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A Mathematical Model for Operating Room Scheduling Considering Limitations on Human Resources Access and Patient Prioritization

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Abstract *– Operating room scheduling is an important task in healthcare sector. This study proposes a Mixed Integer Nonlinear Programming (MINLP) mathematical model for the scheduling of the operating rooms. In the presented model, apart from scheduling the patients' surgery process, shifting of the medical staff is also carried out. The innovation considered in the proposed model is aimed at prioritizing patients in the operation process, according to the priority and level of the patient's emergencies, and operating room treatment processes. Ultimately, the proposed model is assessed with random data, and in addition to scheduling patients based on the level of service delivery priority, the medical staff has been scheduled, as well. Furthermore, the sensitivity analysis results reveal that the proposed model is very sensitive to preoperative preparation times, and this bottleneck needs to be improved in the hospitals. On the other hand, this study presents solutions to improve the operating room scheduling and improving the status of patient services to manage and optimize operating room scheduling that result in satisfied patients.*

Keywords *– Healthcare centers, Modeling, Operating room, Planning, Scheduling.*

I. INTRODUCTION

Due to the aging society and technology advancement, demand for the medical care is on the rise in industrialized countries. Cost reduction and shortage of human resources lead to an increasing pressure upon hospital resources at the same time. This justifies the importance of optimizing the usage of rare hospital resources. Revenue management (RM) is the complex form of demand and supply management which helps the hospitals to maximize their revenue by establishing a balance between pricing and controlling the usage of equipment (Ratcliffeet al., 2011). The existing research studies show that the business model and RM are active in commercial and manufacturing industries and, given the common characteristics between commercial and manufacturing industries, such an approach can be used in service industries including general and specialized hospital management.

Although most hospitals encounter a heavy need for hospital beds, they do not have a scientific method to control the capacity of the beds. All general hospitals use first-come-first-served rule to allocate beds in general and specialized wards. Most research studies on the RM in the health sector discuss capacity control as the nature of health systems. Green et al. (2006) analyzed a patients' scheduling problem and determined threshold policies to manage the patient's demand and allocate the capacity with a limited planning horizon. In addition, Zhuang et al. (2010) showed how the

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distribution of the variable resource consumption affects the expected optimal profit and determines the priority of room order acceptance. Most hospitals use a system called block reserving during planning for surgeries, where a medical specialty, such as urology, is allocated to blocks that show a certain amount of time; for example, one day in an operating room. These blocks can be combined in the program of main scheduling periods in which each block is repeated after a fixed period. Cardeon et al. (2010) investigated the status of the time of operating room allocation to a surgical method using random service time. They studied the problem of controlling the capacity of operating rooms and the sequence of performing operation and scheduling them and then determining the protection level for each class of patients. Based on these studies, the sequencing and scheduling procedure should maximize the expected revenue of the surgical ward for hospitals in the scheduling period. As the surgical ward is one of the most expensive, and at the same time, the most profitable wards of the hospital, the optimal scheduling of operations can provide remarkable savings in this regard, given the type, duration, and limitations of each operation, including the number of rooms, working hours, and surgical groups. Operating rooms are the most expensive resources in hospitals and have received lots of attention from researchers in the area of scheduling (Duran et al., 2017). Research studies show that 15% of cancelations occur due to the bed shortage in the recovery room, under investigation. The duration of the patient's rest in the intensive care unit and the availability and readiness of this unit, affect the scheduling surgery. The duration of surgical operations depends on the given operation. Therefore, sometimes, there is a great difference between the planned duration and what really happens.

The proper management application in hospitals is of paramount importance to properly use resources and provide patients with desirable services. Any performance improvement in this area leads to savings followed by revenue increase (Cardeon et al., 2010). Hence, due to the technical obstacles and limitations of human resources in hospitals (Leelahavarong et al., 2019), a large proportion of patients cannot be immediately treated (Zhu et al., 2019); consequently, the longer the patients wait for treatment, the slower their course of treatment will progress, in addition to decreasing their satisfaction with the hospital services (Samudra et al., 2016). However, other hospitals, and in particular, public hospitals suffer from budget deficits, as well as shortage of work force and appropriate equipment (Molina et al., 2018). This limitation has forced hospitals to increase medical service efficiency in addition to reduce the cost of medical services (Hooshmand et al., 2018). It is worth noting that these hospitals are aimed at nothing more than balancing and integrating budget, scheduling, and equipment (Liu et al., 2018). Given the obtained data, more than 70% of the hospital visits have been for surgical operations (Rowse, 2015), and more than 15% of wasted time in the hospitals have been linked to the operating rooms (Hamid et al., 2019), highlighting the importance of the operating room and its proper use (Zhang et al., 2019) and (Roshanaei et al., 2019). The operating room and the surgical ward in general are of great importance in hospitals (Moosavi, Ebrahimnejad, 2018), considered as the most critical ward in hospitals (Koppka et al., 2018). Therefore, the very little defect or non-compliance with the specific program and predetermined standards will result in problems (Pang et al., 2018). Now, as described above, the hospital operating room may be considered as the hospital engine since many hospital resources, such as operating room scheduling managers, surgeons, anesthesiologists, nurses, etc. who may have different and sometimes contradictory goals, are dependent on the operating room and, ultimately, the operating room scheduling (Belkhamsa et al., 2018).

