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Economic Single-Sampling Plans Based on Different Probability Distributions Considering Inspection Errors

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Abstract *– As a traditional statistical quality control method, acceptance sampling plans are widely applied for quality assurance. In a sampling plan, with the aim of acceptance or rejection of a lot of martial, inspection is carried out to determine adherence to standards. Usually, it is assumed that the inspection of the items is error-free. In the present study, this assumption is relaxed. Using Bayesian inferences and considering inspection errors, three mathematical models are developed for the economic single-sampling plans. First, the model is developed based on Binomial distribution. Regarding the application of Poisson model in approximating Binomial distribution, the second model is developed based on Poisson distribution. The third model is presented considering Negative binomial (which is also known as Pascal Distribution). The models determine the sample size and acceptance number to minimize the expected inspection costs incurred during sampling. A numerical example is presented and sensitivity analyses are carried out.*

Keywords*– Acceptance sampling plan, Bayesian inferences, Inspection errors, Probability distribution*.

I. INTRODUCTION

The application of acceptance sampling plans can be expressed as follows. A company receives much material from a supplier. A sample is randomly taken from the lot, and desired quality characteristics are inspected. According to the sample results, it is decided to accept the lot or reject it. The decision regarding rejection or acceptance of the lot is usually referred to as the lot sentencing. Accepted lot is applied in the manufacturing process. The rejected lot is either returned to the supplier or may be subjected to some additional lot disposition actions.

Usually, there are three approaches for lot sentencing: (1) accept the lot without inspection, (2) inspection of the whole lot, i.e., 100% inspection, and (3) acceptance sampling (Montgomery, 2009). Generally, acceptance sampling has more advantages in comparison with the other two approaches. For example, when the test of items is destructive or when the cost of 100% inspection is exceptionally high.

There are different ways to classify acceptance sampling plans. In an aspect, the plans are classified into attributes and variables. A variable is a quality characteristic that can be measured on a numerical scale. The attribute is a quality

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characteristic that is expressed as a "go" and "no-go" basis. A single-sampling plan is as follows: select a random sample of size *n* and inspect each sample item. If the number of defectives is fewer or equal to a specified number as *c*, then accept the lot. On the other hand, if more than *c* defectives are observed in the sample, then reject the lot.

A double-acceptance sampling plan is complicated. First, a sample is randomly selected from the lot. According to the information obtained from this sample, the management faces three scenarios: (1) accept the lot, (2) reject it, or (3) take another sample. If management decides to take another sample, then the information obtained from both samples is combined to make a final decision. More than two samples may be required to reach the final decision in a multiple-sampling plan. Thus, a multiple-sampling plan can be considered as an extension of a double-sampling plan. A sequential sampling plan is an ultimate extension of the double sampling plan. In this plan, sampling is performed stage by stage. According to the total numbers of defective items and total numbers of items inspected in each stage, it is decided whether to accept the lot, reject it, or continue to sample.

In sampling plans, it is usually assumed that the process of testing or inspecting the items of the sample is perfect. This assumption is rarely met in reality. Inspection errors affect the performance of the sampling plan. The source of these errors can be the operational environment, inspector fatigue, and failure of the inspection tool. Therefore, it is necessary to analyze the statistical and economic influence of inspection errors on the performance measures of a sampling plan. The inspection errors are classified into two categories: Type I and Type II. Type I error yields to the rejection of a conforming item, while Type II error yields to the acceptance of a defective item.

Fallahnezhad et al. (2018 a), based on an absorbing Markov chain, developed a model to determine process parameters while inspection errors were taken into consideration. Fallahnezhad et al. (2018 b) presented an economic acceptance sampling plan based on the Maxima nomination sampling method in the presence of inspection errors. Rasay et al. (2018) analyzed a sequential sampling plan and a double sampling plan in a truncated life test based on Weibull distribution. Fallahnezhad et al. (2017) considered inspection errors in the control chart design for high yield processes. Jamkhaneh et al. (2011) presented a single sampling plan with inspection errors, while the fraction of defective items is considered a fuzzy number. Chattinnawat (2013) used numerical methods to analyze the inspection errors in a single-sampling plan with zero acceptance number

