC	10.6124	11.954	11.8099	9.9864	7.0945	0.63798	0
IC	48	66	80.6667	93.3333	103.3333	112.6667	113.3333
AOQ	0.00072	0.00081	0.00080	0.00067	0.00048	0.000043	0

Fig. 3 shows the quality-related cost components and the expected total cost for the above example when the acceptance number, *a*, is 0. As this figure shows, an increase of sample size reduces the NC rate, so reducing the ENC cost. On the other hand, the total time needed for inspection and therefore, inspection cost inclines linearly as the sample size increases.

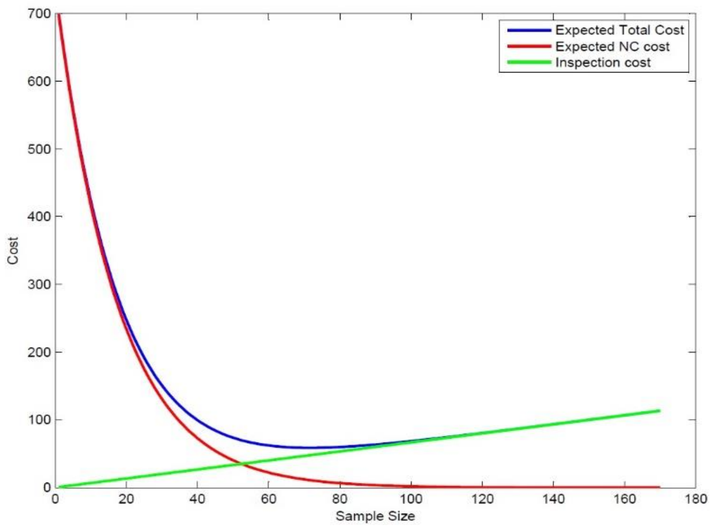

Figure 3. Cost components related to quality and the Expected Total Cost

Fig. 4 displays sample size, IC, ENC, and AOQ at different levels of acceptance number. The sample size and as a result, the IC increase when the acceptance number increases. On the other hand, by increasing the acceptance number the ENC and AOQ decline.

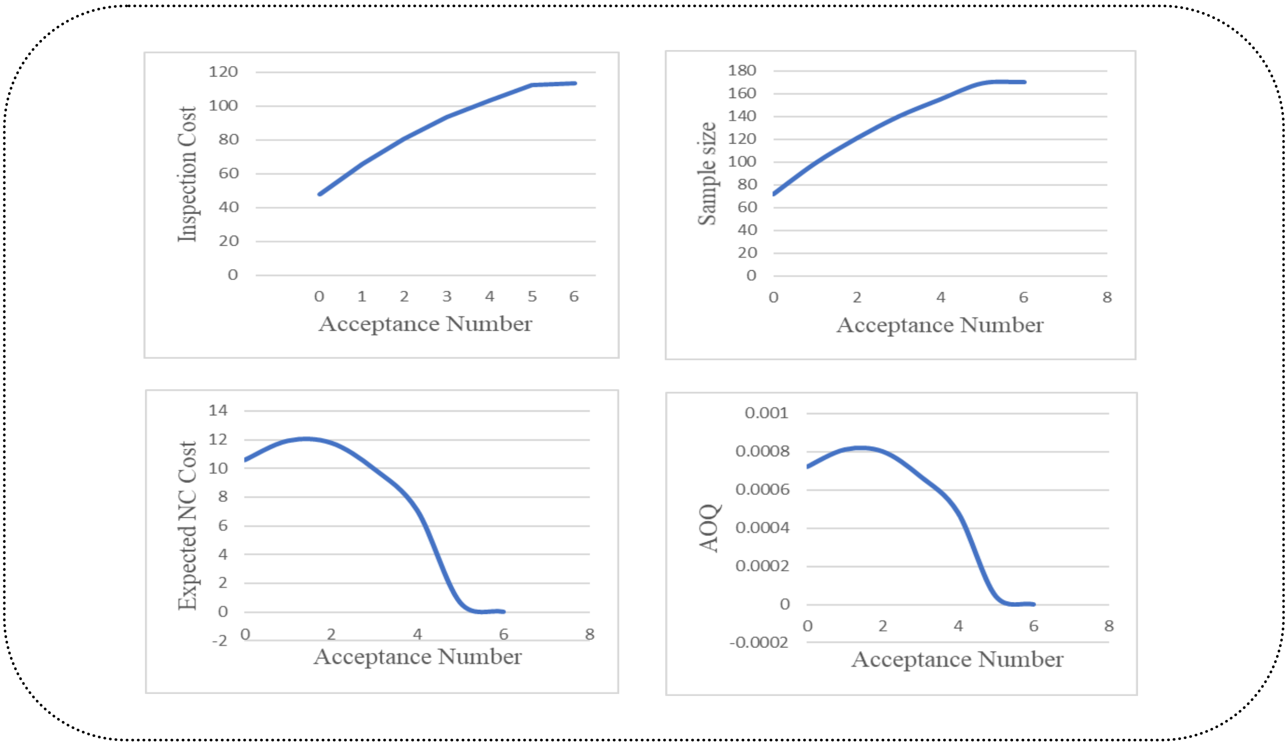

Figure 4. Sample size, IC, ENC, and AOQ at different levels of acceptance number

Based on the solution methodology that was explained in previous section, after generating different single-sampling inspection plans in the first step, the inspection plans should be compared to each other for selecting the right one by SEA and LAM in the second step. So, the decision matrix could be formed as Table III.