In order to schedule the surgery of the selected patients in a hospital, Khalifali et al. (2019) presented an MINLP model to maximize the operating room capacity as well as the number of operations. In this research, the genetic algorithm (GA) was employed to evaluate the model solution on real large data sets. Silva and Souza (2019) presented a scheduling problem of cancer patients in a chemotherapy center. For scheduling and timing the patient visits, they considered random values for patient visits and adapted the new method based on the integration of optimization and scheduling. With this method, the performance of the chemotherapy center increased about 20% compared to its previous performance. In order to improve the scheduling of a health center and provide patients with the best scheduling and timing, Chen et al. (2019) considered specifications including treatment priority, the time of starting the first treatment of each patient, and their longest waiting time duration. Considering the probable time of admission of the patients to the treatment center, the researchers presented a hybrid model of online and random optimizations. The results of the study revealed the satisfactory performance of the hybrid model in real examples. Hamid et al. (2019)

addressed the scheduling of selected surgery patients in orthopedic ward of a hospital. They presented a mathematical model with the aim of using all operating rooms and shortening the operating during to provide patients with the best way of assigning to operating rooms. The authors presented a discrete simulation model to compare the scheduling of the new model with manual scheduling performed by a nurse. Finally, by solving some real examples, they showed that the performance of the presented model is superior to that of the supervisor of the surgical ward. Rezaeiahari and Khasawneh (2020) presented a simulation method based on scheduling problem of patient visits. They used modeling tools and simulation techniques for diagnostic imaging department optimization. The researchers employed the simulated annealing algorithm to optimally schedule the visits. This study is aimed at minimizing the total working time and waiting time of the patients. The obtained results show that the total working time has decreased by 5% and patient waiting time by 38%, on average. Ballestín et al. (2019) presented an approach to solve and test the advanced scheduling and re-scheduling problem. They considered a penalty function in order to assure a certain level of quality presented to the patients. In this problem, the researchers randomly considered how long the new operation lasts and added a new patient to the surgery operation list. To solve this problem, the researchers used the robust optimization model and presented a rotational approach for rescheduling in the desired time horizon. In order to solve the surgical scheduling problem, Varmazyar et al. (2020) proposed an MINLP model that can find optimal solutions to problems of small size. In this study, the researchers simultaneously considered operating rooms, recovery, required surgical resources, as well as the likelihood of emergency surgeries. They transformed the proposed approach into constraint scheduling and presented a meta-heuristic algorithm based on GA and a constructive heuristic algorithm for solving large-size problems.

Considering the above-mentioned problem, this research presents the process of patients' surgery in the hospital with the aim of minimizing the scheduling of performing therapeutic process. It can be seen that most of the previous studies have been carried out regardless of the sequence of operations performed on the patients, without paying attention to the prioritization of the patients to receive operation services. The research gap observed in this section is associated to the patients' priorities for surgery as well as access to resources for preparation and operation. It is known that the process of operating room patients is divided into two groups; patients scheduled to enter the surgery queue based on a prior appointment, and patients admitted to the operating room in the emergencies and with the need to undergo surgery according to their current deterioration. Hence, according to the performed assessment, no study has addressed the classification and prioritization of the patients so far; thus, this research provides innovations in prioritizing operation. On the other hand, resource constraints in hospitals have resulted in the use of medical equipment based on the patient's disease severity, addressed in this study using a sequencing model. Moreover, this study also takes into account enhancing the patients and personnel satisfaction using scheduling modeling approach, given the constraint of lack of access to sufficient human resources, multidimensionality of service provision, access to therapeutic medical equipment, and the sequence of performing surgical operation.

The rest of the paper is organized as follows. Section 2 reviews the previous works and studies. Section 3 presents problem formulation. Implementation of the model with data analyzing are done in Section 4 and the model sensitivity analysis is in section 5. Finally, the conclusion and some future study guidelines are provided in Section 6.

II. LITERATURE REVIEW

Scheduling problem and determining the sequence of the process of patients who visit surgery and recovery is regarded as an important part of presenting services to patients in order to earn income for hospital. Therefore, the study of the problems in the health area has begun about 60 years ago, while a considerable volume of these research studies has been performed since 2000 onward. Management of surgical ward in the healthcare centers is considered an important issue in the management of healthcare centers, recently received remarkable attention from the researchers (Cardeon et al., 2010). Therefore, research studies conducted on the executive nature of scheduling and income-earning in hospital units and operating room scheduling and allocation are presented here. According to a study conducted in

