# Power Allocation Strategies in Block-Fading Two-Way Relay Networks

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Abstract- This paper aims at investigating the superiority of power allocation strategies, based on calculus of variations in a point-to-point two-way relay-assisted channel incorporating the amplify and forward strategy. Single and multilayer coding strategies for two cases of having and not having the channel state information (CSI) at the transmitters are studied, respectively. Using the notion of calculus of variations, the optimum power allocation strategy across code layers for the case of employing Multilayer coding is derived and is shown to surpass the conventional outage approach employing single layer coding strategy in terms of the achievable sum rate of the network. Moreover, for the case of knowing the CSI at the transmitters, an adaptive power control strategy is analytically derived, and its achievable sum-rate is compared to that of the single-layer strategy under peak power constraint.

Index Terms- Multi-layer coding, calculus of variation, power allocation.

#### I. INTRODUCTION

Employing relays is a crucial factor to enhance the reliability and throughput of wireless networks which are steadily serving the emerging data-hungry applications. So, making efficient use of these nodes to widen the network coverage is one of the main challenges for research communities and development workforce [1]. The notion of relaying, which was introduced by [2] is considered in many network topologies.

In [3] a transmission scheme was introduced that admitted the information exchange between two transceivers in the presence of a half-duplex relay in two consecutive time-slots. Soon after, the idea was incorporated by many papers that considered different channel models and objectives for the aforementioned network. In [4], the two-way relay network under gamma fading distribution was considered, where some performance bounds were derived. Many papers like [5], considered the two-way relay (TWR) network in a multiple-antenna basis and proposed many novel transmission schemes, based on signal processing approaches. A common assumption in all of the above works is that the channel state information (CSI) is known at every transceiver node at the network, which turns the full power transmission in each time-slot to a reasonable decision.

There are many papers in the literature like [6], that have considered two-way relay channels in the case of having no access to the channel state information at the transmitter (CSIT). The transmission scheme is based on the notion of the outage approach in these works. The outage approach consists of sending a single-layer channel code in each time-slot, and hoping that the channel strength is above the required threshold for decoding the transmitted information. Hence, the transmitted information is not decodable in the cases that the realized channel strength is lower than the required threshold and an outage event occurs.

In [7], in a block fading point to point channel with no channel state information at the transmitter (non-CSIT), it is shown that by incorporating the multi-layer channel coding, the average achievable rate of a point-to-point communication channel can surpass that of the outage approach. Multi-layer coding consists of transmitting a superposition of channel codewords, called layers, which decoding each of them requires the realized channel strength to be greater than a unique threshold, associated with it. In this way, some layers are decoded in each time-slot and the number of decoded layers varies in each time-slot, according to the channel strength realizations. Moreover, [7] has considered the case of dealing with continuous channel strength distributions, where infinite layer transmission is proposed in accordance to infinite number of possible channel strengths. In the latter case, the power allocation among the code layers is derived by incorporating the notion of calculus of variations [8].

Following this line of thought, there are plenty of papers that have examined the multi-layer coding approach in relay channels, where a relay connects a source to a destination via wireless fading channels [9]–[15]. The pioneering paper [9] has studied the use of multi-layer coding in a relay channel with different types of relaying protocols, including decode-and-forward (DF), compress-and-forward (CF) and amplify-and-forward (AF) methods. The papers [10] and [11] go through some more complicated and extreme cases of having or not having CSI at the transmitters in AF and DF relay networks. [12] has considered a diamond relay channel in which two parallel relays help transmitters communicate with the receivers in a block fading channel. Utilizing the channel with more than one relay, is extended to the case of delay-constrained applications which are using buffer aided relays in [13]. Another example of considering multiple relays in a non-CSIT network is [14], where arbitrary number of relays are incorporated in a network and a central node does the relay selection job, in order to enhance the average achievable rate of the network by employing multi-layer coding strategy.

In what follows, we are going to address the power allocation strategies in a two-way relay network in non-CSIT and full-CSIT fashions. According to the best of the authors' knowledge, the non-CSIT relay-assisted communication together with multi-layer coding is only studied for one-way relay networks. Moreover, there is not any reference, having focused on the use of calculus of variations for power control in the case of full-CSIT communication over a TWR network. In this regard, two novel power allocation strategies are proposed for the TWR network demonstrated in



Fig. 1. The considered two-way communication setup.