Fallahnezhad and Aslam (2013) applied a backward induction of dynamic programming to optimize a sampling plan. Fallahnezhad and Hosseini Nasab (2012) developed a sampling plan in the presence of inspection errors using the negative binomial distribution. Fallahnezhad and Hosseini Nasab (2011) proposed a sampling plan according to a control threshold policy. Khamseh et al. (2008) considered inspection errors in a variable sampling plan. They proposed a double-sampling plan and Taguchi Loss function in their analyses. Khan and Duffuaa (2002) analyzed the effect of the inspection errors on different inspection plans. Markowski and Markowski (2002) showed that it is necessary to consider inspection errors in designing sampling plans. They stated that ignoring the inspection errors led to suboptimal solutions for sampling plans. Duffua (1996) studied the statistical and economic effects of the inspector errors on the sampling plan.

In this paper, a single-attribute acceptance sampling plan is investigated. Three mathematical models are developed considering inspection errors. Binomial, Poisson, and Negative binomial distributions (or Pascal distribution) are employed in developing the models. The models determine the sample size and acceptance number to optimize the expected total cost of the inspection. The rest of the paper is organized as follows: In Section 2, a preliminary is presented regarding the problem. Also, some notations and conditional probabilities are presented in this section. Section 3 presents a mathematical model for the economic sampling plan according to Binomial distribution. Considering the application of the Poisson model in approximating Binomial distribution, Section 4 presents a mathematical model for the economic sampling plan using Poisson distribution. In Section 5, a model is developed based on Negative binomial distribution (also known as Pascal distribution). In Section 6, a numerical example is presented and optimized. Section 7 carries out some sensitivity analyses. Finally, Section 8 concludes the paper.

II. PRELUDE

As a prelude to developing the economic sampling plan, some notations are introduced. Moreover, some conditional probabilities are computed according to the Bayesian Theory.

AQL: Acceptable Quality Level

LTPD: Lot Tolerance Percent Defective

N: Total number of items in a lot

n: sample size

c: acceptance number

p: the defective proportion

k: the number of defective items in the sample of size n

j: the number of defective items in the N-n remaining items

I_1: *the cost of inspecting one item*

I_2: *the cost of replacement or repair for one defective item during the inspection*

I_3: *the cost of classifying one conforming item is defective*

A_2: *the cost of classifying one defective item as a conforming item (Warranty cost)*

α: *Producer's risk*

β: *Consumer's risk*

The procedure of a single-sampling plan is straightforward. Suppose we have a lot of martial, including *N* items. Select a random sample with size *n* from the lot. Each item of the sample is inspected to determine whether it is defective or not. If the total number of defective items in the sample is equal or less than a specified number as *c*, i.e., the acceptance number, then the lot is accepted. On the other hand, if the number of defective items in the sample is more than *c*, then the whole lot is inspected according to the rectifying inspection policy. Two types of errors may occur during the inspection of the items, including type I error and Type II error. Type I error occurs when a conforming item is classified as a defective item. Type II occurs while a defective item is classified as a conforming one. Thus, the probability of type I and Type II errors can be formally stated as follows:

$$e_1 = the\ probablity\ of\ type\ I\ error = P\{the\ item\ is\ calssifed\ as\ defective | the\ item\ is\ conforming\}$$

$$e_2 = the\ probablity\ of\ type\ II\ error = P\{the\ item\ is\ calssifed\ as\ conforming | the\ item\ is\ defective\}$$

The observed defective proportion is denoted by P'. Let define *E* and *F* as in the following:

E = the event that an item is defective

F = the event that an item is classified as defective

Thus, the following equation holds:

$$P' = P(F) = P(E).P(F|E) + P(E^c).P(F|E^c) = P.(1-e_2) + e_1.(1-P) \tag{1}$$

Where

$P = P(E)$, *proper defective fraction*

$P' = P(F)$, *observed defective fraction*

Accordingly, the observed *AQL* and *LTPD* are computed as in the following:

$$AQL' = (1-e_2).AQL + e_1.(1-AQL) \tag{2}$$

$$LTPD' = (1-e_2).LTPD + e_1(1-LTPD) \tag{3}$$

Let define *s* and *d* as in the following:

$s = P\{the\ item\ is\ defective|the\ item\ is\ classifed\ as\ defective)$

$d = P\{the\ item\ is\ defective|the\ item\ is\ calssified\ as\ conforming\}$

According to the Bayesian rule, the following conditional probabilities are computed:

$$s = \frac{P(F|E).P(E)}{P(F|E).P(E) + P(F|E^c).P(E^c)} = \frac{(1-e_2).P}{(1-e_2).P + e_1.(1-P)} \tag{4}$$

$$d = \frac{P(E).P(F^c|E)}{P(E).P(F^c|E) + P(E^c).P(F^c|E^c)} = \frac{e_2.P}{e_2.P + (1-e_1).(1-P)} \tag{5}$$

III. ECONOMIC SAMPLING PLAN BASED ON BINOMIAL DISTRIBUTION

The aim of designing the optimal sampling plan is to determine the sample size, *n*, and the acceptance number, *c*, so that the total cost incurred during the inspection can be minimized. Generally, the inspection costs can be summarized as follows: the cost of inspection if the lot is accepted and the cost of inspection if the lot is rejected. Thus, the economic sampling plan can be formally expressed using the following mathematical model:

$$\begin{aligned} Minimize\ Z(n,c) &= \sum_{k=0}^{n} \binom{n}{k} P'^k (1-P')^{n-k} \sum_{j=0}^{N-n} \binom{N-n}{j} P^j (1-P)^{N-n-j} [I_1.n + I_2.k.s + I_3.k.(1-s) + A_2.(j \\ &+ (n-k).d)] + \sum_{k=c+1}^{n} \binom{n}{k} P'^k (1-P')^{n-k} \sum_{j=0}^{N-n} \binom{N-n}{j} P^j (1-P)^{N-n-j} [I_1.N \\ &+ I_2.\left(k.s + j(1-e_2)\right) + I_3.k.(1-s) + I_3.(N-n).e_1 + A_2.(j + (n-k).d) \end{aligned}$$

Subject to:

$$\sum_{k=0}^{c} \binom{n}{k} AQL'^{k}(1-AQL')^{n-k} \geq 1-\alpha$$

$$\sum_{k=0}^{c} \binom{n}{k} LTPD'^{k}(1-LTPD')^{n-k} \leq \beta$$

In this model, the objective function minimizes the total inspection costs. Decision variables are the sample size, *n*, and acceptance number, *c*. The first and the second constraints of the model guarantee producer's risk and consumer's risk, respectively. Now, we proceed to elucidate each term of the objective function. The objective function includes two general terms. The first one corresponds to the lot acceptance situation, while the second term corresponds to the lot rejection situation. In each situation, four types of costs are incurred, including the cost of inspection of the items, the cost of replacement of defective items, the cost of classification of conforming items as defective, and the cost of classifying the defective items as conforming, i.e., warranty cost.

IV. ECONOMIC SAMPLING PLAN BASED ON POISSON DISTRIBUTION

Let assume the random variable *X* has a binomial distribution as follows:

$$f(x) = \binom{r}{x} p^{x}(1-p)^{r-x} \tag{6}$$

While *r* and *p* are the distribution parameters, the binomial distribution is a model for the number of successes while performing an independent-identical-simple experiment. It is well-known that Poisson distribution can be derived as a limited form of Binomial distribution. If we let *r* approach infinity and p approaches zero so that $\lambda = r.p$ is a constant, then such a binomial variable can be estimated by a Poisson variable with the parameter λ. Usually, for the value of $r \geq 10$ and the value of $p \leq 0.1$, the approximation of Binomial variable by a Poisson variable leads to a proper result. In the following, we use this approximation in designing the economic sampling plan.

As the number of items in a lot is usually an immense value, and the proportion of defective items can be assumed a small value, the Poisson model can be satisfactorily applied. Thus, the mathematical model of the economic sampling plan can be presented as follows:

$Minimize\ Z(n,c)$

$$= \sum_{k=0}^{n} \frac{e^{-\lambda'}\lambda'^{k}}{k!} \sum_{j=0}^{N-n} \frac{e^{-\lambda}\lambda^{j}}{j!} [I_1.n + I_2.k.s + I_3.k.(1-s) + A_2.(j+(n-k).d)]$$

$$+ \sum_{k=c+1}^{n} \frac{e^{-\lambda'}\lambda'^{k}}{k!} \sum_{j=0}^{N-n} \frac{e^{-\lambda}\lambda^{j}}{j!} [I_1.N + I_2.\big(k.s + j(1-e_2)\big) + I_3.k.(1-s)$$

$$+ I_3.(N-n).e_1 + A_2.(j+(n-k).d)$$

Subject to:

$$\sum_{k=0}^{c} \frac{e^{-\lambda'_{AQL}}(\lambda'_{AQL})^{k}}{k!} \geq 1-\alpha$$

$$\sum_{k=0}^{c}\frac{e^{-\lambda'_{LTPD}}(\lambda'_{LTPD})^k}{k!}\leq\beta$$

In this model we have $\lambda'_{AQL} = n.AQL'$ and $\lambda'_{LTPD} = n.LTPD'$. The general structure of the model is like the one developed based on Binomial distribution. Two general terms are observed in the objective function of this model. The first one corresponds to the expected cost if the lot is accepted, while the second corresponds to the expected costs if the lot is rejected. The two constraints of the model ensure producer's and consumer's risks.

V. ECONOMIC SAMPLING PLAN BASED ON PASCAL DISTRIBUTION

Pascal distribution, also known as Negative Binomial distribution, is a model for the number of simple-random-independent experiments until observing a specified number of successes. The idea of using Pascal distribution in sampling plans is studied by Fallahnezhad and Hosseini Nasab (2012). Take a random sample with size *n* from the lot and inspect each item of the sample. If, before ending the inspection of n^{th} item, $c+1$ defective items are observed, then the rest of the items of the lot is inspected according to the rectifying inspection policy. On the other hand, if the number of defective items until inspecting n^{th} item becomes fewer than *c*, then the lot is accepted. According to this policy, the model of economic sampling plan can be formulated as follows:

$$\begin{aligned}Minimize\ Z(n,c) &= \sum_{k=0}^{c}\left\{\sum_{j=0}^{N-n}[I_1n+I_2k.s+I_3k(1-s)\right.\\&\left.+A_2(j+(n-k).d]\binom{N-n}{j}p^j(1-p)^{N-n-j}\right\}\binom{n}{k}p'^k(1-p')^{n-k}(1\\&-\sum_{i=c+1}^{n}\binom{i-1}{c}p'^{c+1}(1-p')^{i-c-1}\\&+\sum_{i=c+1}^{n}\left\{\sum_{j=0}^{N-i}[I_1N+I_2((c+1)s+j(1-e_2))+I_3\big((c+1)(1-s)\big)+(N-i-j)c_1)\right.\\&\left.+A_2(je_2+(i-(c+1)d)]\binom{N-1}{j}p^j(1-p)^{N-i-j}\right\}\binom{i-1}{c}p'^{c+1}(1-p')^{i-(c+1)}\end{aligned}$$

Subject to:

$$\sum_{i=c+1}^{n}\binom{i-1}{c}AQL'^{c+1}(1-AQL')^{i-(c+1)}\leq\alpha$$

$$\sum_{i=c+1}^{n}\binom{i-1}{c}LTPD'^{c+1}(1-LTPD')^{i-(c+1)}\geq 1-\beta$$

Like the model developed based on Binomial distribution, two general terms are observed in the objective function of this model. The first one corresponds to the expected cost if the lot is accepted, while the second corresponds to the expected costs if the lot is rejected. The two constraints of the model ensure producer's and consumer's risk.

VI. NUMERICAL EXAMPLE AND DISCUSSION

In this section, a numerical example is presented to clarify the model performance. Suppose the following data is available for a sampling plan (Fallahnezhad et al., 2013):

I_1=1000, I_2=1500, I_3=3000, A_2=5000, N=90, p=0.05, AQL=0.01, LQL=0.2, α=0.1, β=0.2, $e_1 = 0.001, e_2 = 0.005$.

To optimize the models, a grid search algorithm is applied. MATLAB software is used for the grid search algorithm. Table I indicates the results.