2007, it was found about 60-70% of the hospital admissions belong to the surgical operations (Hans, Nieberg, 2007). Zhuang et al. (2010) studied the problem of income management with several types of quality of service provision where the resource consumption for each group was random and determined only when the patient left the hospital. Their model was developed with the aim of using income management problem. Their studies showed how distribution of the variable resource consumption affects the expected optimal profit. Huang and Hsu (2005) investigated the problem of controlling the operating room capacity and determining protection level for each class of patients in a way that the expected income of the surgical ward is maximized during the scheduling period. As the operating rooms have a close relationship with the other wards of the hospital and the improvement of their efficiency both increases the rate of providing service to patients and their satisfaction and significantly reduces costs and increases the efficiency of the hospital complex, they can be considered as the heart of the hospital and the main center of its cost and income (Hans, Nieberg, 2007). Therefore, even small improvements in the efficiency of the operating room not only can save the life of more humans but also bring about saving and financial revenue and several social advantages. Meanwhile, considering the existing resources, the proper planning and scheduling of the operations related to surgeries in operating rooms are among methods which can increase the efficiency of the operating room without increasing the required hardware resources. Scheduling of the operating room indicates the process of determining the time of surgeries, as well as allocating the post-operation rooms and the related medical team to perform each of these surgeries (Zhuang et al., 2010). Various studies have investigated the problem of surgeries scheduling and allocation of operating rooms. Jebali et al. (2006) conducted a study with the aim to improve the efficiency of operating rooms, given the limitation of the resources for the problem of scheduling the process of hospitalization until the operation room discharge. They investigated the effect of scheduling program on the patients’ expectation, surgeon's waiting time and the operating room overtime work using a probable mixed-integer programming model. Lamiri et al. (2008) presented a model for weekly planning problem of surgeries with the aim to minimize the patients' cost and operating rooms overtime work. They used Monte Carlo simulation method combined with mixed-integer programming model. This model was improved considering the idle cost of operating rooms and the costs related to the patients and operating rooms overtime work. Using a model, Cardeon et al. (2010) investigated the problem of scheduling surgeries given the priority of surgeries. In addition to minimizing overtime works, this model has considered the minimization of priorities violation as objective function. Fügener et al. (2014) proposed the comprehensive surgical scheduling problem given the intensive care unit and the general needs of the patient after leaving the operating room. Considering the dependence of the surgeries time on the surgeons' skill and expertise, Molina-Parient et al. (2015) investigated the problem of scheduling and planning of surgeries in an integrated way. A similar model was presented by Silva et al. (2015) in order to make maximum use of the operating rooms capacity given the limitation of human resources. The increasingly growth of research studies which have investigated the problem of surgery scheduling in the healthcare area indicates the necessity of the subject matter under investigation. A glance at the related literature reveals that minimization of the programs completion time has been always among the common objectives in these problems. Likewise, in the problem of operating room scheduling, attempts were always made to reduce the completion time of surgeries to reduce the costs imposed on healthcare centers. Besides these common indicators, a few research studies have investigated other factors such as priority in performing surgeries, patients waiting time for surgery, the risk related to surgery, desirability of the surgical team, etc., in the scheduling problem.

Considering the importance of operating rooms in the medical services ward, this area has received lots of attention from the researchers. It is worth noting that the researchers differentiate between strategic (long-term), tactical (mid-term), and operational approaches for classifying their planning and scheduling problems. They performed operating rooms scheduling at the tactical level for elective patients. Generally, compared to emergency surgical operations, elective surgical operations are defined as a group of surgical operations which are programmable. Using column production, they modeled the problem mathematically with the aim of maximizing the efficiency of operating room considering the number of required beds in units such as intensive care unit and ward. In order to show the efficiency of the model, they used medical center data. Results showed that the proposed approach performs well for the optimal use of operating room and the required beds in the after surgery- care units (Molina-Parient et al., 2015). Min and Yeh (2010) presented the operational scheduling of elective surgeries which considers the uncertainty during surgery and limitation of the capacity of downstream units such as surgical care unit during several periods. They modeled random surgery scheduling problem, which minimized the sum of costs directly related to the patients, as well as the expected costs of overtime work. They considered the level of operational surgery planning in this research. Wang and Xu (2017) planned and scheduled the operating room by considering the downstream units based on the optimization of particle swarm in two phases. The first phase determines if the patients can undergo operation during planning period. Then the time of their surgery is determined with the aim of maximizing the patients' satisfaction. In the second phase, the surgery schedule, including a sequence of surgeries and related operating rooms is designated as the downstream unit considering the post-anesthesia care unit. The aim of this research was to minimize the constant cost of operating room, cost of overtime work, and cost of recovery. In order to show the efficiency of their method, they used real data and achieved results close to those obtained by CPLEX method, with the difference that the computational time of their method was much less. In another research, Fugener et al. (2014) scheduled the rounds of an operating room block for different specialties. They developed the domain of the downstream resources, needed by the patients after the surgery, such as intensive care unit and general wards. They proposed a random analytical approach that calculates the exact demand distribution for downstream resources in a main scheduling program of surgery. In addition, they introduced some criteria for downstream accrued costs from a main scheduling program of surgery. In another study, Molina-Parient et al. (2015) proposed a random planning model for operating room in which the aim is to minimize the waiting time of operating room and minimize the costs for patients. Among the most important limitations of modeling, they referred to the limited capacity of operating room system. Therefore, for more exact assessment, they used uncertainty in scheduling approach. Finally, Monte Carlo optimization method in sample average approximation mode was used. In addition, the tabu search method was used to solve the model in a greater space. Results obtained by comparing Monte Carlo with tabu search showed that Monte Carlo solutions deviated from those of tabu search by 1%; it was found that meta-heuristic solution is efficient in this area.