Fig. 1, which are based on the notion of calculus of variations. It is assumed that the communication is carried out in two hops. First, the main transceivers send their signals to the relay and in the second hop, the relay amplifies the received signal and broadcasts it to the transceivers.

The main contribution of the current manuscript is two-fold:

- First, for the case of not having any access to the CSI at the transmission phases, the average achievable sum rate of the (TWR) network is formulated under the notion of multi-layer coding approach and the results is compared to that of the single-layer transmission, named as outage approach, showing that the use of multi-layer coding strategy leads to higher average achievable rate in a TWR network.
- Second, for the case of having CSI at transmission ends, a power allocation scheme based on average power constraint at the transmitters is proposed, which is based on the use of calculus of variations and leads to a higher level of achievable rate at the network, when compared with full power transmission in each time-slot.

The rest of the current manuscript is structured as follows. The equivalent channel gain between transceiver ends is derived in Section II. Section III is divided into two parts. Multi-layer coding and the outage approach are characterized in the first subsection, and the adaptive power allocation and full power transmission policies are described in the second subsection. The simulation results in Section IV show the performance of the proposed approaches based on the calculus of variations and section V, summarizes findings.

## II. SYSTEM AND CHANNEL MODEL

Consider two transceivers exchanging their signals, say  $x_1$  and  $x_2$  through the relay-assisted channel of Fig.1, where their corresponding channel coefficients to the relay are  $h_1$  and  $h_2$ , respectively. We consider a block fading environment where the channel gains are constant in two consecutive blocks, named as one transmission block, and vary independently for the next transmission blocks. The communication is carried out in two hops within a transmission block. First, during the first hop, the relay receives the combination of the transmitted signals combined with a complex additive white Gaussian noise CN(0,1), as follows,

$$y_r = h_1 x_1 + h_2 x_2 + n_r, (1)$$

where it is assumed that the transmitted signals  $x_i$ , i = 1, 2 are of power  $P_i$ , i = 1, 2. Second, the relay amplifies the normalized version of its received signal and re-transmits it throughout the second hop. Considering  $s_1$  and  $s_2$  as the channel strength values, associated with  $h_1$  and  $h_2$ , respectively, the relay's received signal power becomes  $s_1P_1 + s_2P_2 + 1$ . Moreover, due to the assumed block fading environment, each of the transceivers can estimate their respective channel coefficient by incorporating a training sequence in the beginning of each transmitted block. Hence, considering the fact that each transceiver knows its transmitted signal in the transmission phase, one can assume to have a complete self-interference cancellation in the reception phase. Therefore, the received signal at the either of transceivers, has the following form,

$$y_{i} = \sqrt{\frac{P_{R}}{s_{1}P_{1} + s_{2}P_{2} + 1} \left(h_{1}h_{2}x_{j} + h_{i}n_{r}\right) + n_{i}},$$
(2)

where in the above formulation we have  $i, j \in \{1, 2\}, i \neq j$ . From now on, we merely concentrate on the first link from the second transmitter to the first receiver, as any finding can be readily extended to the other link. Having (2) and noting that the first receiver tries to decode the second transmitter's information justifies the signal and interference terms. Hence, the mutual information associated with the first link becomes [15],

$$I(x_2; y_1) = \log\left(1 + \frac{s_1 s_2 P_R}{s_1(P_1 + P_R) + s_2 P_2 + 1} P_2\right).$$
(3)

Therefore,  $\gamma_{12} = \frac{s_1 s_2 P_R}{s_1 (P_1 + P_R) + s_2 P_2 + 1}$  can be thought as the equivalent channel strength between

the first link's corresponding endpoints. Noting the particular importance of having the channel strength statistics associated with either of the links, in deriving the power allocation policy for the cases of presence and absence of channel state information, in the sequel we derive a formulation for the equivalent CDF of the channel strength.