Table I. The optimal solution

	Binomial Distribution	*Pascal Distribution*	*Poisson distribution*
n	8	7	8
c	0	0	0
Optimal cost	62590	59930	61913

According to the optimization results, in the models developed based on Poisson and Binomial distributions, a sample with size eight should be randomly selected from the lot, and if the number of defectives is zero, then the lot is accepted. Otherwise, a rectifying inspection is carried out. Applying this policy leads to minimizing the expected sampling cost. In the Binomial model, the optimal cost is 62590, while for Poisson model the cost is 61913. The percent difference between actual optimal cost, 62590, and estimated optimal cost, 61913, is 1.1%. This value indicates the strength of Poisson distribution for estimating the Binomial model. Also, according to the results of the optimizations in the Pascal model, a sample with size seven should be randomly selected from the lot, and if the number of defectives is zero, then the lot is accepted. Otherwise, a rectifying inspection is carried out. Applying this policy leads to minimizing the expected sampling cost. In the Pascal model, the cost is 59930. As can be seen, inspecting, according to Pascal distribution, leads to a decrease in the value of inspection costs.

VII. SENSITIVITY ANALYSIS

In this section, based on the Binomial model, a sensitivity analysis is conducted. First, the effect of the values of producer's and consumer's risks are assessed. Tables II and III show the results.

Table II. Sensitivity analysis for different values of β

The value of β	*Results*
0.15	n=9, c=0, cost=66697
0.2	n=8, c=0, cost=62590
0.25	n=7, c=0, cost=58313

Table III. Sensitivity analysis for different values of α

The value of α	*Results*
0.1	n=8, c=0, cost=62590
0.15	n=14, c=1, cost= 49653
0.2	n=14, c=1, cost=49653

As Tables II and III show, increasing the value of α and β leads to a decrease in the costs of inspection. Also, increasing the values of α and, as expected, relatively leads to an increase in the acceptance rating of the lot (i.e., either the sample size increases or the acceptance number decreases). Table IV shows the results of changes in the value of the proportion of defective items or *p*. An increase in the value of p from 0.05 to 0.15, as expected, leads to an increase in the sampling costs. Nevertheless, the sample size values, *n*, and the acceptance number, *c*, remain unchanged.

Table IV. Sensitivity analysis for different values of *p*

The value of α	*Results*
0.05	*n=8, c=0, cost=62590*
0.1	*n=8, c=0, cost=110900*
0.15	*n=8, c=0, cost=152410*

The effect of change in the value of *LTPD* is shown in Table V. Increasing the value of *LTPD* from 0.2 to 0.3 decreases the cost of the inspection. Also, this change leads to an increase in the acceptance rating.

Table V. Sensitivity analysis for different values of *LTPD*

The value of LTPD	*Results*
0.2	*n=8, c=0, cost=62590*
0.25	*n=6, c=0, cost=53848*
0.3	*n=9, c=1, cost=37230*

The results of the change in the value of warranty cost are shown in Table VI. As this table indicates, increasing the value of warranty cost leads to an increase in objective function value. Also, this change, to some extent, changes the values of sample size and acceptance number. According to our thorough investigation, the sample size and acceptance number are mostly affected by the values of $LTPD$, AQL, α, β, and p. The parameters related to the costs of inspection, i.e. I_1, I_2, I_3, A_2 do not have a significant effect on the values of sample size and acceptance number, although these parameters have a major effect on the objective function and optimal costs.

Table VI. Sensitivity analysis for different values of A_2

The value of A_2	*Results*
5000	*n=8, c=0, cost=62590*
10000	*n=8, c=0, cost=83142*
100000	*n=9, c=0, cost=452450*

VIII. CONCLUSION AND SUGGESTIONS FOR FUTURE RESEARCH

In an acceptance sampling plan, two types of errors may occur during the inspection of items, including type I error and Type II error. Type I error occurs when a conforming item is classified as a defective one. Type II occurs while a defective item is classified as a conforming one. The aim of designing the optimal sampling plan is to determine the sample size, *n*, and the acceptance number, *c*, so that the total cost incurred during the inspection can be minimized. Considering three probability distributions, i.e., Binomial, Pascal, and Poisson distributions, the models are presented for an economic single-sampling plan while inspection errors are taken into account. In the models, the objective function minimizes the total inspection costs. Decision variables are the sample size, *n*, and acceptance number, *c*. The result of the numerical example shows that the Poisson model can well approximate the binomial model. Finally, some sensitivity analyses are carried out. This study can be extended from different directions, such as considering hypergeometric distribution instead of Binomial distribution. Optimizing the average sample number (ASN) and average outgoing quality (AOQ) considering inspection errors is another exciting direction to develop this study.

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