Barrera et al. (2018) assessed and introduced the healthcare environment in Chile. They stated that in the healthcare environment of Chile, patients should go to private centers to treat themselves if medical services are not allocated to them, and therefore, governmental agencies decided to reduce the patients' costs. In this research, a mathematical model was proposed to minimize the patient’s costs and minimize the overtime hours of operational personnel of governmental hospitals. Considering the proposed scheduling model, the authors stated that in virtue of the implementation of modeling and simulating hospital status, the personnel overtime was reduced from 21% to less than 1%, and on the other hand, the waiting time of patients was reduced from 71 patents in queue to 58 patients. Finally, the authors stated that the results of simulation and modeling improved the therapeutic processes of Chile remarkably. Hooshmand et al. (2018) proposed a mathematical model for operating room planning. The time of offering services is expressed as uncertain random time in this model. In order to assess uncertainty, they used the scenario writing approach and stated that as the mathematical model is NP-Hard, he used the genetic meta-heuristic algorithm. As a result, the quality of the solutions and the time to solve the model is far better than exact solution. They also stated that, considering the importance of time in therapeutic decision makings, it is required to use meta-heuristic approaches for planning. Vali-Siar et al. (2018) stated that the effective planning for on-time treatment of patients and improvement of the quality of services and activities of operating room is necessary. In this research, attempts were made to investigate an integrated multi-period and multi-resource planning under uncertainty conditions. To achieve this purpose, a mixed linear planning model was developed to minimize fatigue in surgical operation, overtime, and idle time of resources. Limitations related to human resources, equipment, as well as bed in the preoperative maintenance unit, recovery ward, and intensive care ward and unit were considered. The duration of surgical operation and recovery is uncertain, and in order to manage uncertainty, a powerful optimization method was used. Considering the complexity of the model and the incapability of solving big problems in large scale, genetic algorithm-based meta-heuristic method and a constructive exploratory approach were proposed. After adjusting the solution parameters of using Taguchi method, numerical tests were done based on different samples, and results obtained by solving the mathematical model were compared to the results from exploratory methods. Results showed that the proposed methods have a very good performance and the exploratory approach outperforms the genetic approach, because the objective performance of the proposed constructive exploratory approach is on average better than that of genetic algorithm by 19%. A case study was also performed in a hospital. Results from comparing the proposed methods with hospital planning indicated that overtime work and idle time were improved significantly in the proposed methods. In his research study, Guido and Conforti (2017) presented a multi-objective linear programming model in the area of operating room scheduling management. By comparing the exact algorithms (CPLEX) and genetic metaheuristic, an optimization model was developed to perform operating room scheduling for each surgical specialty. Scheduling the operating room allocated to each surgical team, planning surgery admission using epsilon constraint approach, and making Pareto box present a set of optimal solutions which can be used by hospital managers to efficiently use the resources involved and plan surgeons and surgeries. Considering the conducted research, the efficient genetic algorithm was assessed. In this study, Al-Rafaie et al. (2018) stated that a smart hospital could react to emergency conditions and unexpected events in real time. Emergency conditions often occur in hospitals, and the number of people who receive service should be maximized. As urgent patients interfere with the predetermined programs of operating rooms, the preventive methods to protect the individuals' lives should be maximized. In this study, three optimization models were proposed to optimize the operating room planning in unexpected events and handling urgent patients in the scheduled program. The first model (model I) treats urgent patients in newly constructed rooms, while the second model (model II) transfers urgent patients to the idle rooms. The third model (model III) transfers the urgent patient to the room with the maximum level of free surgery rooms in elective and emergency hospitals. This study presents a real case study to assess the operating room models. Three models were accepted, and then, the results were compared. As a result, it is expected that the proposed models provide the hospital with preventive plans. Additionally, they increase the efficiency of operational rooms. In this research, Wu et al. (2018) investigated the problem of three-stage operating room planning. The pre-operation, post-operation, and recovery after operation stages were assessed and scheduled by the authors. Therefore, they proposed a MIP mathematical model with the aim of minimizing the maximum work time of the operating room. In order to solve the model, standard genetic algorithm (without adjusting parameters) was proposed. A variable neighborhood search was combined with genetic algorithm to improve local search ability. Results of the experiment showed that variable neighborhood search can greatly improve the roulette wheel performance (as we know in genetic algorithm method the rotation takes place randomly and a new child is produced, now by integrating tabu search the roulette wheel performance has improved). Inspired by a real case from a large regional hospital in Argentina, Moreno and Blanco (2018) presented the integer linear programming model (ILP) to plan a patient appointment. As there is considerable uncertainty over the amount of clinical services available per slot, a fuzzy programming method has been developed to solve the multi-objective problem. Considering the obtained results, such an approach awaits natural decision makers. Moreover, Zanda et al. (2018) presented an MIP modeling approach for operating room in one of the Italian hospitals. This mathematical model discussed resource allocation and scheduling. Bagheri et al. (2016) proposed a two-stage stochastic optimization model for heart surgery ward in one of the Iranian hospitals. During the long stay of patients, the patients demand and stay is exposed to uncertainty. Sample average approximation (SAA) method was used to obtain an optimal program to minimize the costs of personnel reappointment and overtime work. Through numerical tests, it shows that convergence of statistical boundaries and average sample size for a given numerical test confirms the results of the model validation. Considering the presented literature review, a research gap is identified on the lack of application of scheduling models and early and tardy approach as well as the lack of investigation of scheduling from different perspectives in modeling. This study tries to address this research gap.