In this regard, one would formalize the probability of random variable  $\Gamma_{12}$  being lower than any non-negative realization like  $\gamma_{12}$ , as follows,

$$F_{\Gamma_{12}}(\gamma_{12}) = \Pr\left(\Gamma_{12} \le \gamma_{12}\right) = \Pr\left(\frac{s_1 s_2 P_R}{s_1 (P_1 + P_R) + s_2 P_2 + 1} \le \gamma_{12}\right).$$
(4)

Simplifying the last inequality in (4) results in,

$$s_1 \left( s_2 P_R - \left( P_1 + P_R \right) \gamma_{12} \right) \le \left( s_2 P_2 + 1 \right) \gamma_{12}.$$
(5)

The inequality of (5) leads in two following disjoint regions of,

$$R_{1}:\left\{s_{2} < \frac{P_{1} + P_{R}}{P_{R}}\gamma_{12}\right\} \cap \left\{s_{1} \ge 0\right\},$$

$$R_{2}:\left\{s_{2} \ge \frac{P_{1} + P_{R}}{P_{R}}\gamma_{12}\right\} \cap \left\{s_{1} \le \frac{\left(s_{2}P_{2} + 1\right)\gamma_{12}}{s_{2}P_{R} - \left(P_{1} + P_{R}\right)\gamma_{12}}\right\}.$$
(6)

Incorporating (6), the equivalent channel strength CDF of (4) is reformulated as,

$$F_{\Gamma_{12}}(\gamma_{12}) = \int_{0}^{\left(1+\frac{R}{P_{R}}\right)\gamma_{12}} f_{S_{2}}(s_{2})ds_{2} + \int_{\left(1+\frac{R}{P_{R}}\right)\gamma_{12}}^{\infty} f_{S_{2}}(s_{2}) \int_{0}^{\frac{(s_{2}P_{2}+1)\gamma_{12}}{s_{2}P_{R}-(P_{1}+P_{R})\gamma_{12}}} f_{S_{1}}(s_{1})ds_{1}ds_{2}.$$
(7)

Simplifying the above equation, gives,

$$F_{\Gamma_{12}}(\gamma_{12}) = F_{S_2}\left(\left(1 + \frac{P_1}{P_R}\right)\gamma_{12}\right) + \int_{\left(1 + \frac{P_1}{P_R}\right)\gamma_{12}}^{\infty} f_{S_2}(s_2) F_{S_1}\left(\frac{(s_2P_2 + 1)\gamma_{12}}{s_2P_R - (P_1 + P_R)\gamma_{12}}\right) ds_2.$$
(8)

where by taking derivative from (8) with respect to  $\gamma_{12}$ , the associated pdf of the equivalent channel gain can be derived.

#### **III. THE PROPOSED POWER ALLOCATION STRATEGIES**

In this section, we propose two power allocation methods for a block fading two-way relay network, both based on the calculus of variations. The multi-layer coding approach in subsection III-A is provided for the case of not having the CSI at the transmitters. This method is further compared with the conventional outage approach. Afterwards, assuming to have the instantaneous CSI at the transmitters, another transmission strategy is proposed in subsection III-B, which is named the adaptive power transmission throughout this paper. To demonstrate the advantage of incorporating the latter, the full power transmission is characterized in the concerned network.

#### A. Power allocation in the lack of CSI

Multilayer coding is a magnificent approach in transmitting information over point-to-point block fading channels, when the channel state information is not available at the transmitter.

According to the notion of multilayer coding in [7], the receiver of a block fading point to point communication channel is cast as a set of virtually ordered receivers in a one-to-one relationship to the set of possible channel strength realizations, which leads to a degraded broadcast channel. In this regard, the transmitted message is the superposition of messages that each of them is designed to be decoded at one of the considered virtual receivers in a successively refinable manner. Explaining more precisely, the first virtual receiver decodes its corresponding message and removes it from the received signal, and then, the second receiver does the same for its corresponding message and the procedure goes on, until the instantaneous channel strength realization does not allow the next virtual

receiver to decode its corresponding message. The allocated power to the transmitted codeword corresponding to each virtual receiver plays the role of interference for the lower ordered receivers and the number of virtual receivers in each transmission block that are able to decode their corresponding messages specifies the instantaneous achievable rate of the considered network. It is worth noting that for any realized channel strength value like  $\gamma_{12}$ , only the virtual receivers associated with channel strength values below  $\gamma_{12}$  are able to decode their corresponding messages.