Table III. Decision matrix for comparing single- sampling inspection plans

Inspection plans	Criteria		
	ENC	IC	AOQ
$Plan_1$: (n= 72, a= 0)	10.6124	48	0.00072
$Plan_2$: (n= 99, a= 1)	11.954	66	0.00081
$Plan_3$: (n= 121, a= 2)	11.8099	80.6667	0.00080
$Plan_4$: (n= 140, a= 3)	9.9864	93.3333	0.00067
$Plan_5$: (n= 155, a= 4)	7.0945	103.3333	0.00048
$Plan_6$: (n= 169, a= 5)	0.63798	112.6667	0.000043
$Plan_7$: (n= 170, a= 6)	0	113.3333	0

As stated in the previous section the criteria in Table 3 do not have the same importance for comparing the single-sampling inspection plans. Therefore, each criterion in decision matrix needs a weight to show its importance. There are some methods for dedication of weight to criteria. As mentioned in previous section, one of these methods that is often used in literature for assessing weight, is SEA. For SEA, first the information in Table 3 should be normalized by Eq. (12) as shown in Table IV.

Table IV. The normalized decision matrix for comparing single- sampling inspection plans

Inspection plans	Criteria		
	ENC	IC	AOQ
$Plan_1$: (n= 72, a= 0)	0.618253937	1.629202191	0.624485232
$Plan_2$: (n= 99, a= 1)	0.879890824	0.899535721	0.883829791
$Plan_3$: (n= 121, a= 2)	0.851788651	0.304991319	0.855013729
$Plan_4$: (n= 140, a= 3)	0.496172325	0.208474976	0.480404921
$Plan_5$: (n= 155, a= 4)	0.067801821	0.613845238	0.067100259
$Plan_6$: (n= 169, a= 5)	1.326942991	0.992193518	1.326362173
$Plan_7$: (n= 170, a= 6)	1.451360926	1.019215499	1.450271241

Based on information in Table 4, E_j and d_j were obtained by Eqs. (15) and (16), respectively, as displayed in Table 5. By applying Eq. (17) the Shannon entropy weight (w_j) was achieved. The last row of Table V belongs to the w_j and shows the relative importance of each criterion.

Table V. The criteria weight by SEA

	ENC	IC	AOQ
E_j	0.08260	0.14236	0.08059
d_j	0.91740	0.85764	0.91941
W_j	0.34048	0.31830	0.34122

After determining weight for each criterion by SEA, the LAM is used to rank the inspection plans. To establish the LAM, first the plans are ranked in accordance with each criterion, as displayed in Table VI.

Table VI. Ranking matrix of alternatives based on each criterion

	ENC	IC	AOQ
1st	$Plan_5$	$Plan_1$	$Plan_5$
2nd	$Plan_7$	$Plan_2$	$Plan_7$
3rd	$Plan_6$	$Plan_3$	$Plan_6$
4th	$Plan_4$	$Plan_4$	$Plan_4$
5th	$Plan_3$	$Plan_5$	$Plan_3$
6th	$Plan_2$	$Plan_6$	$Plan_2$
7th	$Plan_1$	$Plan_7$	$Plan_1$

Based on Table VI, the rank frequency matrix π could be established as displayed in Table VII. The element of this matrix represents the number of times that $plan_i$ is ranked as the kth ranking.

Table VII. The rank frequency matrix π

	1st	2nd	3rd	4th	5th	6th	7th
$Plan_1$	1	0	0	0	0	0	2
$Plan_2$	0	1	0	0	0	2	0
$Plan_3$	0	0	1	0	2	0	0
$Plan_4$	0	0	0	3	0	0	0
$Plan_5$	2	0	0	0	1	0	0
$Plan_6$	0	0	2	0	0	1	0
$Plan_7$	0	2	0	0	0	0	1

By Eq. (19), the weighted rank frequency matrix Π is computed as displayed in Table VIII. The larger the contribution indicated by Π_{ik}, the greater concordance will be resulted from assigning $plan_i$ to the kth ranking.