III. MATHEMATICAL MODELING

In this section, we state the problem and refer to the provided model. In this mathematical model, we seek to schedule the patient's arrival and departure from different hospital wards, in this process, the patient passes the given stages and wards according to the treatment improvement process defined at the beginning of the work. In addition, the sequence of providing service to each patient at different stages as well as regular service queue system are also considered. In the formulated model, the conditions to receive service from hospital personnel has been considered such that hospital personnel do the rounds just one time and present all required services to patient during their visit. Moreover, this model will schedule and plan operating rooms and all hospital wards on a daily basis. One of the main assumptions of this model is that duration of service to patient should be deterministic.

The main difference between the proposed model for operating room scheduling and the industrial model is in the use of hospital staff when performing the therapeutic process. In the operation room mathematical model, in addition to the patient scheduling for operating room, scheduling and staff shifting are performed, as well. On the other hand, the difference between the presented model and the industrial models are the prioritization of patients in order to accelerate the treatment process. This study considers two groups of patients; in the first group, they enter the service providing system and the second group, depending on the critical condition of patients in the operating room therapeutic processes are prioritized, and this has not been addressed in the industrial models so far.

Indices, parameters, and variables of the model are presented as follows:

Table I. Sets and Indices

Sets & index	Description
I	Set of patients
E(i)	Set of common patients
W(i)	Set of special patients
J	Set of all the stages(sections) that the patients pass
K	Set of equipment at each stage
P	Set of hospital personnel
i & i'	Patient index
j & j'	Stages index
k & k'	Equipment index
p	Hospital personnel index

Table II. Parameters

Parameters	Description
PT_{ijk}	Time to prepare patient i at stage j on equipment k (prepared time)
AT_{ij}	The patient's arrival time to stage j (when patient enters the system or arrival time)
ST_{ijk}	Time of service to patient i at stage j on equipment k (service time)
PST_{pijk}	Time of providing service by human resource p for patient i at stage j on equipment k (personnel service time)
WT_i	The waiting time of each patient i from operating time to recovery (waiting time)

Table III. Variables

Variables	Description
C_{ijk}	End time of service activities to patient i at stage j in ward k in shift d
Y_{ijk}	If human resource p provides service k for patient i at stage j, 1, otherwise it is zero
Q_{pijk}	If human resource p provides service k for patient i at stage j it is 1, otherwise it is zero
$X_{ii'jk}$	If patient I after patient i' uses equipment K at stage j it is 1, otherwise it is zero

$$\mathrm{MIN\ C} = \left(\sum_{i}\left(\mathrm{MAX}\sum_{j}\sum_{k} C_{ijk} - AT_{ij}\right)\right) \tag{1}$$

S.t.

$$\sum_{\substack{i\in I\\ i\neq i1}}\sum_{i\prime\in I} X_{ii\prime jk} \leq 1 \quad \forall\, j, k \tag{2}$$

$$\sum_{k\in k_{ij}} y_{ijk} = 1 \quad \forall\, i, j \tag{3}$$

$$\sum_{k\in k_{ij}} Q_{Pijk} = 1 \quad \forall i, j, P \tag{4}$$

$$C_{ijk} \leq M * y_{ijk} \quad \forall i, j\ , k \in k_{ij} \tag{5}$$

$$C_{ijk} \leq M * Q_{pijk} \quad \forall i, j, p, k \in k_{ij} \tag{6}$$

$$C_{ijk} \geq \sum_{k'} C_{ij'k'} + PT_{ijk} + AT_{ij} + ST_{ijk} + \sum_{p} PST_{pijk} - M\left(1 - Y_{ijk}\right) \quad i \in w(i) \tag{7}$$

$$C_{ijk} \geq \sum_{k'} C_{ij'k'} + PT_{ijk} + AT_{ij} + ST_{ijk} + \sum_{p} PST_{pijk} - M\left(1 - Y_{ijk}\right) \quad i \in e(i), j \neq s_3 \tag{8}$$

$$C_{ijk} \geq Y_{ijk} * \left(PT_{ijk} + AT_{ij} + ST_{ijk} + \sum_{p} PST_{pijk}\right) \quad \forall i \in I, \forall\, j \in J, \forall k \in K \tag{9}$$