Incorporating multi-layer coding transmission, the instantaneous achievable rate of the point-topoint block fading channel of the current manuscript, corresponding to the channel strength realization of  $\gamma_{12}$  becomes,

$$R(\gamma_{12}) = \frac{1}{2} \int_{0}^{\gamma_{12}} \frac{\tau g(\tau) d\tau}{1 + \tau G(\tau)}.$$
(9)

The power  $g(\tau)d\tau$  is allocated to the receiver indexed by have  $\tau$ , and  $G(\tau)$  represents the

integration of the allocated power to the layers greater than  $\tau$  that results in  $G(\tau) = \int_{\tau}^{\infty} g(\tau) d\tau$  and

 $g(\tau) = -\frac{d}{d\tau}G(\tau)$ . Taking expectation from (9), it can be demonstrated that the resulting average achievable rate of multi-layer coding can be formulated as,

$$R_{12} = \max_{G(.)} \frac{1}{2} \int_{0}^{\infty} \left( 1 - F_{\Gamma_{12}}(\gamma_{12}) \right) \frac{\gamma_{12}g(\gamma_{12})d\gamma_{12}}{1 + \gamma_{12}G(\gamma_{12})},$$
  
s.t.  $G(0) = \int_{0}^{\infty} g(\gamma_{12})d\gamma_{12}.$  (10)

We aim at maximizing  $R_{12}$  in (10) through finding the optimal form of G(.) and its derivative g(.) in the integral forms of (10). By what follows in (12)-(14), it is worth mentioning that the constraint of (10) is an obvious fact which is always true, and the main purpose of showing it is to get insight regarding the total power constraint regarding the problem of interest and getting the starting point of power allocation interval. The problem can be solved by incorporating the calculus of variations [8]. The Lagrangian dual function associated with (10) has the following form,

$$L = \frac{1}{2} \int_{0}^{\infty} (1 - F_{\Gamma_{12}}(\gamma_{12})) \frac{\gamma_{12}g(\gamma_{12})d\gamma_{12}}{1 + \gamma_{12}G(\gamma_{12})} + \lambda \int_{0}^{\infty} G'(\gamma_{12})d\gamma_{12}$$
  
$$= \int_{0}^{\infty} \left[ \frac{1}{2} (1 - F_{\Gamma_{12}}(\gamma_{12})) \frac{\gamma_{12}g(\gamma_{12})}{1 + \gamma_{12}G(\gamma_{12})} + \lambda G'(\gamma_{12}) \right] d\gamma_{12}.$$
(11)

The integrand of (11) can be named as  $H(\gamma_{12}, G(\gamma_{12}), G'(\gamma_{12}))$  with the following definition,

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$$H(\gamma_{12}, G(\gamma_{12}), G'(\gamma_{12})) = (1 - F_{\Gamma_{12}}(\gamma_{12})) \frac{-\gamma_{12}G'(\gamma_{12})}{1 + \gamma_{12}G(\gamma_{12})} + \lambda G'(\gamma_{12}),$$
(12)

and the optimal form of G(.) is derived from the following Euler equation [9],

$$H_{G} - \frac{d}{d\gamma_{12}} H_{G'} = 0.$$
(13)

In (13),  $H_G$  and  $H_{G'}$  respectively, denote the partial derivation of  $H(\gamma_{12}, G(\gamma_{12}), G'(\gamma_{12}))$  with respect to  $G(\gamma_{12})$  and  $G'(\gamma_{12})$ . Noting the provided discussion regarding the power constraint of (10), it is worth mentioning that the term  $\lambda G'(\gamma_{12})$  is vanished through deriving the components of (13) after taking derivative from  $H_{G'}$  with respect to  $\gamma_{12}$ , and the optimal form of the interference function would have the mathematical form of,

$$G(\gamma_{12}) = \frac{1 - F_{\Gamma_{12}}(\gamma_{12})}{\gamma_{12}^2 f_{\Gamma_{12}}(\gamma_{12})} - \frac{1}{\gamma_{12}},$$
(14)

in an interval like  $[\gamma_1, \gamma_2]$  which its starting point comes from the transmit power constraint of  $G(\gamma_1) = P_1$  and its end point is derived from the positivity of the interference function, i.e.,  $G(\gamma_1) = 0$ . The average achievable sum-rate of the network is derived calculating the average achievable rate of the other link, i.e.,  $R_{12}$  and adding it to  $R_{21}$ .