Table VIII. The weighted rank frequency matrix Π

	1st	2nd	3rd	4th	5th	6th	7th
$Plan_1$	0.3183	0	0	0	0	0	0.68170
$Plan_2$	0	0.3183	0	0	0	0.68170	0
$Plan_3$	0	0	0.3183	0	0.68170	0	0
$Plan_4$	0	0	0	1	0	0	0
$Plan_5$	0.68170	0	0	0	0.3183	0	0
$Plan_6$	0	0	0.68170	0	0	0.3183	0
$Plan_7$	0	0.68170	0	0	0	0	0.3183

The linear assignment model is constructed based on the values in Π_{ik} matrix. So, the linear assignment model can be written as follows:

$$\max z = 0.3183h_{11} + 0.6817h_{17} + 0.3183h_{22} + 0.6817h_{26} + 0.3183h_{33} + 0.6817h_{35} + h_{44} + 0.6817h_{51} + 0.3183h_{55} + 0.6817h_{63} + 0.3183h_{66} + 0.6817h_{72} + 0.3183h_{77}$$

$$\sum_{j=1}^{7} h_{ij} = 1; \quad \forall \quad i$$

$$\sum_{i=1}^{7} h_{ij} = 1; \quad \forall \quad j$$

$$0 \le h_{ij} \le 1; \quad \forall \quad i, j$$

The solution of the above model by Simplex method is as follows:

h_{17}=1, h_{26}=1, h_{35}=1, h_{44}=1, h_{51}=1, h_{63}=1, h_{72}=1

In fact, the optimal permutation matrix H is as follows:

$$H = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The ordering of inspection plans is obtained by multiplying ($Plan_1$, $Plan_2$, $Plan_3$, $Plan_4$, $Plan_5$, $Plan_6$, $Plan_7$) by H. This gives result as the following matrix:

$$(plan_1, plan_2, plan_3, plan_4, plan_5, plan_6, plan_7) \times \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} =$$

$$(plan_5, plan_7, plan_6, plan_4, plan_3, plan_2, plan_1)$$

Therefore, the ranking order of the seven inspection plans are $Plan_5 \succ Plan_7 \succ Plan_6 \succ Plan_4 \succ Plan_3 \succ Plan_2 \succ Plan_1$. Thus, the best choice for single- sampling inspection plan is $plan_5$. It means that sample size, n, should be 155 and acceptance number, a, should be 4 for the proposed example.

VI. COMPARISON

In this section, the proposed method of this study is compared to the literature. For comparison, the result of the numerical example, presented in previous section, is compared to the model which is introduced by Qin et al. (2015). Qin et al. (2015) considered an assembly line that M different parts are coming for inspection. They determine an optimal sample size for each part based on the optimization problem that minimizes the total quality-related cost. It should be considered that in the illustrative example in previous section, we should take M= 1 according to the fact that in this paper, one part comes for inspection.

The solution that the optimization model of Qin et al. (2015) presents for the numerical example of this paper is n= 72 and a= 0. In fact, the solution presented by the model of Qin et al. (2015) is exactly $plan_1$ of the plans submitted by this paper. As the plan obtained by the optimization model of this paper is $plan_5$, n= 155 and a= 4, Table 9 shows these two plans and compares them based on three criteria considered in this paper means ENC, IC, and AOQ.

Table IX. Comparing the result of the paper with the model proposed by Qin et al. (2015)

Inspection plans	**Criteria**		
	ENC	**IC**	**AOQ**
$Plan_1$: (n= 72, a= 0)	10.6124	48	0.00072
$Plan_5$: (n= 155, a= 4)	7.0945	103.3333	0.00048

As Table 9 shows *plan*$_5$ has better performance in criteria of ENC and AOQ, while the plan presented by Qin et al. (2015) has better performance only in criteria of IC. As the analysis of the results in previous section shows, *plan*$_5$ has better ranking in comparison with *paln*$_1$ because the NC cost is much more than the inspection cost in the example. In fact, based on the analysis, *plan*$_5$ is ranked in the first order and *plan*$_1$ is recommended as the worst plan in comparison with six other plans presented in the numerical example.

VII. CONCLUSION

This paper develops an approach to determine an appropriate plan for single- sampling inspection. The proposed approach could determine sample size and acceptance number of the part waiting for inspection. In fact, in this paper, the acceptance number becomes another decision variable in addition to the sample size. According to the fact that the sample size and the acceptance number adversely impact the ENC, this paper formulated a nonlinear integer programming that could obtain the sample size for a given possible acceptance number. In this way, different appropriate inspection plans are generated. Then the inspection plans are ranked for selecting the most appropriate one by techniques of SEA and LAM. In fact, this paper recommends a two-step solution procedure that generates different possible inspection plans by a nonlinear integer programming in the first step and the most appropriate one is ranked and selected by SEA and LAM in the second step.

According to the fact that, the cost of rejection is low for inspection, thus, it was not considered in this paper, but in future research this cost could be considered. Also, this paper focused on single-sampling inspection plan and didnot consider multi-sampling inspection plan, so considering multi-sampling inspection could be another subject for future research. Comparing inspection planes with other techniques of MADM is another area for future research.

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