$$\sum_{k} C_{ijk} + \sum_{k} C_{ij'k} \leq WT_i \quad \forall\, i \in w(i), j = s_2, j' = s_3 \tag{10}$$

$$\sum_{k} C_{ijk} + \sum_{k} C_{ij'k} \leq WT_i \quad \forall\, i \in E(i), j = s_2, j' = s_3 \tag{11}$$

$$X_{ii'jk} + X_{i'ijk} \leq 1 \quad \forall\, i, i < i', j, k \tag{12}$$

$$2X_{ii'jk} \leq Y_{ijk} + Y_{i'jk} \quad \forall\, i, i < i', j, k \tag{13}$$

$$Y_{ijk} + Y_{i'jk} \leq X_{ii'jk} + X_{i'ijk} + 1 \quad \forall\, i, i < i', j, k \tag{14}$$

$$C_{ijk} \geq C_{i'jk} + PT_{ijk} + AT_{ij} + ST_{ijk} - M * X_{i'ijk} - 2M + MY_{ijk} + MY_{i'jk} \quad \forall k, j, i, i', i < i' \tag{15}$$

$$C_{i'jk} \geq C_{ijk} + PT_{i'jk} + AT_{ij} + ST_{i'jk} - M * X_{ii'jk} - 2M + MY_{ijk} + MY_{i'jk} \quad \forall k, j, i, i', i < i' \tag{16}$$

In (1) i.e., the objective function of the problem, we seek to minimize the time of the patient presence in all stages and hospital wards. In fact, relation (2) states that if both patient i and patient i' are present on one of the equipment at a common stage, then one of the patients enters the stage of receiving services and the other patient remains in the waiting queue. Constraint (3) states that according to the "from-to" table defined upon the arrival of the patient, the patient should pass all determined wards. To put it differently, the patients cannot refuse service. Constraint (4) states that in accordance with a pre-considered procedure, if patient is hospitalized, then the patient will certainly receive service from personnel during the therapeutic process. Constraint (5) states that there is a relationship between the time of receiving service and its binary variable. Constraint (6) states that if the patient receives service at each stage, then the corresponding binary variable related to the hospital personnel will gain a value. Constraint (7) states that the end time of receiving a service for each special patient includes the time of receiving services in the previous stages, the time of patient preparation at that stage, the time of receiving services from equipment, and the time of receiving services from personnel, of course with the final condition that the patient needs services and personnel at that stage. Constraint (8) states that the end time of receiving a service for each common patient includes the time of receiving services in the previous stages (except recovery stage), the time of patient preparation at that stage, the time of receiving services from equipment, and the time of receiving services from personnel, of course with the final condition that the patient needs services and personnel at that stage.

Constraint (9) shows that the time of service to the patient at different stages will include the patient's preparation time and the time of providing service by human resources and the time of service and the time of entering into the system if the given stage and ward is defined in the patient treatment description (this constraint checks if the patients' needs service at that stage, then the end time of his/her work will be the sum of preparation time and receiving services). Constraint (10) states that the waiting time to receive recovery services should be less than the fixed value for special patients who have undergone an operation. Constraint (11) states that for general patients who have undergone an operation, the waiting time to receive recovery services should be less than the fixed value. Constraint (12) states that two patients in a ward who ask to receive equipment, only one of the patients receives service at unit time and the other patient is placed in the waiting queue. Constraint (13) states that if a patient is placed in the queue of the previous patient, he/she waits until that equipment is emptied and then receives services. In fact, with this constraint a regular queue is created for each of the equipment. Constraint (14) completes constraints (6) and (7). Constraint (15) states if patient i' receives services sooner than patient i and patient i is in the waiting queue of patient i', then service receiving time for patient i is longer than patient i', and constraint (16) is in contrary to constraint (15).

$$\mathrm{MAX}\sum_{j}\sum_{k} C_{ijkd} = t_i \qquad t_i \geq \mathrm{MAX}\sum_{j}\sum_{k} C_{ijk} - AT_{ij} \tag{17}$$

IV. DATA ANALYSIS

Considering the presented modeling, in order to validate the mathematical model, a hypothetical example was investigated to evaluate the validity of the model. Therefore, the main parameters of the model are as follows:

Table IV. Mathematical model inputs

Parameters	Mathematical model inputs
PT_{ijk}	Uniform [10-20]
AT_{ij}	Uniform [5-15]
ST_{ijk}	Uniform [5-10]
PST_{pijk}	Uniform [10-20]
WT_i	Uniform [10-20]

Considering changes created in the objective function, the above-mentioned problem is a linear integer-programming model implemented by GAMS software. This implementation has reached an optimal solution with a gap of zero under the system with specifications of CPU core I3, RAM 4 for 0.08 s.

By solving the mathematical model using GAMS software and the CPLEX solver, the scheduling results for patients are in Fig. 1.