On the other hand, a conventional approach in establishing a communication in block fading channels, is to optimally set a constant threshold, namely  $\gamma_{th}$ , for the equivalent channel gain and transmit with a fixed rate through the use of a single-layer code, dubbed outage approach. Decoding the received signal, depends on the instantaneous channel realization w.r.t. the adjusted threshold. The average rate of the outage approach, associated with the first link is formulated as follows,

$$R_{12}^{outage} = \max_{\gamma_{th}} \frac{1}{2} \log\left(1 + \gamma_{th} P_1\right) \times \left(1 - F_{\Gamma_{12}}\left(\gamma_{th}\right)\right),\tag{15}$$

and the total average sum rate of the network comes from calculating  $R_{21}^{outage}$  and adding it to the result of (15).

### B. Power allocation in the presence of CSI

In the case of having the instantaneous CSI at the transmitter, a wise decision is to make use of this information in setting the transmission power in each block. Considering the first transmitter is constrained with the average power  $P_1$ , the average rate of  $R_{12}$  is formulated as follows,

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$$R_{12} = \max_{p(.)} \frac{1}{2} \int_{0}^{\infty} f_{\Gamma_{12}}(\gamma_{12}) \log(1 + \gamma_{12}p(\gamma_{12})) d\gamma_{12},$$
  
s.t. 
$$\int_{0}^{\infty} f_{\Gamma_{12}}(\gamma_{12}) p(\gamma_{12}) d\gamma_{12} = P_{1}.$$
 (16)

In order to find the optimal function p(.) maximizing the achievable rate of (16), the following Lagrangian dual function can be proposed,

$$L = \int_{0}^{\infty} f_{\Gamma_{12}}(\gamma_{12}) \left( \frac{1}{2} \log \left( 1 + \gamma_{12} p(\gamma_{12}) \right) - \lambda p(\gamma_{12}) \right) d\gamma_{12} + \lambda P_{1}.$$
(17)

Further, considering the notion of calculus of variations the variational function of  $H(\gamma_{12}, p(\gamma_{12}), p'(\gamma_{12}))$  can be defined as,

$$H(\gamma_{12}, p(\gamma_{12}), p'(\gamma_{12})) = f_{\Gamma_{12}}(\gamma_{12}) \left(\frac{1}{2} \log(1 + \gamma_{12} p(\gamma_{12})) - \lambda p(\gamma_{12})\right),$$
(18)

and the optimal form of instantaneous transmission power would come from solving the following Euler equation,

$$H_{p} - \frac{d}{d\gamma_{12}} H_{p'} = 0.$$
(19)

The factor  $\lambda$  in (18) is the Lagrange multiplier. The solution of the above optimization is,

$$p(\gamma_{12}) = \frac{1}{2\lambda} - \frac{1}{\gamma_{12}}, \qquad \gamma_{12} \ge 2\lambda, \tag{20}$$

where  $\gamma_{12} \ge 2\lambda$  comes from the positivity of the power allocation in each transmission block, and  $\lambda$  is derived by solving the following equation,

$$\int_{2\lambda}^{\infty} f_{\Gamma_{12}}\left(\gamma_{12}\right) \left(\frac{1}{2\lambda} - \frac{1}{\gamma_{12}}\right) d\gamma_{12} = P_1,\tag{21}$$

which satisfies the transmitter's average power constraint.

Another way to transmit in the case of having instantaneous CSI at the transmitter, is to change the transmission rate according to the current fading realization, while using the transmitter's full power in each transmission block. Concurrently, the average achievable rate at either of the receivers is derived by taking two-dimensional expectation from (3) as follows,

$$R_{12}^{CSIT} = \frac{1}{2} E_{s_1, s_2} \left[ \log \left( 1 + \frac{s_1 s_2 P_R}{s_1 \left( P_1 + P_R \right) + s_2 P_2 + 1} P_2 \right) \right].$$
(22)

Again, it is worth mentioning that computing the average sum rate of the network in the above two schemes requires calculation of the average achievable rate of the other link and adding it to the one, addressed in the current subsection.