Section / Equipment	S1			S2			S3		
Patient	eq1	eq2	eq3	eq4	eq5	eq6	eq7	eq8	eq9
P1	6			10					
P2					8				
P3			6			8		10	
P4					6				12
P5		8				10			

Fig. 1. The scheduling results for patients

As observed in the above figure, the deadline for the completion of the operating room process for all patients was considered 12 hours; accordingly, all patients were completed within the time limit of their surgical procedure, and the cycle of therapeutic process was as above. The status of assigning human resources to patients is in Fig. 2.

Section	S1														
Equipment	eq1	eq2	eq3	eq1	eq2	eq3	eq1	eq2	eq3	eq1	eq2	eq3	eq1	eq2	eq3
Patient / Staff	P1			P2			P3			P4			P5		
nurse1	■													■	
nurse2									■						
nurse3											■				

Section	S2														
Equipment	eq4	eq5	eq6	eq4	eq5	eq6	eq4	eq5	eq6	eq4	eq5	eq6	eq4	eq5	eq6
Patient / Staff	P1			P2			P3			P4			P5		
dr1					■				■						■
dr2	■										■				
dr3															

Fig. 2. The status of the human resources allocation

Section	**S3**														
Equipment	**eq7**	**eq8**	**eq9**	**eq7**	**eq8**	**eq9**	**eq7**	**eq8**	**eq9**	**eq7**	**eq8**	**eq9**	**eq7**	**eq8**	**eq9**
Patient / Staff	**P1**			**P2**			**P3**			**P4**			**P5**		
rec1								■							
rec2												■			
rec3															

Continue Fig. 2. The status of the human resources allocation

As observed, in the human resources assignment in the operating room ward, the human resources assignment to the treatment departments of the patients is clear. Hence, the most appropriate human resources assignment to patients has been implemented in this process. Therefore, the validation of the mathematical model in different dimensions is as described in Table 5.

Table V. Mathematical model result

GAMS		**no. of equipment at each section**	**no. of section**	**no. of patients**	**no**
t(s)	f_{opt}				
1	6400	4	3	10	**1**
1	8460	4	3	15	**2**
5	9550	4	3	20	**3**
8	31500	5	4	25	**4**
10	38700	5	4	30	**5**
12	75030	10	4	40	**6**
32	102560	15	4	50	**7**
496	203480	15	4	60	**8**

V. MODEL SENSITIVITY ANALYSIS

Considering the input data in the mathematical model, the sensitivity analysis is carried out on key parameters of the model and changes in the model objective function are assessed.

Considering the model sensitivity analysis at the time of providing service to the patient, by reducing the intervals of providing service to patient time, the duration of activities undergoes no change and only by an increase of 15%, the service to patient time has increased by 2% during the whole period, indicating the existence of vacant capacity in the model designed to serve the patients.