Fig. 2. The equivalent channel gain CDF between the second transmitter and first receiver, for different transmitter and relay powers in Rayleigh flat fading environment.

## IV. NUMERICAL RESULTS

In order to simulate the resulting average achievable rate, Rayleigh block fading environment is considered. In this case, the equivalent channel gain CDF between the second transmitter and the first receiver is simulated by inserting (23) into (8),

$$F_{s_1}(s_1) = 1 - \exp(-s_1),$$
  

$$F_{s_1}(s_2) = 1 - \exp(-s_2).$$
(23)

Incorporating (8), the equivalent channel strength CDF of the considered network has the following form,

$$F_{\Gamma_{12}}(\gamma_{12}) = 1 - \exp\left(\left(1 + \frac{P_1}{P_R}\right)\gamma_{12}\right) + \int_{\left(1 + \frac{P_1}{P_R}\right)\gamma_{12}}^{\infty} \exp\left(-s_2\right) \left(1 - \exp\left(\frac{(s_2 P_2 + 1)\gamma_{12}}{s_2 P_R - (P_1 + P_R)\gamma_{12}}\right)\right) ds_2.$$
(24)

The equivalent channel strength CDF of (24) is depicted in Fig. 2, where it can be seen that increasing the transmit power of main transceivers, as well as the relay, leads the equivalent CDF to a smoother curve. Therefore, the probability of experiencing lower equivalent channel gains is decreased and the probability of experiencing higher channel strength is increased.

The average achievable rate of the considered network is considered in Fig. 3. The curves, corresponding the power allocation strategies are derived for setting a fixed power at the relay, sweeping the transmitters power from 0dBW to 35dBW.



Fig. 3. The average achievable sum-rate of the two-way relay network for different power allocation strategies. Continuous and dashed lines, respectively, denote setting the relay's power equal to 20 dBW and 15 dBW.



Fig. 4. The average achievable sum-rate of the two-way relay network for different power allocation strategies, under 100Watts total power Constraint.

In this regard, the average achievable rate of the considered network using the outage approach and multilayer coding, in the case of lacking CSI at the transmitters, as well as the result of employing full power and adaptive power transmission in the case of having full-CSI at the transmitters are depicted in Fig. 3.

When the whole network is facing a total power constraint, allocating the available power to the transmitter changes the resulting average sum rate of the network. Noting the symmetry of the main transceivers, allocating similar transmit power to the first and second relay is reasonable.



Fig. 5. The average achievable sum-rate of the two-way relay network for different power allocation strategies and total power constraint.

Following this idea, the average achievable rate of the network is simulated for total power constraint of 10Watts and the results are depicted in Fig.4. The results of this figure suggest that the allocating one quarter of the available power to each main transceiver and spending the rest at the relay results in having the highest average achievable rate.

The resulting average sum-rate of the discussed power allocation strategies are depicted in Fig. 5 for the total power constraint of the network. The results are conducted for the case of allocating onequarter of the total power to either of the main transceivers and letting the relay consume the remained half. It can be seen that the resulting average rate in case of incorporating the power allocation policies taking out of the calculus of variations, surpass the conventional approaches.

#### V. CONCLUSION

A two-way relay communication system under block fading characteristics is studied throughout this manuscript. Two novel approaches, based on the calculus of variations are provided in case of having and lacking CSI at the transmitters. Multilayer coding, known to be a merit approach in the lack of CSI at the transmitter, is characterized in the considered setup and its corresponding optimal power allocation function among the code layers and the average achievable sum-rate are derived. In case of having channel state information at the transmitter, an adaptive power allocation is provided, which adaptively changes its transmission power with respect to the channel fading strength variations. The two proposed methods are compared with outage approach - in the lack of CSIT - and the adaptive rate transmission - in the presence of CSIT - and it is shown that the proposed methods outperform the conventional methods in terms of the average achievable sum-rate.

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