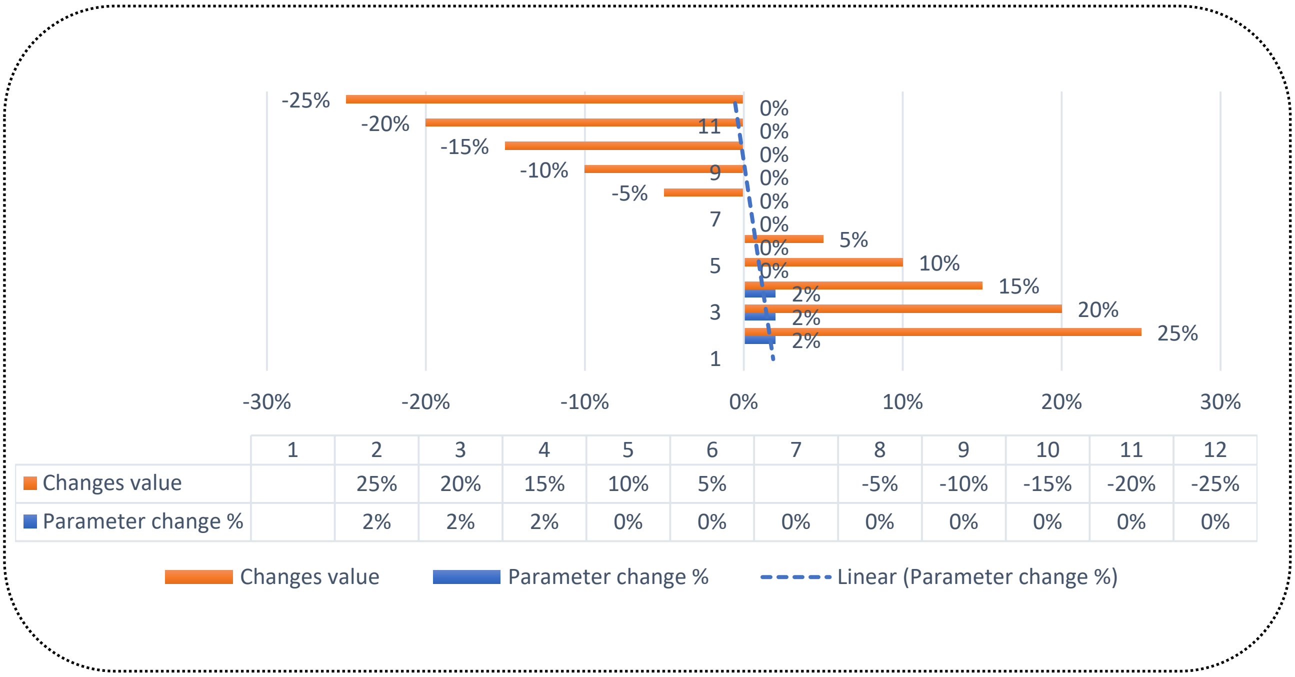

Fig. 3. Sensitivity analysis of service to patient time

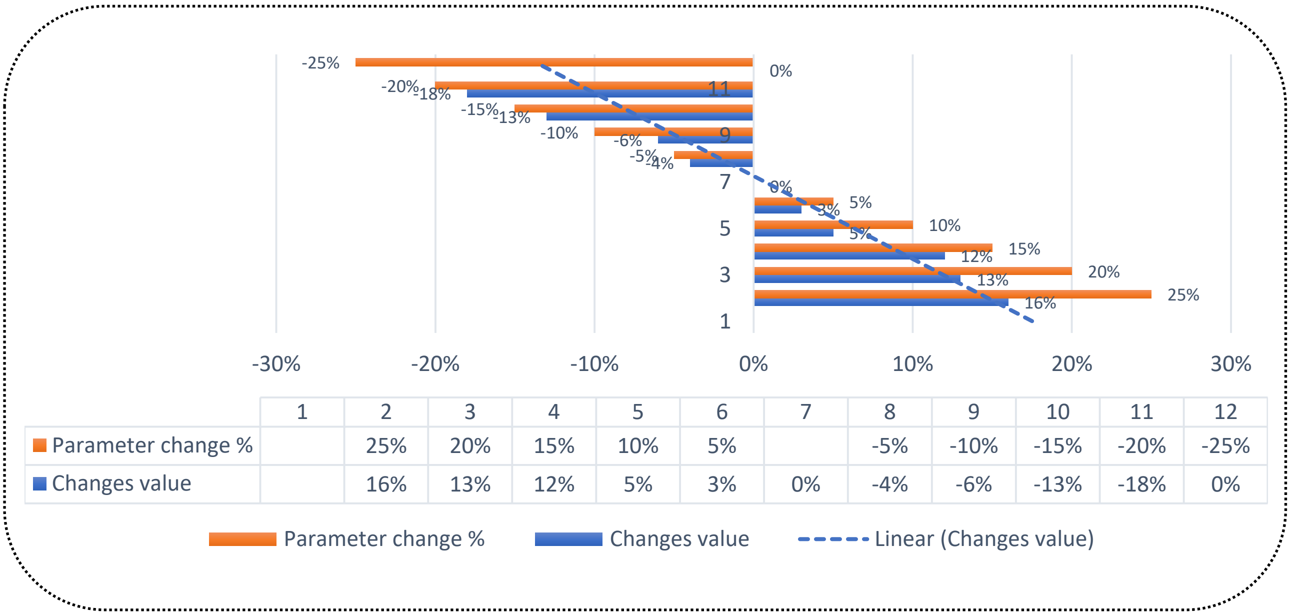

Fig. 4. Sensitivity analysis of preparation time

As shown in the above sensitivity analysis, the effect of model changes has been very sensitive at all stages compared to the patient's preparation time and changes of this time interval have a direct effect on the model. Therefore, planning hospital resources such as nurses is of paramount importance and scheduling medical personnel is very effective on the proposed model.

VI. CONCLUSION AND RECOMMENDATIONS

Operating room is one of the important wards of the hospital whose most vital task is to provide medical emergency services. Patients referring to this ward are in critical condition and treating them as soon as possible and with the highest quality is included among the duties of medical and nursing staff working in this ward. A survey of the Iranian health care services has revealed that the patient's average waiting time in the operating room ward is high, indicating problems in the operating room health care system. The increasing number of referrals to the operating room as well as the waiting time of patients, and the complexity of service provision, on the other hand, necessitate changing, modifying, or upgrading this highly sensitive system. Patients waiting time and costs are one of the important factors that should be considered in the management and organization of the healthcare sector. Hence, in the present study, a mathematical model was presented for scheduling and assigning work in operating room for patients' operating process. The proposed approach optimizes patients operating room scheduling as well as the use of hospital equipment and medical staff, simultaneously. The results of the present study suggest that in assessment of the proposed model, significant improvement in terms of sequencing of surgery according to patient's prioritization has been made and the best path of operating the patients has been selected based on the description of treatment.

It is recommended to follow the present study by applying the uncertainty approach based on patient's parameters and other parameters that can have effects on the results and happen in scheduling of operating room health services, and to evaluate and analyze the larger dimensions of the proposed model using metaheuristic or heuristic algorithm approaches and new exact approach for solving the model and for comprehensive review it is suggested to develop this studies model in specific health center and solving this model with real data in health centers and finally scheduling and planning and develop new approach for achieve to real result.

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